Scaling Law of Feedback Bits for Distributed Antenna Systems with Limited Feedback

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***Abstract*—In this paper, we study a feedback bit allocation algorithm with signal-to-leakage plus noise ratio maximizing beamforming (FA-SMB) for distributed antenna systems (DAS) presented in work [1]. We first investigate a scaling law of feedback bits for both the FA-SMB scheme and equal bit allocation. Through this analysis, we confirm that the required number of feedback bits to satisfy the maximum allowable rate gap between DAS with perfect channel state information (CSI) and limited feedback linearly increases with signal-to-noise ratio. Also, it is verified that the FA-SMB scheme saves the feedback bits by up to 30% over the equal bit allocation with the same rate gap at SNR $= 40$ dB for three-user DAS. Moreover, we show that the FA-SMB scheme substantially reduces the computational complexity compared to exhaustive search. Finally, we provide simulation results to demonstrate the efficacy of the FA-SMB scheme.**

I. INTRODUCTION

In recent years, distributed antenna systems (DAS) have gained interests because of its ability to extend the cell coverage and increase the system capacity. The DAS was first introduced for indoor wireless communication systems to enhance coverage [2]. Unlike conventional centralized antenna systems (CAS) where all antennas are co-located at the cell center, distributed antenna (DA) ports in the DAS are separated geographically within a cell. Thus, the DAS can reduce the access distance along with the transmit power and co-channel interference, which results in improved cell-edge performance [3] [4]. Recent works on the DAS have concentrated on the performance analysis [5] [6] and antenna location designs [7].

In practical limited feedback systems, each receiver quantizes instantaneous channel states and feeds back the index of the quantized channel to the transmitter based on a codebook whose size is determined by the allocated number of feedback bits, and each user should properly allocate the number of feedback bits to all links in order to improve system performance. In work [1], by applying signal-to-leakage plus noise ratio (SLNR) maximizing beamforming [8] [9], a feedback bit allocation algorithm which minimizes an upper bound of a mean rate loss was introduced for multi-user downlink DAS where pairing between DA ports and users is considered based on distance. We refer to this scheme as feedback bit allocation with the SLNR maximizing beamforming (FA-SMB).

In this paper, we evaluate the FA-SMB for the multi-user DAS. First, for both the FA-SMB and the equal bit allocation scheme, we examine the scaling law of feedback bits. In [10], Jindal introduced the scaling law of feedback bits for limited feedback CAS with zero forcing beamforming (ZFBF). It was shown that it is sufficient to scale the number of feedback bits according to signal-to-noise ratio (SNR) in order to achieve the same multiplexing gain of the system with perfect channel state information (CSI).

In this paper, we verify that the required number of feedback bits which satisfies the maximum allowable rate gap compared to the perfect CSI case is also proportional to SNR for both schemes in the limited feedback DAS with the SLNR maximizing beamforming. In addition, we compare the computational complexity of the FA-SMB scheme with several feedback bit allocation methods. Through analyses and simulations, it is confirmed that the FA-SMB scheme is more efficient than other schemes in terms of performance and complexity.

Throughout this paper, bold lower case letters denote vectors, and the superscripts $(\cdot)^H$ and $(\cdot)^{-1}$ stand for Hermitian and the inverse operation, respectively. Also, $\mathcal{E}(\cdot)$ represents expectation.

II. SYSTEM MODEL

We consider a downlink single-cell DAS where N DA ports equipped with M antennas supports K single antenna users as illustrated in Figure 1. In this configuration, for the pairing among DA ports and users, each user first selects the nearest DA port as the serving DA port. Then, the chosen DA ports transmit the signal to the corresponding users with SLNR maximizing beamforming [8] [9] under per-DA port power constraint P, while the DA ports which is not selected by any user are turned off. Also, it is assumed that DA ports do not share CSI or users' data.

Throughout this paper, we employ the composite channel model which encompasses not only small scale fading (i.e. Rayleigh fading) but also large scale fading (i.e. path loss). It is assumed that each user has perfect knowledge of the linked

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010- 0017909).

