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Closed-form Linear Transceiver Designs for MIMO AF Relaying Systems with Direct Link

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***Abstract*—In this paper, we investigate the minimum mean squared error based relay transceiver design in amplify-and-forward multiple antenna relay systems in the presence of direct link. Instead of the conventional optimal design which requires iterative methods, we propose a simple and near optimal closed-form solution. The proposed method exploits the decomposable property of the error covariance matrix to simplify the problem. Then, we impose a structural constraint on the non-convex problem to attain a simple closed-form solution. Through numerical simulations, we confirm that the proposed solution has almost no performance loss compared to the optimal iterative design with much reduced complexity.**

I. Introduction

It has been recognized that multiple-input and multiple-output (MIMO) wireless systems can potentially provide the improved link performance [1]. Recently, wireless relaying techniques have also garnered a significant interest, since the cell coverage can be substantially extended by supporting shadowed areas. These benefits make MIMO relaying systems be considered as a powerful candidate for next generation wireless networks [2]–[7].

In practical relay networks, one of the most popular relay protocol is amplify-and-forward (AF) which amplifies the signal received from the source and forward it to the destination [8]. In the AF MIMO relaying systems, many works have been studied to derive the optimal relay amplifying matrices. Without consideration on direct link from the source to the destination, information theoretic approaches have been reported in [4] and [5]. Running parallel with this, minimum mean squared error (MMSE) based studies have also been investigated in [6] [7] and references therein. However, in the presence of the direct link, all these works become obviously suboptimal.

In fact, the source-to-destination direct link can provide a valuable spatial diversity [8]. Thus, in case where the direct link is available, we need to optimize the relay amplifying matrix considering the direct link. For a single stream transmission, the direct link does not affect the MMSE based optimal relay matrix design, where the problem is simply convex [9] [10]. However, in case of multiple streams, it is generally difficult to solve because the direct link incurs non-convexity of the problem. Recently, the authors in [11] attempted to find the optimal solution using iterative methods such as the gradient descent algorithm.

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In this paper, we introduce a new MMSE relay filter design strategy for relaying systems with direct link which approaches the optimal performance with a closed-form solution. Instead of the conventional canonical form based methods [6] [11], we address the problem with a different approach using the error decomposition property. More specifically, utilizing the fact that the MMSE optimal relay matrix is expressed as a combination of the relay receiver and the precoder, we prove that the error covariance matrix at the destination can be represented as a sum of two individual covariance matrices, which allows us to considerably simplify the problem and to obtain valuable insights on the relaying system.

Since the decomposed problem is still non-convex, we employ a relaxation technique imposing a structural constraint on the relay precoder to establish a strictly convex optimization problem [12]. Then we identify a simple closed-form solution which yields the mean squared error (MSE) upper bound of the global optimal point. Note that the optimal single stream beamforming structure [9] [10] can be characterized as a special case in our design strategy. The proposed closed-form solution also includes the existing works in [6] and [7] without direct link. Although our solution is theoretically suboptimal, numerical results in Section V confirm that it provides almost identical performance to the optimal design [11] with much reduced complexity.

Throughout this paper, normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. $\mathbb{C}$ and $\mathbb{R}_+$ denote the set of complex numbers and the set of non-negative real numbers, respectively. The superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ stand for transpose, conjugate transpose, and element-wise conjugate, respectively. $E[\cdot]$ denotes the expectation operator, $\mathbf{I}_N$ indicates an $N \times N$ identity matrix, and $\mathrm{Tr}\,(\mathbf{A})$ represents the trace of a matrix $\mathbf{A}$. Accounting for a complex matrix $\mathbf{A}$, $\Re\{\mathbf{A}\}$ and $\mathrm{vec}(\mathbf{A})$ denote the real part of $\mathbf{A}$ and the stacked columns of $\mathbf{A}$, respectively.

II. System Model

As shown in Figure 1, we consider a three node MIMO relaying system where one relay node helps the communication between the source and the destination. We assume that both the relay and the destination know global channel state information (CSI) including the channel matrices $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{T}$, but no CSI is allowed at the source node. The source, relay and destination nodes are equipped with N_t, N_r and N_d antennas, respectively. As we consider a spatial multiplexing system which transmits N_t data streams simultaneously, we assume $N_t \leq \min(N_r, N_d)$. We also assume that all noises

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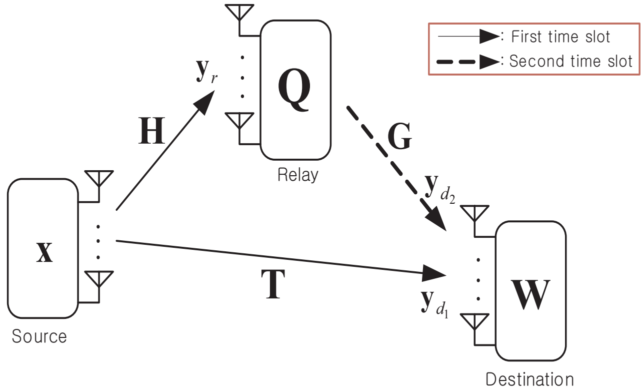

Fig. 1. System description for an AF MIMO relay network with direct link

are additive white Gaussian with zero mean and unit variance. Due to the loop interference in the relay node, it is assumed that data transmission occurs in two separate time slots.

