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# Degrees of Freedom on MIMO Multi-Link Two-Way Relay Channels

Kwangwon Lee, Seok-Hwan Park, Jin-Sung Kim and Inkyu Lee
School of Electrical Eng., Korea University, Seoul, Korea
Email: {kwangwonlee, psk1501, jinsungkim, inkyu}@korea.ac.kr

***Abstract*—In this paper, we introduce multi-link two-way relay channels where multiple two-way relay systems are interfering with each other. We study the capacity of this system by investigating the degree of freedom (DOF) with various message settings. Specifically, we consider two cases of two-way relay interference channels and two-way relay X channels. We show that the two-way relay interference channel where all nodes have $M$ antennas obtains the DOF of $2M$ and compare with multi-pair two-way relay channels. Next, we introduce general message settings for two-way relay X channels. For the case where each user is equipped with 3 antennas and relays have 4 antennas, we prove that the DOF of 8 is achieved by employing network coding.**

## I. Introduction

Two way relay channels have been studied to overcome a loss of spectral efficiency in relaying-aided communication systems [1] [2] [3]. The two-way relay channel consists of two phases: the multiple access (MAC) phase and the broadcast (BC) phase. In the MAC phase, two users simultaneously send their messages to an intermediate relay terminal. After that, the relay retransmits the information obtained in the MAC phase to two users during the BC phase. By exploiting the knowledge of their own transmitted information, each user is able to cancel self-interference and decode the intended message. In this two-way relay channel, amplify-and-forward (AF) and decode-and-forward (DF) can be applied for the relay operation [2]. In the context of network coding, analog network coding (ANC) [4] and physical-layer network coding (PNC) [5] have been proposed lately. Both schemes allow simultaneous transmission of two users by performing joint detection. At the BC phase, the ANC performs symbol-wise re-encoding, whereas bit-wise encoding is applied for the PNC.

Two-way relay channels can be generalized to support a multi-pair setting where a relay helps the communication between multiple pairs of users [6] [7] [8]. In [6], the authors proposed a jointly demodulate-and-XOR forward relaying scheme for code-division multiple access systems under interference limited environments. Also, the authors in [7] and [8] characterized the capacity of multi-pair two-way relay channels in the deterministic channel and the Gaussian channel, respectively.

More general scenarios than multi-pair two-way relay channels were investigated in [9] [10]. In [9], the authors considered multiple interfering clusters of users which communicate simultaneously with the help of a relay, where the users within the same cluster exchange messages among themselves. They studied the achievable rate region of this multi-way relay channel according to different relaying schemes. By taking into account multiple transmit messages per user, multiple-input multiple-output (MIMO) Y channels with three users and a single intermediate relay were introduced in [10], where each user wants to unicast independent messages for different two users via the relay. They proposed an efficient scheme to deal with multiple interference signals in multi-user bi-directional communication systems. Also, using signal space alignment for network coding in the MAC and a nulling method in the BC, it was shown that degree of freedom (DOF) of 6 for MIMO Y channels is achieved where users have 2 antennas and the relay is equipped with 3 antennas [10].

In this paper, we introduce MIMO multiple-link two-way relay channels where multiple two-way relay links are simultaneously operating. The motivation of this channel comes from a practical scenario in wireless networks, where two user pairs want to simultaneously exchange their information in the same frequency band via two relays. Hence, if all users concurrently transmit their own signals in the multi-link two-way channel, the relays suffer from a severe interference problem, and the issue on interference management becomes critical. In such a case, our fundamental question is how each user pair and relays should cooperate to maximize the transmission rate on the network. Also our setup is interesting from the information theoretical perspective, since this multi-link two-way relay channel can be considered as a combination of two-way relay channels and interference channels [11]. Thus, we term this model as *two-way relay interference channels*. In addition, we generalize the message settings similar to X channels [12] such that each user unicasts data to multiple users at the other side through relays. We name this model as *two-way relay X channels* and provide an example to gain insight on the network coding scheme.

Throughout the paper, the transpose, conjugate transpose, inverse and trace of a matrix are represented by $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and $tr\{\cdot\}$, respectively. span$(\mathbf{A})$ is the column space spanned by the columns of $\mathbf{A}$ and $\mathbb{E}[\cdot]$ indicates the expectation operator. The organization of the paper is as follows: In Section II, we introduce the multi-link two-way relay channel model. Section III derives the DOF of this system, and discusses the system characteristics. In Section IV, we study two-way relay X channels. Finally the paper is terminated with conclusions in Section V.