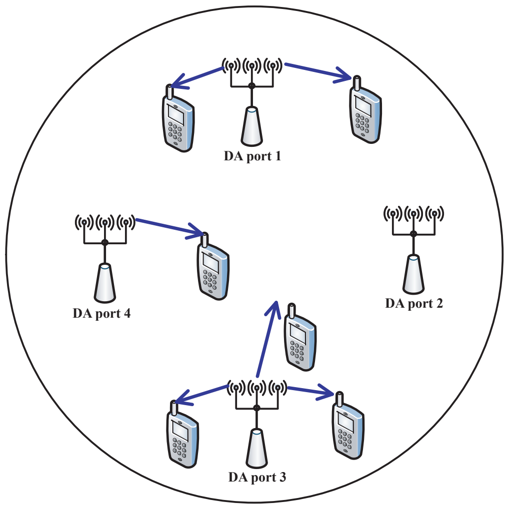

Fig. 1. System model for single-cell DAS with $N = 4$ DA ports and $K = 6$ users

channel state and quantizes channel direction information (CDI) of the small scale fading based on a codebook. Here, the codebook vectors are independently determined by utilizing random vector quantization (RVQ) [11] [12]. Then, the index of the quantized CDI is fed back to the DA ports. Moreover, we assume that DA ports know distances between the users and all linked channels by measuring the received signal strength in the uplink phase.

Let us denote the channel column vector of length M between the n-th DA port and the k-th user as $d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}$ where $d_{n,k}$ stands for the distance between the n-th DA port and the k-th user, α indicates the path loss exponent and $\mathbf{h}_{n,k}$ equals the channel column vector which accounts for small scale fading. The elements of $\mathbf{h}_{n,k}$ are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. Let us define $\mathcal{D}$ and $\mathcal{U}_n$ as the set of DA ports selected by users and the set of users served by the n-th DA port, respectively. Assuming that the k-th user chooses the j-th DA port as the serving DA port, the received signal for the k-th user can be written as

$$\begin{aligned} y_k &= \sqrt{P_j} d_{j,k}^{-\alpha/2} \mathbf{h}_{j,k}^H \hat{\mathbf{w}}_{j,k} s_{j,k} + \sum_{i\in\mathcal{U}_j, i\neq k} \sqrt{P_j} d_{j,k}^{-\alpha/2} \mathbf{h}_{j,k}^H \hat{\mathbf{w}}_{j,i} s_{j,i} \\ &\quad + \sum_{n\in\mathcal{D}, n\neq j} \sum_{i\in\mathcal{U}_n} \sqrt{P_n} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,i} s_{n,i} + z_k, \end{aligned}$$

where P_n is defined as $P_n \triangleq \frac{P}{|\mathcal{U}_n|}$, $\hat{\mathbf{w}}_{n,i}$ stands for the i-th user beamforming column vector of length M with unit norm ($\|\hat{\mathbf{w}}_{n,i}\| = 1$) which is computed by utilizing the quantized channel information at the n-th DA port, $s_{n,i}$ represents the desired signal of the i-th user transmitted from the n-th DA port with $\mathcal{E}\left[|s_{n,i}|^2\right] = 1$ and z_k indicates the additive complex Gaussian noise variable with zero mean and unit variance.

The k-th user beamforming vector which maximizes the SLNR is given by [8] [9]

$$\hat{\mathbf{w}}_{j,k} = \max \mathrm{ev}\left(\left(\mathbf{I} + P_j \sum_{i=1, i\neq k}^{K} d_{j,i}^{-\alpha} \hat{\mathbf{h}}_{j,i} \hat{\mathbf{h}}_{j,i}^H \right)^{-1} P_j d_{j,k}^{-\alpha} \hat{\mathbf{h}}_{j,k} \hat{\mathbf{h}}_{j,k}^H \right)$$

