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Beamforming Design Based on Virtual SINR Maximization for Interference Networks

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***Abstract*—In this paper, we propose beamforming techniques based on virtual signal-to-interference-plus-noise ratio (VSINR) maximization for weighted sum-rate (WSR) maximization in multiple-input single-output (MISO) interference channels. In the earlier work by Zakhour and Gesbert, it was shown that all Pareto-optimal beamformers can be expressed as a solution to the VSINR problem. However, how to choose the weight coefficients in the VSINR expression is not addressed when solving the WSR maximization problem. Thus, we provide a method of computing the weight terms to achieve a certain desired WSR maximizing point. Since the beamforming vectors in the proposed scheme should be computed as a function of global channel state information (CSI), we also propose a decentralized approach which shows a performance close to the centralized scheme with a significant reduction in the CSI exchange overhead.**

I. Introduction

Since cellular systems and ad-hoc networks are inevitably interference-limited, research on interference channels (IC) has become an important and timely issue [1] [2]. However, the capacity region of the IC is not completely characterized even for the 2-user case [3]. The best achievable scheme until now is based on the rate-splitting which considers the possibility of multi-user detection (MUD) at each receiver. Since optimizing the transmit strategy under the MUD is quite complicated, it is reasonable to design the input signals under the assumption of single-user detection (SUD) where each receiver treats the interference signals as noise. If we restrict each receiver as the SUD in the multiple-input single-output (MISO) IC, it was shown in [4] that the transmit beamforming combined with scalar Gaussian coding can achieve all Pareto boundary points of the achievable rate region.

In this paper, we study the beamforming design in the MISO IC. The authors in [5] showed that one can achieve all Pareto-optimal points with a solution to the virtual signal-to-interference-plus-noise ratio (VSINR) maximization problem if we properly choose the weight coefficients in the VSINR expression. The VSINR is different from the actual signal-to-interference-plus-noise (SINR) in a sense that it is a function of a single beamforming vector. However, in [5], only necessary conditions for achieving the Pareto-optimal points are provided, i.e., how to choose the parameters to guarantee some desired boundary points is not completely addressed.

To solve this issue, we provide a method of adapting the weight coefficients to achieve a desired weighted sum-rate (WSR) maximizing point. By deriving the gradient expressions for the WSR and VSINR maximization problems, it is shown that the weight terms for the equivalence between two problems depend on the beamforming vectors. Thus, we propose a two step algorithm. For fixed beamformers, the weight coefficients are updated such that two problems have the same gradient expression. Then, the beamforming vectors are computed as a solution to the VSINR problem with the updated weights. Since the weight terms and the beamforming vectors depend on each other, these two steps should be repeated until convergence. Also, we propose a decentralized scheme based on the local channel state information (CSI). From numerical results, it is confirmed that the proposed VSINR-based schemes exhibit the near-optimal WSR performance and the decentralized scheme provides performance close to the centralized method with a significant reduction in the CSI exchange overhead.

Throughout this paper, we will use the following notations. The transpose, Hermitian transpose and Euclidean 2-norm are represented by $(\cdot)^T$, $(\cdot)^H$ and $||\cdot||$, respectively. An identity matrix is denoted by $\mathbf{I}$. A set of all N-dimensional column vectors is given by $\mathbb{C}^N$.

The organization of the paper is as follows: Section II presents the system model for MISO IC. In Section III, the previous works on the VSINR-based parameterization for Pareto-optimal beamformers are reviewed. Section IV presents the proposed VSINR-based beamforming scheme and a distributed implementation of the proposed VSINR approach is investigated in Section V. Through the simultion results in Section VI, we confirm the effectiveness of our proposed algorithms. Finally, this paper is terminated with conclusions in Section VII.