## II. System Model

In this section, we describe a system model for multi-link two-way relay networks as shown in Fig. 1, where multiple

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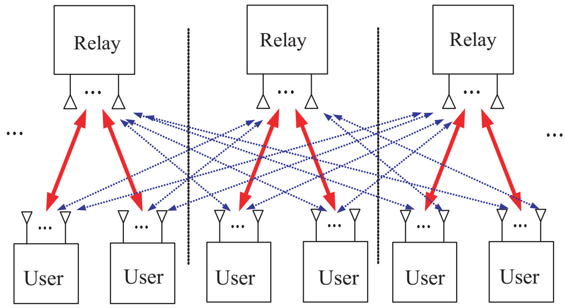


Fig. 1. MIMO multi-link two-way relay channels

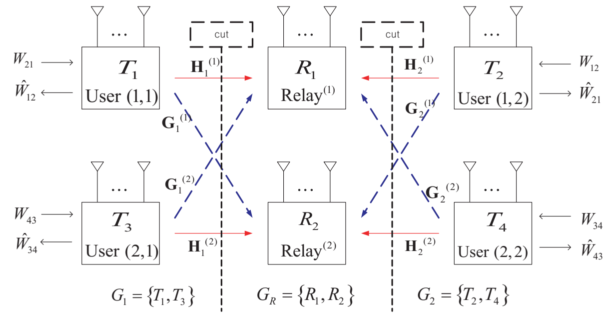


Fig. 2. MAC phase of two-link two-way relay channels and its cut-set region

two-way relay links are interfering with each other. It is assumed that there is no direct link between two user terminals. This scenario corresponds to the case where users are physically separated far away and the signal power received from each other is weak due to a large path loss. Thus an intermediate relay supports bi-directional communications between two users. For simplicity, we assume the case with two links as shown in Fig. 2. In this channel configuration, two pairs of users want to exchange their information via two intermediate relays which support bi-directional communication without data cooperation. We assume that the user nodes and the relay nodes are equipped with $M$ and $N$ antennas, respectively. We refer to the $l$-th user in the $i$-th link as user $(i, l)$ $(i, l = 1, 2)$. For simple notation of message settings, we will also denote user $(1, 1)$, $(1, 2)$, $(2, 1)$ and $(2, 2)$ as terminal $T_1$, $T_2$, $T_3$, and $T_4$, respectively. Using this terminal index, we indicate the messages transmitted from $T_l$ to $T_k$ as $W_{kl}$.

During the MAC phase, each user simultaneously transmits the signals to its relay node as shown in Fig. 2. In this case, the two-link two-way relay system is mathematically described as

$$\mathbf{y}_R^{(i)} = \mathbf{H}_1^{(i)}\mathbf{x}_1^{(i)} + \mathbf{H}_2^{(i)}\mathbf{x}_2^{(i)} + \mathbf{G}_1^{(\bar{i})}\mathbf{x}_1^{(\bar{i})} + \mathbf{G}_2^{(\bar{i})}\mathbf{x}_2^{(\bar{i})} + \mathbf{n}^{(i)} \quad (1)$$

for $i = 1, 2$, where $\mathbf{x}_l^{(i)}$ is the $M \times 1$ vectors of the transmitted signal of user $(i, l)$, $\mathbf{n}^{(i)}$ denotes the additive white Gaussian noise with unit variance, $\mathbf{y}_R^{(i)}$ indicates the $N \times 1$ received signal at the relay of the $i$-th link, $\mathbf{H}_l^{(i)}$ stands for the $N \times M$ channel matrix from user $(i, l)$ to the $i$th link relay node and $\mathbf{G}_l^{(i)}$ represents the interfering channel matrix from user $(i, l)$ to the other $\bar{i}$th link relay. Here, we define $\bar{1} = 2$ and $\bar{2} = 1$. It is assumed that the channel elements are generated from independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance.

After the MAC phase, each relay generates new transmitting signals $\mathbf{x}_R^{(i)}$ and broadcasts them to users in the BC phase. Then, the received signal vector of user $(i, l)$ is simply represented as

$$\mathbf{y}_l^{(i)} = \mathbf{D}_l^{(i)}\mathbf{x}_R^{(i)} + \mathbf{Z}_l^{(\bar{i})}\mathbf{x}_R^{(\bar{i})} + \mathbf{n}_l^{(i)}, \quad (2)$$

where $\mathbf{D}_l^{(i)}$ and $\mathbf{Z}_l^{(i)}$ are the channel matrices from the $i$-th relay to its desired link user $(i, l)$ and to the other link user $(\bar{i}, l)$, respectively. Although the channels $\mathbf{D}_l^{(i)}$ and $\mathbf{Z}_l^{(i)}$ are equal to $\mathbf{H}_l^{(i)H}$ and $\mathbf{G}_l^{(\bar{i})H}$, respectively, in the time division duplex mode, we consider that each channel matrix is generally different. Throughout this paper, a full-duplex mode is assumed during the two phases so that all nodes can transmit and receive simultaneously. Also, we assume that channel state information (CSI) is perfectly known at all nodes.

