

3D Beamforming Designs for Single User MISO Systems

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Abstract—In this paper, we study a transmit beamforming technique for multiple input single output downlink single-user systems with three dimensional antennas where a transmit antenna gain is determined in three dimensional coordinates. In general, the transmit antenna gain is controlled by adjusting the boresight of antennas in directional antennas. To derive the optimal tilting angles for the directional antenna systems, we provide the probability density functions (PDF) of the three dimensional user distribution. Furthermore, based on the PDF, the analysis for the average rates of passive and active antenna systems is presented. Simulation results verify the accuracy of the performance analysis.

I. INTRODUCTION

One of the most important design considerations in next generation cellular networks is to support the explosive growth of demand for the data rate. Several approaches have been investigated to tackle this challenge. Among promising solutions, multiple-input multiple-output (MIMO) methods [1]–[7] and frequency reuse techniques through a cell architecture planning using directional antennas [8]–[10] have been highlighted in the past.

Traditionally, MIMO precoding² schemes which achieve high data rate transmission have been intensively investigated assuming isotropic or omni-directional antennas. In the meantime, the optimization of the cell architecture planning or the directional antenna pattern settings has been considered as a deployment issue. For example, the directional antenna pattern has been normally determined based on field tests or the cell modeling in practice. However, as the cell site architecture evolves, amplifiers and transceivers get located closer to passive antenna elements. Eventually, it is expected that most of the radio will be co-located with the antenna element, as is called an active antenna system [11] [12]. In the active antenna array, each antenna element can be connected to a separate transceiver component, and thus the active antenna system can support an electronic beam-tilt feature by controlling the phase, amplitude and delay of individual antenna elements [11].

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²To reduce confusion, we employ the terminology “precoding” for closed-loop MIMO systems, while “beamforming” is adopted for directional antenna pattern designs.

In this paper, we investigate how to optimize the antenna pattern for three dimensional (3D) antenna systems, and provide efficient solutions for the single user multiple input single output (MISO) system. We first derive the probability density function (PDF) of the vertical angle distribution for 3D channel model [13]. Then, we offer the optimal vertical beamforming solutions for the active and passive antenna systems with maximum ratio transmission as a MISO precoding technique. Moreover, we analyze the average rates of the active antenna system and the passive antenna system. The accuracy of the analysis will be verified by numerical results. To our best knowledge, there is no reported work for the optimal vertical beamforming method or directional antenna settings for the active and passive antenna systems.

The rest of the paper is organized as follows: In Section II, we introduce a wireless downlink system with 3D antennas. We derive the PDF of the vertical angel distribution for 3D channel model in Section III. Then, in Section IV, we provide the optimal solutions and its analysis for the active and passive antenna systems. Section V presents the simulation results of the proposed scheme. Section VI concludes the paper.

The following notations are used throughout the paper. \mathbf{A}^H denotes complex-transpose for any general matrix \mathbf{A} . $\mathbb{E}[\cdot]$ and $\|\mathbf{a}\|^2$ indicate the expectation operation and the Euclidean 2-norm of a vector \mathbf{a} , respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wireless downlink system with 3D antennas. Unlike isotropic antenna systems where an antenna gain for all directions is the same, the antenna gain in directional antenna systems changes according to the transmit antenna pattern. Typically, the transmit antenna pattern can be divided into a horizontal and vertical part [13]. Fig. 1 illustrates the 3D system which consists of a base station and a user. We define x and y as relative distances between the user and the base station in the x and y coordinate, respectively, and denote Δz as the height difference between the user and the base station.

