Coordinated SINR Balancing Methods for Multi-Cell Downlink Systems

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***Abstract*—In this paper, we consider coordinated beamforming techniques where multiple base stations jointly design a transmission strategy by sharing channel state information. Particularly, we tackle the signal-to-interference-plus-noise ratio (SINR) balancing problem to maximize the worst-user rate for multi-cell downlink systems. To solve this problem, both symmetric complex (SC) and asymmetric complex (AC) signaling methods are investigated. First, we present the SINR balancing algorithm with the SC signaling. Due to residual interference, the worst-user rate in the SC signaling is saturated at high signal-to-noise ratio (SNR). To alleviate this issue, we also propose the SINR balancing technique based on the AC signaling which combines both interference alignment and power control methods. Simulation results confirm that the AC signaling outperforms the SC signaling scheme over all SNR range.**

I. Introduction

In cellular networks, interference mitigation is an important issue since inter-cell interference seriously limits the overall system performance. A network multiple input multiple output (MIMO) technique has been recognized as a good candidate for solving this issue, since a coordination among neighboring cells can effectively reduce inter-cell interference [1]. Depending on the base station (BS) cooperation level, the network MIMO can be classified into two categories [2]. One is joint processing where BSs share both users' messages and channel state information (CSI). The other is coordinate beamforming (CB) where BSs design their transmission strategy by sharing only users' CSI. Since the backhaul link among the BSs has the limited capacity, this paper focuses on the CB system which can reduce the system complexity.

For the CB system, several transmission strategies to optimize the network throughput has been proposed in [3] and [4]. Along with the sum-rate maximization metric, signal-to-interference-plus-noise ratio (SINR) balancing methods which maximize the worst-user rate has also been widely investigated, since providing quality-of-service (QoS) for each user is an important issue [5] [6]. In this context, we have established optimization algorithms which solve the SINR balancing problem in 2-cell environments in our prior work [7].

In this paper, extending our earlier work to multi-cell downlink systems, which can be modeled as 3-user interference channels (IFC), we investigate two SINR balancing techniques: symmetric complex (SC) and asymmetric complex (AC) signaling[1]. First, we develop the SINR balancing algorithm with the SC signaling. By means of alternating optimization which iteratively finds a solution, the proposed scheme provides the worst-user rate performance almost identical to the optimal performance obtained from exhaustive search. However, the worst-user rate of the SC signaling is saturated as signal-to-noise ratio (SNR) grows due to residual interference.

To improve this issue, we also propose the transmission strategy with the AC signaling which exhibits advantages in terms of the degrees of freedom (DOF), which represents the characteristic of the capacity in the high SNR region [9], for IFC with constant channel coefficients [8]. In our design strategy with the AC signaling, interference alignment (IA) proposed in [10] and the power control method are employed. The key idea of the IA is to align all interferences from undesired transmitters to the half of the receiver signal space, so that we can recover the desired signal with a simple zero-forcing (ZF) filter. Unlike 2-user single input single output (SISO) IFC, the perfect IA is impossible without symbol extension due to the lack of the receiver signal space in 3-user environments [8] [10]. Thus, we only allow two users to have the aligned interference in our strategy. As a result, the worst-user rate performance is dominated by the link whose interference terms are not perfectly aligned. To solve this problem, the power control method to maximize the worst-user rate is introduced after IA beamforming. The simulation result confirms that the proposed AC signaling outperforms the SC signaling for overall SNR region.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices, lowercase boldface for vectors and normal letters for scalar quantities. $\mathbf{A}^T$, $\mathbf{A}^H$ and det($\mathbf{A}$) represent transpose, conjugate transpose and determinant for any matrix $\mathbf{A}$, respectively. Additionally, $\mathbf{I}_d$ indicates an identity matrix of size d, $E[\cdot]$ accounts for expectation and $\mathbb{R}^{M\times N}$ denotes a set of real matrices of size M-by-N.

II. System Model

In this section, we present a general description of multi-cell downlink systems which can be modeled as 3-user SISO IFC illustrated in Fig. 1. There are 3 BSs which transmit the message W_i $(i = 1, 2, 3)$ to its corresponding receiver where all nodes are equipped with a single antenna. In Figure 1, the solid line indicates the desired signal and the dashed line represents the interference signal. For practical implementation issues, we consider 3-user SISO IFC with constant channel coefficient and no symbol extension of the

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[1] In the AC signaling, the inputs are chosen to be complex but not circularly symmetric [8].

