Achievable Rate Regions for Two-Way MIMO AF Multiple-Relay Channels

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***Abstract*—This paper investigates the achievable rate region of two-way amplify-and-forward (AF) relaying systems with multiple relays, where all nodes are equipped with multiple antennas. First, we find linear processing filters to optimize the weighted sum-rate for the two-way channels with sum power constraints. The proposed algorithm achieves the maximum weighted sum-rate by deriving the gradient expressions and iteratively applying the gradient descent method. Consequently, the proposed scheme outperforms the conventional methods in terms of the weighted sum-rate. Also, the achievable rate region is obtained by adjusting the weights in the proposed algorithm. It is observed from the ergodic rate region that channel reciprocity can improve the achievable sum-rate in two-way relay systems unlike one-way channels.**

I. Introduction

Relay based wireless networks have been studied with a lot of interest for extending the cell coverage or increasing the system performance [1]–[11]. In the relay network, multiple antennas can be employed for obtaining a similar performance benefit observed in point-to-point multiple-input multiple-output (MIMO) systems. An amplify-and-forward (AF) relaying method exhibits advantages of simple implementation compared to decode-and-forward techniques in practical systems [1]. The capacity and the optimal precoder of the MIMO AF relay systems have also been studied in [2]–[4]. Furthermore, the capacity scaling analysis in [5] shows that the capacity of multiple relay systems grows with the number of relay nodes. Most relay systems are assumed to operate in the half-duplex mode where relay nodes do not transmit and receive signals simultaneously to avoid loop interference [12] in the relay nodes. Such half-duplex relay systems suffer from a substantial loss in terms of spectral efficiency due to the pre-log factor $\frac{1}{2}$, which dominates the capacity at high signal-to-noise ratio (SNR).

Two-way relaying protocol has been proposed to overcome such a spectral efficiency loss in the half-duplex one-way system [6]–[11]. In [6], linear relay precoders have been introduced to eliminate interference in two-way systems. Also, the analog network coding (ANC) based on *self interference* canceling has been employed for improving the performance of the two-way system in [7] and [8]. Linear optimization of MIMO two-way AF systems with the ANC has been recently investigated for single-relay and multiple-relay cases in [10] and [11], respectively. The optimization algorithm in [11] employs the iterative gradient descent method [13], and thus identifies the source and relay precoders. In addition, the asymptotic optimality of the sum-rate maximization has been demonstrated for two-way multiple-relay systems [11], where the prefixed transmit power at each node is assumed according to the individual power constraint.

This work was supported by Seoul R&BD Program (ST090852).

In this paper, we consider sum power constraints for jointly optimizing the source and relay filters which maximize the weighted sum-rate in multiple-relay MIMO systems for two-way protocols. For battery-limited systems such as a sensor network, it can be an important issue to allocate transmit power among terminals on the sum power constraint [14] [15]. The proposed algorithm on the sum power constraints provides the achievable rate region by adaptively optimizing the power distribution between both terminals and all relay nodes. We derive gradient expressions of the weighted sum-rate by taking into account the sum power constraints. Simulation results demonstrate that the channel reciprocity can enhance the sum-rate performance for bidirectional channels. Also, the proposed scheme outperforms the conventional methods in terms of the weighted sum-rate.

This paper is organized as follows: Section II describes the system model for two-way MIMO multiple relay channels. In Section III, we present the proposed optimization algorithm on the sum power constraint. Section IV illustrates the channel reciprocity issue through the numerical results. Finally, the paper is terminated with conclusions in Section V.

Throughout this paper, the superscripts $(\cdot)^T$, $(\cdot)^\dagger$ and $(\cdot)^*$ stand for transpose, conjugate transpose, and element-wise conjugate, respectively. $\mathcal{E}(\cdot)$ denotes the expectation, and $\mathbf{I}_N$ indicates an $N \times N$ identity matrix. $\text{Tr}(\mathbf{A})$, $|\mathbf{A}|$ and $\text{vec}(\mathbf{A})$ represent the trace, the determinant and the stacked columns of a matrix $\mathbf{A}$, respectively, and the Frobenius norm of $\mathbf{A}$ is defined as $\|\mathbf{A}\|_F^2 = \text{Tr}(\mathbf{A}\mathbf{A}^\dagger)$.

II. System Model

In this section, we describe a system model for two-way multiple relay networks shown in Fig. 1. We consider that two terminal nodes $\mathcal{T}_A$ and $\mathcal{T}_B$ want to communicate to each other, and K relay nodes $\mathcal{R}_1, \cdots, \mathcal{R}_K$ help the communication between two terminals. The terminal nodes $\mathcal{T}_A$ and $\mathcal{T}_B$ are equipped with M_A antennas and M_B antennas, respectively, and the k-th relay node $\mathcal{R}_k$ has N_k antennas. In this paper, it is assumed that the channel knowledge of all MIMO links is available at both terminals $\mathcal{T}_A$ and $\mathcal{T}_B$, and the optimized relay filters are broadcasted to the corresponding relay nodes.

