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Regularized Interference Alignment based on Weighted Sum-MSE Criterion for MIMO Interference Channels

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***Abstract*— The original interference alignment (IA) scheme provides poor sum-rate performance compared to simple orthogonal access schemes such as time-division multiple access (TDMA) in low-to-medium SNR under total power constraint. In this paper, we address this problem by proposing a method of regularizing the IA scheme with a criterion of minimizing the weighted sum of the mean square error (WMSE) function. To perform the regularization process efficiently, the weight terms in the WMSE metric should be computed from the optimal zero-forcing (ZF) schemes. Thus, we first prove the optimality of the enhanced IA algorithm introduced in our previous work in the ZF sense. From simulation results, it is shown that the proposed scheme outperforms the TDMA in overall SNR regime. We can further improve the performance by repeating the proposed regularization process iteratively. Moreover, we propose a modified design that provides robustness in the presence of channel uncertainty.**

I. Introduction

In theory, multiple-input multiple-output (MIMO) techniques enable spatial multiplexing which achieves extremely high spectral efficiency by transmitting independent data streams simultaneously. Nevertheless, their effectiveness may become much limited in next generation cellular systems since all well designed cellular systems are interference-limited [1]. Therefore, it is quite important to study cellular networks as MIMO interference channels (IFC). However, the capacity characterization of the IFC is still an open problem even for two user and single antenna cases [2].

The degree of freedom (DOF) has emerged as an alternative measure since the sum capacity of communication systems at high signal-to-noise ratio (SNR) is dominated by the DOF [3]–[5]. Especially, the authors in [3] provide a closed-form solution for achieving the maximum achievable DOF in 3-user MIMO IFC where all nodes are equipped with the equal number of antennas. They used an idea of interference alignment (IA) to maximize the dimension of the desired signals. In [6], the enhanced interference alignment (E-IA) algorithm was proposed to improve the sum-rate performance of the conventional IA scheme. However, from a sum-rate point of view, both the conventional IA and E-IA scheme suffer from a performance loss in comparison to a simple time-division multiple access (TDMA) scheme at low SNR.

To address this problem, in this paper, we propose a method of regularizing the E-IA scheme based on the weighted sum of mean square errors (MSE) criterion. The idea of minimizing the weighted sum of MSE (WMSE) was originally developed for multi-user MIMO downlink channels [7] with a goal of regularizing a zero-forcing (ZF) scheme. We apply this concept of regularization to MIMO IFC for further enhancing the performance. To execute the regularization process efficiently, the weight terms in the WMSE metric should be computed as the effective channel gains of the optimal ZF scheme. We first prove that the E-IA scheme in [6] achieves the optimal sum-rate performance in terms of the ZF criteria. Then, the proposed regularized IA (R-IA) algorithm attempts to minimize the WMSE function whose weight term is obtained from the E-IA scheme. It is shown from simulations that the proposed scheme outperforms both the TDMA scheme and the conventional IA over all SNR regime with small additional complexity. In addition, we provide a modified R-IA scheme which is more robust to the channel estimation error.

The following notations are used for description. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The trace, conjugate, Hermitian transpose and the column space of a matrix or vector are represented by $\mathrm{Tr}(\cdot)$, $(\cdot)^*$, $(\cdot)^H$ and $\mathcal{C}(\cdot)$, respectively. $||\cdot||^2$ indicates the 2-norm of a vector and an identity matrix of size n is denoted by $\mathbf{I}_n$.

II. System Model

In this paper, we consider a K-user MIMO IFC where all transmit and receive nodes are equipped with M antennas. Assuming that M is even, the i-th transmitter attempts to send the information vector $\mathbf{s}^{(i)} \in \mathbb{C}^{\frac{M}{2}}$ to the i-th receiver ($i = 1, \cdots, K$). Denoting $\mathbf{y}^{(k)}$ as the signal received by the k-th receiver, $\mathbf{y}^{(k)}$ can be written as

$$\mathbf{y}^{(k)} = \mathbf{H}^{(kk)}\mathbf{x}^{(k)} + \sum_{l \neq k} \mathbf{H}^{(kl)}\mathbf{x}^{(l)} + \mathbf{n}^{(k)} \qquad (1)$$

where $\mathbf{x}^{(i)} \in \mathbb{C}^M$ stands for the signal vector transmitted from the i-th transmitter, $\mathbf{n}^{(i)}$ denotes the additive Gaussian noise vector at receiver i with covariance matrix $\sigma^2\mathbf{I}_M$ and $\mathbf{H}^{(kl)} \in \mathbb{C}^{M\times M}$ indicates the channel matrix from transmitter l to receiver k. It is assumed that the channel elements are sampled from independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance so that the probability of event that the channel is rank-deficient converges to zero. Also, all channel realizations are assumed to be perfectly known at all nodes. In (1), $\mathbf{x}^{(i)}$ is related to $\mathbf{s}^{(i)}$ as $\mathbf{x}^{(i)} = \mathbf{V}^{(i)}\mathbf{s}^{(i)}$, where $\mathbf{V}^{(i)} \in \mathbb{C}^{M\times\frac{M}{2}}$ is a precoding matrix post-multiplied to the information vector

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$\mathbf{s}^{(i)}$. From the dimensions of $\mathbf{V}^{(i)}$ and $\mathbf{s}^{(i)}$, we can see that each user is served with the maximum available DOF of $\frac{M}{2}$ [3]. Throughout the paper, we focus on the case of $K = 3$.[1] Although the elements of $\mathbf{H}^{(kk)}$ are generally distributed with power larger than those of $\mathbf{H}^{(kl)} (k \neq l)$ due to the path loss in cellular networks, we consider the most challenging case where all of them have unit power. This situation arises for users located in cell boundaries.