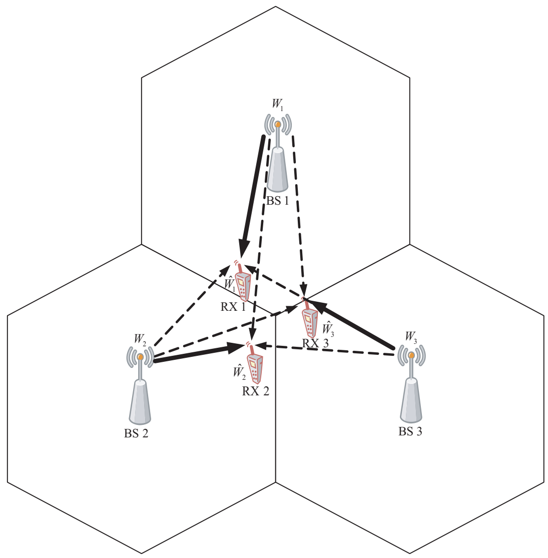

Fig. 1. Schematic diagram of 3-cell downlink systems

channel [8] [9]. Then, under frequency-flat fading channels, the received signal at receiver i is given by

$$y_i = h_{i,i}x_i + \sum_{j\neq i, j=1}^{3} h_{i,j}x_j + n_i \tag{1}$$

where x_i is the transmitted signal at BS i, $h_{i,j}$ stands for the channel coefficient form BS j to receiver i, and n_i denotes the additive complex Gaussian noise with zero mean and covariance σ_n^2 at receiver i. It is assumed that the CSI is globally available. Although the desired channel coefficient $h_{i,i}$ generally has power greater than that of the interference channel coefficient $h_{i,j}$ $(i \neq j)$ due to path loss, we consider the most challenging case where users are located in cell boundaries so that both $h_{i,i}$ and $h_{i,j}$ have the same power. Also, due to high implementation complexity of multi-user detection, the interference terms are treated as noise [11]. In addition, we consider per-BS power constraint as $E[|x_i|^2] \leq P_{\max}$ for $i = 1, 2, 3$ where $P_{\max}$ is the maximum transmit power, since each BS has its own power amplifier.

III. Coordinated SINR Balancing with SC Signaling

In this section, we first establish the SINR balancing technique with SC signaling for 3-user SISO IFC. Since beamforming with the SC signaling is not available, we develop the power control algorithm to maximize the worst-user rate by means of alternating optimization which finds a solution iteratively. In (1), the transmit signal x_i is related to s_i as $x_i = \sqrt{P_i}s_i$ where $s_i \sim CN(0,1)$ indicates the data symbol intended for receiver i and P_i represents the transmit power at BS i. Then, the achievable rate of link k for $k = 1, 2, 3$ is a function of the power levels $\{P_i\}$ as $R_k^{SC}(\{P_i\}) = \log_2\left(1 + \gamma_k^{SC}(\{P_i\})\right)$ where $\gamma_k^{SC}(\{P_i\}) = \frac{g_{k,k}P_k}{\sigma_n^2 + \sum_{j\neq k} g_{k,j}P_j}$ is the individual SINR for link k and $g_{k,j} = |h_{k,j}|^2$ denotes the instantaneous channel gain.

As a result, the optimization problem for the SINR balancing can be formulated as

$$\left(P_1^{\text{opt}}, P_2^{\text{opt}}, P_3^{\text{opt}}\right) = \arg\max_{(P_1,P_2,P_3)\in S} \min_k \left\{\gamma_k^{SC}(\{P_i\})\right\} \tag{2}$$

where $S = \{(P_1, P_2, P_3)|0 \leq P_i \leq P_{\max}, i = 1, 2, 3\}$. In this case, it was shown in [7, Theorem 1] that we can reduce a three-dimension set S into two-dimension one $\tilde{S}$, since at least one BS can employ full power transmission without loss of optimality. This means that instead of searching in S in (2), it suffices to search in

$$\tilde{S} = \{(P_{\max}, P_2^{(1)}, P_3^{(1)}), (P_1^{(2)}, P_{\max}, P_3^{(2)}), (P_1^{(3)}, P_2^{(3)}, P_{\max})\} \tag{3}$$

where $P_j^{(i)}$ represents the optimized power level at transmitter j for given $P_i = P_{\max}$.

To illustrate the key idea of the SC signaling method, we describe how to obtain $P_1^{(3)}$ and $P_2^{(3)}$, i.e., optimization of P_1 and P_2 for fixed $P_3 = P_{\max}$. Since the individual SINR is a function of both P_1 and P_2, a solution can be obtained via joint optimization. However, it is somewhat complicated to derive a closed form solution due to non-convexity of the formulated problem (2). Instead, we employ an alternating optimization method which iteratively finds a local optimal solution. In this algorithm, we first identify the optimal P_1 for fixed P_2 and P_3. Then, P_2 is optimized for fixed P_1 and P_3. This procedure is repeated until convergence occurs.