In the first time slot, the source signal vector $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ with $E[\mathbf{x}\mathbf{x}^H] = \sigma_x^2 \mathbf{I}_{N_t}$ is transmitted to both the relay and the destination nodes. Note that we have $\sigma_x^2 = P_T/N_t$ where P_T is the total transmit power used by the source. Then the received signal at the relay $\mathbf{y}_r \in \mathbb{C}^{N_r \times 1}$ and the received signal at the destination $\mathbf{y}_{d_1} \in \mathbb{C}^{N_d \times 1}$ are respectively given by

$$\mathbf{y}_r = \mathbf{H}\mathbf{x} + \mathbf{n}_r \quad \text{and} \quad \mathbf{y}_{d_1} = \mathbf{T}\mathbf{x} + \mathbf{n}_{d_1},$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{T} \in \mathbb{C}^{N_d \times N_t}$ denote the source-to-relay and the source-to-destination (direct link) channel matrix, respectively, and $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{n}_{d_1} \in \mathbb{C}^{N_d \times 1}$ indicate the noise vectors at the relay and the destination, respectively.

In the second time slot, the received signal $\mathbf{y}_r$ at the relay is precoded by the relay transceiver $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ and transmitted to the destination. Then, the received signal at the destination is written by

$$\mathbf{y}_{d_2} = \mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{x} + \mathbf{n}_{d_2},$$

where $\mathbf{n}_{d_2} \triangleq \mathbf{G}\mathbf{Q}\mathbf{n}_r + \mathbf{n}_d$ designates the effective noise vector in the second time slot with covariance matrix $\mathbf{R}_{n_{d_2}} \triangleq \mathbf{G}\mathbf{Q}\mathbf{Q}^H\mathbf{G}^H + \mathbf{I}_{N_d}$. In this case, the relay matrix $\mathbf{Q}$ needs to satisfy the relay power constraint P_R as $E[\|\mathbf{Q}\mathbf{y}_r\|^2] \leq P_R$.

As a result, combining two signals received at the destination over two consecutive time slots, we have the signal vector $\mathbf{y}_d \in \mathbb{C}^{2N_d \times 1}$ at the destination as

$$\mathbf{y}_d = \begin{bmatrix} \mathbf{y}_{d_1} \\ \mathbf{y}_{d_2} \end{bmatrix} = \begin{bmatrix} \mathbf{T} \\ \mathbf{G}\mathbf{Q}\mathbf{H} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_{d_1} \\ \mathbf{n}_{d_2} \end{bmatrix}.$$

When a linear receiver $\mathbf{W} \in \mathbb{C}^{N_t \times 2N_d}$ is employed at the destination, the estimated signal waveform $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ is expressed as $\mathbf{s} = \mathbf{W}\mathbf{y}_d = \mathbf{W}_1\mathbf{y}_{d_1} + \mathbf{W}_2\mathbf{y}_{d_2}$ where $\mathbf{W}_1$ and $\mathbf{W}_2$ are defined as the $N_t \times N_d$ submatrices of $\mathbf{W}$.

III. Problem Formulation

In this section, we formulate the problem optimizing the relay transceiver $\mathbf{Q}$ under the MMSE criterion. We first derive the error covariance matrix as a function of $\mathbf{Q}$, and then show that from the MMSE point of view, it can be represented as a sum of two individual covariance matrices. Using this property, we can substantially simplify the problem.

Defining the error vector as $\mathbf{e} \triangleq \mathbf{s} - \mathbf{x} = \mathbf{W}\mathbf{y}_d - \mathbf{x}$, the joint optimization problem for minimizing the MSE is mathematically expressed as

$$\begin{aligned} &\min_{\mathbf{W},\mathbf{Q}} \mathrm{Tr}\left(\mathbf{R}_e(\mathbf{W},\mathbf{Q})\right) \\ &s.t. \quad \mathrm{Tr}\left(\mathbf{Q}(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})\mathbf{Q}^H\right) \leq P_R \end{aligned} \tag{1}$$

where $\mathbf{R}_e(\mathbf{W},\mathbf{Q}) \triangleq E[\mathbf{e}\mathbf{e}^H]$ denotes the error covariance matrix as a function of $\mathbf{W}$ and $\mathbf{Q}$. It is easy to verify that this problem is strictly convex with respect to each of $\mathbf{W}$ and $\mathbf{Q}$, although it is generally non-convex in the joint optimization perspective.