At receiver k, the receive filter $\mathbf{U}^{(k)}$ is post-multiplied to $\mathbf{y}^{(k)}$ to yield

$$\hat{\mathbf{s}}^{(k)} = \mathbf{U}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}^{(k)}\mathbf{s}^{(k)} + \mathbf{U}^{(k)H}\sum_{l\neq k}\mathbf{H}^{(kl)}\mathbf{V}^{(l)}\mathbf{s}^{(l)} + \mathbf{U}^{(k)H}\mathbf{n}^{(k)}$$

where $\hat{\mathbf{s}}^{(k)}$ is the estimate vector for $\mathbf{s}^{(k)}$. Under the assumption of $E\left[\mathbf{s}^{(i)}\mathbf{s}^{(i)H}\right] = \mathbf{I}_{\frac{M}{2}}$, the achievable sum-rate with single-user detection is given by

$$R_\Sigma = \sum_{k=1}^{K}\sum_{i=1}^{\frac{M}{2}} \log_2\left(1 + \frac{|\mathbf{u}_i^{(k)}\mathbf{H}^{(kk)}\mathbf{v}_i^{(k)}|^2}{\sum_{(l,j)\neq(k,i)} |\mathbf{u}_i^{(k)}\mathbf{H}^{(kl)}\mathbf{v}_j^{(l)}|^2 + \sigma^2||\mathbf{u}_i^{(k)}||^2}\right)$$

where $\mathbf{u}_i^{(k)}$ and $\mathbf{v}_i^{(k)}$ denote the i-th column of $\mathbf{U}^{(k)}$ and $\mathbf{V}^{(k)}$, respectively.

For the sake of fair comparison among various transmission techniques, we consider the total power constraint as $\sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{V}^{(k)}\mathbf{V}^{(k)H}\right) = P$. In this MIMO IFC scenario, the simplest method of managing inter-user interference is an orthgonal access technique such as TDMA where at each time slot only one link operates with the other links being turned off. Especially, we consider the TDMA scheme with round-robin scheduling where each active MIMO link operates with singular-value decomposition (SVD)-based beamforming combined with water-filling power allocation [8].

III. Optimality of the Enhanced IA under ZF Condition

In this section, we briefly review the E-IA scheme in [6] and show that the E-IA is optimal in the ZF criteria.

A. Review of Enhanced IA Scheme

The feasibility condition for IA [3] is given as

$$\mathcal{C}\left(\mathbf{H}^{(ij)}\mathbf{V}^{(j)}\right) = \mathcal{C}\left(\mathbf{H}^{(ik)}\mathbf{V}^{(k)}\right), \text{ for all } j \neq i, k \neq i. \quad (2)$$

We notice that the precoders of all ZF schemes (e.g., conventional IA [3] and E-IA [6]) should satisfy the above conditions for the inter-user interference to be completely eliminated at the receiver side.

The E-IA scheme [6] employs the two-step precoding and decoding process as

$$\mathbf{V}^{(i)} = \mathbf{V}_{\text{MU}}^{(i)}\mathbf{V}_{\text{SU}}^{(i)} \quad \text{and} \quad \mathbf{U}^{(i)} = \mathbf{U}_{\text{MU}}^{(i)}\mathbf{U}_{\text{SU}}^{(i)}, \quad \text{for } i = 1,2,3$$

where the inter-user interference is eliminated by the multi-user precoders $\mathbf{V}_{\text{MU}}^{(i)} \in \mathbb{C}^{M\times\frac{M}{2}}$ and the decoders $\mathbf{U}_{\text{MU}}^{(i)} \in \mathbb{C}^{M\times\frac{M}{2}}$. This means that accomplishing the feasibility conditions are related to the design of $\mathbf{V}_{\text{MU}}^{(i)}$. With an aid of $\mathbf{V}_{\text{MU}}^{(i)}$ and $\mathbf{U}_{\text{MU}}^{(i)}$, the MIMO IFC is decoupled into 3 parallel single-user MIMO systems and we can maximize the single-user capacity by adjusting the single-user precoders $\mathbf{V}_{\text{SU}}^{(i)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$ and the decoders $\mathbf{U}_{\text{SU}}^{(i)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$.