In the first channel phase, two terminals $\mathcal{T}_A$ and $\mathcal{T}_B$ precode their signal vectors $\mathbf{s}_A \in \mathbb{C}^{M_A}$ and $\mathbf{s}_B \in \mathbb{C}^{M_B}$ by the transmit

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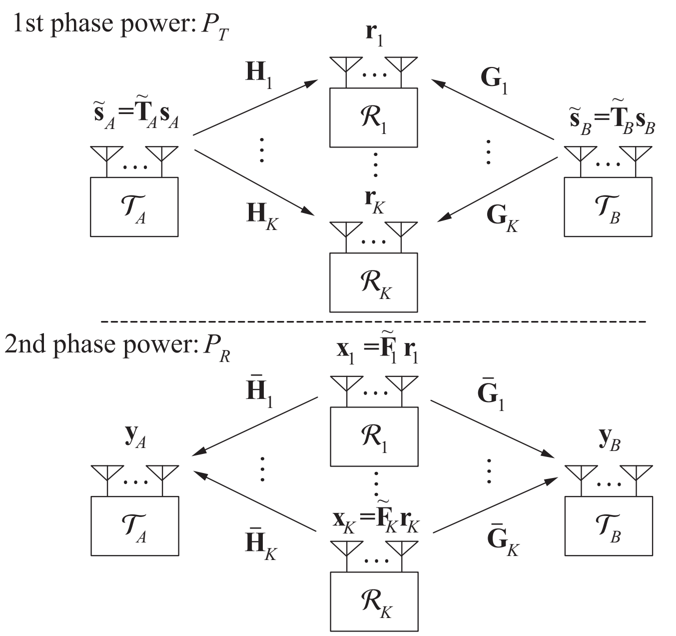

Fig. 1. Schematic diagram of MIMO relaying systems in the two-way protocol

filters $\tilde{\mathbf{T}}_A \in \mathbb{C}^{M_A\times M_A}$ and $\tilde{\mathbf{T}}_B \in \mathbb{C}^{M_B\times M_B}$, respectively, and then simultaneously transmit the precoded signals $\tilde{\mathbf{s}}_A = \tilde{\mathbf{T}}_A\mathbf{s}_A$ and $\tilde{\mathbf{s}}_B = \tilde{\mathbf{T}}_B\mathbf{s}_B$ to K relay nodes $\mathcal{R}_1, \cdots, \mathcal{R}_K$ as shown in Fig. 1, where $\mathbf{s}_A$ and $\mathbf{s}_B$ are assumed to have $\mathcal{E}(\mathbf{s}_A\mathbf{s}_A^\dagger) = \mathbf{I}_{M_A}$ and $\mathcal{E}(\mathbf{s}_B\mathbf{s}_B^\dagger) = \mathbf{I}_{M_B}$, respectively.

At the k-th relay node, the received signal vector $\mathbf{r}_k \in \mathbb{C}^{N_k}$ for $k = 1, \cdots, K$ is given as

$$\begin{aligned}\mathbf{r}_k &= \mathbf{H}_k\tilde{\mathbf{s}}_A + \mathbf{G}_k\tilde{\mathbf{s}}_B + \mathbf{n}_k \\ &= \mathbf{H}_k\tilde{\mathbf{T}}_A\mathbf{s}_A + \mathbf{G}_k\tilde{\mathbf{T}}_B\mathbf{s}_B + \mathbf{n}_k\end{aligned}$$

where $\mathbf{H}_k \in \mathbb{C}^{N_k\times M_A}$ and $\mathbf{G}_k \in \mathbb{C}^{N_k\times M_B}$ are the channel matrices for links $\mathcal{T}_A \to \mathcal{R}_k$ and $\mathcal{T}_B \to \mathcal{R}_k$, respectively, and $\mathbf{n}_k$ denotes the additive complex Gaussian noise vector with zero mean and $\mathcal{E}(\mathbf{n}_k\mathbf{n}_k^\dagger) = \sigma_{\mathbf{n}}^2\mathbf{I}_{N_k}$.

In the second channel phase, the received signal $\mathbf{r}_k$ is multiplied by the relay filter matrix $\tilde{\mathbf{F}}_k \in \mathbb{C}^{N_k\times N_k}$ at the k-th relay node. Then, the signal vector $\mathbf{x}_k \in \mathbb{C}^{N_k}$ at the relay node is computed by

$$\begin{aligned}\mathbf{x}_k &= \tilde{\mathbf{F}}_k\mathbf{r}_k \\ &= \tilde{\mathbf{F}}_k\mathbf{H}_k\tilde{\mathbf{T}}_A\mathbf{s}_A + \tilde{\mathbf{F}}_k\mathbf{G}_k\tilde{\mathbf{T}}_B\mathbf{s}_B + \tilde{\mathbf{F}}_k\mathbf{n}_k.\end{aligned}$$

Now, the processed signals $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K$ are transmitted from the relay nodes to two terminal nodes $\mathcal{T}_A$ and $\mathcal{T}_B$. We denote the MIMO channel matrices for links $\mathcal{R}_k \to \mathcal{T}_A$ and $\mathcal{R}_k \to \mathcal{T}_B$ at the second channel phase as $\bar{\mathbf{H}}_k \in \mathbb{C}^{M_A\times N_k}$ and $\bar{\mathbf{G}}_k \in \mathbb{C}^{M_B\times N_k}$, respectively. Since terminal nodes know their own symbols transmitted at the previous channel phase as well as the corresponding effective channels, these back-propagating *self-interference* terms can be canceled when received at both terminals [7].