where $\max \mathrm{ev}(\mathbf{A})$ stands for the eigenvector corresponding to the largest eigenvalue of $\mathbf{A}$ and $\hat{\mathbf{h}}_{n,k}$ equals the quantized channel between the k-th user and the n-th DA port. Note that this SLNR maximizing beamforming is known to converge to ZFBF, which is optimal for achieving the maximum sum rate in a high SNR regime. Let us denote the set of codewords for the k-th user and the n-th DA port as

$$\mathcal{C}_{n,k} = \left\{ \mathbf{c}_{n,k,1}, \mathbf{c}_{n,k,2}, \cdots, \mathbf{c}_{n,k,2^{B_{n,k}}} \right\},$$

where each element is a unit norm column vector of length M and $B_{n,k}$ represents the allocated number of feedback bits for the channel between the k-th user and the n-th DA port. Also, defining the CDI of $\mathbf{h}_{n,k}$ as $\tilde{\mathbf{h}}_{n,k} \triangleq \mathbf{h}_{n,k}/\|\mathbf{h}_{n,k}\|$, $\hat{\mathbf{h}}_{n,k}$ is obtained as

$$\hat{\mathbf{h}}_{n,k} = \arg \max_{\mathbf{c}\in\mathcal{C}_{n,k}} |\tilde{\mathbf{h}}_{n,k}^H \mathbf{c}|^2.$$

A. *Review of the Feedback Bit Allocation Method for DAS*

Now, we review the FA-SMB scheme presented in work [1]. This algorithm determines feedback bit allocation which minimizes an upper bound of a mean rate loss for the downlink DAS with limited feedback. In [1], it is assummed that the j-th DA port supports user k. Then, the mean rate loss for the k-th user is upper-bounded by [1]

$$\begin{aligned} \Delta R_k &< -\log_2 \frac{1 - 2^{-\frac{B_{j,k}}{M-1}}}{1 + \frac{|\mathcal{U}_j|-1}{|\mathcal{U}_j|} P d_{j,k}^{-\alpha} M 2^{-\frac{B_{j,k}}{M-1}} + \sum_{n\in\mathcal{D}, n\neq j} P d_{n,k}^{-\alpha} M 2^{-\frac{B_{n,k}}{M-1}}} \\ &\triangleq -\Omega_k. \end{aligned}$$

Thus, the bit allocation problem for the k-th user with $n \in \mathcal{D}$ is formulated as

$$\begin{aligned} B_{n,k}^{\text{real}} &= \arg \max_{\{B_{n,k}\}_{n\in\mathcal{D}}} \Omega_k, \\ s.t. &\sum_{n\in\mathcal{D}} \left\lfloor B_{n,k}^{\text{real}} \right\rceil = B_k^t \end{aligned}$$

where B_k^t represents the total number of feedback bits for the k-th user, $B_{n,k}^{\text{real}} \in [0, B_k^t]$ is a real number and $\lfloor\cdot\rceil$ indicates the round operation.

Then, for the k-th user, the feedback bits which maximize Ω_k are expressed by

$$B_{l,k}^{\text{real}} = \left((M-1) \log_2 \frac{-C_2 + \sqrt{C_2^2 + 2^{\frac{B_k^t - a_{j,l}}{M-1}} (C_1 + C_3) C_2}}{C_1 + C_3} \right)^+ \quad \text{for} \quad l \in \mathcal{D} \setminus \{j\} \quad (1)$$

$$B_{l,k}^{*} = \left\lfloor B_{l,k}^{\text{real}} \right\rceil, \quad B_{j,k}^{*} = B_k^t - \sum_{l\in\mathcal{D}\setminus\{j\}} B_{l,k}^{*}, \quad (2)$$

where $(A)^+$ denotes $\max(0, A)$. Here, C_1, C_2 and C_3 are defined as

$$\begin{aligned} C_1 &= 1 + \sum_{n \in \mathcal{D}, n \neq j, l} P d_{n,k}^{-\alpha} 2^{-\frac{B_{n,k}}{M-1}} M, \\ C_2 &= P d_{l,k}^{-\alpha} M, \\ C_3 &= \left(|\mathcal{U}_j| - 1\right) P_j d_{j,k}^{-\alpha} M, \end{aligned}$$

respectively.