According to the message settings, we classify two-link two-way relay channels into *two-way relay interference channels* and *two-way relay X channels*. In the following sections, we will study the DOF of these channel models. Since it is hard to identify an exact capacity region in the interference environments, an alternative metric such as the DOF is needed to evaluate the system capacity performance. The DOF is an expression of the pre-log factor on the capacity [11] defined as

$$\eta \triangleq \lim_{\rho \to \infty} \frac{C_\Sigma(\rho)}{\log \rho}$$

where $C_\Sigma(\rho)$ denotes the sum capacity at SNR $\rho$. Since the sum capacity of communication networks at the high SNR regime is determined by the DOF, it is a crucial metric for characterizing the capacity behavior.

## III. Degree of Freedom of Two-Way Relay Interference Channels

In this section, we will characterize an exact expression of the DOF for MIMO two-way relay interference channels where relays also have $M$ antennas. We will show that the DOF is equal to $2M$, i.e.,

$$d_{12}^{(1)} + d_{21}^{(1)} + d_{12}^{(2)} + d_{21}^{(2)} = 2M \quad (3)$$

where $d_{\bar{l}l}^{(i)}$ indicates the DOF for information transfer from user $(i, l)$ to user $(i, \bar{l})$. The converse result and the achievability of (3) are given in the following subsections.

### A. Converse

In this subsection, the converse result is briefly derived on the basis of the DOF result of MIMO interference channels in [11] and the cut-set theorem in [13]. In [11], the authors studied two user Gaussian interferences channel where base station $i$ $(i = 1, 2)$ with $M_i$ antennas supports user $i$ with $N_i$ antennas. They showed that the exact number of the spatial DOF for the interference channel with the $(M_1, N_1, M_2, N_2)$ antenna configuration is $\min\{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\}$. Since all nodes have $M$ antennas in our scenario, the spatial DOF of the interference channel simply becomes $M$.

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First, let us consider a situation where $T_2$ and $T_4$ do not have any message $W_{12}$ and $W_{34}$ for the other user $T_1$ and $T_3$, respectively, i.e., $W_{12} = W_{34} = \phi$, which can be seen as two-link one-way relay channels. For a brief explanation, we classify all nodes into three groups as $G_1 = \{T_1, T_3\}$, $G_2 = \{T_2, T_4\}$, and $G_R = \{R_1, R_2\}$ as shown in Fig. 2. In this case, a $G_R$ receives only $G_1$'s messages during the MAC phase and transmits its messages to $G_2$ during the BC phase. Then, each phase can be considered as interference channels. We can find from the Jafar's result [11] that the DOF in the MAC phase is $M$. Denoting the sum-rate of interference channels from group A to group B as $R_{BA}$, we can express the DOF from $G_1$ to $G_R$ as $\lim_{\rho\to\infty} \frac{R_{G_R G_1}(\rho)}{\log(\rho)} = M$. Also, relays can transfer the messages to the users in $G_2$ by the DOF of $M$, i.e., $\lim_{\rho\to\infty} \frac{R_{G_R G_2}(\rho)}{\log(\rho)} = M$.

Applying the cut-set theorem on each phase as described in Fig. 2, the sum-rate of one-way relay interference channels from $G_1$ to $G_2$ becomes $R_{G_2G_1} = \min\{R_{G_RG_1}, R_{G_2G_R}\}$. Then, the total DOF for information transfer in this case equals

$$\begin{aligned} d_{21}^{(1)} + d_{21}^{(2)} &= \lim_{\rho\to\infty} \frac{R_{G_2G_1}(\rho)}{\log(\rho)} \\ &= \min\left\{\lim_{\rho\to\infty} \frac{R_{G_RG_1}(\rho)}{\log(\rho)}, \lim_{\rho\to\infty} \frac{R_{G_2G_R}(\rho)}{\log(\rho)}\right\} \\ &= M. \end{aligned} \quad (4)$$

Similarly, by considering the case where only users in $G_2$ have messages ($W_{21} = W_{43} = \phi$), the DOF from $G_2$ to $G_1$ is given by

$$d_{12}^{(1)} + d_{12}^{(2)} = M. \quad (5)$$

If both of the above one-way communications happen at the same time, the sum of each DOF becomes an upper bound. Thus, from (4) and (5), the upper bound on the DOF of MIMO two-way relay interference channels is obtained as

$$d_{12}^{(1)} + d_{21}^{(1)} + d_{12}^{(2)} + d_{21}^{(2)} \leq 2M.$$

### *B. Achievability*

In order for two bi-directional links to operate simultaneously with the same frequency band via two relays, we need to manage the interference signals. Before we prove the achievability of (3), we briefly review the signal space alignment for the network coding scheme introduced in [10]. In this scheme, the beamforming vectors are designed such that two desired signal vectors coming from two users are aligned. For example, user $(i,l)$ transmits its message to the relay using the codeword symbol $s_l^{(i)}$ and the beamforming vector $\mathbf{v}_l^{(i)}$. Each user computes the beamforming vector to receive two desired signals at the relay at the same spatial dimension i.e., $\text{span}(\mathbf{H}_1^{(i)}\mathbf{v}_1^{(i)}) = \text{span}(\mathbf{H}_2^{(i)}\mathbf{v}_2^{(i)})$, for $i = 1, 2$. Then, the relay is able to jointly detect and encode the signals using the ANC [4] or the PNC [5].