First, we illustrate how to determine $\mathbf{V}_{\text{MU}}^{(i)}$ and $\mathbf{U}_{\text{MU}}^{(i)}$ in the E-IA scheme. The multi-user precoder has a form of $\mathbf{V}_{\text{MU}}^{(i)} = \mathbf{Q}\left(\mathbf{B}^{(i)}\right)$ where the columns of $\mathbf{Q}(\mathbf{X})$ consists of orthonormal basis vectors of $\mathcal{C}(\mathbf{X})$. Defining $\mathbf{E}, \mathbf{F}$ and $\mathbf{G}$ as

$$\mathbf{E} = (\mathbf{H}^{(31)})^{-1}\mathbf{H}^{(32)}(\mathbf{H}^{(12)})^{-1}\mathbf{H}^{(13)}(\mathbf{H}^{(23)})^{-1}\mathbf{H}^{(21)},$$
$$\mathbf{F} = (\mathbf{H}^{(32)})^{-1}\mathbf{H}^{(31)} \text{ and } \mathbf{G} = (\mathbf{H}^{(23)})^{-1}\mathbf{H}^{(21)},$$

the feasibility conditions can be satisfied by setting $\mathbf{B}^{(i)}$ to

$$\mathbf{B}^{(1)} = \left[\mathbf{e}_{i_1} \cdots \mathbf{e}_{i_{\frac{M}{2}}}\right], \text{ for } 1 \leq i_1 < \cdots < i_{\frac{M}{2}} \leq M, \quad (3)$$
$$\mathbf{B}^{(2)} = \mathbf{F}\mathbf{B}^{(1)} \text{ and } \mathbf{B}^{(3)} = \mathbf{G}\mathbf{B}^{(1)},$$

where $\mathbf{e}_1, \cdots, \mathbf{e}_M$ are the eigenvectors of $\mathbf{E}$. Once the feasibility conditions are met, the inter-user interference can be nulled at the receiver side by consituting the columns of $\mathbf{U}_{\text{MU}}^{(i)}$ as $\frac{M}{2}$ left singular vectors of $\mathbf{H}^{(ij)}\mathbf{V}_{\text{MU}}^{(j)}$ $(j \neq i)$ corresponding to zero singular values. As any choice of the indexes $1 \leq i_1 < i_2 < \cdots < i_{\frac{M}{2}} \leq M$ satisfies the feasibility conditions, it was proposed in [6] to choose the best $\frac{M}{2}$ indexes out of $\{1, \cdots, M\}$ corresponding to the maximum sum-rate performance with the search size $\binom{M}{\frac{M}{2}}$.

For given multi-user transceivers, the optimal single-user precoders and decoders can easily be found as [6]

$$\mathbf{V}_{\text{SU}}^{(i)} = \bar{\mathbf{V}}^{(i)}\mathbf{\Phi}^{(i)} \quad \text{and} \quad \mathbf{U}_{\text{SU}}^{(i)} = \bar{\mathbf{U}}^{(i)}, \quad \text{for } i = 1,2,3$$

where $\bar{\mathbf{V}}^{(i)}$ and $\bar{\mathbf{U}}^{(i)}$ are the right and left singular matrices of $\mathbf{U}_{\text{MU}}^{(i)H}\mathbf{H}^{(ii)}\mathbf{V}_{\text{MU}}^{(i)}$, respectively, and the power allocation matrix $\mathbf{\Phi}^{(i)}$ is a real diagonal matrix whose diagonal elements can be optimized according to the water-filling algorithm [8] subject to the total power constraint $\sum_i \mathrm{Tr}\left(\mathbf{\Phi}^{(i)2}\right) = P$.

B. Optimality of Enhanced IA Scheme

In this subsection, we show that from the ZF criteria, the precoders and decoders of the E-IA scheme achieve the optimal sum-rate performance. To this end, we need to confirm the joint optimality of the multi-user and single-user transceivers of the E-IA scheme. Since it is obvious that the single-user transceiver designs in Subsection III-A are optimal for given multi-user transceivers, it is sufficient to check the optimality of the multi-user precoders and decoders of the E-IA scheme. Recognizing that the E-IA selects the best $\frac{M}{2}$ indexes $i_1, \cdots, i_{\frac{M}{2}}$ out of $\{1, \cdots, M\}$ in (3) corresponding to the maximum sum-rate, the proof is completed by showing that if $(\mathbf{V}^{(1)}, \mathbf{V}^{(2)}, \mathbf{V}^{(3)}) \in \mathcal{S}$ where $\mathcal{S}$ is a set of all precoding matrices satisfying the feasibility conditions (2), then $\mathbf{B}^{(1)}, \mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ should satisfy

$$\mathcal{C}\left(\mathbf{B}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{e}_{i_1} \cdots \mathbf{e}_{i_{\frac{M}{2}}}\right]\right), \text{for } 1 \leq i_1 < \cdots < i_{\frac{M}{2}} \leq M, \quad (4)$$
$$\mathcal{C}\left(\mathbf{B}^{(2)}\right) = \mathcal{C}\left(\mathbf{F}\mathbf{B}^{(1)}\right) \text{ and } \mathcal{C}\left(\mathbf{B}^{(3)}\right) = \mathcal{C}\left(\mathbf{G}\mathbf{B}^{(1)}\right). \quad (5)$$

As (5) can be directly derived, we concentrate on the proof of (4).

[1]The proposed regularization can be applied to any MIMO IFC systems where a solution for ZF precoder and decoder is given.