In the case of $|\mathcal{D}| \leq 2$, the feedback bits can be directly determined as a closed-form solution by (1) and (2). However, when $|\mathcal{D}| \geq 3$, to compute the number of feedback bits for all links, an iterative procedure is required since $B_{l,k}$ calculated from (1) affects the solution for other channel links. Please refer to [1] for further details of the FA-SMB scheme.

III. Scaling Law of Feedback Bits

In this section, we compare the scaling law of feedback bits for the FA-SMB and the equal bit allocation scheme for DAS with limited feedback. Then, from the derived scaling laws, we verify that the number of required feedback bits for the FA-SMB is within a constant rate gap compared to the perfect CSI case. We provide the scaling laws for DAS in the following theorem.

Theorem 1: To satisfy the maximum allowable rate gap of $\log_2 b$ bps/Hz per user, the total number of feedback bits B_k^t for the FA-SMB and equal bit allocation scheme are expressed respectively as (3) and (4).

Proof: We first prove the scaling law of the FA-SMB scheme. In order to ensure the maximum allowable rate gap of $\log_2 b$ bps/Hz per user, $-\Omega_k$ should be upper-bounded by

$$\log_2 \frac{1 - 2^{-\frac{B_{j,k}}{M-1}}}{1 + \frac{|\mathcal{U}_j| - 1}{|\mathcal{U}_j|} P d_{j,k}^{-\alpha} M 2^{-\frac{B_{j,k}}{M-1}} + \sum_{\substack{n \in \mathcal{D} \\ n \neq j}} P d_{n,k}^{-\alpha} M 2^{-\frac{B_{n,k}}{M-1}}} \leq \log_2 b. \quad (5)$$

After some manipulations, $B_{j,k}$ is expressed as

$$\begin{aligned} B_{j,k} &\geq (M-1) \log_2 \frac{PM \frac{|\mathcal{U}_j| - 1}{|\mathcal{U}_j|} d_{j,k}^{-\alpha} + b}{b - 1 - PM \sum_{\substack{n \in \mathcal{D} \\ n \neq j}} d_{n,k}^{-\alpha} 2^{-\frac{B_{n,k}}{M-1}}} \\ &\geq (M-1) \log_2 \frac{PM \frac{|\mathcal{U}_j| - 1}{|\mathcal{U}_j|} d_{j,k}^{-\alpha} + b}{b - 1}. \quad (6) \end{aligned}$$

Here, the result in (6) is obtained from $PM \sum_{n \in \mathcal{D}, n \neq j} d_{n,k}^{-\alpha} 2^{-\frac{B_{n,k}}{M-1}} \geq 0$. Note that when the number of feedback bits is sufficiently large, this bound becomes accurate since we have $PM \sum_{n \in \mathcal{D}, n \neq j} d_{n,k}^{-\alpha} 2^{-\frac{B_{n,k}}{M-1}} \ll 1$ with a large $B_{n,k}$ for $n \in \mathcal{D}, n \neq j$.

Similarly, we have

$$C_1 = 1 + \sum_{n \in \mathcal{D}, n \neq j, l} PM d_{n,k}^{-\alpha} 2^{-\frac{B_{n,k}}{M-1}} \simeq 1.$$

Removing the operation $(\cdot)^+$ in (1) for a large number of feedback bits, $B_{l,k}$ for $l \in \mathcal{D}, l \neq j$ is given by

$$\begin{aligned} B_{l,k} &\simeq (M-1) \log_2 \frac{PM d_{l,k}^{-\alpha}}{1 + PM \frac{|\mathcal{U}_j| - 1}{|\mathcal{U}_j|} d_{j,k}^{-\alpha}} \\ &\quad + (M-1) \log_2 \left(2^{\frac{B_{j,k}}{M-1}} - 2 \right) \\ &\simeq (M-1) \log_2 \frac{PM d_{l,k}^{-\alpha}}{1 + PM \frac{|\mathcal{U}_j| - 1}{|\mathcal{U}_j|} d_{j,k}^{-\alpha}} + B_{j,k}, \quad (7) \end{aligned}$$