After removing the self-interferences, the received signals $\hat{\mathbf{y}}_A \in \mathbb{C}^{M_A}$ and $\hat{\mathbf{y}}_B \in \mathbb{C}^{M_B}$ are then given as

$$\hat{\mathbf{y}}_A = \sum_{k=1}^{K}\bar{\mathbf{H}}_k\tilde{\mathbf{F}}_k\mathbf{G}_k\tilde{\mathbf{T}}_B\mathbf{s}_B + \sum_{k=1}^{K}\bar{\mathbf{H}}_k\tilde{\mathbf{F}}_k\mathbf{n}_k + \mathbf{z}_A \tag{1}$$

and

$$\hat{\mathbf{y}}_B = \sum_{k=1}^{K}\bar{\mathbf{G}}_k\tilde{\mathbf{F}}_k\mathbf{H}_k\tilde{\mathbf{T}}_A\mathbf{s}_A + \sum_{k=1}^{K}\bar{\mathbf{G}}_k\tilde{\mathbf{F}}_k\mathbf{n}_k + \mathbf{z}_B \tag{2}$$

where $\mathbf{z}_A \in \mathbb{C}^{M_A}$ and $\mathbf{z}_B \in \mathbb{C}^{M_B}$ denote the complex white Gaussian noise vectors with zero mean and variance $\sigma_{\mathbf{z}}^2$. In [11], the sum-rate for the two-way channels in (1) and (2) is optimized with the prefixed transmit power at the source and the relay nodes. In the following section, the weighted sum-rate maximization algorithm with adaptive transmit power based on the sum power constraint is presented.

III. Weighted Sum-rate Maximization on Sum Power Constraints

A main objective in this paper is to maximize the weighted sum-rate of the bidirectional links in (1) and (2). In this section, we propose a linear processing scheme for solving this problem on the sum power constraint.

A. Problem Formulation

Assuming the sum power constraint, the weighted sum-rate maximization problem in (1) and (2) can be expressed as

$$\max_{\mathbf{T}_A,\mathbf{T}_B,\mathbf{F}_1,\cdots,\mathbf{F}_K} \quad R_{\text{sum}} \triangleq w_A R_A + w_B R_B \tag{3}$$

$$\text{subject to} \quad \sum_{l=A,B}\mathcal{E}\{||\tilde{\mathbf{s}}_l||^2\} \le P_T, \tag{4}$$

$$\sum_{k=1}^{K}\mathcal{E}\{||\mathbf{x}_k||^2\} \le P_R \tag{5}$$

where the individual rates for $\mathcal{T}_A \to \mathcal{T}_B$ and $\mathcal{T}_B \to \mathcal{T}_A$ are defined as [2]

$$\begin{aligned}R_A \triangleq\ & \frac{1}{2}\log_2\Bigg|\mathbf{I}_{M_A} + \left(\sum_{k=1}^{K}\bar{\mathbf{G}}_k\tilde{\mathbf{F}}_k\mathbf{H}_k\tilde{\mathbf{T}}_A\right)^\dagger \\ &\times\left(\sigma_n^2\sum_{k=1}^{K}\bar{\mathbf{G}}_k\tilde{\mathbf{F}}_k\tilde{\mathbf{F}}_k^\dagger\bar{\mathbf{G}}_k^\dagger + \sigma_z^2\mathbf{I}_{M_B}\right)^{-1}\left(\sum_{k=1}^{K}\bar{\mathbf{G}}_k\tilde{\mathbf{F}}_k\mathbf{H}_k\tilde{\mathbf{T}}_A\right)\Bigg|, \\ R_B \triangleq\ & \frac{1}{2}\log_2\Bigg|\mathbf{I}_{M_B} + \left(\sum_{k=1}^{K}\bar{\mathbf{H}}_k\tilde{\mathbf{F}}_k\mathbf{G}_k\tilde{\mathbf{T}}_B\right)^\dagger \\ &\times\left(\sigma_n^2\sum_{k=1}^{K}\bar{\mathbf{H}}_k\tilde{\mathbf{F}}_k\tilde{\mathbf{F}}_k^\dagger\bar{\mathbf{H}}_k^\dagger + \sigma_z^2\mathbf{I}_{M_A}\right)^{-1}\left(\sum_{k=1}^{K}\bar{\mathbf{H}}_k\tilde{\mathbf{F}}_k\mathbf{G}_k\tilde{\mathbf{T}}_B\right)\Bigg|,\end{aligned}$$

and w_A and w_B are nonnegative weight constants. Here the pre-log factor 1/2 is caused by the half-duplex mode. Also, the sum power constraints (4) and (5) are specified for two terminals and all K relay nodes, respectively.