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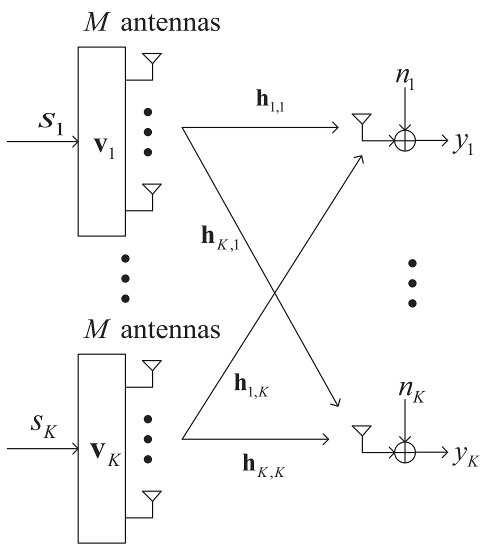

Fig. 1. Transmit beamforming model in K-user MISO IC

II. System Model

We consider K-user MISO IC where each transmitter i equipped with M transmit antennas performs transmit beamforming to support receiver i ($i = 1, \cdots, K$) as illustrated in Figure 1. For simplicity, $M \geq K$ is assumed as in [6]. Then, the received signal at receiver k can be written as

$$y_k = \mathbf{h}_{k,k}^H \mathbf{v}_k s_k + \sum_{j \neq k} \mathbf{h}_{k,j}^H \mathbf{v}_j s_j + n_k,$$

where $\mathbf{h}_{k,j} \in \mathbb{C}^M$ is the channel vector from transmitter j to receiver k, $n_k \sim \mathcal{CN}(0, N_0)$ denotes the additive white Gaussian noise at receiver k, $s_k \sim \mathcal{CN}(0,1)$ is the data symbol intended for receiver k and $\mathbf{v}_k$ indicates the beamforming vector at transmitter k. The beamformer $\mathbf{v}_k$ is subject to $||\mathbf{v}_k||^2 \leq 1$ to satisfy the per-transmitter power constraint. We will call the pair of transmitter k and receiver k as user k.

The individual rate of user k under the assumption of the SUD is given as $R_k(\{\mathbf{v}_l\}) = \log(1 + \text{SINR}_k(\{\mathbf{v}_l\}))$ where $\text{SINR}_k(\{\mathbf{v}_l\})$ represents the individual SINR as

$$\text{SINR}_k(\{\mathbf{v}_l\}) = \frac{|\mathbf{h}_{k,k}^H \mathbf{v}_k|^2}{N_0 + \sum_{j \neq k} |\mathbf{h}_{k,j}^H \mathbf{v}_j|^2}.$$

In this paper, our goal is to find the beamforming vectors maximizing the WSR defined as $R_{\sum}(\{\mathbf{v}_l\}) = \sum_{k=1}^K w_k R_k(\{\mathbf{v}_l\})$ where the weight term w_k is determined depending on the required quality of service for applications. Then, the problem can be mathematically written as

$$\max_{\mathbf{v}_1, \cdots, \mathbf{v}_K} R_{\sum}(\{\mathbf{v}_l\}) \ \text{ s.t. } ||\mathbf{v}_i||^2 \leq 1 \ \forall i. \tag{1}$$

III. Review of VSINR-based Parameterization

Solving (1) is quite complicated due to the non-convexity. Since it is obvious that the optimal WSR occurs at the Pareto boundary of the achievable rate region, we provide the detailed definition of the rate region and the Pareto boundary as follows:

Definition 1: The achievable rate region for a given channel realization denoted by $\mathcal{R}\left(\{\mathbf{h}_{i,j}, \forall i, j\}\right)$ is defined as

$$\mathcal{R}\left(\{\mathbf{h}_{i,j}, \forall i, j\}\right) = \bigcup_{||\mathbf{v}_k||^2 \leq 1, \forall k} \Big\{(R_1(\{\mathbf{v}_l\}), \cdots, R_K(\{\mathbf{v}_l\}))\Big\}.$$

Definition 2: The Pareto boundary consists of Pareto-optimal points where a rate tuple $(r_1, \cdots, r_K) \in \mathcal{R}\left(\{\mathbf{h}_{i,j}, \forall i, j\}\right)$ is called Pareto-optimal if there is no other rate tuple $(q_1, \cdots, q_K)$ such that $(q_1, \cdots, q_K) \geq (r_1, \cdots, r_K)$. Here, $\geq$ indicates the element-wise inequality.