Defining the horizontal angle between the base station antenna boresight and the user by $\theta_U = \tan^{-1} \frac{y}{x}$, the horizontal antenna attenuation can be written in dB scale as [13]

$$A_H(\theta_U) = \min \left(12 \left(\frac{\theta_U}{\theta_{3dB}} \right)^2, A_m \right)$$

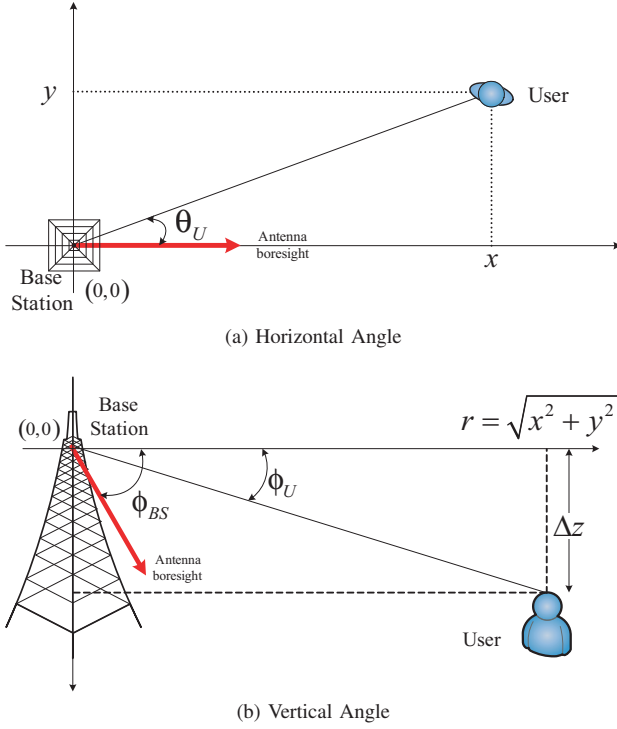


Fig. 1. Three dimensional antenna configuration

where θ_{3dB} and A_m represent the horizontal 3 dB beamwidth and the maximum attenuation of the base station antennas, respectively.

Similarly, the vertical antenna attenuation can be expressed in dB scale as [13]

$$A_V(\phi_U) = \min \left(12 \left(\frac{\phi_U - \phi_{BS}}{\phi_{3dB}} \right)^2, A_m \right)$$

where $\phi_U = \tan^{-1} \frac{\Delta z}{\sqrt{x^2 + y^2}}$ indicates the vertical angle with respect to the horizontal plane at the user location, ϕ_{BS} equals the vertical boresight angle of the base station (also known as the tilting angle), and ϕ_{3dB} stands for the vertical 3 dB beamwidth of the base station antennas.

Let us denote G_{\max} as the maximum antenna gain at the antenna boresight. Then, after combining the antenna attenuations and the maximum antenna gain, the resultant antenna gain in dB scale for the user with the horizontal angle θ_U and the vertical angle ϕ_U can be formulated as [13]

$$G(\theta_U, \phi_U) = G_{\max} - \min(A_H(\theta_U) + A_V(\phi_U), A_m).$$

To simplify the analysis, we assume $A_m = \infty$ and $A_H(\theta_U) = 0$. Here, the first assumption is valid when $A_H(\theta_U) + A_V(\phi_U) \leq A_m$, and the second assumption holds when θ_{3dB} is relatively large.³ These conditions are well matched with typical cell deployments, and all work in this paper can be simply extended to the case of non-zero $A_H(\theta_U)$. Thus, we can ignore the horizontal angle θ_U from now on.

³For example, the antenna model in [13], θ_{3dB} is 70° while ϕ_{3dB} is 15° .

With these assumptions, we can rewrite the antenna gain in linear scale for the user as

$$g(\phi_U) = g_{\max} a_V(\phi_U) = g_{\max} 10^{-1.2(\phi_U - \phi_{BS})^2 / \phi_{3dB}^2} \quad (1)$$

where $g_{\max} = 10^{G_{\max}/10}$ denotes the maximum antenna gain at the boresight, and $a_V(\phi_U)$ indicates the vertical antenna attenuation at ϕ_U . In what follows, we will refer to ϕ_U and ϕ_{BS} as the vertical angle and the tilting angle, respectively.