Now, we will show the lower bound on the available DOF for two-way interference relay channels. For simple presentation, we assume that $M$ is even . Then we can argue that for every users, the DOF of $\frac{M}{2}$ is achieved, i.e., $\left(d_{12}^{(1)}, d_{21}^{(1)}, d_{12}^{(2)}, d_{21}^{(2)}\right) = \left(\frac{M}{2}, \frac{M}{2}, \frac{M}{2}, \frac{M}{2}\right)$. In the MAC phase, user $(i,l)$ transmits the message to user $(i,\bar{l})$ using $\frac{M}{2}$ independent streams multiplied by the beamforming matrix $\mathbf{V}_l^{(i)} = \left[\mathbf{v}_{l,1}^{(i)}\mathbf{v}_{l,2}^{(i)}, \cdots, \mathbf{v}_{l,\frac{M}{2}}^{(i)}\right]$ as

$$\mathbf{x}_l^{(i)} = \sum_{m=1}^{\frac{M}{2}} \mathbf{v}_{l,m}^{(i)} s_{l,m}^{(i)}.$$

In order to simultaneously decode each $M/2$ stream at the relays, the beamforming vectors need to satisfy the following condition

$$\text{span}\left(\mathbf{H}_1^{(i)}\mathbf{V}_1^{(i)} + \mathbf{H}_2^{(i)}\mathbf{V}_2^{(i)}\right) \subset \text{span}(\mathbf{U}_i^{(i)}), \quad (6)$$

where $\mathbf{U}_i^{(i)}$ is the $M \times \frac{M}{2}$ unitary basis of the intersection subspace of the $i$th link users' signals for the $i$th relay.

Also, the interfering signal space to the $\bar{i}$th relay by user $(i,1)$ and user $(i,2)$ is aligned in order not to exceed the $\frac{M}{2}$ signal space dimension for the $\bar{i}$th link. Thus we have

$$\text{span}\left(\mathbf{G}_1^{(i)}\mathbf{V}_1^{(i)} + \mathbf{G}_2^{(i)}\mathbf{V}_2^{(i)}\right) \subset \text{span}(\mathbf{U}_i^{(\bar{i})}), \quad (7)$$

where $\mathbf{U}_i^{(\bar{i})}$ is the $M \times \frac{M}{2}$ unitary basis of the intersection subspace of the interfering signals to the $\bar{i}$th relay from the $i$th link users. Consequently, equations (6) and (7) can be rewritten as

$$\text{span}(\mathbf{H}_1^{(i)}\mathbf{V}_1^{(i)}) \cap \text{span}(\mathbf{H}_2^{(i)}\mathbf{V}_2^{(i)}) = \text{span}\left(\mathbf{U}_i^{(i)}\right), \quad (8)$$

$$\text{span}(\mathbf{G}_1^{(i)}\mathbf{V}_1^{(i)}) \cap \text{span}(\mathbf{G}_2^{(i)}\mathbf{V}_2^{(i)}) = \text{span}\left(\mathbf{U}_i^{(\bar{i})}\right). \quad (9)$$

In this constraint, $\mathbf{V}_1^{(i)}$ and $\mathbf{V}_2^{(i)}$ should be given by

$$\begin{aligned} \text{span}(\mathbf{V}_1^{(i)}) &= \text{span}(\mathbf{H}_1^{(i)-1}\mathbf{H}_2^{(i)}\mathbf{V}_2^{(i)}), \\ \text{span}(\mathbf{G}_2^{(i)-1}\mathbf{G}_1^{(i)}\mathbf{V}_1^{(i)}) &= \text{span}(\mathbf{V}_2^{(i)}). \end{aligned} \quad (10)$$

Then, it follows

$$\text{span}(\mathbf{V}_2^{(i)}) = \text{span}(\mathbf{G}_2^{(i)-1}\mathbf{G}_1^{(i)}\mathbf{H}_1^{(i)-1}\mathbf{H}_2^{(i)}\mathbf{V}_2^{(i)}). \quad (11)$$

If we denote $\mathbf{G}_2^{(i)-1}\mathbf{G}_1^{(i)}\mathbf{H}_1^{(i)-1}\mathbf{H}_2^{(i)}$ as $\mathbf{H}_{eff}^{(i)}$, we can choose $\frac{M}{2}$ beamforming vectors for $\mathbf{V}_2^{(i)} = \left[\mathbf{v}_{2,1}^{(i)}, \cdots, \mathbf{v}_{2,\frac{M}{2}}^{(i)}\right]$ among the eigenvectors of $\mathbf{H}_{eff}^{(i)}$. After designing $\mathbf{V}_2^{(i)}$, we can get $\mathbf{V}_1^{(i)}$ from the equation (10) for $i = 1, 2$.