Since $\mathbf{H}^{(ij)}$ is invertible almost surely, it is straightforward to see that if $(\mathbf{V}^{(1)}, \mathbf{V}^{(2)}, \mathbf{V}^{(3)}) \in \mathcal{S}$, then $\mathcal{C}\left(\mathbf{E}\mathbf{V}^{(1)}\right) = \mathcal{C}\left(\mathbf{V}^{(1)}\right)$. As $\mathbf{e}_1, \cdots, \mathbf{e}_M$ span the whole $\mathbb{C}^M$, we can express $\mathbf{V}^{(1)}$ as

$$\mathbf{V}^{(1)} = [\mathbf{e}_1 \cdots \mathbf{e}_M]\, \mathbf{C}$$

where $\mathbf{C} \in \mathbb{C}^{M \times \frac{M}{2}}$ is a coefficient matrix. Then, it follows from $\mathcal{C}\left(\mathbf{E}\mathbf{V}^{(1)}\right) = \mathcal{C}\left(\mathbf{V}^{(1)}\right)$ that

$$\mathcal{C}\left(\mathbf{E}\left[\mathbf{e}_1 \cdots \mathbf{e}_M\right]\mathbf{C}\right) = \mathcal{C}\left(\left[\mathbf{e}_1 \cdots \mathbf{e}_M\right]\mathbf{C}\right). \tag{6}$$

Let $\lambda_1, \cdots, \lambda_M$ denote the eigenvalues of $\mathbf{E}$. Then, $\mathbf{E}\left[\mathbf{e}_1 \cdots \mathbf{e}_M\right]$ in the left-hand side of (6) is computed as

$$\mathbf{E}\left[\mathbf{e}_1 \cdots \mathbf{e}_M\right] = \left[\mathbf{e}_1 \cdots \mathbf{e}_M\right] \mathrm{diag}(\lambda_1, \cdots, \lambda_M).$$

Then, (6) is equivalent to

$$\mathcal{C}\left(\mathrm{diag}\left(\lambda_1, \cdots, \lambda_M\right)\mathbf{C}\right) = \mathcal{C}\left(\mathbf{C}\right) \tag{7}$$

as $\left[\mathbf{e}_1 \cdots \mathbf{e}_M\right]$ is invertible with probability converging to one.

Since λ_i's are distinct almost surely, (7) is true only when there are at most $\frac{M}{2}$ nonzero vectors in $\mathbf{c}_1, \cdots, \mathbf{c}_M$ where $\mathbf{c}_i$ is the i-th row vector of $\mathbf{C}$. Denoting the nonzero vectors by $\mathbf{c}_{i_1}, \cdots, \mathbf{c}_{i_{\frac{M}{2}}}$, $\mathbf{V}^{(1)}$ can be expressed as $\mathbf{V}^{(1)} = \sum_{l=1}^{M} \mathbf{e}_l \mathbf{c}_l = \sum_{l=1}^{\frac{M}{2}} \mathbf{e}_{i_l} \mathbf{c}_{i_l}$ which tells us that

$$\mathcal{C}\left(\mathbf{V}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{e}_{i_1} \cdots \mathbf{e}_{i_{\frac{M}{2}}}\right]\right).$$

Since $\mathcal{C}\left(\mathbf{V}^{(1)}\right) = \mathcal{C}\left(\mathbf{B}^{(1)}\right)$, the proof for (4) is completed.

IV. Proposed Regularized Interference Alignment

The main limitation of the IA algorithm is its poor performance at low SNR due to the restriction in the ZF scheme. In this section, we present our proposed R-IA algorithm which regularizes the precoders and decoders of the E-IA scheme. The proposed scheme achieves the regularization by minimizing the WMSE defined as

$$\sum_{k=1}^{K} E\left[||\mathbf{\Lambda}_{\text{E-IA}}^{(k)}\mathbf{s}^{(k)} - \hat{\mathbf{s}}^{(k)}||^2\right]$$

where $\mathbf{\Lambda}_{\text{E-IA}}^{(k)} = \mathbf{U}_{\text{E-IA}}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}_{\text{E-IA}}^{(k)}$ is the effective channel gain matrix for user pair k, and $\mathbf{V}_{\text{E-IA}}^{(k)}$ and $\mathbf{U}_{\text{E-IA}}^{(k)}$ indicate the precoder and decoder matrices computed from the E-IA algorithm. $\mathbf{\Lambda}_{\text{IA}}^{(k)}$ is introduced to prevent weaker subchannels from being assigned more power which is undesirable in terms of the sum-rate performance [8]. Actually, if we adapt the weight matrix more intelligently according to [9], we would obtain the near-optimal sum-rate performance. However, we use the weight of $\mathbf{\Lambda}_{\text{E-IA}}^{(k)}$ to achieve the improved performance with minimum additional complexity.