where (7) comes from the fact that $2^{\frac{B_{j,k}}{M-1}} - 2 \simeq 2^{\frac{B_{j,k}}{M-1}}$ for a large number of feedback bits. Combining (6) and (7), we can determine the required total feedback bits for the FA-SMB as $B_k^t = B_{j,k} + \sum_{n \in \mathcal{D}, n \neq j} B_{n,k} \geq B_{\text{FA-SMB}}^{t,\text{scaled}}$. For the equal bit allocation case where feedback bits are evenly allocated for each channel link, it is easy to show that the scaling law is obtained as $B_{\text{equal}}^{t,\text{scaled}}$ in (4) by plugging $B_{n,k} = B_k^t / |\mathcal{D}|$ for $n \in \mathcal{D}$ into (5). This completes the proof. ■

Theorem 1 implies that for both the FA-SMB and the equal bit allocation scheme, we should increase the total number of feedback bits B_k^t as SNR P grows to keep the constant rate gap. However, it can be shown that the equal bit allocation scheme requires more bits than the FA-SMB for retaining the same rate gap. We will separately examine the case of $|\mathcal{U}_j| = 1$ and $|\mathcal{U}_j| > 1$. We first consider the case of $|\mathcal{U}_j| = 1$. Let us denote $P_{\text{dB}} = 10 \log_{10} P$ as the SNR in dB scale. Then, in high SNR regime, we can easily check that the scaling laws for the FA-SMB and equal bit allocation scheme are given by $B_{\text{FA-SMB}}^{t,\text{scaled}} \geq (|\mathcal{D}| - 1) \frac{M-1}{3} P_{\text{dB}}$ and $B_{\text{equal}}^{t,\text{scaled}} \geq |\mathcal{D}| \frac{M-1}{3} P_{\text{dB}}$, respectively. Therefore, when $|\mathcal{U}_j| = 1$, the equal bit allocation method requires additional $\frac{M-1}{3} P_{\text{dB}}$ bits compared to the FA-SMB, and it is emphasized that this additional bit increases as SNR grows.

In contrast, for the case of $|\mathcal{U}_j| > 1$, both schemes have the same scaling law $|\mathcal{D}| \frac{M-1}{3} P_{\text{dB}}$. However, it can be verified that the FA-SMB can save feedback bits compared to the equal bit allocation scheme. Defining $g_{j,k} = (|\mathcal{U}_j| - 1)/|\mathcal{U}_j| d_{j,k}^{-\alpha}$ and $g_{n,k} = d_{n,k}^{-\alpha}$ for $n \in \mathcal{D}, n \neq j$, we have

$$\begin{aligned} B_{\text{equal}}^{t,\text{scaled}} &\geq |\mathcal{D}| \frac{M-1}{3} P_{\text{dB}} + |\mathcal{D}| (M-1) \log_2 \sum_{n \in \mathcal{D}} g_{n,k} \\ &\quad - |\mathcal{D}| (M-1) \log_2 (b-1) \\ &\geq |\mathcal{D}| \frac{M-1}{3} P_{\text{dB}} + |\mathcal{D}| (M-1) \log_2 |\mathcal{D}| \prod_{n \in \mathcal{D}} g_{n,k}^{1/|\mathcal{D}|} \\ &\quad - |\mathcal{D}| (M-1) \log_2 (b-1) \quad (8) \\ &\simeq B_{\text{FA-SMB}}^{t,\text{scaled}} + |\mathcal{D}| (M-1) \log_2 |\mathcal{D}|, \quad (9) \end{aligned}$$

where (8) comes from the relation between the arithmetic and geometric means and (9) results from a high SNR approximation. Hence, we confirm that the FA-SMB is more efficient than the equal bit allocation case in terms of the required feedback bits.