Now, the received signal at the relays in (1) can be represented as

$$\mathbf{y}_R^{(i)} = \left[\mathbf{U}_i^{(i)} \quad \mathbf{U}_{\bar{i}}^{(i)}\right] \begin{bmatrix} \tilde{\mathbf{s}}_i^{(i)} \\ \tilde{\mathbf{s}}_{\bar{i}}^{(i)} \end{bmatrix} + \mathbf{n}^{(i)}, \qquad \text{for } i = 1, 2, \quad (12)$$

where $\tilde{\mathbf{s}}_j^{(i)}$ is the sum of the $j$th link users' symbols with their channel gains received at the $i$th relay. Specifically, $\tilde{\mathbf{s}}_i^{(i)}$ and $\tilde{\mathbf{s}}_{\bar{i}}^{(i)}$ can be written as

$$\begin{aligned} \tilde{\mathbf{s}}_i^{(i)} &= \left[a_{1,1}^{(i)}s_{1,1}^{(i)} + a_{2,1}^{(i)}s_{2,1}^{(i)}, \cdots, a_{1,\frac{M}{2}}^{(i)}s_{1,\frac{M}{2}}^{(i)} + a_{2,\frac{M}{2}}^{(i)}s_{2,\frac{M}{2}}^{(i)}\right]^T, \\ \tilde{\mathbf{s}}_{\bar{i}}^{(i)} &= \left[b_{1,1}^{(\bar{i})}s_{1,1}^{(\bar{i})} + b_{2,1}^{(\bar{i})}s_{2,1}^{(\bar{i})}, \cdots, b_{1,\frac{M}{2}}^{(\bar{i})}s_{1,\frac{M}{2}}^{(\bar{i})} + b_{2,\frac{M}{2}}^{(\bar{i})}s_{2,\frac{M}{2}}^{(\bar{i})}\right]^T \end{aligned}$$

where $a_{j,k}^{(i)}$ and $b_{j,k}^{(i)}$ are the normalized channel gains determined from $a_{j,k}^{(i)}\mathbf{u}_{i,k}^{(i)} = \mathbf{H}_j^{(i)}\mathbf{v}_{j,k}^{(i)}$ and $b_{j,k}^{(\bar{i})}\mathbf{u}_{i,k}^{(\bar{i})} = \mathbf{G}_j^{(i)}\mathbf{v}_{j,k}^{(i)}$,

For the odd $M$ case, we can prove using time extension in [12] respectively. In other words, $\tilde{\mathbf{s}}_i^{(i)}$ and $\tilde{\mathbf{s}}_{\bar{i}}^{(i)}$ are the $i$th and $\bar{i}$th link signal received at the $i$th relay, respectively.

Note that $\mathbf{u}_{i,k}^{(i)}$ and $\mathbf{u}_{\bar{i},k}^{(i)}$ are the basis vectors of the intersection subspace $\mathbf{U}_i^{(i)}$ and $\mathbf{U}_{\bar{i}}^{(i)}$ spanned by (8) and (9), respectively. Since $\mathbf{H}_l^{(i)}$ and $\mathbf{G}_l^{(i)}$ for $\forall i, l$ are generated by a continuous distribution, $\mathbf{u}_{i,k}^{(i)}$ and $\mathbf{u}_{\bar{i},k}^{(i)}$ for $m = 1, \cdots, \frac{M}{2}$ are linearly independent. Namely, the event that a basis vector in the intersection space spanned by $\mathbf{H}_1^{(i)}$ and $\mathbf{H}_2^{(i)}$ lies in the other intersection space spanned by $\mathbf{G}_1^{(i)}$ and $\mathbf{G}_2^{(i)}$ does not occur and vice versa. Consequently, $\left[\mathbf{U}_i^{(i)} \quad \mathbf{U}_{\bar{i}}^{(i)}\right]$ in (12) has full rank, and thus the $i$th relay can detect its link symbol $\tilde{\mathbf{s}}_i^{(i)}$. After detecting the symbols, the $i$th relay can generate the network coded messages. For example, the ANC yields $\tilde{\mathbf{s}}_i^{(i)}$ itself, while the PNC creates $\tilde{W}_{12} = W_{21} \oplus W_{12}$ [5]. Note that the $i$th relay is also able to detect the other link symbol $\tilde{\mathbf{s}}_{\bar{i}}^{(i)}$. Thus these symbols could be transmitted in various ways.