Since the cost function R_{sum} is not generally convex or concave with respect to $\mathbf{T}_A, \mathbf{T}_B, \mathbf{F}_1, \cdots, \mathbf{F}_K$, this problem in (3) is difficult to solve analytically. Hence, we optimize the transmit and relay filters by deriving the gradients of the weighted sum-rate and applying the gradient descent algorithm. In order to exploit the gradient method, we first convert the problem in (3) into an unconstrained maximization problem. Since the maximum sum-rate is achieved through full power transmission at each the transmit phase, without loss of optimality, we define $\tilde{\mathbf{T}}_l = \rho_T \mathbf{T}_l$ for $l = A, B$ and $\tilde{\mathbf{F}}_k = \gamma_R \mathbf{F}_k$ for $k = 1, \cdots, K$ from (4) and (5), where the normalization factors ρ_T and γ_R are expressed by

$$\rho_T = \sqrt{\frac{P_T}{\text{Tr}(\mathbf{T}_A\mathbf{T}_A^\dagger) + \text{Tr}(\mathbf{T}_B\mathbf{T}_B^\dagger)}} \quad \text{and} \tag{6}$$

$$\gamma_R = \sqrt{\frac{P_R}{\sum_{k=1}^K \text{Tr}\Big\{\mathbf{F}_k\Big(\rho_T^2\mathbf{H}_k\mathbf{T}_A\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger + \rho_T^2\mathbf{G}_k\mathbf{T}_B\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger + \sigma_n^2\mathbf{I}_{N_k}\Big)\mathbf{F}_k^\dagger\Big\}}}. \tag{7}$$

Then, using these normalization factors, the unconstrained problem which computes the optimum filter matrices for maximizing the weighted sum-rate can be formulated as

$$\{\hat{\mathbf{T}}_A, \hat{\mathbf{T}}_B, \hat{\mathbf{F}}_1, \cdots, \hat{\mathbf{F}}_K\} = \arg \max_{\mathbf{T}_A, \mathbf{T}_B, \mathbf{F}_1, \cdots, \mathbf{F}_K} \tilde{R}_{\text{sum}}. \tag{8}$$

Here, $\tilde{R}_{\text{sum}}$ is defined by

$$\tilde{R}_{\text{sum}} = \frac{w_A}{2}\log_2\left|\mathbf{\Pi}_A^{-1}\right| + \frac{w_B}{2}\log_2\left|\mathbf{\Pi}_B^{-1}\right| \tag{9}$$

where $\mathbf{\Pi}_l$ for $l = A$ and B is denoted as

$$\mathbf{\Pi}_l = \left(\mathbf{I}_{M_l} + \rho_T^2\mathbf{\Sigma}_l^\dagger\mathbf{\Omega}_l\mathbf{\Sigma}_l\right)^{-1},$$

and $\mathbf{\Sigma}_A$, $\mathbf{\Sigma}_B$, $\mathbf{\Omega}_A$, and $\mathbf{\Omega}_B$ are expressed by

$$\mathbf{\Sigma}_A = \sum_{k=1}^K \bar{\mathbf{G}}_k\mathbf{F}_k\mathbf{H}_k\mathbf{T}_A, \;\; \mathbf{\Sigma}_B = \sum_{k=1}^K \bar{\mathbf{H}}_k\mathbf{F}_k\mathbf{G}_k\mathbf{T}_B,$$

$$\mathbf{\Omega}_A = \left(\sigma_n^2\sum_{k=1}^K \bar{\mathbf{G}}_k\mathbf{F}_k\mathbf{F}_k^\dagger\bar{\mathbf{G}}_k^\dagger + \frac{\sigma_z^2}{\gamma_R^2}\mathbf{I}_{M_B}\right)^{-1}, \text{ and}$$

$$\mathbf{\Omega}_B = \left(\sigma_n^2\sum_{k=1}^K \bar{\mathbf{H}}_k\mathbf{F}_k\mathbf{F}_k^\dagger\bar{\mathbf{H}}_k^\dagger + \frac{\sigma_z^2}{\gamma_R^2}\mathbf{I}_{M_A}\right)^{-1}.$$

B. Precoder Design using Gradients

Now we derive the gradients of the weighted sum-rate $\tilde{R}_{\text{sum}}$ in (9) with respect to $\mathbf{T}_A, \mathbf{T}_B, \mathbf{F}_1, \cdots, \mathbf{F}_K$. Since $\tilde{R}_{\text{sum}}$ is a real-valued function, the gradient is written as $\nabla_{\mathbf{X}}\tilde{R}_{\text{sum}} = 2\partial\tilde{R}_{\text{sum}}/\partial\mathbf{X}^*$ [16]. To obtain the derivative of the weighted sum-rate function $\partial\tilde{R}_{\text{sum}}/\partial\mathbf{F}_k^*$, we first compute the differential of the function. Using the differential rule $d\ln|\mathbf{Y}| = \text{Tr}(\mathbf{Y}^{-1}d\mathbf{Y})$ [17], the partial differential of the weighted sum-rate in (9) for $\mathbf{F}_k^*$ or $\mathbf{T}_l^*$ is given by

$$\begin{aligned} d\tilde{R}_{\text{sum}} = &\frac{w_A}{2\ln 2}\text{Tr}\{\mathbf{\Pi}_A d(\rho_T^2\mathbf{\Sigma}_A^\dagger\mathbf{\Omega}_A\mathbf{\Sigma}_A)\} \\ &+ \frac{w_B}{2\ln 2}\text{Tr}\{\mathbf{\Pi}_B d(\rho_T^2\mathbf{\Sigma}_B^\dagger\mathbf{\Omega}_B\mathbf{\Sigma}_B)\}. \end{aligned} \tag{10}$$