$$B_k^t \geq |\mathcal{D}|(M-1)\log_2\left(PM\frac{|\mathcal{U}_j|-1}{|\mathcal{U}_j|}d_{j,k}^{-\alpha}+b\right)+\sum_{\substack{n\in\mathcal{D}\\ n\neq j}}(M-1)\log_2 PMd_{n,k}^{-\alpha}$$
$$-(|\mathcal{D}|-1)(M-1)\log_2\left(PM\frac{|\mathcal{U}_j|-1}{|\mathcal{U}_j|}d_{j,k}^{-\alpha}+1\right)-|\mathcal{D}|(M-1)\log_2(b-1)\triangleq B_{\text{FA-SMB}}^{t,\text{scaled}} \quad (3)$$

$$B_k^t \geq |\mathcal{D}|(M-1)\log_2\left(PM\left(\frac{|\mathcal{U}_j|-1}{|\mathcal{U}_j|}d_{j,k}^{-\alpha}+\sum_{\substack{n\in\mathcal{D}\\ n\neq j}}d_{n,k}^{-\alpha}\right)+b\right)-|\mathcal{D}|\,(M-1)\log_2(b-1)\triangleq B_{\text{equal}}^{t,\text{scaled}} \quad (4)$$

IV. Computational Complexity

In this section, we present the computational complexity of the FA-SMB scheme compared with the equal bit allocation and exhaustive search which determines the optimal bit allocation in terms of the sum rate by searching for all possible bit allocation cases. For simple analysis, we assume that $d_{n,k}^{-\alpha}$ is given for all n and k, and neglect the multiplication operations when computing $\log_2 a$ and 2^a. First, we analyze the computational complexity of the FA-SMB algorithm. For the k-th user, we need $\mathcal{O}(|\mathcal{D}|^2)$ multiplications in each iteration, since we calculate $B_{l,k}^{\text{real}}$ for $l\in\mathcal{D}\setminus\{j\}$ and $\mathcal{O}(|\mathcal{D}|)$ multiplications are required to compute an arbitrary $B_{l,k}^{\text{real}}$. Defining β as the required number of iterations for each user to obtain a converged solution, the FA-SMB scheme performs $\mathcal{O}(\beta K|\mathcal{D}|^2)$ multiplications to determine the number of feedback bits for all users. Simulations show that the FA-SMB scheme converges within 4 iterations in most cases, and thus β is negligible with large K and $|\mathcal{D}|$.

On the other hand, the equal bit allocation scheme can compute the feedback bits with only $\mathcal{O}(K)$ multiplications. However, this scheme does not guarantee reliable performance. Note that the FA-SMB scheme has a closed-form solution for the case of $|\mathcal{D}|\leq 2$, and thus the computational complexity of the FA-SMB scheme is similar to that of the equal bit allocation in this case. In the simulation section, we will show that the FA-SMB scheme provides a large performance gain compared to the equal bit allocation scheme with similar complexity.

Now, we investigate the computational complexity of the exhaustive search. The required multiplications for determining each user's beamforming vector and the individual user rate are $\mathcal{O}(M^3)+\mathcal{O}(KM^2)$ and $\mathcal{O}(KM)$, respectively, since a matrix inversion and eigenvalue decomposition of an $a\times a$ matrix need $\mathcal{O}(a^3)$ computations and a matrix multiplication between an $a\times b$ matrix and a $b\times c$ matrix requires $\mathcal{O}(abc)$ calculations [13]. Also, the number of all possible cases of feedback bit allocation for each user equals $\gamma=\binom{|\mathcal{D}|+B^t}{B^t}$ by assuming $B_k^t=B^t$ for all k. As a result, the exhaustive search needs $\gamma^K(\mathcal{O}(KM^3)+\mathcal{O}(K^2M^2))$ multiplications, as we should compare the sum rate for all possible cases and choose the largest one. Thus, the computational complexity of this case may become prohibitive with large B^t and K. We will confirm in the following simulation section that the FA-SMB scheme exhibits the performance very close to the exhaustive search with substantially reduced complexity.