Therefore, for given $\mathbf{Q}$, the optimum receive filter $\hat{\mathbf{W}}$ is simply obtained as

$$\hat{\mathbf{W}}(\mathbf{Q}) = \left(\mathbf{H}^H\mathbf{Q}^H\mathbf{G}^H\mathbf{R}_{n_{d_2}}^{-1}\mathbf{G}\mathbf{Q}\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1}\mathbf{H}_S^H$$

where $\mathbf{R}_T \triangleq (\mathbf{T}^H\mathbf{T} + \sigma_x^{-2}\mathbf{I}_{N_t})^{-1}$ and $\mathbf{H}_S \triangleq \left[\mathbf{T}^T \quad (\mathbf{R}_{n_{d_2}}^{-1}\mathbf{G}\mathbf{Q}\mathbf{H})^T\right]^T$. Then, substituting $\hat{\mathbf{W}}(\mathbf{Q})$ into $\mathbf{R}_e(\mathbf{W},\mathbf{Q})$ and invoking the matrix inversion lemma [7], it can be rephrased as a function of $\mathbf{Q}$ as

$$\mathbf{R}_e(\mathbf{Q}) = \left(\mathbf{H}^H\mathbf{Q}^H\mathbf{G}^H\mathbf{R}_{n_{d_2}}^{-1}\mathbf{G}\mathbf{Q}\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1}. \tag{2}$$

Now, we look at the problem (1) in terms of the relay matrix $\mathbf{Q}$. Once $\mathbf{W}$ is given, the problem is also convex with respect to $\mathbf{Q}$, and thus can be efficiently solved by a Lagrangian method as shown in the following Lemma.

Lemma 1: For fixed $\mathbf{W}$, the optimal relay matrix $\mathbf{Q}$ is expressed as $\hat{\mathbf{Q}} = \mathbf{B}\mathbf{L}_R$ where $\mathbf{B} \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{L}_R \in \mathbb{C}^{N_t \times N_r}$ stand for the relay precoder and the receiver, respectively, and are computed as

$$\begin{aligned} \mathbf{B} &= \left(\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_2\mathbf{G} + \mu\mathbf{I}_{N_r}\right)^{-1}\mathbf{G}^H\mathbf{W}_2^H(\mathbf{I} - \mathbf{W}_1\mathbf{T}) \\ &\quad \times \left(\mathbf{I} + (\mathbf{H}^H\mathbf{H} + \sigma_x^{-2}\mathbf{I}_{N_t})^{-1}\mathbf{T}^H\mathbf{T}\right) \\ \mathbf{L}_R &= \left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1}\mathbf{H}^H. \end{aligned}$$

Here μ is chosen to satisfy the relay constraint in (1).

Proof: See appendix A. ■

Note that if we have no direct link (i.e., $\mathbf{T} = \mathbf{0}$), $\mathbf{B}$ and $\mathbf{L}_R$ in Lemma 1 are given by simple transmit and receive Wiener filter structures, respectively [7]. Again, plugging the result in Lemma 1 into (2), we obtain the equivalent error covariance matrix as a function of $\mathbf{B}$ as illustrated in the following lemma.

Lemma 2: For the given optimal structure of the relay matrix $\hat{\mathbf{Q}} = \mathbf{B}\mathbf{L}_R$, $\mathbf{R}_e(\mathbf{Q})$ in (2) can be decomposed into two individual covariance matrices as

$$\mathbf{R}_e(\mathbf{B}) = \left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1} + \left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}, \tag{3}$$

where $\mathbf{\Omega} \triangleq \mathbf{R}_T\mathbf{H}^H\mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1} \in \mathbb{C}^{N_t \times N_t}$.

Proof: See appendix B. ■

We now see from Lemma 2 that the first term in (3) is independent of $\mathbf{Q}$ or $\mathbf{W}$, and thus we only need to optimize the second term of (3) with respect to $\mathbf{B}$. Finally, the original joint optimization problem in (1) can be transformed to a simple problem which finds the optimal relay precoder $\mathbf{B}$ as

$$\min_{\mathbf{B}} \text{Tr}\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B}+\mathbf{\Omega}^{-1}\right)^{-1}$$
$$s.t.\quad \text{Tr}\left(\mathbf{B}\mathbf{R}_y\mathbf{B}^H\right)\leq P_R, \tag{4}$$

where $\mathbf{R}_y \triangleq \mathbf{L}_R\left(\sigma_x^2\mathbf{H}\mathbf{H}^H+\mathbf{I}_{N_t}\right)^{-1}\mathbf{L}_R^H$ denotes the covariance matrix of the relay receiver output signal, i.e., $\mathbf{L}_R\mathbf{y}_r$. It is remarkable to note that the problem (4) is considerably simplified without any optimality loss compared to (1).