First, we explain the optimization of P_1 for fixed P_2 and P_3. For the ease of explanation, we denote $\bar{\gamma}_k^{l,m}(P_i)$ as the SINR of link k for fixed P_l and P_m where $\{l, m\} = \{1, 2, 3\}\backslash\{i\}$. Then, finding the optimal P_1^* can be formulated as

$$P_1^* = \arg\max_{0\leq P_1\leq P_{\max}} \min\left\{\bar{\gamma}_1^{2,3}(P_1), \bar{\gamma}_2^{2,3}(P_1), \bar{\gamma}_3^{2,3}(P_1)\right\}. \tag{4}$$

To solve the above problem, the following useful properties are used. First, $\bar{\gamma}_1^{2,3}(0)$ is smaller than $\min\{\bar{\gamma}_2^{2,3}(0), \bar{\gamma}_3^{2,3}(0)\}$. Second, $\bar{\gamma}_1^{2,3}(P_1)$ is monotonically increasing and $\bar{\gamma}_2^{2,3}(P_1)$ and $\bar{\gamma}_3^{2,3}(P_1)$ are monotonically decreasing with respect to P_1. From these properties, if $\bar{\gamma}_1^{2,3}(P_{\max}) \geq \min\{\bar{\gamma}_2^{2,3}(P_{\max}), \bar{\gamma}_3^{2,3}(P_{\max})\}$, we can figure out that there always exists at least one crossover point within $0 \leq P_1 \leq P_{\max}$. Thus, a solution for (4) is given by $P_1^* = \min\{P_1^{1,2}, P_1^{1,3}\}$ where $P_1^{1,2}$ and $P_1^{1,3}$ represent the power level P_1 which satisfies $\bar{\gamma}_1^{2,3}(P_1) = \bar{\gamma}_2^{2,3}(P_1)$ and $\bar{\gamma}_1^{2,3}(P_1) = \bar{\gamma}_3^{2,3}(P_1)$, respectively. Otherwise, $P_1^* = P_{\max}$ is the solution for (4) obviously.

The optimal P_2^* for fixed P_1 and P_3 can be calculated in a similar fashion. As a result, P_1^* and P_2^* are obtained as (5) at the top of the next page where we define $\bar{1} = 2$, $\bar{2} = 1$, $\delta_i = g_{i,i}\left(\sigma_n^2 + g_{i,\bar{i}}P_{\bar{i}} + g_{i,3}P_{\max}\right)$, $\beta_{i,\bar{i}} = g_{i,i}\left(\sigma_n^2 + g_{\bar{i},3}P_{\max}\right)$ and $\beta_{i,3} = g_{i,i}\left(\sigma_n^2 + g_{3,\bar{i}}P_{\bar{i}}\right)$. This alternating optimization procedure is repeated until convergence occurs.

In a general alternating optimization method, $P_i(n)$ for $i = 1, 2$ is updated simply by P_i^* where $P_i(n)$ indicates the power level of BS i at the n-th iteration. Since P_i^* is determined between $\min\{P_i^{i,\bar{i}}, P_i^{i,3}\}$ and $P_{\max}$, a relation between the first and second worst-user SINR after one iteration becomes one of following 3 cases: $\hat{\gamma}_1 = \hat{\gamma}_2$, $\hat{\gamma}_2 = \hat{\gamma}_3$ and $\hat{\gamma}_1 = \hat{\gamma}_3$ where $\hat{\gamma}_k$ represents the instantaneous SINR of link k. If $\hat{\gamma}_1 = \hat{\gamma}_2$, $P_i(n)$'s cannot be changed due to a decrease of $\hat{\gamma}_1$ or $\hat{\gamma}_2$. Thus, the general approach converges to a local optimal point with only one iteration. Since the formulated problem is non-convex in general, the one-shot solution of general approach may be a local optimal point whose worst-user rate is poor. Otherwise, i.e, $\hat{\gamma}_2 = \hat{\gamma}_3$ or $\hat{\gamma}_1 = \hat{\gamma}_3$, $P_i(n)$'s are changed because of an increase of the

$$P_i^* = \begin{cases} \min\limits_{j=\bar{i},3} \left(\frac{-\beta_{i,j} + \sqrt{\beta_{i,j}^2 + 4\delta_i g_{j,i} g_{j,j} P_j}}{2 g_{i,i} g_{j,i}} \right) & \text{, if } \bar{\gamma}_i^{\bar{i},3}(P_{\max}) \geq \min\{\bar{\gamma}_{\bar{i}}^{\bar{i},3}(P_{\max}), \bar{\gamma}_3^{\bar{i},3}(P_{\max})\} \\ P_{\max} & \text{, otherwise} \end{cases} \tag{5}$$

worst-user SINR. The convergence for these cases will be shown in a theorem below.