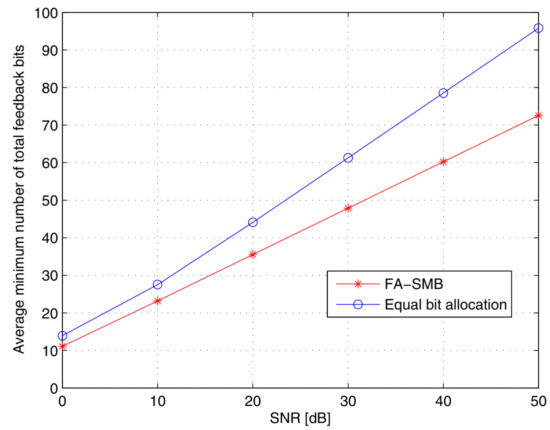

Fig. 2. Average minimum number of total feedback bits for $N=7$, $M=3$ and $K=3$

V. Simulation Results

In this section, we present simulation results to demonstrate the efficacy of the FA-SMB scheme. In the simulation, the cell radius and the path loss exponent are set to $R=1$ and $\alpha=3.75$, respectively, and SNR is defined as P. In addition, we assume that all users are uniformly distributed within a cell in each channel realization, and the locations of DA ports are determined by the algorithm in [7]. Figure 2 illustrates the average minimum number of total feedback bits which guarantees a rate loss to be no larger than the maximum allowable rate gap compared to the full CSI case for both the FA-SMB scheme and the equal bit allocation with $N=7$, $M=3$ and $K=3$. In this figure, the maximum allowable rate gap per user is set to be $\log_2 10$, i.e., $b=10$. As expected from the analysis in Section III, we can see that the required number of feedback bits linearly increases with SNR for both schemes, however, the slope of the equal bit allocation is steeper than that of the FA-SMB scheme. It is because that the required feedback bits of the equal bit allocation is always affected by $|\mathcal{D}|$, while B^t in the FA-SMB is determined by $|\mathcal{D}|-1$ and $|\mathcal{D}|$ for the case of $|\mathcal{U}_j|=1$ and $|\mathcal{U}_j|>1$, respectively, as we discussed earlier. We can check that the FA-SMB scheme saves the feedback bits up to 30% compared to the equal bit

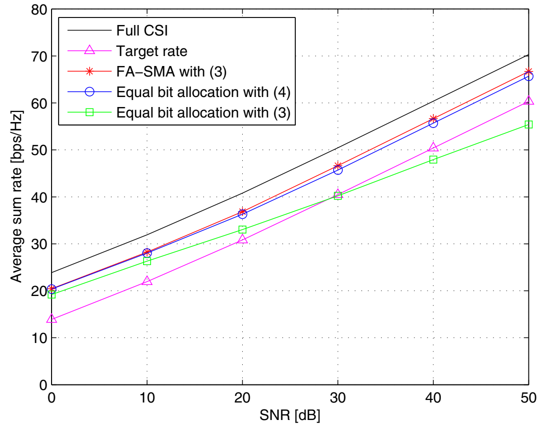

Fig. 3. Average sum rate with a scaled B^t for $N = 7$, $M = 3$ and $K = 3$

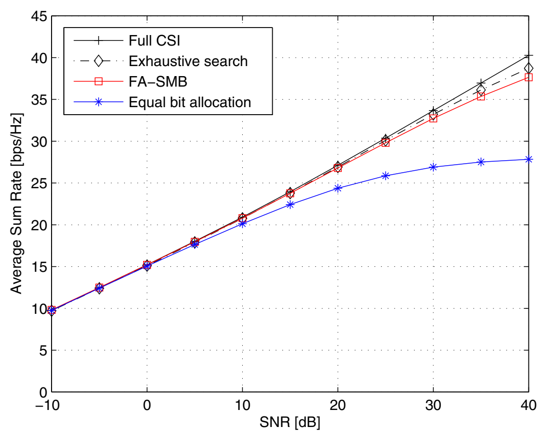

Fig. 4. Average sum rate with $B^t = 16$ for $N = 7$, $M = 2$ and $K = 2$

allocation at SNR $= 40$ dB.