III. PROBABILITY DENSITY FUNCTION OF THE VERTICAL ANGLE

In this section, we derive the PDF of the vertical angle for 3D channel model. Let $r = \sqrt{x^2 + y^2}$ be the distance between the user and the base station. We assume that users are uniformly distributed over three dimensional space with $r_0 \leq r \leq r_m$ and $0 \leq z_0 \leq \Delta z \leq z_m$ where r_0 and r_m represent the minimum distance and the cell radius, respectively, and z_0 and z_m denote the minimum and the maximum height difference between the base station and the user, respectively. Since the number of users with the distance r is proportional to the circumference of a circle with the radius r , the PDF of the random variable r is given by $f_r(\gamma) = 2\gamma/(r_m^2 - r_0^2)$ for $r_0 \leq \gamma \leq r_m$.⁴

Now, we derive the PDF of a random variable $\phi = \tan^{-1} \frac{\Delta z}{r}$ as follows. First of all, the cumulative distribution function (CDF) of ϕ is computed as

$$\begin{aligned} F_\phi(\phi) &= \Pr \left\{ \tan^{-1} \frac{\Delta z}{r} \leq \phi \right\} \\ &= \Pr \{ \Delta z \leq r \tan \phi \} \\ &= \mathbb{E} [\Pr \{ \Delta z \leq \gamma \tan \phi \} | r = \gamma]. \end{aligned} \quad (2)$$

In order to obtain the above CDF, we need to examine the following six cases.

- i) For $r_m \tan \phi < z_0$: $F_\phi(\phi) = 0$.
- ii) For $r_0 \tan \phi < z_0$ and $z_0 < r_m \tan \phi < z_m$:

$$\begin{aligned} F_\phi(\phi) &= ab \int_{\frac{z_0}{\tan \phi}}^{r_m} \gamma (\gamma \tan \phi - z_0) d\gamma \\ &= \frac{ab}{3} r_m^3 \tan \phi + \frac{ab}{6 \tan^2 \phi} z_0^3 - \frac{ab}{2} z_0 r_m^2, \end{aligned}$$

where $a = \frac{1}{z_m - z_0}$ and $b = \frac{2}{r_m^2 - r_0^2}$.

- iii) For $r_0 \tan \phi < z_0$ and $r_m \tan \phi > z_m$:

$$\begin{aligned} F_\phi(\phi) &= ab \int_{\frac{z_0}{\tan \phi}}^{\frac{z_m}{\tan \phi}} \gamma (\gamma \tan \phi - z_0) d\gamma + b \int_{\frac{z_m}{\tan \phi}}^{r_m} \gamma d\gamma \\ &= \left(\frac{ab}{3} z_m^3 - \frac{ab}{2} z_m^2 x_0 + \frac{ab}{6} x_0^3 - \frac{b}{2} x_m^2 \right) \frac{1}{\tan^2 \phi} \\ &\quad + \frac{b}{2} r_m^2. \end{aligned}$$

⁴The density of r can be determined by $f_r(\gamma) d\gamma = \Pr \{ \gamma < r \leq \gamma + d\gamma \} = \iint_{\Delta D_\gamma} f_{x,y}(x,y) dx dy$ where ΔD_γ is the region of the xy plane such that $\gamma < \sqrt{x^2 + y^2} \leq \gamma + d\gamma$ and $f_{x,y}(x,y)$ equals the joint distribution of x and y which is a constant within the region.

iv) For $z_0 < r_0 \tan \phi < z_m$ and $z_0 < r_m \tan \phi < z_m$:

$$\begin{aligned} F_\phi(\phi) &= ab \int_{r_0}^{r_m} \gamma(\gamma \tan \phi - z_0) d\gamma \\ &= \frac{ab}{3}(r_m^3 - r_0^3) \tan \phi - \frac{ab}{2} z_0(r_m^2 - r_0^2). \end{aligned}$$

v) For $z_0 < r_0 \tan \phi < z_m$ and $r_m \tan \phi > z_m$:

$$\begin{aligned} F_\phi(\phi) &= ab \int_{r_0}^{\frac{z_m}{\tan \phi}} \gamma(\gamma \tan \phi - z_0) d\gamma + b \int_{\frac{z_m}{\tan \phi}}^{r_m} \gamma d\gamma \\ &= \left(\frac{ab}{3} x_m^3 - \frac{ab}{2} x_0 x_m^2 - \frac{b}{2} x_m^2 \right) \frac{1}{\tan^2 \phi} + \frac{b}{2} r_m^2 \\ &\quad + \frac{ab}{2} z_0 r_0^2 - \frac{ab}{3} r_0^3 \tan \phi. \end{aligned}$$

vi) For $r_0 \tan \phi > z_m$:

$$F_\phi(\phi) = b \int_{r_0}^{r_m} \gamma d\gamma = 1.$$

Then, the PDF of a random variable ϕ can be calculated by differentiating the CDF which results in (3) for $z_0/r_0 > z_m/r_m$ or (4) for $z_0/r_0 \leq z_m/r_m$ at the top of the next page.