To avoid falling into a bad local optimal point, we update $P_i(n)$ as $(P_i^* + P_i(n-1))/2$ for $i = 1, 2$ in our algorithm. By doing this, our algorithm identifies a solution by comparing multiple points which include the solution of the above general approach. Thus, we can further improve the worst-user SINR compared to the general approach. The proposed algorithm of finding $P_1^{(3)}$ and $P_2^{(3)}$ for fixed $P_3 = P_{\max}$ is summarized below.

Initialization
1) Set $P_1 \leftarrow P_{\max}$ and $P_2 \leftarrow P_{\max}$.
Main Loop
2) Compute P_1^* using (5), and update $P_1 \leftarrow \frac{P_1^* + P_1}{2}$.
3) Compute P_2^* using (5), and update $P_2 \leftarrow \frac{P_2^* + P_2}{2}$.
4) Go back to step 2 until convergence.

The convergence proof of our proposed scheme is presented in the following theorem.

Theorem 1: The proposed SINR balancing algorithm with SC signaling always converges.

Proof: Unlike the general alternating approach, we have 6 possible states of the individual SINRs: $\hat{\gamma}_3 \leq \hat{\gamma}_2 \leq \hat{\gamma}_1$, $\hat{\gamma}_3 \leq \hat{\gamma}_1 \leq \hat{\gamma}_2$, $\hat{\gamma}_1 \leq \hat{\gamma}_2 \leq \hat{\gamma}_3$, $\hat{\gamma}_1 \leq \hat{\gamma}_3 \leq \hat{\gamma}_2$, $\hat{\gamma}_2 \leq \hat{\gamma}_1 \leq \hat{\gamma}_3$ and $\hat{\gamma}_2 \leq \hat{\gamma}_3 \leq \hat{\gamma}_1$. In our scheme, if $\hat{\gamma}_3 \leq \hat{\gamma}_2 \leq \hat{\gamma}_1$ or $\hat{\gamma}_3 \leq \hat{\gamma}_1 \leq \hat{\gamma}_2$, $P_1(n)$ and $P_2(n)$ are decreased which leads to an improvement of the minimum SINR $\hat{\gamma}_3$. Also, $P_1(n)$ is increased and $P_2(n)$ is decreased for an increase of $\hat{\gamma}_1$ when $\hat{\gamma}_1 \leq \hat{\gamma}_2 \leq \hat{\gamma}_3$ or $\hat{\gamma}_1 \leq \hat{\gamma}_3 \leq \hat{\gamma}_2$. Similarly, if $\hat{\gamma}_2 \leq \hat{\gamma}_1 \leq \hat{\gamma}_3$ or $\hat{\gamma}_2 \leq \hat{\gamma}_3 \leq \hat{\gamma}_1$, $P_1(n)$ is decreased and $P_2(n)$ is increased. As a result, the worst-user SINR is improved at the expense of a decrease of other user's SINR, and the state of the individual SINRs is changed to one of 6 cases. This means that the gap between the first and second worst-user SINRs approaches zero as the iteration goes. As a result, we reach at a stop criterion which is given as $\hat{\gamma}_1 = \hat{\gamma}_2 \leq \hat{\gamma}_3$. In this case, $P_1(n)$ and $P_1(n)$ cannot be increased (or decreased) due to a decrease of $\hat{\gamma}_2$ (or $\hat{\gamma}_1$). Thus, the algorithm converges, and this concludes the proof. ■

In summary, we can obtain $P_1^{(3)}$ and $P_2^{(3)}$ using the above algorithm. $P_1^{(2)}$, $P_3^{(2)}$, $P_2^{(1)}$ and $P_3^{(1)}$ are computed in a similar fashion. Finally, we have a solution of (2) by choosing the best one among three candidates in (3). It is straightforward to extend our proposed algorithm to the general K-user SISO IFC. As will be shown in the simulation section, the worst-user rate of the SC signaling is almost identical to the optimal performance in SC signaling. However, it is saturated as the SNR grows due to residual interference.

IV. Improved SINR Balancing with AC signaling

In this section, to improve the worst-user rate of the SC signaling, we establish the efficient SINR balancing algorithm based on the AC signaling which combines both the IA algorithm proposed in [10] and the power control method for 3-user SISO IFC.

Employing the real-valued representation, the system equation (1) can be equivalently expressed as

$$\mathbf{y}_i = \mathbf{H}_{i,i}\mathbf{x}_i + \sum_{j \neq i, j=1}^{3} \mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i, \tag{6}$$

where $\mathbf{y}_i = [\Re\{y_i\}\ \Im\{y_i\}]^T$, $\mathbf{x}_i = [\Re\{x_i\}\ \Im\{x_i\}]^T$, $\mathbf{n}_i = [\Re\{n_i\}\ \Im\{n_i\}]^T$ and

$$\mathbf{H}_{i,j} = \begin{bmatrix} \Re\{h_{i,j}\} & -\Im\{h_{i,j}\} \\ \Im\{h_{i,j}\} & \Re\{h_{i,j}\} \end{bmatrix}.$$

Here, $\Re\{x\}$ and $\Im\{x\}$ indicate the real and the imaginary part of x, respectively.