Denoting $\mathbf{U}^{(i)} = \mu^{-1}\tilde{\mathbf{U}}^{(i)}$, the WMSE minimization problem can be formulated as

$$\min_{\mathbf{V}^{(i)}\text{'s}, \tilde{\mathbf{U}}^{(i)}\text{'s}} \sum_{k=1}^{K} E\left[||\mathbf{\Lambda}_{\text{E-IA}}^{(k)}\mathbf{s}^{(k)} - \mu^{-1}\tilde{\hat{\mathbf{s}}}^{(k)}||^2\right] \tag{8}$$

$$\text{subject to } \sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{V}^{(k)}\mathbf{V}^{(k)H}\right) = P$$

where $\tilde{\hat{\mathbf{s}}}^{(k)} = \tilde{\mathbf{U}}^{(k)H}\left(\sum_{l=1}^{K}\mathbf{H}^{(kl)}\mathbf{V}^{(l)}\mathbf{s}^{(l)} + \mathbf{n}^{(k)}\right)$. Here, a positive real number μ is introduced to consider the gain of the transmit filters [10]. To solve this problem, we form the Lagrangian function as

$$\begin{aligned} L\left(\mathbf{V}^{(i)}\text{'s}, \mathbf{U}^{(i)}\text{'s}, \mu, \lambda\right) &= \sum_{k=1}^{K} E\left[\left|\left|\mathbf{\Lambda}_{\text{E-IA}}^{(k)}\mathbf{s}^{(k)} - \mu^{-1}\tilde{\mathbf{U}}^{(k)H}\mathbf{y}^{(k)}\right|\right|^2\right] \\ &+ \lambda\left(\sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{V}^{(k)}\mathbf{V}^{(k)H}\right) - P\right) \end{aligned}$$

where λ is the Lagrangian multiplier.

The expectation over $\mathbf{s}^{(i)}$'s and $\mathbf{n}^{(i)}$'s can be computed as

$$\begin{aligned} &L\left(\mathbf{V}^{(i)}\text{'s}, \mathbf{U}^{(i)}\text{'s}, \mu, \lambda\right) \\ &= \sum_{k=1}^{K}\mathrm{Tr}\Big\{\left(\mathbf{\Lambda}_{\text{E-IA}}^{(k)} - \mu^{-1}\tilde{\mathbf{U}}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}^{(k)}\right) \times \\ &\qquad \left(\mathbf{\Lambda}_{\text{E-IA}}^{(k)H} - \mu^{-1}\mathbf{V}^{(k)H}\mathbf{H}^{(kk)H}\tilde{\mathbf{U}}^{(k)}\right)\Big\} \\ &+\mu^{-2}\sigma^2\sum_{k=1}^{K}\mathrm{Tr}\left(\tilde{\mathbf{U}}^{(k)H}\tilde{\mathbf{U}}^{(k)}\right) + \lambda\left(\sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{V}^{(k)}\mathbf{V}^{(k)H}\right) - P\right) + \Omega. \end{aligned}$$

Here, Ω has two expressions as

$$\begin{aligned} \Omega &= \mu^{-2}\sum_{k=1}^{K}\mathrm{Tr}\left\{\mathbf{V}^{(k)H}\left(\sum_{l\neq k}\mathbf{H}_{\text{U}}^{(lk)H}\mathbf{H}_{\text{U}}^{(lk)}\right)\mathbf{V}^{(k)}\right\} \\ &= \mu^{-2}\sum_{k=1}^{K}\mathrm{Tr}\left\{\tilde{\mathbf{U}}^{(k)H}\left(\sum_{l\neq k}\mathbf{H}_{\text{V}}^{(kl)}\mathbf{H}_{\text{V}}^{(kl)H}\right)\tilde{\mathbf{U}}^{(k)}\right\} \end{aligned} \tag{9}$$

where $\mathbf{H}_{\text{U}}^{(lk)} \triangleq \tilde{\mathbf{U}}^{(l)H}\mathbf{H}^{(lk)}$ and $\mathbf{H}_{\text{V}}^{(kl)} \triangleq \mathbf{H}^{(kl)}\mathbf{V}^{(l)}$.

Setting the derivatives of $L(\cdot)$ with respect to $\mathbf{V}^{(k)*}$ and $\tilde{\mathbf{U}}^{(k)*}$ to zero results in

$$\mathbf{V}^{(k)} = \mu\left(\sum_{l=1}^{K}\mathbf{H}_{\text{U}}^{(lk)H}\mathbf{H}_{\text{U}}^{(lk)} + \omega\mathbf{I}_M\right)^{-1}\mathbf{H}_{\text{U}}^{(kk)H}\mathbf{\Lambda}_{\text{E-IA}}^{(k)}, \tag{10}$$

$$\tilde{\mathbf{U}}^{(k)} = \mu\left(\sum_{l=1}^{K}\mathbf{H}_{\text{V}}^{(kl)}\mathbf{H}_{\text{V}}^{(kl)H} + \sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{\text{V}}^{(kk)}\mathbf{\Lambda}_{\text{E-IA}}^{(k)} \tag{11}$$

where ω includes the Lagrangian multiplier as $\omega = \lambda\mu^2$. Since finding $\mathbf{V}^{(i)}$'s and $\tilde{\mathbf{U}}^{(i)}$'s that have a form of (10) and (11) needs an iterative algorithm, we instead propose to execute the following two-step process to attain the improved performance in a non-iterative way.

1) $\mathbf{V}^{(i)}$'s, μ and ω are optimized for fixed $\tilde{\mathbf{U}}^{(i)}$'s.
2) $\tilde{\mathbf{U}}^{(i)}$'s are optimized for fixed $\mathbf{V}^{(i)}$'s, μ and ω.