From the definitions, we can see that the Pareto boundary is the outer boundary of the rate region $\mathcal{R}\left(\{\mathbf{h}_{i,j}, \forall i, j\}\right)$. Thus, it is enough to consider only Pareto-optimal points for the WSR maximization. The following theorems give one of parameterization methods for Pareto-optimal beamformers [5] [6].

Theorem 1: As long as we have $M \geq K$ and the set $\{\mathbf{h}_{1,k}, \cdots, \mathbf{h}_{K,k}\}$ consists of K linearly independent vectors for all $k = 1, \cdots, K$, we can assume full power transmissions at all transmitters, i.e., $||\mathbf{v}_k||^2 = 1$, $\forall k$ without any loss of optimality.

Proof: See [6]. ■

Since there is no need to apply power control due to Theorem 1, our remaining task is the optimization of beamforming directions. Regarding the parameterization of the beamforming directions, the following theorem is useful.

Theorem 2: Defining the VSINR for user k as [5]

$$\text{VSINR}_k = \frac{|\mathbf{h}_{k,k}^H \mathbf{v}_k|^2}{N_0 + \sum_{j \neq k} \alpha_{j,k} |\mathbf{h}_{j,k}^H \mathbf{v}_k|^2},$$

we can achieve all Pareto-optimal points by adopting the following VSINR maximizing beamformers

$$\begin{aligned} \mathbf{v}_k &= \arg \max_{||\mathbf{v}_k||^2 = 1} \text{VSINR}_k \\ &= \frac{\left(N_0 \mathbf{I} + \sum_{j \neq k} \alpha_{j,k} \mathbf{h}_{j,k} \mathbf{h}_{j,k}^H\right)^{-1} \mathbf{h}_{k,k}}{\left|\left| \left(N_0 \mathbf{I} + \sum_{j \neq k} \alpha_{j,k} \mathbf{h}_{j,k} \mathbf{h}_{j,k}^H\right)^{-1} \mathbf{h}_{k,k} \right|\right|} \end{aligned} \tag{2}$$

if we properly choose weight coefficients $\alpha_{j,k}$ ($j \neq k$).

Proof: See [5]. ■

However, a way of choosing the weight terms $\alpha_{j,k}$ is not addressed for the general WSR maximization problem in [5]. In the following sections, we provide the beamforming designs for the WSR maximization problem.

IV. Proposed VSINR Approach

In this section, we propose a beamforming scheme based on the VSINR maximization which consists of the following two steps: 1) For fixed beamformers, the weight terms $\alpha_{j,k}$

are updated such that the WSR and VSINR maximization problems have the same gradient expression. 2) the beamforming vectors are computed as a solution to the VSINR maximization problem with the updated weights. Since the weight terms for the equivalence between two problems are a function of the beamforming vectors, the above two steps should be performed iteratively until convergence. For the remainder of the section, we present the gradient expressions for the WSR and VSINR maximization problems to identify the condition of both two gradients having the zero value at the same point.