Assuming that the input signal is chosen from a Gaussian codebook at each BS, the achievable individual rate R_i is determined by the input covariance matrices represented by $\mathbf{Q}_i = E[\mathbf{x}_i\mathbf{x}_i^H]$ for $i = 1, 2, 3$. Using eigenvalue decomposition, $\mathbf{Q}_i$ can be rewritten as

$$\mathbf{Q}_i = \mathbf{J}(\phi_i)\mathrm{diag}(\lambda_i, P_i - \lambda_i)\mathbf{J}(-\phi_i) \tag{7}$$

where $0 \leq P_i \leq P_{\max}$, $0 \leq \lambda_i \leq P_i$ and $\mathbf{J}(\phi)$ is a unitary rotation matrix defined by

$$\mathbf{J}(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}.$$

Here, λ_i controls the degree of asymmetry. For example, the distribution of $\mathbf{x}_i$ is circularly symmetric when $\lambda_i = \frac{P_i}{2}$ and the degree of asymmetry grows as $|\lambda_i - \frac{P_i}{2}|$ increases [7]. Therefore, the SC signaling is a special case of the AC signaling with $\lambda_i = \frac{P_i}{2}$.

Using a rotation matrix, the channel matrix $\mathbf{H}_{i,j}$ in (6) can be decomposed into its magnitude and phase as $\mathbf{H}_{i,j} = A_{i,j}\mathbf{J}(\theta_{i,j})$ where $A_{i,j} = |h_{i,j}|$ and $\theta_{i,j} = \angle h_{i,j}$. Then, equation (6) can be given as

$$\mathbf{y}_i = A_{i,i}\mathbf{J}(\theta_{i,i})\mathbf{x}_i + \sum_{j \neq i, j=1}^{3} A_{i,j}\mathbf{J}(\theta_{i,j})\mathbf{x}_j + \mathbf{n}_i. \tag{8}$$

Since the optimal DOF for 3-user SISO IFC is equal to 1.5 [9], we assume a rank-1 transmission for each BS which corresponds to an extremely asymmetric distribution of $\mathbf{x}_i$, i.e., $\lambda_i = P_i$. Then, the transmit signal vector $\mathbf{x}_i$ is related to u_i as $\mathbf{x}_i = \mathbf{v}_i u_i$ where $u_i \sim N(0, 1)$ indicates the data symbol intended for receiver i and $\mathbf{v}_i \in \mathbb{R}^{2\times 1}$ denotes the transmit beamformer.

Thus, the individual rate for link i is calculated as (9) at the top of the next page. Here, the pre-log factor $\frac{1}{2}$ is caused by employing real-valued data symbols. Consequently, the problem of maximizing the worst-user rate in AC signaling is formulated as

$$\max_{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3} \min\{R_1^{AC}, R_2^{AC}, R_3^{AC}\} \tag{10}$$
$$\text{subject to } \|\mathbf{v}_i\|^2 \leq P_{\max}, \forall i = 1, 2, 3.$$

$$R_i^{AC} = \frac{1}{2}\log_2 \frac{\det\left(E\left[\mathbf{y}_i\mathbf{y}_i^T\right]\right)}{\det\left(E[(\mathbf{y}_i - A_{i,i}\mathbf{J}(\theta_{i,i})\mathbf{v}_i u_i)\,(\mathbf{y}_i - A_{i,i}\mathbf{J}(\theta_{i,i})\mathbf{v}_i u_i)^T]\right)} \quad (9)$$

Due to non-convexity of the formulated problem, identifying the optimal solution of (10) is somewhat complicated. Thus, we propose an one-shot suboptimal algorithm employing the IA and the power control approach.

In our proposed algorithm, the beamforming vector for link i is designed to have the form of $\mathbf{v}_i = \sqrt{P_i}\mathbf{j}(\phi_i)$ where $\mathbf{j}(\phi)$ is defined as $\mathbf{j}(\phi) \triangleq [\cos(\phi)\ \sin(\phi)]^T$ and ϕ_i determines the direction of $\mathbf{v}_i$. Then, (8) can be rewritten as

$$\mathbf{y}_i = \sqrt{P_i}A_{i,i}\mathbf{j}(\theta_{i,i}+\phi_i)u_i + \sum_{j\neq i, j=1}^{3}\sqrt{P_j}A_{i,j}\mathbf{j}(\theta_{i,j}+\phi_j)u_j + \mathbf{n}_i. \quad (11)$$

Note that $\mathbf{J}(x)$ and $\mathbf{j}(x)$ have the properties of $\mathbf{J}(x)\mathbf{j}(y) = \mathbf{j}(x+y)$ and $\mathbf{j}(x)^T\mathbf{j}(y) = \cos(-x+y)$.