Since the step 2) is nothing but a computation of (11), we explain how to perform the step 1), i.e., the optimization of $\mathbf{V}^{(i)}$'s, μ and ω for fixed $\tilde{\mathbf{U}}^{(i)}$'s.

After some manipulations, we can express $\mathbf{V}^{(k)} = \mu\tilde{\mathbf{V}}^{(k)}(\omega)$ as a function of a single variable ω where $\tilde{\mathbf{V}}^{(k)}(\omega)$ is defined as

$$\tilde{\mathbf{V}}^{(k)}(\omega) = \left(\sum_{l=1}^{K}\mathbf{H}_{\text{U}}^{(lk)H}\mathbf{H}_{\text{U}}^{(lk)} + \omega\mathbf{I}_M\right)^{-1}\mathbf{H}_{\text{U}}^{(kk)H}\mathbf{\Lambda}_{\text{E-IA}}^{(k)}. \tag{12}$$

The total power constraint is satisfied by associating μ with $\tilde{\mathbf{V}}^{(k)}(\omega)$ as

$$\mu = \sqrt{\frac{P}{\sum_{k=1}^{K} \text{Tr}\left(\tilde{\mathbf{V}}^{(k)}(\omega)\tilde{\mathbf{V}}^{(k)}(\omega)^H\right)}}. \tag{13}$$

As a result, for fixed $\tilde{\mathbf{U}}^{(i)}$'s, the optimization of $\mathbf{V}^{(i)}$'s reduces to a single variable quasi-convex problem with respect to ω formulated as

$$\min_{\omega} \sum_{k=1}^{K} E\left[\left|\left|\mathbf{\Lambda}_{\text{E-IA}}^{(k)}\mathbf{s}^{(k)} - \sum_{l=1}^{K} \mathbf{H}_{\text{U}}^{(kl)}\tilde{\mathbf{V}}^{(l)}(\omega)\mathbf{s}^{(l)} - \mu\tilde{\mathbf{U}}^{(k)H}\mathbf{n}^{(k)}\right|\right|^2\right] \tag{14}$$

where μ depends on ω as in (13). By making the derivatives of the cost function to zero, we arrive at the optimal ω as

$$\omega = \frac{\sigma^2 \sum_{k=1}^{K} \text{Tr}\left(\tilde{\mathbf{U}}^{(k)H}\tilde{\mathbf{U}}^{(k)}\right)}{P}. \tag{15}$$

Consequently, we can execute the step 1) by comptuting $\mathbf{V}^{(i)}$'s, μ and ω in the order of $\omega \rightarrow \mu \rightarrow \mathbf{V}^{(i)}$'s. The whole algorithm of the proposed R-IA scheme is summarized as follows:

1) Compute $\mathbf{\Lambda}_{\text{E-IA}}^{(i)}$ and $\mathbf{U}_{\text{E-IA}}^{(i)}$, $\forall i$.
2) $\tilde{\mathbf{U}}^{(i)} \leftarrow \mathbf{U}_{\text{E-IA}}^{(i)}$, $\forall i$.
3) Compute ω using (15).
4) Construct $\mathbf{V}^{(k)} = \mu\tilde{\mathbf{V}}^{(k)}(\omega)$ using (12) and (13), $\forall k$.
5) Compute $\tilde{\mathbf{U}}^{(k)}$ using (11), $\forall k$.

It should be noted that we can further improve the performance by repeating from step 3) to step 5) iteratively at the expense of increased computational complexity.

We briefly compare the proposed R-IA with the iterative signal-to-interference-plus-noise ratio (SINR) maximization scheme in [11]. In the SINR maximing algorithm, the transmit and receive filters are alternately updated to improve the streamwise SINR. The main advantage of the proposed R-IA over the SINR maximization scheme is the complexity savings since the R-IA needs the inverse operation of an M-by-M matrix only once to update $\mathbf{V}^{(i)}$(or $\mathbf{U}^{(i)}$) at each iteration, whereas the SINR maximization algorithm requires $\frac{M}{2}$ times.

V. Regularized Interference Alignment with Imperfect Channel State Information

In practical scenarios, the mismatch between the true channel $\mathbf{H}^{(ij)}$ and the estimated channel $\bar{\mathbf{H}}^{(ij)}$ is inevitable due to an estimation error or time varying channels [12]. In this section, we provide a method for mitigating the performance degradation caused by the channel mismatch. We assume that $\bar{\mathbf{H}}^{(ij)}$ is related to $\mathbf{H}^{(ij)}$ as $\bar{\mathbf{H}}^{(ij)} = \mathbf{H}^{(ij)} + \mathbf{E}^{(ij)}$ where the elements of $\mathbf{E}^{(ij)}$ are i.i.d. complex Gaussian random variables of variance σ_e^2 [12]. Then, the received signal can be rewritten as

$$\mathbf{s}^{(k)} = \mathbf{U}^{(k)H}\sum_{l}\left(\bar{\mathbf{H}}^{(kl)} - \mathbf{E}^{(kl)}\right)\mathbf{V}^{(l)}\mathbf{s}^{(l)} + \mathbf{U}^{(k)H}\mathbf{n}^{(k)}$$

where $\mathbf{V}^{(i)}$ and $\mathbf{U}^{(i)}$ are computed from $\bar{\mathbf{H}}^{(ij)}$ being unaware of $\mathbf{E}^{(ij)}$.