The differential with respect to $\mathbf{F}_k^*$ yields $d\mathbf{\Sigma}_A^\dagger = \gamma_R\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger d\mathbf{F}_k^\dagger\bar{\mathbf{G}}_k^\dagger$ and $d(\mathbf{\Omega}_A^{-1}) = \sigma_{\mathbf{n}}^2\bar{\mathbf{G}}_k\mathbf{F}_k d\mathbf{F}_k^\dagger\bar{\mathbf{G}}_k^\dagger + d\frac{1}{\gamma_R^2}\mathbf{I}_{M_B}$ where we have $d\frac{1}{\gamma_R^2} = \frac{1}{P_R}\text{Tr}\{\mathbf{F}_k(\rho_T^2\mathbf{H}_k\mathbf{T}_A\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger + \rho_T^2\mathbf{G}_k\mathbf{T}_B\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger + \sigma_{\mathbf{n}}^2\mathbf{I}_{N_k})d\mathbf{F}_k^\dagger\}$. Here, it should be noted that γ_R is a function of $\mathbf{F}_k^*$ through (7). After some matrix manipulations using the above results in (10), the differential with respect to $\mathbf{F}_k^*$ is obtained as

$$\begin{aligned} d\tilde{R}_{\text{sum}} = &\Big[\frac{\rho_T^2 w_A}{\ln 2}\text{vec}\Big\{\bar{\mathbf{G}}^\dagger\mathbf{\Omega}_A\mathbf{\Sigma}_A\mathbf{\Pi}_A(\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger - \sigma_n^2\mathbf{\Sigma}_A^\dagger\mathbf{\Omega}_A\bar{\mathbf{G}}_k\mathbf{F}_k)\Big\}^T \\ &+ \frac{\rho_T^2 w_B}{\ln 2}\text{vec}\Big\{\bar{\mathbf{H}}^\dagger\mathbf{\Omega}_B\mathbf{\Sigma}_B\mathbf{\Pi}_B(\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger - \sigma_n^2\mathbf{\Sigma}_B^\dagger\mathbf{\Omega}_B\bar{\mathbf{H}}_k\mathbf{F}_k)\Big\}^T \\ &- \frac{\rho_T^2\sigma_z^2}{P_R\ln 2}\sum_{l=A,B} w_l\text{Tr}(\mathbf{\Sigma}_l^\dagger\mathbf{\Omega}_l^2\mathbf{\Sigma}_l\mathbf{\Pi}_l) \\ &\times\text{vec}\Big\{\mathbf{F}_k(\rho_T^2\mathbf{H}_k\mathbf{T}_A\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger + \rho_T^2\mathbf{G}_k\mathbf{T}_B\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger + \sigma_n^2\mathbf{I}_{N_k})\Big\}^T\Big] \\ &\times d\text{vec}(\mathbf{F}_k^*). \end{aligned}$$

Finally, since the derivative $\partial\tilde{R}_{\text{sum}}/\partial\mathbf{F}_k^*$ can be directly derived from the coefficients of $d\text{vec}(\mathbf{F}_k^*)$ in the above differential, the gradient of the weighted sum-rate with respect to the k-th relay filter $\mathbf{F}_k$ is written by

$$\begin{aligned} &\nabla_{\mathbf{F}_k}\tilde{R}_{\text{sum}} \\ &= \frac{\rho_T^2 w_A}{\ln 2}\bar{\mathbf{G}}^\dagger\mathbf{\Omega}_A\mathbf{\Sigma}_A\mathbf{\Pi}_A(\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger - \sigma_n^2\mathbf{\Sigma}_A^\dagger\mathbf{\Omega}_A\bar{\mathbf{G}}_k\mathbf{F}_k) \\ &\quad+ \frac{\rho_T^2 w_B}{\ln 2}\bar{\mathbf{H}}^\dagger\mathbf{\Omega}_B\mathbf{\Sigma}_B\mathbf{\Pi}_B(\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger - \sigma_n^2\mathbf{\Sigma}_B^\dagger\mathbf{\Omega}_B\bar{\mathbf{H}}_k\mathbf{F}_k) \\ &\quad- \frac{\rho_T^2\sigma_z^2}{P_R\ln 2}\sum_{l=A,B} w_l\text{Tr}(\mathbf{\Sigma}_l^\dagger\mathbf{\Omega}_l^2\mathbf{\Sigma}_l\mathbf{\Pi}_l) \\ &\quad\times\mathbf{F}_k(\rho_T^2\mathbf{H}_k\mathbf{T}_A\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger + \rho_T^2\mathbf{G}_k\mathbf{T}_B\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger + \sigma_n^2\mathbf{I}_{N_k}). \end{aligned} \tag{11}$$