In the following section, based on the derived distributions, we examine two antenna technologies: the active antenna case where ϕ_{BS} is determined instantaneously based on instant channel realizations, and the passive antenna case where ϕ_{BS} is obtained statistically. From now on, we refer to the former case as an active beamforming scheme, while we call the latter case as a passive beamforming scheme.

IV. SINGLE-USER VERTICAL BEAMFORMING

In this section, we consider a system which consists of a base station with M transmit antennas and a single user equipped with a single receive antenna. For frequency-flat fading channels, the received signal $y \in \mathbb{C}$ with the vertical angle ϕ_U at the distance r is given by

$$y = \sqrt{r^{-\alpha} g(\phi_U) P} \mathbf{h}^H \mathbf{s} + n \quad (5)$$

where α and P equal the pathloss exponent and the transmit power of the base station, respectively, $\mathbf{h} \in \mathbb{C}^{M \times 1}$ stands for the channel vector from the base station to the user whose entry is an independent circular-symmetric complex Gaussian random variable $\mathcal{CN}(0, 1)$, i.e. Rayleigh fading, $\mathbf{s} \in \mathbb{C}^{M \times 1}$ represents the transmit signal vector, and n indicates the complex additive white Gaussian noise (AWGN) at the user with variance σ_n^2 .

The transmit signal vector \mathbf{s} is expressed as $\mathbf{s} = \mathbf{w}d$ where $\mathbf{w} \in \mathbb{C}^{M \times 1}$ and d denote the transmit precoding vector with $\|\mathbf{w}\|^2 = 1$ and the transmit data symbol with unit variance, respectively. We further assume that ϕ and \mathbf{h} are independent of each other. With perfect channel state information (CSI) at the transmitter, it is well known that maximum ratio transmission (MRT), i.e. $\mathbf{w} = \mathbf{h}/\|\mathbf{h}\|$, is the optimal scheme for the single user single stream precoding

[14]. The resultant received signal-to-noise ratio (SNR) at the user can be written as

$$\text{SNR} = \frac{r^{-\alpha} g(\phi_U) P \|\mathbf{h}\|^2}{\sigma_n^2} = \frac{r^{-\alpha} g_{\max} a_V(\phi_U) P \|\mathbf{h}\|^2}{\sigma_n^2}. \quad (6)$$

Now we want to find the optimal tilting angle ϕ_{BS}^* for the active and passive beamforming cases which maximize the average rate as

$$\phi_{BS}^* = \arg \max_{\phi_{BS}} \mathbb{E}_{r, \phi, \mathbf{h}} [\log_2(1 + \text{SNR})]. \quad (7)$$

Obviously, for the case of the active beamforming, the optimal tilting angle is obtained by setting the tilting angle ϕ_{BS}^* as the actual vertical angle ϕ_U . In this case, the corresponding vertical antenna attenuation becomes $a_V(\phi_U) = 1$. In contrast, identifying the optimal tilting angle for the passive beamforming is not trivial. Since it is difficult to directly optimize ϕ_{BS} over all SNR region, we focus on the high SNR region. The following theorem addresses the optimal tilting angle ϕ_{BS}^* for the passive beamforming in the high SNR region.

Theorem 1: For the single user passive beamforming case, the optimal tilting angle ϕ_{BS}^* for the high SNR region is the mean value of the vertical angle ϕ , i.e. $\phi_{BS}^* = \mathbb{E}[\phi]$.