In this setup, we mitigate the effect of the channel mismatch by minimizing the weighted sum-MSE metric averaged over $\mathbf{E}^{(ij)}$'s as

$$\sum_{k=1}^{K} E_{\mathbf{E}^{(kl)}}\left[||\mathbf{\Lambda}_{\text{E-IA}}^{(k)}\mathbf{s}^{(k)} - \hat{\mathbf{s}}^{(k)}||^2\right] \tag{16}$$

where the expectation with respect to $\mathbf{s}^{(i)}$ and $\mathbf{n}^{(i)}$ is included implicitly. As in Section IV, the modified Lagrangian function $\bar{L}\left(\mathbf{V}^{(i)}\text{'s}, \mathbf{U}^{(i)}\text{'s}, \mu, \lambda\right)$ can be computed by replacing $\mathbf{H}^{(ij)} \rightarrow \bar{\mathbf{H}}^{(ij)}$ and $\Omega \rightarrow \bar{\Omega}$ where $\bar{\Omega}$ is expressed as

$$\begin{aligned}\bar{\Omega} &= \mu^{-2}\sum_{k=1}^{K}\text{Tr}\Bigg\{\mathbf{V}^{(k)H}\Bigg[\sum_{l\neq k}\bar{\mathbf{H}}_{\text{U}}^{(lk)H}\bar{\mathbf{H}}_{\text{U}}^{(lk)} + \\ &\qquad \sigma_e^2\left(\sum_{l=1}^{K}\text{Tr}\left(\mathbf{D}_{\text{U}}^{(l)}\right)\right)\mathbf{I}_M\Bigg]\mathbf{V}^{(k)}\Bigg\} \\ &= \mu^{-2}\sum_{k=1}^{K}\text{Tr}\Bigg\{\tilde{\mathbf{U}}^{(k)H}\Bigg[\sum_{l\neq k}\bar{\mathbf{H}}_{\text{V}}^{(kl)}\bar{\mathbf{H}}_{\text{V}}^{(kl)H} + \\ &\qquad \sigma_e^2\left(\sum_{l=1}^{K}\text{Tr}\left(\mathbf{D}_{\text{V}}^{(l)}\right)\right)\mathbf{I}_M\Bigg]\tilde{\mathbf{U}}^{(k)}\Bigg\}.\end{aligned}$$

Here, $\mathbf{D}_{\text{U}}^{(l)}$ and $\mathbf{D}_{\text{V}}^{(l)}$ are diagonal matrices whose elements consist of the eigenvalues of $\tilde{\mathbf{U}}^{(l)}\tilde{\mathbf{U}}^{(l)H}$ and $\mathbf{V}^{(l)}\mathbf{V}^{(l)H}$, respectively. We have also used the following equalities:

$$\begin{aligned}E\left[\mathbf{E}^{(kl)}\mathbf{V}^{(l)}\mathbf{V}^{(l)H}\mathbf{E}^{(kl)H}\right] &= \sigma_e^2\text{Tr}\left(\mathbf{D}_{\text{V}}^{(l)}\right)\mathbf{I}_M, \\ E\left[\mathbf{E}^{(kl)H}\tilde{\mathbf{U}}^{(k)}\tilde{\mathbf{U}}^{(k)H}\mathbf{E}^{(kl)}\right] &= \sigma_e^2\text{Tr}\left(\mathbf{D}_{\text{U}}^{(l)}\right)\mathbf{I}_M.\end{aligned}$$

Since $\bar{L}(\cdot)$ differs from (9) only in the last term, $\partial\bar{L}(\cdot)/\left(\partial\mathbf{V}^{(k)*}\right) = \partial\bar{L}(\cdot)/\left(\partial\tilde{\mathbf{U}}^{(k)*}\right) = \mathbf{0}$ occurs when

$$\mathbf{V}^{(k)} = \mu\left[\sum_{l=1}^{K}\bar{\mathbf{H}}_{\text{U}}^{(lk)H}\bar{\mathbf{H}}_{\text{U}}^{(lk)} + \sigma_e^2\left(\sum_{l=1}^{K}\text{Tr}\left(\mathbf{D}_{\text{U}}^{(l)}\right)\right)\mathbf{I}_M + \omega\mathbf{I}_M\right]^{-1} \times \bar{\mathbf{H}}_{\text{U}}^{(kk)H}\mathbf{\Lambda}_{\text{E-IA}}^{(k)}, \tag{17}$$

$$\tilde{\mathbf{U}}^{(k)} = \mu\left[\sum_{l=1}^{K}\bar{\mathbf{H}}_{\text{V}}^{(kl)}\bar{\mathbf{H}}_{\text{V}}^{(kl)H} + \sigma_e^2\left(\sum_{l=1}^{K}\text{Tr}\left(\mathbf{D}_{\text{V}}^{(l)}\right)\right)\mathbf{I}_M + \sigma^2\mathbf{I}_M\right]^{-1} \times \bar{\mathbf{H}}_{\text{V}}^{(kk)}\mathbf{\Lambda}_{\text{E-IA}}^{(k)}. \tag{18}$$

As a result, the second term inside the inverse in (17) and (18) is newly added compared to (10) and (11). Now, we can optimize $\tilde{\mathbf{U}}^{(i)}$, $\mathbf{V}^{(i)}$, μ and ω in a similar fashion as in Section IV by using (17) and (18) instead of (10) and (11), respectively.