IV. Near Optimal Relay Transceiver Designs

In this section, we solve the problem in (4) and derive a new closed-form solution for the relay transceiver $\mathbf{Q}$. Let us first introduce the following eigenvalue decompositions

$$\mathbf{G}^H\mathbf{G}=\mathbf{V}_g\mathbf{\Lambda}_g\mathbf{V}_g^H \quad\text{and}\quad \mathbf{\Omega}=\mathbf{U}_\omega\mathbf{\Lambda}_\omega\mathbf{U}_\omega^H,$$

where $\mathbf{V}_g\in\mathbb{C}^{N_r\times N_r}$ and $\mathbf{U}_\omega\in\mathbb{C}^{N_t\times N_t}$ are unitary matrices, and $\mathbf{\Lambda}_g$ and $\mathbf{\Lambda}_\omega$ represent square diagonal matrices with diagonal elements $\lambda_{g,k}\in\mathbb{R}_+$ for $k=1,\ldots,N_r$ and $\lambda_{\omega,k}\in\mathbb{R}_+$ for $k=1,\ldots,N_t$, respectively. We assume that all eigenvalues are arranged in a descending order.

Without loss of generality, we can write $\mathbf{B}$ in (4) in a general form as $\mathbf{B}=\mathbf{V}_g\mathbf{\Phi}\mathbf{U}_\omega^H$ where $\mathbf{\Phi}=\left[\mathbf{\Phi}_1^T \quad \mathbf{\Phi}_2^T\right]^T$ with $\mathbf{\Phi}_1\in\mathbb{C}^{N_t\times N_t}$ and $\mathbf{\Phi}_2\in\mathbb{C}^{(N_r-N_t)\times N_t}$. Through some deductions, it is easy to show that the setting $\mathbf{\Phi}_2=\mathbf{0}$ has no impact on the objective function in (4) while the power consumption is reduced [4] [6]. Thus, it follows

$$\hat{\mathbf{B}}=\widetilde{\mathbf{V}}_g\mathbf{\Phi}_1\mathbf{U}_\omega^H, \tag{5}$$

where $\widetilde{\mathbf{V}}_g\in\mathbb{C}^{N_r\times N_t}$ denotes the matrix constructed by the first N_t columns of $\mathbf{V}_g$.

Then, substituting (5) into (4), the modified problem determines the optimal $\mathbf{\Phi}_1$ as

$$\hat{\mathbf{\Phi}}_1=\arg\min_{\mathbf{\Phi}_1}\ f_o(\mathbf{\Phi}_1)$$
$$s.t.\quad f_p\left(\mathbf{\Phi}_1\right)\leq P_R, \tag{6}$$

where $f_o(\mathbf{\Phi}_1)\triangleq\text{Tr}\left(\mathbf{\Phi}_1^H\widetilde{\mathbf{\Lambda}}_g\mathbf{\Phi}_1+\mathbf{\Lambda}_\omega^{-1}\right)^{-1}$ and $f_p(\mathbf{\Phi}_1)\triangleq\text{Tr}(\mathbf{\Phi}_1\mathbf{R}_\omega\mathbf{\Phi}_1^H)$. Here $\widetilde{\mathbf{\Lambda}}_g$ is defined as the $N_t\times N_t$ upper left submatrix of $\mathbf{\Lambda}_g$ and $\mathbf{R}_\omega\triangleq\mathbf{U}_\omega^H\mathbf{R}_y\mathbf{U}_\omega$.

For $N_t\times N_t$ positive definite matrices $\mathbf{A}$ and $\mathbf{B}$, it is known [13] that $\text{Tr}(\mathbf{A}\mathbf{B})\geq\sum_{i=1}^{N_t}\lambda_i(\mathbf{A})\lambda_{N_t-i+1}(\mathbf{B})$ and $\text{Tr}(\mathbf{A}^{-1})\geq\sum_{i=1}^{N_t}(A_{k,k})^{-1}$ where $\lambda_k(\mathbf{A})$ and $A_{k,k}$ stand for the the k-th largest eigenvalue and the k-th diagonal element of $\mathbf{A}$, respectively. From these facts, we can check that the optimum solution for (6) is obtained when the matrices inside the trace in (6), i.e., $\left(\mathbf{\Phi}_1^H\widetilde{\mathbf{\Lambda}}_g\mathbf{\Phi}_1+\mathbf{\Lambda}_\omega^{-1}\right)^{-1}$ and $\mathbf{\Phi}_1\mathbf{R}_\omega\mathbf{\Phi}_1^H$ are simultaneously diagonalized. However, unlike the case with no direct link (i.e., $\mathbf{R}_\omega=\mathbf{\Lambda}_\omega$) [7], there exists no such a case for all $\mathbf{\Phi}_1$ due to the non-diagonal structure of $\mathbf{R}_\omega$, which makes the problem in (6) generally non-convex. To overcome this difficulty and provide a closed-form solution, we impose the following structural constraint on $\mathbf{\Phi}_1$.