Proof: For large x , we have $\log_2(1 + x) \approx \log_2 x$. We can write the average rate as $\mathbb{E}_{r, \phi, \mathbf{h}} [\log_2(1 + \text{SNR})] \approx \mathbb{E}_{r, \phi, \mathbf{h}} [\log_2 \text{SNR}]$ for high SNR. Then the optimal ϕ_{BS}^* can be determined as

$$\begin{aligned} \phi_{BS}^* &= \arg \max_{\phi_{BS}} \mathbb{E}_{r, \phi, \mathbf{h}} \left[\log_2 \frac{r^{-\alpha} g_{\max} 10^{-1.2(\phi - \phi_{BS})^2 / \phi_{3dB}^2} P \|\mathbf{h}\|^2}{\sigma_n^2} \right]. \end{aligned}$$

This is equivalent to

$$\phi_{BS}^* = \arg \min_{\phi_{BS}} \mathbb{E} \left[1.2 \frac{(\phi - \phi_{BS})^2}{\phi_{3dB}^2} \log_2 10 \right]. \quad (8)$$

Since the second derivative of the expectation value in (8) with respect to ϕ_{BS} is always a positive value, i.e. $\frac{d^2}{d\phi_{BS}^2} \mathbb{E} \left[1.2 \frac{(\phi - \phi_{BS})^2}{\phi_{3dB}^2} \log_2 10 \right] = \frac{2.4 \log_2 10}{\phi_{3dB}^2} \int f_\phi(\phi) d\phi = \frac{2.4 \log_2 10}{\phi_{3dB}^2}$, we can compute the optimal ϕ_{BS}^* from

$$\begin{aligned} \frac{d}{d\phi_{BS}} \int (\phi - \phi_{BS})^2 f_\phi(\phi) d\phi &= \\ 2\phi_{BS} \int f_\phi(\phi) d\phi - 2 \int \phi f_\phi(\phi) d\phi &= 0. \end{aligned} \quad (9)$$

Then a solution of equation (9) is obtained as $\phi_{BS}^* = \int \phi f_\phi(\phi) d\phi = \mathbb{E}[\phi]$. Thus, the optimal ϕ_{BS}^* is the mean value of the vertical angle ϕ for the high SNR region. ■

Now, $\mathbb{E}[\phi]$ can be calculated using (3) or (4) depending on the ratio of the minimum height difference to the minimum distance z_0/r_0 and that of the maximum height to the maximum distance z_m/r_m . After applying the trigonometry function equality [15], for both $z_0/r_0 > z_m/r_m$ and $z_0/r_0 \leq$

$$f_\phi(\phi) = \begin{cases} \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \left(\frac{r_m^3}{\cos^2 \phi} - \frac{z_0^3}{\tan \phi \sin^2 \phi} \right) & \text{for } \tan^{-1} \frac{z_0}{r_m} \leq \phi \leq \tan^{-1} \frac{z_m}{r_m} \\ \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \frac{z_m^3 - z_0^3}{\tan \phi \sin^2 \phi} & \text{for } \tan^{-1} \frac{z_m}{r_m} \leq \phi \leq \tan^{-1} \frac{z_0}{r_0} \\ \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \left(\frac{z_m^3}{\tan \phi \sin^2 \phi} - \frac{r_0^3}{\cos^2 \phi} \right) & \text{for } \tan^{-1} \frac{z_0}{r_0} \leq \phi \leq \tan^{-1} \frac{z_m}{r_0} \\ 0 & \text{else.} \end{cases} \quad (3)$$

$$f_\phi(\phi) = \begin{cases} \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \left(\frac{r_m^3}{\cos^2 \phi} - \frac{z_0^3}{\tan \phi \sin^2 \phi} \right) & \text{for } \tan^{-1} \frac{z_0}{r_m} \leq \phi \leq \tan^{-1} \frac{z_0}{r_0} \\ \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \frac{r_m^3 - r_0^3}{\cos^2 \phi} & \text{for } \tan^{-1} \frac{z_0}{r_0} \leq \phi \leq \tan^{-1} \frac{z_m}{r_m} \\ \frac{2}{3(z_m - z_0)(r_m^2 - r_0^2)} \left(\frac{z_m^3}{\tan \phi \sin^2 \phi} - \frac{r_0^3}{\cos^2 \phi} \right) & \text{for } \tan^{-1} \frac{z_m}{r_m} \leq \phi \leq \tan^{-1} \frac{z_m}{r_0} \\ 0 & \text{else.} \end{cases} \quad (4)$$