After some manipulations [7], it can be shown that the gradient expressions for two problems are given as

$$\begin{aligned}\nabla_{\mathbf{v}_l}\left[R_{\sum}(\{\mathbf{v}_k\})\right] &= \frac{2w_l}{I_l+D_l}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\mathbf{v}_l \\ &\quad -2\sum_{k\neq l}\frac{w_k D_k}{I_k\left(I_k+D_k\right)}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\mathbf{v}_l, \qquad (3)\end{aligned}$$

$$\begin{aligned}\nabla_{\mathbf{v}_l}\left[\log\left(\mathrm{VSINR}_l\right)\right] &= \frac{2}{D_l}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\mathbf{v}_l \\ &\quad -2\sum_{k\neq l}\frac{\alpha_{k,l}}{N_0+\sum_{j\neq l}\alpha_{j,l}|\mathbf{h}_{j,l}^H\mathbf{v}_l|^2}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\mathbf{v}_l \qquad (4)\end{aligned}$$

where $I_k = N_0+\sum_{j\neq k}|\mathbf{h}_{k,j}^H\mathbf{v}_j|^2$ and $D_k = |\mathbf{h}_{k,k}^H\mathbf{v}_k|^2$ are the interference-plus-noise power and the desired signal power at receiver k, respectively.

Comparing (3) and (4), we can see that two expressions become zero at the same point if

$$\frac{w_k D_k(I_l+D_l)}{w_l I_k(I_k+D_k)} = \frac{\alpha_{k,l}D_l}{N_0+\sum_{j\neq l}\alpha_{j,l}|\mathbf{h}_{j,l}^H\mathbf{v}_l|^2} \quad \text{for } k\neq l.$$

This equation can be converted to a linear equation with respect to $\alpha_{m,l}$ $(m\neq l)$ as

$$N_0 = \frac{w_l D_l}{I_l+D_l}\frac{I_k(I_k+D_k)}{w_k D_k}\alpha_{k,l} - \sum_{j\neq l}|\mathbf{h}_{j,l}^H\mathbf{v}_l|^2\alpha_{j,l} \quad \text{for } k\neq l.$$

By stacking the above equations over all $k\neq l$, we obtain a compact matrix expression as

$$N_0\mathbf{1}_{K-1} = \mathbf{A}_l\bar{\alpha}_l$$

where $\mathbf{1}_{K-1}$ is the all-one vector of length $K-1$ and $\mathbf{A}_l$ and $\bar{\alpha}_l$ are defined as

$$\begin{aligned}\mathbf{A}_l &= \frac{w_l D_l}{I_l+D_l}\mathrm{diag}\left(\left\{\frac{I_k(I_k+D_k)}{w_k D_k}, k\neq l\right\}\right) \\ &\quad - \mathbf{1}_{K-1}\Big[|\mathbf{h}_{1,l}^H\mathbf{v}_l|^2\cdots|\mathbf{h}_{l-1,l}^H\mathbf{v}_l|^2\ |\mathbf{h}_{l+1,l}^H\mathbf{v}_l|^2\cdots|\mathbf{h}_{K,l}^H\mathbf{v}_l|^2\Big], \\ \bar{\alpha}_l &= \left[\alpha_{1,l}\ \cdots\ \alpha_{l-1,l}\ \alpha_{l+1,l}\ \cdots\ \alpha_{K,l}\right]^T.\end{aligned}$$

Thus, at step 1), the weight coefficients $\alpha_{j,k}$'s are updated according to $\bar{\alpha}_l = N_0\mathbf{A}_l^{-1}\mathbf{1}_{K-1}$ for all $l=1,\cdots,K$. At the subsequent step, we compute each beamforming vector using (2) for the given weight terms. The whole algorithm of the proposed approach based on the VSINR is summarized as follows:

1. Initialize $\mathbf{v}_l$ for $l=1,\cdots,K$.
2. Compute $D_l, I_l, \mathbf{A}_l$ for $l=1,\cdots,K$.
3. Update the weight coefficients using $\bar{\alpha}_l \leftarrow N_0\mathbf{A}_l^{-1}\mathbf{1}_{K-1}$ for $l=1,\cdots,K$.
4. Compute the beamforming vectors $\mathbf{v}_l$ maximizing VSINR_l according to (2) for $l=1,\cdots,K$.
5. Go back to step 2 until convergence.