We denote $\mathbf{r}_j^{(i)}$ as the network coded symbols obtained from decoding the received symbols $\tilde{\mathbf{s}}_j^{(i)}$. In the BC phase, consider that the relay multicasts each link's network coded information $\mathbf{r}_i^{(i)}$ using $\frac{M}{2}$ independently streams precoded by the beamforming matrix $\mathbf{V}_R^{(i)} = \left[\mathbf{v}_{R,1}^{(i)}, \cdots, \mathbf{v}_{R,\frac{M}{2}}^{(i)}\right]$ as $\mathbf{x}_R^{(i)} = \sum_{m=1}^{\frac{M}{2}} \mathbf{v}_{R,m}^{(i)} r_{i,m}^{(i)}$, where $\mathbf{x}_R^{(i)}$ is normalized by the power constraint as $\mathbb{E}\left[\mathrm{tr}\left\{\mathbf{x}_R^{(i)} \mathbf{x}_R^{(i)H}\right\}\right] = \rho$. Then, the received signal vector of user $(i, l)$ becomes

$$\begin{aligned} \mathbf{y}_l^{(i)} &= \mathbf{D}_l^{(i)} \mathbf{x}_R^{(i)} + \mathbf{Z}_l^{(\bar{i})} \mathbf{x}_R^{(\bar{i})} + \mathbf{n}_l^{(i)} \\ &= \mathbf{D}_l^{(i)} \mathbf{V}_R^{(i)} \mathbf{r}_i^{(i)} + \mathbf{Z}_l^{(\bar{i})} \mathbf{V}_R^{(\bar{i})} \mathbf{r}_{\bar{i}}^{(\bar{i})} + \mathbf{n}_l^{(i)}. \end{aligned} \quad (13)$$

Note that relays transmit the total $M$ data streams and each user has enough dimensions. Thus, the relays are able to design the beamforming vectors which make $\mathbf{D}_l^{(i)} \mathbf{V}_R^{(i)}$ and $\mathbf{Z}_l^{(\bar{i})} \mathbf{V}_R^{(\bar{i})}$ linearly independent for $\forall l, i$, and then each user can detect its symbol $\mathbf{r}_i^{(i)}$.

Finally, user $(1, 1)$ can extract the desired message $W_{12}$ using its own information for self interference canceling for the ANC. Also for the PNC, $W_{12}$ is obtained as $\hat{W}_{12} = \tilde{W}_{12} \oplus W_{21} = (W_{12} \oplus W_{21}) \oplus W_{21} = W_{12} \oplus (W_{21} \oplus W_{21})$. Therefore, each user can exchange $\frac{M}{2}$ independent data streams with the opposite user. As a result, the total $2M$ DOF is achieved in MIMO two-way relay interference channels. We can make the following observations.

*Remark 1:* Comparing two-way relay interference channels with multi-pair relay channels where only one relay supports multi-pair users' communication, we can see from equation (12) that the $i$th link relay is able to detect both information messages of two link network coded messages $\tilde{\mathbf{s}}_i^{(i)}$ and $\tilde{\mathbf{s}}_{\bar{i}}^{(i)}$. Even if the $\bar{i}$th link relay is not used, which can be interpreted as two-pair two-way relay channels, the $i$th relay can still broadcast both network coded messages, and thus it is possible to obtain the DOF of $2M$. Therefore, the DOF of two-pair two-way relay channels where all nodes have $M$ antennas becomes $2M$.

*Remark 2:* Even though one more relay is added to support communications in two-pair two-way relay channels, which become two-way relay interference channels, we can observe that there is no gain in terms of the DOF. In multi-link two-way relay systems, however, relays are able to choose a communication method among various schemes. Each relay multicasts only one link symbol (cooperative relay multicasting) or one relay broadcasts both link symbols while the other relay is kept silent (cooperative relay silencing). Also both relays can broadcast both link symbols by considering all users' effective channels (cooperative relay broadcasting). After relays evaluate the system performance such as the outage probability of network coded messages, they can select a scheme which has better performance for their link.