$z_m/r_m, \phi_{BS}^*$ is given as

$$\begin{aligned} \phi_{BS}^* = & \frac{1}{3(z_m - z_0)(r_m^2 - r_0^2)} \left((z_0^2 - z_m^2)(r_0 - r_m) + \right. \\ & z_0^3 \left(\tan^{-1} \frac{z_0}{r_0} - \tan^{-1} \frac{z_0}{r_m} \right) - z_m^3 \left(\tan^{-1} \frac{z_m}{r_0} - \tan^{-1} \frac{z_m}{r_m} \right) \\ & + r_m^3 \ln \frac{z_0^2 + r_m^2}{z_m^2 + r_m^2} - r_0^3 \ln \frac{z_0^2 + r_0^2}{z_m^2 + r_0^2} + 3r_m^2 z_m \tan^{-1} \frac{z_m}{r_m} + \\ & \left. 3r_0^2 z_0 \tan^{-1} \frac{z_0}{r_0} - 3r_m^2 z_0 \tan^{-1} \frac{z_0}{r_m} - 3r_0^2 z_m \tan^{-1} \frac{z_m}{r_0} \right). \end{aligned}$$

From this result, for the single user system, the average rate in the high SNR region can be derived as

$$\begin{aligned} R & \approx \mathbb{E}_{r, \phi, \mathbf{h}} [\log_2 \text{SNR}] \\ & = \mathbb{E}_{R, \phi, \mathbf{h}} [\log_2 (r^{-\alpha} g_{\max} a_V(\phi) P ||\mathbf{h}||^2)] \\ & = \log_2 P + \mathbb{E} [\log_2 ||\mathbf{h}||^2] + \mathbb{E} [\log_2 a_V(\phi)] + \log_2 g_{\max} \\ & \quad - \alpha \mathbb{E} [\log_2 r]. \end{aligned}$$

Using the optimal tilting angle ϕ_{BS}^* for each beamforming case, we can calculate the average rate of the passive and the active beamforming. Since $a_V(\phi_U) = 1$ in the active beamforming for any ϕ_U and $\mathbb{E} [\log_2 ||\mathbf{h}||^2] = \psi(M)/\ln 2$ [16] where $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$ is the digamma function, the average rate of the active beamforming is expressed as

$$R_{\text{active}} \approx \log_2 P + \frac{\psi(M)}{\ln 2} + \log_2 g_{\max} - \alpha \frac{2r_m^2 \ln r_m - 2r_0^2 \ln r_0 - r_m^2 + r_0^2}{2(r_m^2 - r_0^2) \ln 2}. \quad (10)$$

For the passive beamforming, after plugging $\phi_{BS}^* = \mathbb{E}[\phi]$

into $a_V(\cdot)$, $\mathbb{E} [\log_2 a_V(\phi)]$ can be obtained by

$$\begin{aligned} \mathbb{E} [\log_2 a_V(\phi)] & = \mathbb{E} \left[\log_2 10^{-1.2(\phi - \phi_{BS}^*)^2 / \phi_{3dB}^2} \right] \\ & = -\frac{1.2 \log_2 10}{\phi_{3dB}^2} \left(\int \phi^2 f_\phi(\phi) d\phi - (\mathbb{E}[\phi])^2 \right) \\ & = -\frac{1.2 \sigma_\phi^2 \log_2 10}{\phi_{3dB}^2} = -3.986 \frac{\sigma_\phi^2}{\phi_{3dB}^2}. \end{aligned} \quad (11)$$