As will be shown in Section VI, the proposed VSINR approach shows the near-optimal WSR performance with one initial point and provides a significant performance gain over the simple VSINR scheme which operates with $\alpha_{j,k}=1$ for all $j\neq k$ [5]. Developing a beamforming method with local CSI is also an important issue for practical implementation. Thus, in the following section, we propose a distributed scheme based on the VSINR maximization.

V. Distributed Implementation of the Proposed VSINR Scheme

In this section, we propose a distributed beamforming technique which operates with only local CSI. From a viewpoint of transmitter l, the local CSI means the channel vectors connected to transmitter l, i.e., $\mathbf{h}_{k,l}$ for all $k=1,\cdots,K$. In order to enable the computation of $\mathbf{v}_l$ using only $\{\mathbf{h}_{k,l},\forall k\}$, we decouple the WSR problem in (1) into K separate problems by applying high SINR and signal-to-noise ratio (SNR) approximations. Especially for the two-user case ($K=2$), a high SINR approximation is enough to obtain fully decoupled problems.

First, we apply the high SINR approximation as

$$\begin{aligned}R_{\sum}(\{\mathbf{v}_k\}) &= \sum_{k=1}^K w_k\log\left(1+\frac{|\mathbf{h}_{k,k}^H\mathbf{v}_k|^2}{N_0+\sum_{j\neq k}|\mathbf{h}_{k,j}^H\mathbf{v}_j|^2}\right) \\ &\approx \sum_{k=1}^K w_k\log\left(\frac{|\mathbf{h}_{k,k}^H\mathbf{v}_k|^2}{N_0+\sum_{j\neq k}|\mathbf{h}_{k,j}^H\mathbf{v}_j|^2}\right).\end{aligned}$$

Collecting the terms dependent only on $\mathbf{v}_l$ results in

$$\begin{aligned}f_l(\mathbf{v}_l) &= w_l\log\left(|\mathbf{h}_{l,l}^H\mathbf{v}_l|^2\right) \\ &\quad - \sum_{k\neq l}w_k\log\left(N_0+|\mathbf{h}_{k,l}^H\mathbf{v}_l|^2+\sum_{j\neq k,l}|\mathbf{h}_{k,j}^H\mathbf{v}_j|^2\right).\end{aligned}$$

It is observed that $f_l(\mathbf{v}_l)$ depends only on $\mathbf{v}_l$ for the two user case since $\sum_{j\neq k,l}|\mathbf{h}_{k,j}^H\mathbf{v}_j|^2=0$ when $K=2$. In order to obtain the decoupled problems for general configurations of $K\geq 3$, we apply further approximation. Notice that at high SNR, the zero-forcing (ZF) beamforming exhibits the near optimal WSR performance, since the performance impairments mainly come from inter-user interference. Thus, at high SNR, the terms $\sum_{j \neq k,l} |\mathbf{h}_{k,j}^H \mathbf{v}_j|^2$ should be very small to achieve the optimal WSR performance. Motivated by this observation, $f_l(\mathbf{v}_l)$ can be approximated to

$$f_l(\mathbf{v}_l) \approx w_l \log\left(|\mathbf{h}_{l,l}^H \mathbf{v}_l|^2\right) - \sum_{k \neq l} w_k \log\left(N_0 + |\mathbf{h}_{k,l}^H \mathbf{v}_l|^2\right)$$

which depends only on $\mathbf{v}_l$ and the local CSIs $\{\mathbf{h}_{k,l}, \forall k\}$.