Now, considering the partial differential with respect to $\mathbf{T}_A^*$ in (10), we obtain $d\mathbf{\Sigma}_A^\dagger = d\mathbf{T}_A^\dagger\sum_{k=1}^K\mathbf{H}_k^\dagger\mathbf{F}_k^\dagger\bar{\mathbf{G}}_k^\dagger$ and $d(\mathbf{\Omega}_A^{-1}) = \sigma_{\mathbf{z}}^2 d\frac{1}{\gamma_R^2}\mathbf{I}_{M_B}$, where $d\frac{1}{\gamma_R^2} = \frac{1}{P_R}d\rho_T^2\sum_{k=1}^K\text{Tr}\{\mathbf{F}_k(\mathbf{H}_k\mathbf{T}_A\mathbf{T}_A^\dagger\mathbf{H}_k^\dagger + \mathbf{G}_k\mathbf{T}_B\mathbf{T}_B^\dagger\mathbf{G}_k^\dagger)\mathbf{F}_k^\dagger\} + \frac{\rho_T^2}{P_R}\sum_{k=1}^K\text{Tr}(\mathbf{H}_k^\dagger\mathbf{F}_k^\dagger\mathbf{F}_k\mathbf{H}_k\mathbf{T}_A d\mathbf{T}_A^\dagger)$ and $d\rho_T^2 = -\frac{\rho_T^4}{P_T}\text{Tr}(\mathbf{T}_A d\mathbf{T}_A^\dagger)$, since ρ_T and γ_R are functions of $\mathbf{T}_A^*$ in (6) and (7), respectively. Then, using the same method in (11), the gradient of $\tilde{R}_{\text{sum}}$ with respect to the transmit filter $\mathbf{T}_l$ for $l = A$ and B is computed by

$$\begin{aligned} &\nabla_{\mathbf{T}_l}\tilde{R}_{\text{sum}} \\ &= \frac{\rho_T^2 w_l}{\ln 2}\mathbf{\Sigma}_l^\dagger\mathbf{\Omega}_l\mathbf{\Sigma}_l\mathbf{\Pi}_l - \frac{\rho_T^4}{P_T\ln 2}\sum_{i=A,B} w_i\text{Tr}(\mathbf{\Sigma}_i^\dagger\mathbf{\Omega}_i\mathbf{\Sigma}_i\mathbf{\Pi}_i)\mathbf{T}_l \\ &\quad- \frac{\rho_T^4}{P_T P_R\ln 2}\sum_{i=A,B} w_i\text{Tr}(\mathbf{\Sigma}_i^\dagger\mathbf{\Omega}_i^2\mathbf{\Sigma}_i\mathbf{\Pi}_i) \\ &\quad\times\sum_{k=1}^K\left\{P_T\mathbf{\Gamma}_{l,k} - \rho_T^2\sum_{i=A,B}\text{Tr}(\mathbf{T}_i^\dagger\mathbf{\Gamma}_{i,k}\mathbf{T}_i)\,\mathbf{I}_{M_i}\right\}\mathbf{T}_l \end{aligned} \tag{12}$$

where $\mathbf{\Gamma}_{l,k}$ for $l = A$ and B are denoted as

$$\mathbf{\Gamma}_{A,k} = \mathbf{H}_k^\dagger\mathbf{F}_k^\dagger\mathbf{F}_k\mathbf{H}_k \;\text{ and }\; \mathbf{\Gamma}_{B,k} = \mathbf{G}_k^\dagger\mathbf{F}_k^\dagger\mathbf{F}_k\mathbf{G}_k.$$

Finally, with the derived gradient expressions, we now apply a gradient descent method [13] to solve the problem (8). This algorithm exploits a fact that the weighted sum-rate increases fastest when $\mathbf{T}_l$ or $\mathbf{F}_k$ moves in a direction of the corresponding gradient. Therefore, the proposed algorithm iteratively searches the optimized filter while other filters are fixed. The proposed iterative algorithm for maximizing the weighted sum-rate (8) is given as follows.

1) Initialize $\mathbf{T}_A, \mathbf{T}_B, \mathbf{F}_1, \cdots, \mathbf{F}_K$
2) for $k = 1$ to K
    Calculate the gradient $\nabla_{\mathbf{F}_k} \tilde{R}_{\text{sum}}$ in (11)
    Update $\mathbf{F}_k \leftarrow \mathbf{F}_k + \delta_k \cdot \nabla_{\mathbf{F}_k} \tilde{R}_{\text{sum}}$
  end
3) for $l = A$ and B
    Calculate the gradient $\nabla_{\mathbf{T}_l} \tilde{R}_{\text{sum}}$ in (12)
    Update $\mathbf{T}_l \leftarrow \mathbf{T}_l + \delta_l \cdot \nabla_{\mathbf{T}_l} \tilde{R}_{\text{sum}}$
  end
4) Repeat steps 2) and 3) until convergence

Several line search methods are introduced in [13] to efficiently determine the step sizes δ_k and δ_l. We employ a method called Armijo's rule [13] which provides provable convergence as in [11]. Then, the proposed algorithm obtains a non-decreasing weighted sum-rate value with respect to the number of iterations. Therefore, it can be shown that the proposed iterative algorithms achieve at least a local optimum solution.[1] Consequently, with the proper w_A and w_B, the proposed iterative algorithm maximizes the weighted sum-rate and provides the achievable rate region. Here, the transmission power at both terminals and K relay nodes can be adaptively determined on the sum power constraints P_T and P_R, respectively.