Thus, the average rate of the passive beamforming is written as

$$\begin{aligned} R_{\text{passive}} & \approx \log_2 P + \frac{\psi(M)}{\ln 2} - 3.986 \frac{\sigma_\phi^2}{\phi_{3dB}^2} + \log_2 g_{\max} \\ & \quad - \alpha \frac{2r_m^2 \ln r_m - 2r_0^2 \ln r_0 - r_m^2 + r_0^2}{2(r_m^2 - r_0^2) \ln 2}. \end{aligned} \quad (12)$$

By observing the difference between (10) and (12), we can see that the average rate gain of the active beamforming over the passive beamforming in the high SNR region is equal to $3.986 \frac{\sigma_\phi^2}{\phi_{3dB}^2}$. Thus, we expect the average rate gain is proportional to $\frac{\sigma_\phi^2}{\phi_{3dB}^2}$. This result will be verified in the simulation section. Note that the average rate gain does not depend on the number of transmit antennas, since \mathbf{h} and ϕ are independent.

V. SIMULATION RESULTS

In this section, we demonstrate the efficiency of the active beamforming scheme compared to the passive beamforming scheme through Monte-Carlo simulations. It is assumed that the maximum antenna gain at the antenna boresight G_{\max} is 17 dB. We adopt the micro cell environment in [13] with slight modifications as listed in Table I if not stated otherwise. Here we denote h_{BS} , $h_{U, \min}$, and $h_{U, \max}$ by the base station antenna height, the minimum user antenna height, and the maximum user antenna height, respectively. In this case, the minimum and the maximum height difference are $z_0 = h_{BS} - h_{U, \max}$

TABLE I
SIMULATION SETTINGS

Maximum antenna gain	$G_{\max} = 17$ dB
Vertical 3 dB beamwidth	$\phi_{3dB} = 10^\circ$
Base station antenna height	$h_{BS} = 10$ m
Minimum user antenna height	$h_{U,\min} = 1.5$ m
Maximum user antenna height	$h_{U,\max} = 9$ m
Minimum distance between a user and a base station	$r_0 = 10$ m
Cell radius	$r_m = 50$ m
Number of transmit antennas at a base station	$M = 2$ or 4
Number of receive antennas at a user	1
Pathloss exponent	3.4

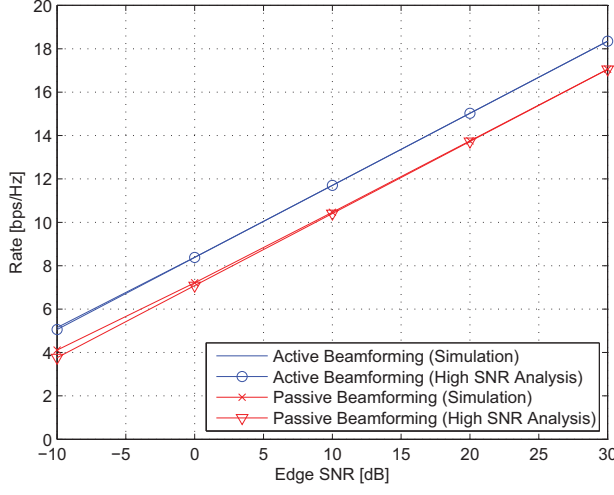


Fig. 2. Performance for single user systems ($M = 2$, $\phi_{3dB} = 10^\circ$)

and $z_m = h_{BS} - h_{U,\min}$, respectively. Moreover, we define the edge SNR as $\frac{r_m^{-\alpha} P}{\sigma_n^2}$.

Fig. 2 shows simulation results for systems with two transmit antennas and a single user ($M = 2$ and $N = 1$). The variance of the vertical angle becomes $\sigma_\phi^2 = 0.0099$ according to the simulation setting in Table I. In the figure, we include the high SNR analysis for the active beamforming in (10) as well as the analysis for the passive beamforming in (12). We can see that the analytic curves agree well with the simulation results as SNR increases, since the analysis is based on the high SNR assumption. It can be observed that our analysis of the active beamforming case compared to that of the passive beamforming is tighter, since the SNR of the user applied with the active beamforming becomes higher than that of the passive beamforming at the same edge SNR. From the plot, we confirm that the active beamforming outperforms the passive beamforming. Particularly, a 1.3 bps/Hz gain is observed at medium to high SNR which accurately matches with the analysis in (11), i.e. $3.986 \frac{\sigma_\phi^2}{\phi_{3dB}^2} = 1.3$ bps/Hz.