First, we apply the IA scheme. To decode the desired information symbol in (11), two interference terms should be aligned. To this end, we adjust ϕ_i's to satisfy the following IA constraints for link 1 and 2 as

$$\text{span}(\mathbf{j}(\theta_{1,2}+\phi_2)) = \text{span}(\mathbf{j}(\theta_{1,3}+\phi_3)) \quad (12)$$
$$\text{span}(\mathbf{j}(\theta_{2,1}+\phi_1)) = \text{span}(\mathbf{j}(\theta_{2,3}+\phi_3)) \quad (13)$$

where span$(\mathbf{x})$ indicates the signal space spanned by a column vector $\mathbf{x}$.

Then, both (12) and (13) are satisfied if $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are expressed as

$$\begin{aligned}\mathbf{v}_1 &= \sqrt{P_1}\mathbf{j}(\theta_{2,3}-\theta_{2,1}+\phi_3)\\ \mathbf{v}_2 &= \sqrt{P_2}\mathbf{j}(\theta_{1,3}-\theta_{1,2}+\phi_3)\\ \mathbf{v}_3 &= \sqrt{P_3}\mathbf{j}(\phi_3). \end{aligned} \quad (14)$$

Since the choice of ϕ_3 does not change the individual rate, we can set $\phi_3 = 0$ without loss of optimality [10].

With $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ in (14), link i for $i = 1, 2$ has the perfectly aligned interference as

$$\begin{aligned}\mathbf{y}_i = &\sqrt{P_i}A_{i,i}\mathbf{j}(\theta_{i,i}+\theta_{\bar{i},3}-\theta_{\bar{i},i})u_i\\ &+(\sqrt{P_{\bar{i}}}A_{i,\bar{i}}u_{\bar{i}} + \sqrt{P_3}A_{i,3}u_3)\mathbf{j}(\theta_{i,3}) + \mathbf{n}_i. \end{aligned} \quad (15)$$

Hence, we can easily remove the interference terms by adopting a ZF receiver $\mathbf{w}_i = \mathbf{j}(\theta_{i,3} - \frac{\pi}{2})^T$. Then, the ZF filter output of link i, $i = 1, 2$, is given as

$$\hat{u_i} = \sqrt{P_i}A_{i,i}\cos(\alpha_{i,i}) + \tilde{n}_i \quad (16)$$

where we have $\alpha_{i,i} = \theta_{i,i} + \theta_{\bar{i},3} - \theta_{\bar{i},i} - \theta_{i,3} + \frac{\pi}{2}$ and $\tilde{n}_i = \mathbf{w}_i\mathbf{n}_i \sim N(0, \frac{\sigma_n^2}{2})$ is the filtered noise.

On the other hand, the received signal at link 3 contains interference signal which is not completely aligned and is expressed as

$$\mathbf{y}_3 = \sqrt{P_3}A_{3,3}\mathbf{j}(\theta_{3,3})u_3 + \sum_{i=1}^{2}\sqrt{P_i}A_{i,i}\mathbf{j}(\beta_{3,i})u_i + \mathbf{n}_3 \quad (17)$$

where we define $\beta_{3,i} = \theta_{3,i} + \theta_{\bar{i},3} - \theta_{\bar{i},i}$ for $i = 1, 2$. Since the condition of IA for link 3, span$(\mathbf{j}(\beta_{3,1}))$ = span$(\mathbf{j}(\beta_{3,2}))$, is measured as a zero event due to channel randomness, we cannot eliminate the interference terms in (17). Note that a perfect IA scheme for 3-user SISO IFC with constant channel coefficients and no symbol extension does not exist [8]–[10].

For this reason, we employ a receive filter which eliminates only one out of two interference terms for link 3 in our algorithm. We assume that the interference from transmitter 1 is eliminated using the receive filter $\mathbf{w}_3 = \mathbf{j}(\alpha_{3,1} - \frac{\pi}{2})^T$ for the ease of explanation. The other case of interference nulling can be similarly explained. Then, the output of the filter $\mathbf{w}_3$ is written as

$$\begin{aligned}\hat{u_3} = &\sqrt{P_3}A_{3,3}\cos(\theta_{3,3}-\beta_{3,1}+\frac{\pi}{2})u_3\\ &+\sqrt{P_2}A_{3,2}\cos(\beta_{3,2}-\beta_{3,1}+\frac{\pi}{2})u_2 + \tilde{n}_3. \end{aligned} \quad (18)$$

From (16) and (18), we have the following SINR expressions as

$$\gamma_1^{AC} = \frac{G_{1,1}P_1}{\sigma_n^2/2},\ \gamma_2^{AC} = \frac{G_{2,2}P_2}{\sigma_n^2/2},\ \gamma_3^{AC} = \frac{G_{3,3}P_3}{\sigma_n^2/2 + G_{3,2}P_2} \quad (19)$$

where $G_{i,j} = (A_{i,j})^2\cos^2(\alpha_{i,j}), \alpha_{3,3} = \theta_{3,3} - \beta_{3,1} + \frac{\pi}{2}$ and $\alpha_{3,2} = \beta_{3,2} - \beta_{3,1} + \frac{\pi}{2}$.