IV. Simulation Results

In this section, we present numerical results for the proposed scheme on two-way MIMO multiple relay channels. In our simulation, we assume $P_T = P_R = P$ and $\sigma_{\mathbf{n}}^2 = \sigma_{\mathbf{z}}^2 = 1$, and the SNR is defined as $P/\sigma_{\mathbf{n}}^2$. It is assumed that the elements of the channel matrices $\mathbf{H}_k$ and $\bar{\mathbf{G}}_k$ for the link of $\mathcal{T}_A \rightarrow \mathcal{T}_B$ are an independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. For the time-division duplex (TDD) mode, we assume reciprocity of bidirectional channels, i.e., the channel matrices $\bar{\mathbf{H}}_k$ and $\mathbf{G}_k$ for $\mathcal{T}_B \rightarrow \mathcal{T}_A$ are given as $\bar{\mathbf{H}}_k = \mathbf{H}_k^T$ and $\mathbf{G}_k = \bar{\mathbf{G}}_k^T$ [7], while the frequency-division duplex (FDD) mode employs the i.i.d. channel matrices for all MIMO links. We use a notation of $M_A \times \{N_1, \cdots, N_K\} \times M_B$ for representing antenna configurations in this paper. Also, the sum-rate weights are determined as $w_A + w_B = 2$.

Fig. 2 plots the achievable rate region of an instantaneous channel in two-way $2 \times 2 \times 2$ relay systems at a SNR of 20 dB by exploiting both the proposed optimization algorithm and the method in [11].[2] Here, the method in [11] generates the rate region curve with the fixed transmit power by adjusting the non-negative weights w_A and w_B, and then the achievable rate region is illustrated as the convex hull of the rate region points with any transmit power satisfying the sum power constraint.

[1]The global optimality of the proposed method can be shown if assuming an asymptotic antenna dimension as in [11].

[2]In all simulations, both the iterative algorithms employ a single initial point with identity matrices.

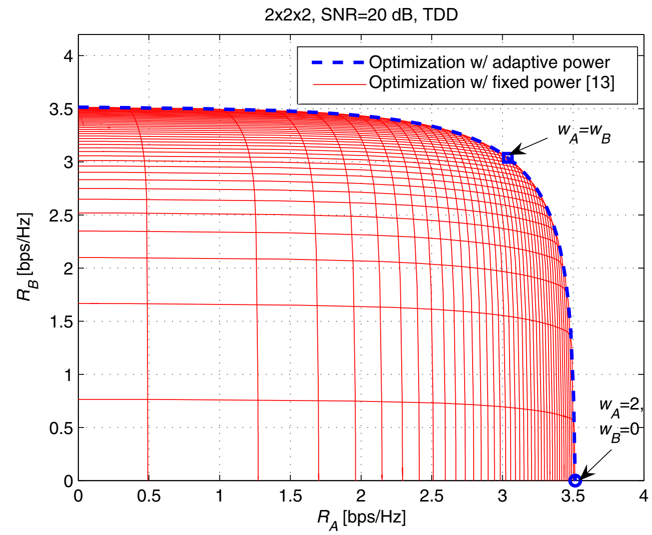

Fig. 2. Instantaneous rate region for two-way AF relay systems

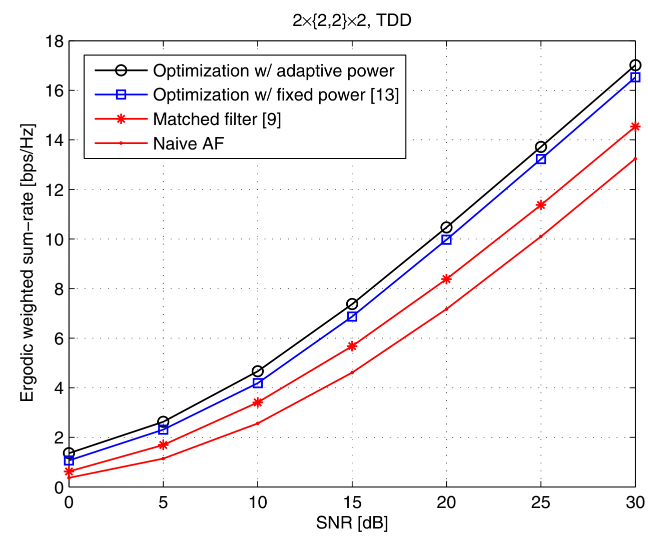

Fig. 3. Comparison of ergodic weighted sum-rates as a function of SNR in two-way AF relay systems

Since the proposed algorithm adaptively determines the power distribution with the sum power constraint unlike the fixed-power approach, the achievable rate region boundary points are directly obtained from various weights w_A and w_B as shown in Fig. 2.