To capture the effect of the vertical 3 dB beamwidth of the base station antennas ϕ_{3dB} , Fig. 3 evaluates the active beamforming and the passive beamforming for different ϕ_{3dB} , and plot the rate gain of the active beamforming over the

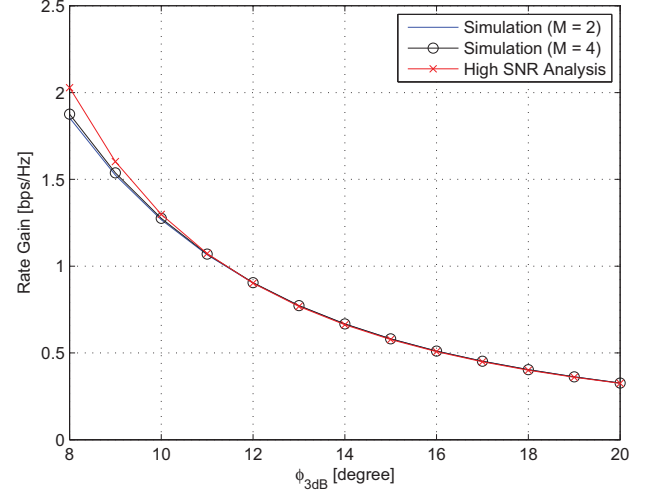


Fig. 3. The rate gain with different ϕ_{3dB} with Edge SNR = 20 dB

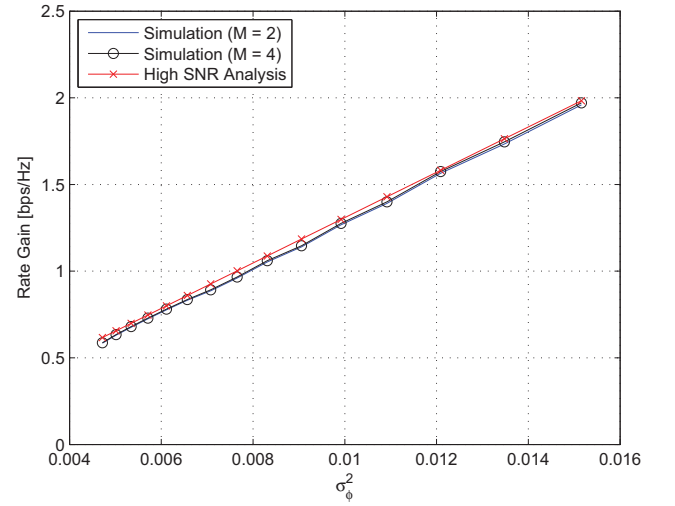


Fig. 4. The rate gain with different σ_ϕ^2

passive beamforming at the edge SNR of 20 dB. From Fig. 3, we can see that the rate gain is proportional to $\frac{1}{\phi_{3dB}^2}$. Similarly, Fig. 4 depicts the effect of the variance of the vertical angle σ_ϕ^2 . The simulation results support the rate gain is proportional to σ_ϕ^2 . The performance results in figures 3 and 4 confirm that the rate gain $3.986 \frac{\sigma_\phi^2}{\phi_{3dB}^2}$ bps/Hz derived in (11) is accurate. Moreover, from the plot, we verify that the rate gain is independent of the number of transmit antennas in the single user case. Note that the accuracy of the analysis is increased as ϕ_{3dB} grows and σ_ϕ^2 decreases. The reason for this is that higher ϕ_{3dB} and lower σ_ϕ^2 lead to a higher SNR environment for the user in the passive beamforming case.

VI. CONCLUSION

In this paper, we have studied a transmit beamforming technique for MISO downlink single systems in three dimensional antennas. We have provided the PDF of the vertical angle and the optimal tilting angle for the active and passive antenna systems accordingly. Moreover, we have derived the average rate for the active and passive antenna systems for a single user case, which is verified by simulation results.

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