Finally, for given $\{\mathbf{v}_i\}$, the SINR balancing problem in (10) is changed to a simple power allocation problem as

$$(P_1^*, P_2^*, P_3^*) = \arg\max_{0\le P_i\le P_{\text{max}},\ i=1,2,3}\min\{\gamma_1^{AC}, \gamma_2^{AC}, \gamma_3^{AC}\}. \quad (20)$$

Since increasing P_1 and P_3 does not decrease the other user's SINR in (19), we set $P_1^* = P_3^* = P_{\text{max}}$ without loss of generality. Then, γ_2^{AC} and γ_3^{AC} are a monotonically increasing and monotonically decreasing function with respect to P_2, respectively, and γ_1^{AC} becomes a constant value. Thus, we only adjust P_2 to maximize the worst-user SINR. For simplifying explanation of deriving P_2^*, we define $\bar{\gamma}_i^{AC}(P_2) = \gamma_i^{AC}|_{P_1=P_3=P_{\text{max}}}$ for $i = 1, 2, 3$. Then, if $\bar{\gamma}_2^{AC}(P_{\text{max}}) > \bar{\gamma}_3^{AC}(P_{\text{max}})$, the minimum of γ_i^{AC}'s is maximized when $\bar{\gamma}_2^{AC}(P_2) = \bar{\gamma}_3^{AC}(P_2)$ since there always exists a cross-over point within $0 \le P_2 \le P_{\text{max}}$. Otherwise, we set $P_2^* = P_{\text{max}}$ as a solution of (20). As a result, P_2^* is computed as in (21) at the top of the next page.

In the above illustration, beamforming vectors $\{\mathbf{v}_k\}$ are designed to have the perfectly aligned interference only for link 1 and 2. Thus, link 3 has misaligned interference terms from BS 1 and 2, and we only eliminate the interference from BS 1 using receive filters $\{\mathbf{w}_k\}$. In general, the beamforming vectors $\{\mathbf{v}_k\}$ can be designed so that misaligned interference remains at either link 1,2 or 3. Also, between interference terms from two aligned links, we choose one to apply the interference nulling at misaligned link. Thus, there are 6 possible designs in total, and the worst-user rate can be further improved if we choose the best one out of 6 possible cases.

For simplifying explanation, let us denote $\gamma_{\text{max-min}}^{AC}(i,j,k)$ as the maximized worst-user SINR corresponding to a system where beamformers perfectly align interference terms in link i and j $(i, j \in \{1,2,3\},\ i < j)$ and receivers null out the interference from BS k $(k \in \{i,j\})$ for the link with misaligned interference. Then, the optimal solution $\hat{\gamma}^{AC}$ is

$$P_2^* = \begin{cases} \frac{-G_{2,2}+\sqrt{(\sigma_n^2/2)^2 G_{2,2}^2+4(\sigma_n^2/2)G_{2,2}G_{3,2}G_{3,3}P_{\max}}}{2G_{2,2}G_{3,2}} & \text{, if } \bar{\gamma}_2^{AC}(P_{\max}) > \bar{\gamma}_3^{AC}(P_{\max}) \\ P_{\max} & \text{, otherwise} \end{cases} \quad (21)$$

computed as

$$\hat{\gamma}^{AC} = \max_{\substack{i,j\in\{1,2,3\},\ i\neq j, \\ k\in\{i,j\}}} \gamma_{\text{max-min}}^{AC}(i,j,k). \quad (22)$$

As a result, an additional selection diversity is expected by choosing the best one of 6 possible cases, and this will be confirmed a the following section.

V. Simulation Results

In this section, we present simulation results to demonstrate the efficiency of the proposed SINR balancing algorithm for 3-user SISO IFC systems. In our simulation, it is assumed that the channel coefficients are sampled from independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance.