In Fig. 3, we present the ergodic weighted sum-rates with the fixed weights $w_A = 8/5$ and $w_B = 2/5$ for two-way multiple relay channels with $2 \times \{2, 2\} \times 2$ systems. In this figure, we can see that the proposed algorithm provides the SNR gains of about 3.5 and 6 dB at 10 bps/Hz over the conventional matched filter method using $\mathbf{F}_k = \bar{\mathbf{G}}_k^\dagger \mathbf{H}_k^\dagger + \bar{\mathbf{H}}_k^\dagger \mathbf{G}_k^\dagger$ in [9] and the naive AF scheme with $\mathbf{F}_k = \mathbf{I}_{N_k}$. Furthermore, the proposed algorithm achieving the optimal power provides a weighted sum-rate gain over the conventional optimization method with equally allocated power at each node.

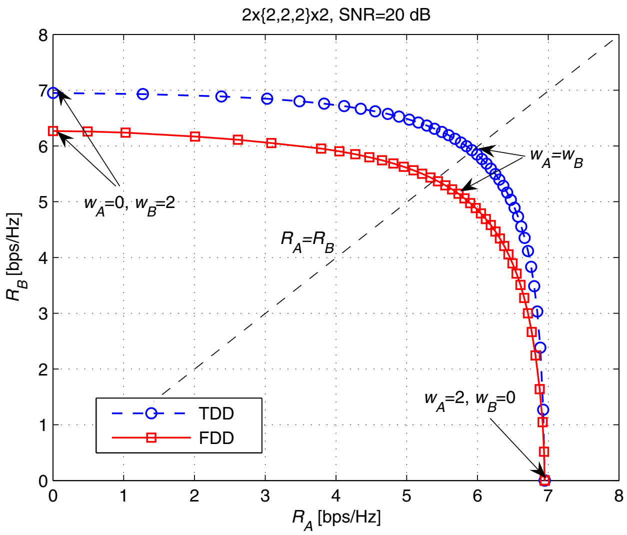

Fig. 4. Comparison of instantaneous rate regions with the TDD and the FDD modes for two-way AF relay systems

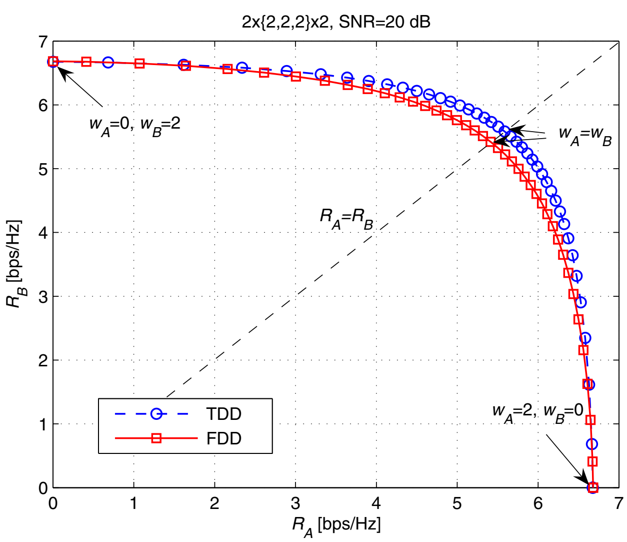

Fig. 5. Comparison of ergodic rate regions with the TDD and the FDD modes for two-way AF relay systems

Finally, by employing the proposed algorithm on the TDD and the FDD modes, the instantaneous and ergodic rate regions are compared in Fig. 4 and Fig. 5, respectively. Fig. 4 shows that the rate region in the TDD mode is symmetric while the FDD mode has an unsymmetrical rate region. This observation can be explained by the fact that both the bidirectional channels on the TDD mode are symmetrically inter-related such as $\bar{\mathbf{G}}_k\mathbf{F}_k\mathbf{H}_k = \mathbf{G}_k^T\mathbf{F}_k\mathbf{H}_k$ and $\bar{\mathbf{H}}_k\mathbf{F}_k\mathbf{G}_k = \mathbf{H}_k^T\mathbf{F}_k\mathbf{G}_k$ due to the perfect channel reciprocity.

As a result, in Fig. 5, the TDD mode exhibits better ergodic rates than the FDD case especially around the points with $w_A = w_B$, while on the one-way cases (i.e., $w_A = 2$, $w_B = 0$ and $w_A = 0$, $w_B = 2$), the ergodic rates of the TDD and the FDD modes are identical. These results imply that for two-way relay systems unlike one-way channels, the TDD protocol with the channel reciprocity may be preferred rather than the FDD in terms of the sum-rate performance.

V. Conclusion

In this paper, we have presented the achievable rate region for two-way protocol systems with the sum power constraints. By exploiting the gradient descent algorithm, we obtain the transmit and relay filter matrices which maximizes the weighted sum-rate. The gradient expression for the weighted sum-rate is derived for the proposed algorithm. We adaptively optimize the power distribution among the terminals or the relay nodes with the sum power constraints. Consequently, the proposed algorithm allows us to generate the achievable rate region for two-way MIMO multiple relay channels. It is interesting to observe from the rate region that for the two-way systems, the TDD mode with the channel reciprocity exhibits a sum-rate gain over the FDD, which is not common in one-way systems.

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