Let us define a diagonal matrix $\mathbf{\Phi}_d\in\mathbb{C}^{N_t\times N_t}$ with diagonal entries $\phi_1,\ldots,\phi_{N_t}$. Then, we can always find a proper $\mathbf{\Phi}_d$ such that $f_o(\mathbf{\Phi}_d)=f_o(\hat{\mathbf{\Phi}}_1)$, since $\mathbf{\Phi}_d$ is the optimum structure for the objective function $f_o(\cdot)$. For the problem (6), however, $\mathbf{\Phi}_d$ is obviously suboptimal because it may increase the power consumption, i.e., $f_p(\mathbf{\Phi}_d)\geq f_p(\hat{\mathbf{\Phi}}_1)$ with the same MSE. Nevertheless, a diagonal structure can provide a simple convex problem, and thus in this paper, we consider $\mathbf{\Phi}_d$ as a solution. Then, by imposing the structural constraint $\mathbf{\Phi}_1=\mathbf{\Phi}_d$ on the problem (6), we attain the following strictly convex optimization problem[1] as

$$\hat{\mathbf{\Phi}}_d=\arg\min_{\mathbf{\Phi}_d} f_o(\mathbf{\Phi}_d)$$
$$s.t.\quad f_p\left(\mathbf{\Phi}_d\right)\leq P_R. \tag{7}$$

The solution for (7) can be found efficiently by using the Lagrangian multiplier ν.

Finally, combining with the relay receiver $\mathbf{L}_R$ in Lemma 1, a closed-form solution for the relay transceiver $\mathbf{Q}$ is obtained as

$$\mathbf{Q}_{cf}=\widetilde{\mathbf{V}}_g\hat{\mathbf{\Phi}}_d\mathbf{U}_\omega^H\mathbf{L}_R,$$

where

$$|\hat{\phi}_k|^2=\frac{1}{\lambda_{\omega,k}\lambda_{g,k}}\left(\sqrt{\frac{\lambda_{\omega,k}^2\lambda_{g,k}}{r_k\nu}}-1\right)^+,\ \text{for } k=1,\ldots,N_t.$$

Here $(x)^+$ is defined as $\max(x,0)$, r_k designates the k-th diagonal element of $\mathbf{R}_\omega$, and ν is chosen to satisfy the power constraint in (7). Note that when $\mathbf{R}_\omega$ is a diagonal matrix, the derived solution is globally optimal (i.e., $\hat{\mathbf{\Phi}}_1=\hat{\mathbf{\Phi}}_d$). Thus, our solution includes the existing works in [6] and [7] without the direct link. For the same reason, the optimal single stream beamforming (i.e. $N_t=1$) [9] [11] can also be specified as a special case of our solution, because in this case, we have scalar $\mathbf{R}_\omega\in\mathbb{R}_+$.

In what follows, we give some interpretations for our design strategy to provide more insights. Let us consider an arbitrary $\mathbf{\Phi}_d$ that consumes the same power with the global optimal solution $\hat{\mathbf{\Phi}}_1$, i.e., $f_p(\mathbf{\Phi}_d)=f_p(\hat{\mathbf{\Phi}}_1)$. In this case, it is always true that $f_o(\mathbf{\Phi}_d)\geq f_o(\hat{\mathbf{\Phi}}_1)$. Therefore, our closed-form solution $\hat{\mathbf{\Phi}}_d$ in (7) minimizes the upper-bound of the objective function in the original problem (6). On the contrary, assuming that both $\hat{\mathbf{\Phi}}_1$ and an arbitrary $\mathbf{\Phi}_d$ achieve the same MSE, i.e., $f_o(\mathbf{\Phi}_d)=f_o(\hat{\mathbf{\Phi}}_1)$, it is also clear that $f_p(\mathbf{\Phi}_d)\geq f_p(\hat{\mathbf{\Phi}}_1)$. This observation indicates that our strategy establishes a convex optimization problem (7) by contracting the feasible set of the original non-convex problem in (6). For these reasons, our solution does not guarantee the optimality in terms of the MSE. Nevertheless, numerical results in the following section show that the proposed closed-form solution achieves almost identical performance with the optimal design based on the iterative algorithm [11] with much reduced complexity.

V. Numerical Results

In this section, we compare the bit error rate (BER) and MSE performance of the proposed solution with those of the optimal design. We use a notation $N_t\times N_r\times N_d$ to denote the

[1] Strictly speaking, the problem (7) is convex with respect to $\mathbf{\Phi}_d\mathbf{\Phi}_d^H$, since the phase of each element in $\mathbf{\Phi}_d$ has no impact on both $f_o(\mathbf{\Phi}_d)$ and $f_p(\mathbf{\Phi}_d)$.