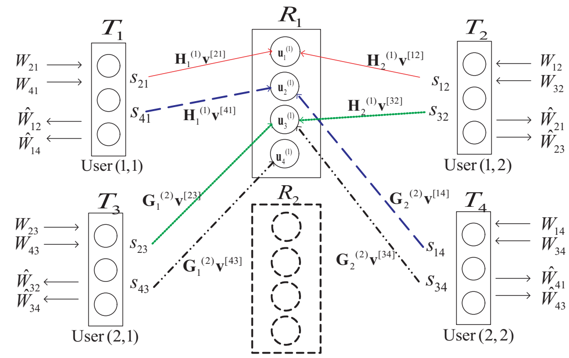


Fig. 3. Two-way relay X channels with signal space alignment during the MAC phase

## IV. MIMO Two-Way Relay X Channels

Until now, we have focused on the message settings for single unicast in two-way relay interference channels. In this section, we introduce two-way relay X channels where multiple unicast message settings are assumed. As shown in Fig. 3, user $(i, l)$ wants to transmit the messages not only to user $(i, \bar{l})$ but also to user $(\bar{i}, \bar{l})$ for $\forall i, l$. We will show that the signal space alignment scheme combined with the interference nulling beamforming [10] provides higher DOF gains for this channel by illustrating a simple example where each user and the relay have $M = 3$ and $N = 4$ antennas, respectively. For simplicity, we consider the cooperative relay silencing system where a relay helps multi-directional communication. The following theorem is the main result in this section.

*Theorem 1:* The DOF of 8 is achieved for two-way relay X channels with $M = 3$ and $N = 4$.

*Proof:* Let us consider a system where each user transmits 2 data streams simultaneously. Then, there are eight independent messages in MIMO two-way relay X channels, and they are $W_{12}$, $W_{14}$, $W_{21}$, $W_{23}$, $W_{32}$, $W_{34}$, $W_{41}$ and $W_{43}$. Then, $W_{lk}$ are mapped to the symbol $s_{lk}$. During the MAC phase, each user terminal $T_l$ transmits the message $W_{kl}$ to $T_k$ using the beamforming vector $\mathbf{v}^{[kl]}$. Since there are enough intersection spaces between any two channels, $T_l$ and $T_k$ can design the beamforming vectors such that its signal space for $W_{kl}$ and $W_{lk}$ is aligned in one dimension as

$$\mathrm{span}(\mathbf{H}_1^{(1)} \mathbf{v}^{(21)}) \cap \mathrm{span}(\mathbf{H}_2^{(1)} \mathbf{v}^{(12)}) = \mathrm{span}\left(\mathbf{u}_1^{(1)}\right),$$

$$\mathrm{span}(\mathbf{H}_1^{(1)} \mathbf{v}^{(41)}) \cap \mathrm{span}(\mathbf{G}_2^{(2)} \mathbf{v}^{(14)}) = \mathrm{span}\left(\mathbf{u}_2^{(1)}\right),$$

$$\mathrm{span}(\mathbf{G}_1^{(2)} \mathbf{v}^{(23)}) \cap \mathrm{span}(\mathbf{H}_2^{(1)} \mathbf{v}^{(32)}) = \mathrm{span}\left(\mathbf{u}_3^{(1)}\right),$$

$$\mathrm{span}(\mathbf{G}_1^{(2)} \mathbf{v}^{(43)}) \cap \mathrm{span}(\mathbf{G}_2^{(2)} \mathbf{v}^{(34)}) = \mathrm{span}\left(\mathbf{u}_4^{(1)}\right),$$

where $\mathbf{u}_j^{(i)}$ is a unitary column vector in the intersection subspace $(j = 1, \ldots, N)$ as depicted in Fig. 3.

Then, the received signal at the relay becomes

$$\begin{aligned}\mathbf{y}_R^{(1)} &= \left[\mathbf{u}_1^{(1)}\ \mathbf{u}_2^{(1)}\ \mathbf{u}_3^{(1)}\ \mathbf{u}_4^{(1)}\right]\begin{bmatrix}\alpha_{12}s_{12}+\alpha_{21}s_{21}\\ \alpha_{14}s_{14}+\alpha_{41}s_{41}\\ \alpha_{23}s_{23}+\alpha_{32}s_{32}\\ \alpha_{34}s_{34}+\alpha_{43}s_{43}\end{bmatrix} + \mathbf{n}_R^{(i)}\\ &= \mathbf{U}^{(1)}\mathbf{s} + \mathbf{n}_R^{(i)},\end{aligned}$$

where $\alpha_{kl}$ is the normalized channel gain determined by the channel matrix and the beamforming vector as $\alpha_{12}\mathbf{u}_1^{(1)} = \mathbf{H}_1^{(1)}\mathbf{v}^{(21)}$. Note that channel matrices are generated from a continuous distribution and therefore $\mathbf{u}_j^{(1)}$ for $\forall j$ are linearly independent with probability one. Since the relay has the $N = 4$ dimensional signal space, the relay can perform joint detection for the symbol vector $\mathbf{s}$ almost surely. Then, relay obtains four network coded messages $\tilde{W}_{12} = W_{12} \oplus W_{21}$, $\tilde{W}_{14} = W_{14} \oplus W_{41}$, $\tilde{W}_{23} = W_{23} \oplus W_{32}$ and $\tilde{W}_{34} = W_{34} \oplus W_{43}$, and those messages are mapped to the symbols $\tilde{s}_{12}$, $\tilde{s}_{14}$, $\tilde{s}_{23}$ and $\tilde{s}_{34}$, respectively.