As a result, transmitter l can compute $\mathbf{v}_l$ using only the local CSI as a solution to

$$\mathbf{v}_l = \arg \max_{||\mathbf{v}_l||^2=1} f_l(\mathbf{v}_l).$$

We solve the above problem by employing the VSINR approach. To this end, the gradient of $f_l(\mathbf{v}_l)$ can be obtained as

$$\begin{aligned} \nabla_{\mathbf{v}_l}\left[f_l(\mathbf{v}_l)\right] &= \frac{2w_l}{D_l}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\mathbf{v}_l \\ &- 2\sum_{k \neq l} \frac{w_k}{N_0 + |\mathbf{h}_{k,l}^H \mathbf{v}_l|^2}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\mathbf{v}_l. \quad (5) \end{aligned}$$

To make the connection between $f_l(\mathbf{v}_l)$ and VSINR$_l$, we observe that (4) and (5) become zero at the same point if

$$N_0 \mathbf{1}_{K-1} = \hat{\mathbf{A}}_l \bar{\alpha}_l$$

where $\hat{\mathbf{A}}_l$ is defined as

$$\begin{aligned} \hat{\mathbf{A}}_l &= w_l \,\mathrm{diag}\left(\left\{\frac{N_0 + |\mathbf{h}_{j,l}^H \mathbf{v}_l|^2}{w_j}, j \neq l\right\}\right) \\ &- \mathbf{1}_{K-1}\Big[|\mathbf{h}_{1,l}^H \mathbf{v}_l|^2 \cdots |\mathbf{h}_{l-1,l}^H \mathbf{v}_l|^2 \; |\mathbf{h}_{l+1,l}^H \mathbf{v}_l|^2 \cdots |\mathbf{h}_{K,l}^H \mathbf{v}_l|^2\Big]. \end{aligned}$$

Thus, in the distributed scheme, we update the weight coefficients using $\bar{\alpha}_l \leftarrow N_0 \hat{\mathbf{A}}_l^{-1} \mathbf{1}_{K-1}$. Compared to the centralized scheme presented in the previous section, $\mathbf{A}_l$ is replaced by $\hat{\mathbf{A}}_l$ which depends only on $\mathbf{v}_l$ and $\{\mathbf{h}_{k,l}, \forall k\}$.

Now, we compare the proposed distributed scheme with the conventional distributed algorithms in [8] and [9]. In these schemes, transmitter l computes its beamformer as a function of $\{\mathbf{h}_{k,l}, P_k^I, \forall k\}$ where $P_k^I = \sum_{k \neq l} |\mathbf{h}_{k,l}^H \mathbf{v}_l|^2$ is the interference power at receiver k. Since P_k^I is a function of all beamformers, the receivers should report their interference power to the transmitters until convergence. In contrast, in our proposed scheme, as each transmitter l can calculate its beamformer using its own local CSIs $\{\mathbf{h}_{k,l}, \forall k\}$, there is no need for additional information exchange during iterations. Thus, the overhead for the information exchange is lower in our distributed scheme compared to the conventional schemes in [8] and [9].

VI. Simulation Results

In this section, we confirm the effectiveness of the proposed scheme by observing the average WSR performance. In all simulations, we assume spatially uncorrelated Rayleigh fading channels with unit variance. For comparison, the performance of the following schemes are displayed.

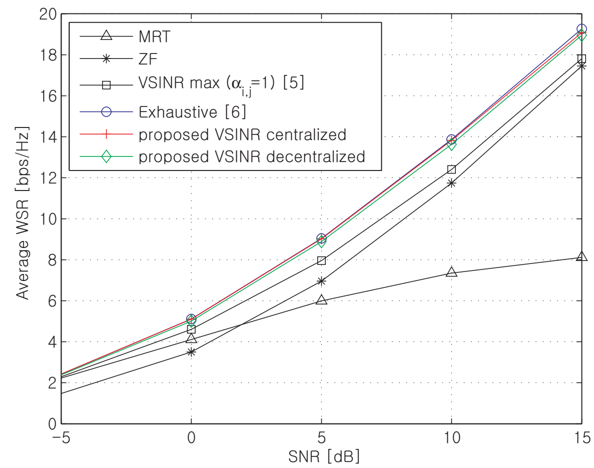

Fig. 2. Average WSR performance with $[w_1\,, w_2] = [3\,, 1]$ in 2-user MISO IC with $M = 2$