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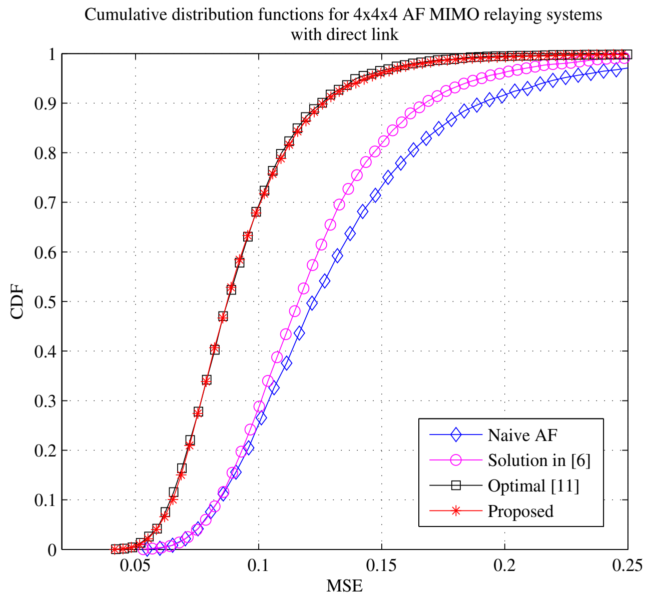

Fig. 2. CDFs of the instantaneous MSE for various relay transceiver designs at $P_0 = 20$ dB

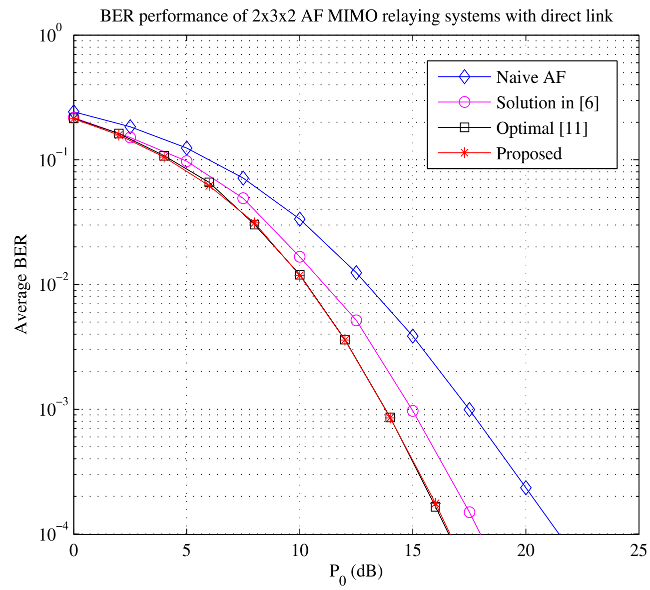

Fig. 4. BER performance comparison for various relay transceiver designs with QPSK constellation as a function of P_0

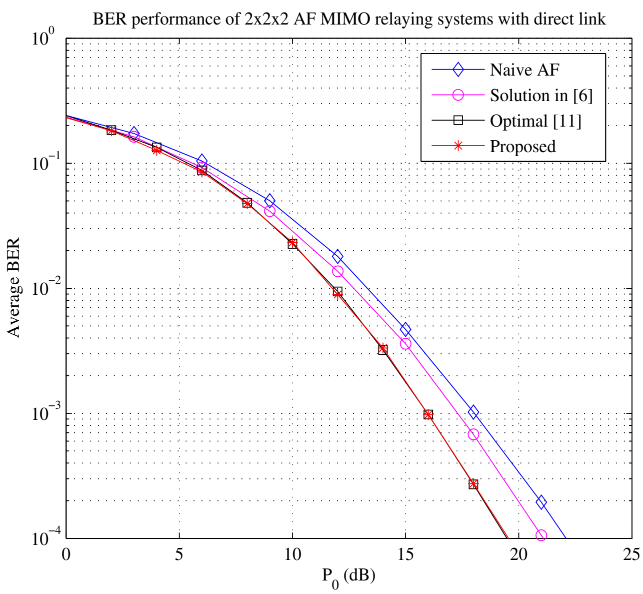

Fig. 3. BER performance comparison for various relay transceiver designs with QPSK constellation as a function of P_0

system with N_t source, N_r relay and N_d destination antennas. The total transmit power is defined as P_0 assuming that $P_T = P_R = P_0/2$. Also we assume that the elements of the channel matrices $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{T}$ have an independent and identically distributed complex Gaussian distribution with zero mean and unit variance.