In the BC phase, the relay broadcasts those four symbols by multiplying the beamforming vectors $\mathbf{f}_{12}$, $\mathbf{f}_{14}$, $\mathbf{f}_{23}$, and $\mathbf{f}_{34}$. Among the four network coded data, each user needs only two data and the remainder becomes interference. For example, $\tilde{s}_{12}$ is the interfering signal to $T_3$ and $T_4$, since it only contains the information $W_{12}$ and $W_{21}$. As each user has the $M = 3$ dimensions to receive data, it is capable of dealing with one interference signal. However, the remaining second interference should be eliminated from the relay so that only one interference signal can be reached to each user at a time. Note that the $3 \times 4$ channel matrix from the relay to each user has the null space of one dimension. Then we are able to select the beamforming vector for $\tilde{s}_{12}$ from the null space of the channels from the relay to $T_3$ or from the relay to $T_4$. For all symbols $\tilde{s}_{kl}$, we carefully choose the beamforming vectors $\mathbf{f}_{kl}$ not to overlap the null space of the channels as

$$\begin{aligned}\mathbf{f}_{12} \in \text{null}\left(\mathbf{Z}_2^{(1)}\right) \quad &, \quad \mathbf{f}_{14} \in \text{null}\left(\mathbf{Z}_1^{(1)}\right),\\ \mathbf{f}_{23} \in \text{null}\left(\mathbf{D}_1^{(1)}\right) \quad &, \quad \mathbf{f}_{34} \in \text{null}\left(\mathbf{D}_2^{(1)}\right),\end{aligned}$$

where null $(\mathbf{A})$ indicates null space of $\mathbf{A}$, e.g., $\mathbf{D}_1^{(1)}\mathbf{f}_{23} = \mathbf{0}$.

Consequently, one of the interfering signals is eliminated from all users' received signals using the above beamforming vectors. Then, each user receives two desired data and one non-intended data. For example, the received signal at $T_1$ becomes

$$\begin{aligned}\mathbf{y}_1^{(1)} &= \mathbf{D}_1^{(1)}\left[\mathbf{f}_{12}\ \mathbf{f}_{14}\ \mathbf{f}_{23}\ \mathbf{f}_{34}\right]\tilde{\mathbf{s}} + \mathbf{n}_1^{(1)}\\ &= \mathbf{D}_1^{(1)}\left[\mathbf{f}_{12}\ \mathbf{f}_{14}\ \mathbf{f}_{34}\right]\tilde{\mathbf{s}}' + \mathbf{n}_1^{(1)}, \end{aligned} \tag{14}$$

where $\tilde{\mathbf{s}} = [\tilde{s}_{12}\ \tilde{s}_{14}\ \tilde{s}_{23}\ \tilde{s}_{34}]^T$ and $\tilde{\mathbf{s}}' = [\tilde{s}_{12}\ \tilde{s}_{14}\ \tilde{s}_{34}]^T$. Since the effective channel matrix $\mathbf{D}_1^{(1)}[\mathbf{f}_{12}\ \mathbf{f}_{14}\ \mathbf{f}_{34}]$ in (14) has full rank almost surely, $T_1$ is able to detect its desired symbols. From the detected symbols $\hat{s}_{12}$ and $\hat{s}_{14}$, it can extract the desired messages $W_{12}$ and $W_{14}$ using the side information. The other terminals are also able to get their messages transmitted from two users. Finally, we can obtain the DOF of 8 by using the signal space alignment for network coding at the MAC phase and the interference nulling method at the BC phase. ■

Note that in the $M = 3$ and $N = 4$ case, the DOF of 6 can be obtained if bidirectional time-division multiple access (TDMA) where one link maintains silencing during the other links' information change is applied for two-way relay X channels. In contrast, we show that the signal space alignment for network coding and the interference nulling beamforming for network coding achieves additional DOF of 2.

## V. Conclusions

In this paper, multi-link MIMO two-way relay channels have been investigated. Motivated by the earlier work, we have characterized the effect of the interference and network coding on the DOF. We have studied how the network coding gain is attained through efficient exploitation of the signal space, even though two-way relay links are interfering with each other. According to different network message exchange scenarios, we have shown that the DOF of $2M$ is achieved for two-way relay interference channels where all nodes are equipped with $M$ antennas by the signal space alignment for network coding. Furthermore, we have verified that the DOF of 8 is achieved in two-way relay X channels for the case $M = 3$ and $N = 4$. Characterization of the DOF for general configurations in terms of users and relays remains as a future work.

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