- Maximal-ratio transmission (MRT): its own channel gain is maximized neglecting the interference.
- ZF: beamformers are optimized while eliminating the interference to other users.
- VSINR maximization: the VSINR is maximized with the non-weighted coefficients, i.e., $\alpha_{i,j} = 1\ \forall i, j$ [5].
- Exhaustive: exhaustive search is performed for the WSR maximization using paremeterization in [6].
- Proposed schemes: the proposed centralized and distributed methods based on the VSINR maximization with the ZF beamforming as an initial point.

In Figure 2, the average WSR performance with $[w_1\,, w_2] = [3\,, 1]$ is plotted as a function of the system SNR $\frac{1}{N_0}$ for the 2-user MISO IC with $M = 2$. Although the proposed schemes start with only one initial point, it is observed that the proposed centralized scheme shows the near-optimal performance and the proposed distributed method provides performance almost identical to the centralized scheme.

Figure 3 presents the WSR performance for $[w_1\,, w_2] = [5\,, 1]$. We can see that as the ratio $\max(w_1, w_2)/\min(w_1, w_2)$ increases, a performance gain over the VSINR maximization scheme in [5] grows.

Figure 4 depicts the performance for the 3-user IC with $M = 3$ and $[w_1\,, w_2\,, w_3] = [10\,, 5\,, 1]$. In this case, it is hard to implement the exhaustive schemes since we should perform exhaustive search over $K(K-1) = 6$ real variables and $K^2 = 9$ complex variables for the parameterizations in [5] and [6], respectively. It should be emphasized that the proposed distributed scheme in Section V shows a performance gain of about 3 dB compared to the VSINR maximization scheme in [5] without any additional CSI exchange overhead.

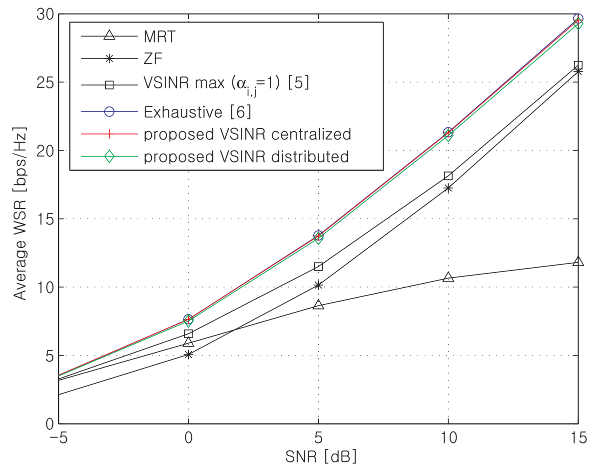

Fig. 3. Average WSR performance with $[w_1\,,w_2] = [5\,,1]$ in 2-user MISO IC with $M = 2$

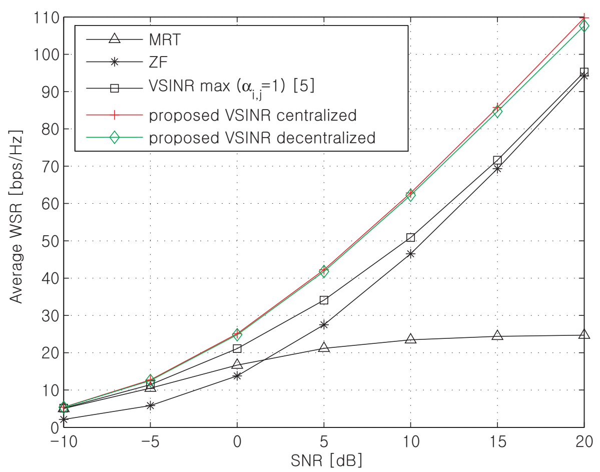

Fig. 4. Average WSR performance with $[w_1\,,w_2\,,w_3] = [10\,,5\,,1]$ in 3-user MISO IC with $M = 3$

In Figure 5, the effect of the number of iterations on the WSR performance is presented for the proposed distributed scheme. Checking the effect of the number of iterations is important in the distributed scheme, since each transmitter cannot compute the rate increase at each iteration due to the lack of global CSI. From the figure, we can see that the proposed distributed algorithm converges within 10 iterations.