Figure 2 shows the cumulative distribution functions (CDF) of the instantaneous MSE for $4 \times 4 \times 4$ AF relaying systems with the fixed transmit power $P_0 = 20$ dB. The *Naive AF* indicates the most simple scheme where only the power normalizing operation is performed at the relay, i.e., $\mathbf{Q} = \sqrt{P_R/\text{Tr}(\sigma_x^2 \mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})}\, \mathbf{I}_{N_r}$. From this figure, we see that *Naive AF* exhibits the worst performance, since it does not exploit the CSI of the system. In addition, the proposed solution clearly outperforms the algorithm in [6], because the CSI of the direct link (i.e., $\mathbf{T}$) is exploited at the relay while only the source-relay-destination link is optimized in [6]. This figure also demonstrates that our closed-form solution has no difference in the MSE compared to the optimal design based on the iterative algorithm in [11].

In Figures 3 and 4, we present Monte Carlo simulations for the BER performance of $2 \times 2 \times 2$ and $2 \times 3 \times 2$ systems as a function of P_0. For both cases of $N_r = 2$ and 3, the proposed solution obtains about 2 dB gain at a BER of 10^{-4} over the solution in [6]. Also, similar to the previous results, we can check that the proposed closed-form solution has the identical BER performance with the optimal design with substantially reduced complexity. Note that although not presented in this paper, we obtain similar consequences for other constellations.

VI. Conclusion

In this paper, we have developed an MMSE-based near-optimal closed form solution for the relay matrix in AF MIMO relaying systems with direct link. First, we have shown that the optimal relay matrix consists of the relay receiver and the precoder. Using this fact, the decomposable property of the error covariance matrix has been exploited to simplify the problem. Then, to solve non-convexity of the problem and derive a simple closed-form solution, we impose a structural constraint on the solution. Finally, from numerical simulations, we have demonstrated that the proposed closed-form solution has no performance loss compared to the optimal iterative solution with substantially lower complexity.

APPENDIX

A. Proof of Lemma 1

First, using the Lagrangian multiplier μ, we set up the cost function $\mathcal{C}$ as

$$\begin{aligned}\mathcal{C} &= \mathrm{Tr}\left(\mathbf{R}_e(\mathbf{W},\mathbf{Q})\right) + \mu \mathrm{Tr}\left(\mathbf{Q}^H(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})\mathbf{Q}\right)\\ &= \mathrm{Tr}\Big(2\Re[\sigma_x^2\mathbf{W}_2\mathbf{G}\mathbf{Q}\mathbf{H}(\mathbf{T}^H\mathbf{W}_1^H + \mathbf{I}_{N_t})]\\ &\quad +\mathbf{W}_2\mathbf{G}\mathbf{Q}(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})\mathbf{Q}^H\mathbf{G}^H\mathbf{W}_2^H\\ &\quad +\mu\mathbf{Q}(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})\mathbf{Q}^H + \mathbf{C}\Big),\end{aligned}$$

where the matrix $\mathbf{C}$ indicates the constant terms independent of $\mathbf{Q}$. Then, we can find $\hat{\mathbf{Q}}$ by setting the gradient with respect to $\mathbf{Q}$ equal to zero as

$$\begin{aligned}\frac{\partial \mathcal{C}}{\partial \mathbf{Q}^*} &= \sigma_x^2\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_1\mathbf{T}\mathbf{H}^H + \sigma_x^2\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_2\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{H}^H\\ &\quad +\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_2\mathbf{G}\mathbf{Q} - \sigma_x^2\mathbf{G}_H\mathbf{W}_2^H\mathbf{H}^H\\ &\quad +\mu(\mathbf{Q}\mathbf{H}\mathbf{H}^H + \mathbf{Q}) = 0. \end{aligned} \tag{8}$$

This result can be verified using some rules such as $d\mathrm{tr}\,(\mathbf{Y}) = \mathrm{tr}\,(d\mathbf{Y})$, $\mathrm{vec}(d\mathbf{X}) = d\mathrm{vec}(\mathbf{X})$, $\mathrm{tr}(\mathbf{X}^T\mathbf{Y}) = \mathrm{vec}(\mathbf{X})^T\mathrm{vec}(\mathbf{Y})$.