In Fig. 2, we illustrate the average worst-user rate of both SC and AC signaling as a function of $\frac{P_{\max}}{\sigma_n^2}$. The non-cooperative scheme indicates full power transmission at all BSs, i.e., $P_1 = P_2 = P_3 = P_{\max}$, which is well known to approach Nash equilibrium [12], and the exhaustive search represents the optimal solution in SC signaling obtained by searching every possible power levels. Besides, the AC signaling without selection diversity indicates the proposed AC signaling which considers only one possible transceiver design in (22), i.e., the selection diversity is not exploited. It is observed that the proposed SC signaling scheme provides the performance almost identical to the optimal exhaustive search method in terms of the worst-user rate and outperforms the non-cooperative scheme. As shown in the plot, the transmission schemes based on the SC signaling does not exhibit a linear increase with respect to $\frac{P_{\max}}{\sigma_n^2}$, since the number of data streams is 3 and this is greater than the optimal degree of freedom (DOF) which is given by 1.5 [9]. On the other hand, the worst-user rate of the SINR balancing algorithms with the AC signaling increases linearly. Although the worst-user rate of AC signaling is dominated by the imperfect IA link due to residual interference at high SNR region, the optimal power control (21) allows the worst-user rate to increase linearly. Also, the proposed AC signaling exploits selection diversity which leads to an additional gain of 7.5 dB over the AC signaling without selection diversity at 1.5 bps/Hz, and as a result, the proposed AC signaling outperforms the SC signaling algorithms for overall SNR region.

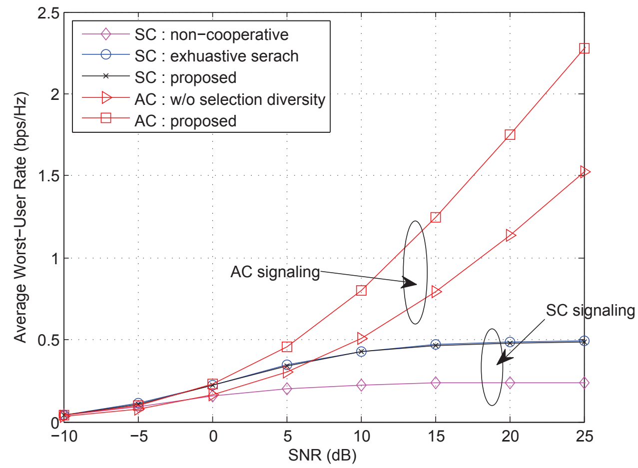

Fig. 2. Average worst-user rate for 3-user SISO IFC systems

VI. Conclusion

In this paper, we have addressed the problem of SINR balancing algorithms for multi-cell downlink systems. First, we have developed the SINR balancing algorithms with SC signaling. Since they transmit streams more than the optimal DOF, the worst-user rate is saturated as SNR increases. To solve this, we have also proposed SINR balancing algorithms based on the AC signaling which combines both the IA and the power control method. Simulation results confirm that the proposed AC signaling outperforms the SC signaling for overall SNR region.

References

[1] G. J. Foschini, K. Karakayali, and R. A. Valenzuela, "Coordinating Multiple Antenna Cellular Networks to Achieve Enormous Spectral Efficiency," *IEE Proc. Communications*, vol. 153, pp. 548–555, August 2006.

[2] J. Zhang and J. G. Andrews, "Adaptive Spatial Intercell Interference Cancellation in Multicell Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 28, pp. 1455–1468, December 2010.

[3] R. Zhang and S. Cui, "Cooperative Interference Management With MISO Beamforming," *IEEE Transactions on Signal Processing*, vol. 58, pp. 5450–5458, October 2010.

[4] H. Sung, S.-H. Park, K.-J. Lee, and I. Lee, "Linear Precoder Designs for K-user Interference Channels," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 291–301, January 2010.

[5] M. Schubert and H. Boche, "Solution of the Multiuser Downlink Beamforming Problem With Individual SINR Constraints," *IEEE Transactions on Signal Processing*, vol. 53, pp. 18 – 28, January 2004.

[6] A. Tölli, H. Pennanen, and P. Komulainen, "SINR Balancing with Coordinated Multi-cell Transmission," in *Proc. IEEE WCNC '09*, April 2009.

[7] S.-H. Park, H. Park, and I. Lee, "Coordinated SINR Balancing Techiniques for Multi-Cell Downlink Transmission," *in Proc. IEEE VTC '10*, September 2010.

[8] V. R. Cadambe, S. A. Jafar, and C. Wang, "Interference Alignment With Asymmetric Complex Signaling - Settling the Høst-Madsen-Nosratinia Conjecture," *IEEE Transactions on Information Theory*, vol. 56, pp. 4552–4565, September 2010.

[9] V. R. Cadambe and S. A. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Transactions on Information Theory*, vol. 54, pp. 3425–3441, August 2008.

[10] H.-Y. Shin, S.-H. Park, H. Park, and I. Lee, "Interference Alignment with Asymmetric Complex Signaling and Multiuser Diversity," in *Proc. IEEE Globecom '10*, December 2010.

[11] R. Zakhour, Z. K. M. Ho, and D. Gesbert, "Distributed Beamforming Coordination in Multicell MIMO Channels," in *Proc. IEEE VTC '09*, April 2009.

[12] J. Huang, R. A. Berry, and M. L. Honig, "Distributed Interference Compensation for Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, pp. 1074–1084, May 2006.