VII. Conclusion

We have proposed a beamforming scheme for the WSR maximization based on the VSINR. Compared to the earlier work in [5], our proposed scheme operates with adaptive weight terms in the VSINR expression. Since the implementation with the local CSI is a very important issue, the distributed implementation of the proposed VSINR approach is also proposed. From the simulation results, we have confirmed that the proposed schemes provide near-optimal WSR performance for simulated configurations. Also, the distributed scheme exhibits the performance very close to the centralized scheme with a significant reduction in the CSI exchange overhead.

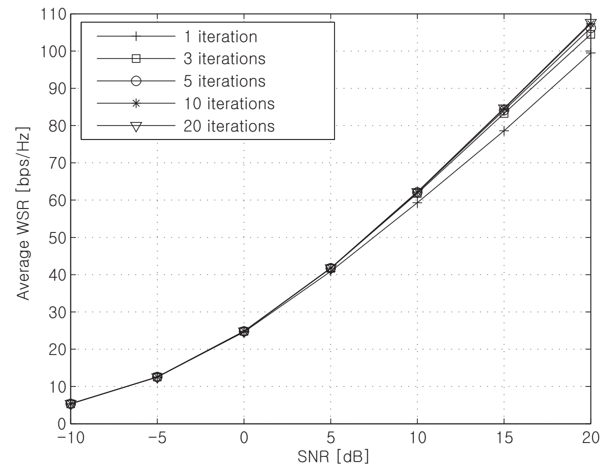

Fig. 5. The effect of the number of iterations on the WSR performance with $[w_1\,,w_2\,,w_3] = [10\,,5\,,1]$ in 3-user IC with $M = 3$

References

[1] J. G. Andrews, W. Choi, and R. W. Heath, "Overcoming Interference in Spatial Multiplexing MIMO Cellular Networks," *IEEE Wireless Communications*, vol. 14, pp. 95–104, December 2007.

[2] H. Sung, S.-H. Park, K.-J. Lee, and I. Lee, "Linear Precoder Designs for K-user Interference Channels," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 291–301, January 2010.

[3] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian Interference Channel Capacity to within One Bit," *IEEE Transactions on Information Theory*, vol. 54, pp. 5534–5562, December 2008.

[4] X. Shang, B. Chen, and H. V. Poor, "On the Optimality of Beamforming for Multi-User MISO Interference Channels with Single-User Detection," in *Proc. IEEE Globecom '09*, November 2009.

[5] R. Zakhour and D. Gesbert, "Coordination on the MISO Interference Channel using the Virtual SINR Framework," in *Proc. ITG/IEEE Workshop on Smart Antennas '09*, February 2009.

[6] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of pareto boundary for the MISO interference channel," *IEEE Transactions on Signal Processing*, vol. 56, pp. 5292–5296, October 2008.

[7] K.-J. Lee, H. Sung, E. Park, and I. Lee, "Joint Optimization for One and Two-Way MIMO AF Multiple-Relay Systems," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 3671 – 3681, December 2010.

[8] Z. K. M. Ho and D. Gesbert, "Spectrum Sharing in Multiple-Antenna Channels: A Distributed Cooperative Game Theoretic Approach," in *Proc. IEEE PIMRC '08*, September 2008.

[9] J. Lindblom and E. Karipidis, "Cooperative Beamforming for the MISO Interference Channel," in *Proc. IEEE European Wireless Conference '10*, pp. 631–638, April 2010.