Solving the equation (8), we obtain the optimal $\mathbf{Q}$ as

$$\begin{aligned}\hat{\mathbf{Q}} &= \left(\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_2\mathbf{G} + \mu\mathbf{I}_{N_r}\right)^{-1}\mathbf{G}^H\mathbf{W}_2^H\left(\mathbf{I} - \mathbf{W}_1\mathbf{T}\right)\\ &\quad \times(\mathbf{H}^H\mathbf{H} + \sigma_x^{-2}\mathbf{I}_{N_t})^{-1}\mathbf{H}^H\\ &= \left(\mathbf{G}^H\mathbf{W}_2^H\mathbf{W}_2\mathbf{G} + \mu\mathbf{I}_{N_r}\right)^{-1}\mathbf{G}^H\mathbf{W}_2^H\left(\mathbf{I} - \mathbf{W}_1\mathbf{T}\right)\\ &\quad \times\left(\mathbf{I} + (\mathbf{H}^H\mathbf{H} + \sigma_x^{-2}\mathbf{I}_{N_t})^{-1}\mathbf{T}^H\mathbf{T}\right)\\ &\quad \times\left(\mathbf{H}^H\mathbf{H} + \mathbf{T}^H\mathbf{T} + \sigma_x^{-2}\mathbf{I}_{N_t}\right)^{-1}\mathbf{H}^H\end{aligned}$$

and the proof is completed.

B. Proof of Lemma 2

By employing Lemma 1, $\mathbf{R}_e(\mathbf{Q})$ in (2) can be rewritten as

$$\mathbf{R}_e = \left(\mathbf{H}^H\mathbf{L}_R^H\mathbf{B}^H\mathbf{G}^H\mathbf{R}_{n_{d_2}}^{-1}\mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1}.$$

Then, using the matrix inversion lemma, we have

$$\begin{aligned}\mathbf{R}_e &= \mathbf{R}_T - \mathbf{R}_T\widetilde{\mathbf{H}}^H\left(\widetilde{\mathbf{H}}\mathbf{R}_T\widetilde{\mathbf{H}}^H + \mathbf{R}_{n_{d_2}}\right)^{-1}\widetilde{\mathbf{H}}\mathbf{R}_T\\ &= \mathbf{R}_T - \mathbf{R}_T\widetilde{\mathbf{H}}^H\left(\mathbf{G}\mathbf{B}\mathbf{\Omega}\mathbf{B}^H\mathbf{G}^H + \mathbf{I}_{N_d}\right)^{-1}\widetilde{\mathbf{H}}\mathbf{R}_T,\end{aligned} \tag{9}$$

where $\widetilde{\mathbf{H}} \triangleq \mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{H}$ and the positive definite matrix $\mathbf{\Omega}$ is calculated as

$$\begin{aligned}\mathbf{\Omega} &\triangleq \mathbf{L}_R\left(\mathbf{H}\mathbf{R}_T\mathbf{H}^H + \mathbf{I}_{N_r}\right)\mathbf{L}_R^H\\ &= \mathbf{R}_T\mathbf{H}^H\mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1}.\end{aligned}$$

Again, applying the matrix inversion lemma to the equation (9) and employing some mathematical manipulations, we obtain

$$\begin{aligned}\mathbf{R}_e &= \mathbf{R}_T - \mathbf{R}_T\widetilde{\mathbf{H}}^H\\ &\quad \times\left(\mathbf{I}_{N_d} - \mathbf{G}\mathbf{B}\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\mathbf{B}^H\mathbf{G}^H\right)\widetilde{\mathbf{H}}\mathbf{R}_T\\ &= \mathbf{R}_T - \mathbf{R}_T\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{R}_T\\ &\quad +\mathbf{R}_T\widetilde{\mathbf{H}}^H\mathbf{G}\mathbf{B}\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\\ &\quad \times\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1} - \mathbf{\Omega}^{-1}\right)\mathbf{L}_R\mathbf{H}\mathbf{R}_T\\ &= \mathbf{R}_T - \mathbf{R}_T\widetilde{\mathbf{H}}^H\mathbf{G}\mathbf{B}\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\\ &\quad \times\mathbf{\Omega}^{-1}\mathbf{L}_R\mathbf{H}\mathbf{R}_T\\ &= \mathbf{R}_T - \mathbf{R}_T\mathbf{H}^H\mathbf{L}_R^H\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1} - \mathbf{\Omega}^{-1}\right)\\ &\quad \times\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\mathcal{J}\\ &= \mathbf{R}_T - \mathbf{R}_T\mathbf{H}^H\mathbf{L}_R^H\mathcal{J} + \mathcal{J}\left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\mathcal{J}\end{aligned}$$

where $\mathcal{J} \triangleq \mathbf{\Omega}^{-1}\mathbf{L}_R\mathbf{H}\mathbf{R}_T$. We can easily show that $\mathcal{J}$ equals identity matrix $\mathbf{I}_{N_t}$. Therefore, it follows

$$\begin{aligned}\mathbf{R}_e &= \mathbf{R}_T - \mathbf{R}_T\mathbf{H}^H\mathbf{L}_R^H + \left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\\ &= \left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_T^{-1}\right)^{-1} + \left(\mathbf{B}^H\mathbf{G}^H\mathbf{G}\mathbf{B} + \mathbf{\Omega}^{-1}\right)^{-1}\end{aligned}$$

and we complete the proof.

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