VI. Numerical Results

In this section, we provide numerical results evaluating the sum rate performance of the proposed R-IA scheme. In Figure 1, the average sum-rate performance is presented for several transmission techniques in the MIMO IFC with $M = 2$. Although the curve of the E-IA shows the slope steeper than the TDMA scheme, the sum-rate performance at low SNR is poor compared to the TDMA. On the other hand, the proposed

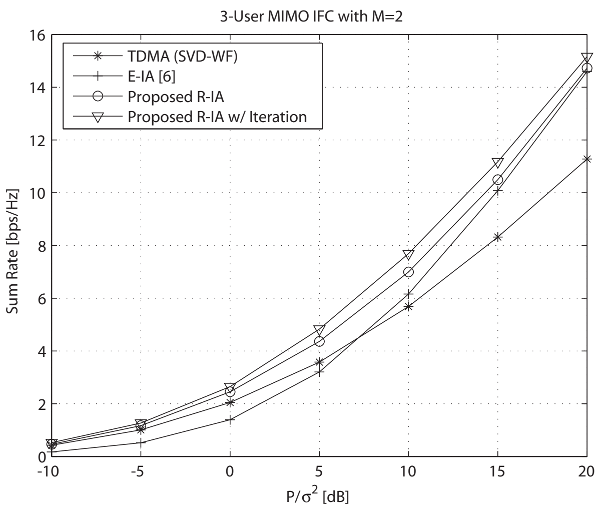

Fig. 1. Average sum-rate performance for 3-user MIMO IFC with $M = 2$

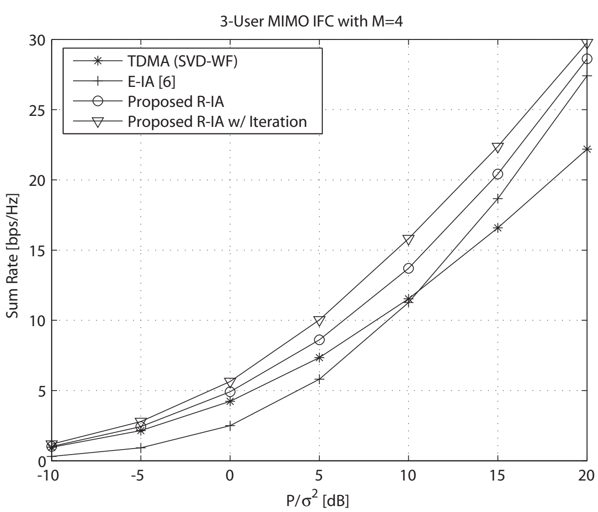

Fig. 2. Average sum-rate performance for 3-user MIMO IFC with $M = 4$

R-IA scheme outperforms the TDMA scheme over all SNR regime. Also, as shown in the plot, the further improvement is attained by allowing iterations in the proposed scheme.

Figure 2 demonstrates the sum-rate results for the case of $M = 4$. We observe that the effect of the regularization on the E-IA scheme tends to be more significant when the number of data streams is increased.

Lastly, from Figure 3, we demonstrate the effectiveness of the modified design in Section V compared to the original R-IA scheme in the presence of channel uncertainty. We note that the mismatch compensation becomes more significant for a large number of antennas.

VII. Conclusion

In this paper, we have proposed a regularized version of [6] and accomplished a goal of achieving good sum-rate performance in overall SNR regime. To enhance the sum-rate performance with minimum additional complexity, we have utilized the WMSE metric whose weight term is computed from the E-IA scheme. To justify our selection of weight matrix, it is shown that the E-IA scheme is the optimal ZF scheme in terms of sum-rate performance. Also, a modified version of the proposed scheme has been provided for compensating the channel estimation error and its effectiveness is confirmed through simulation results.

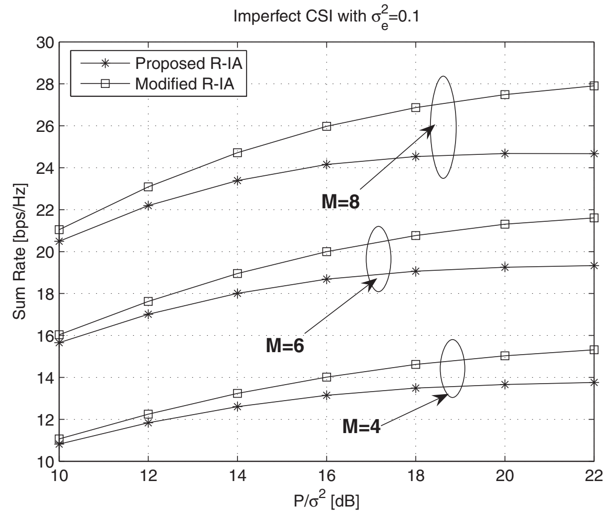

Fig. 3. Sum-rate performance with imperfect channel knowledge

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