In Figure 3, we plot the average sum rate curves as a function of SNR with a scaled B^t for $N = 7$, $M = 3$ and $K = 3$. In this figure, the target rate means the average sum rate which has a rate loss of $\log_2 10$ bps/Hz per user compared to the full CSI case. We can see that the average sum rates for the FA-SMB scheme with (3) and the equal bit allocation with (4) satisfy the maximum allowable rate gap, and the sum rate performance of two bit allocation schemes are similar. However, as shown in Figure 2, the equal bit allocation requires much more bits than the FA-SMB as shown in Figure 2. It is emphasized that the target rate cannot be achieved when the equal bit allocation scheme is applied with (3).

Figure 4 presents the average sum rate curves as a function of SNR with $B^t = 16$ for $N = 7$, $M = 2$ and $K = 2$. This figure shows that the performance of the FA-SMB scheme is quite close to cases with the full CSI and exhaustive search. Note that the exhaustive search needs much higher computational complexity than the FA-SMB scheme as discussed in Section IV. Also, the FA-SMB algorithm exhibits an average sum rate gain of 35% compared to the equal bit allocation scheme at SNR $= 40$ dB. It should be emphasized that the FA-SMB scheme generates a closed-form solution in this system configuration, and thus the computational complexity of the FA-SMB scheme is similar to that of the equal bit allocation scheme. Thus, we can conclude that the FA-SMB scheme efficiently allocates the feedback bits and guarantees good performance.

VI. Conclusion

In this paper, we have analyzed the performance of the FA-SMB [1] by presenting the scaling law of feedback bits and the computational complexity analysis. From the scaling law, we have verified that in order to achieve the same rate gap performance, the equal bit allocation requires more feedback bits than the FA-SMB. In addition, by analyzing the computational complexity, it has been shown that the FA-SMB reduces the number of multiplication compared to the numerical exhaustive search method with a negligible performance loss. Simulation results shows that the FA-SMB can save feedback bits by up to 30% over the equal bit allocation scheme in the three-user DAS.

References

[1] E. Park, H. Kim, H. Park, and I. Lee, "Feedback Bit Allocation Schemes for Multi-User Distributed Antenna Systems," *IEEE Communications Letters*, vol. 17, pp. 99–102, January 2013.

[2] A. A. M. Saleh, A. J. Rustako, and R. Roman, "Distributed Antennas for Indoor Radio Communications," *IEEE Transactions on Communications*, vol. 35, pp. 1245–1251, December 1987.

[3] S. Zhou, M. Zhao, X. Xu, J. Wang, and Y. Yao, "Distributed Wireless Communication System: A New Architecture for Future Public Wireless Access," *IEEE Communication Magazine*, pp. 108–113, April 2003.

[4] W. Choi and J. G. Andrews, "Downlink Performance and Capacity of Distributed Antenna Systems in a Multicell Environment," *IEEE Transactions on Wireless Communications*, vol. 6, pp. 69–73, January 2007.

[5] S.-R. Lee, S.-H. Moon, J.-S. Kim, and I. Lee, "Capacity Analysis of Distributed Antenna Systems in a Composite Fading Channel," *IEEE Transactions on Wireless Communications*, vol. 11, no.3, pp. 1076–1086, March 2012.

[6] H. Kim, S.-R. Lee, K.-J. Lee, and I. Lee, "Transmission Schemes based on Sum Rate Analysis in Distributed Antenna Systems," *IEEE Transactions on Wireless Communications*, vol. 11, no.3, pp. 1201–1209, March 2012.

[7] E. Park, S.-R. Lee, and I. Lee, "Antenna Placement Optimization for Distributed Antenna Systems," *IEEE Transactions on Wireless Communications*, vol. 11, pp. 2468–2477, July 2012.

[8] M. Sadek, A. Tarighat, and A. H. Sayed, "A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels," *IEEE Transactions on Wireless Communications*, vol. 6, pp. 1711–1721, May 2007.

[9] B. O. Lee, H. W. Je, O.-S. Shin, and K. B. Lee, "A Novel Uplink MIMO Transmission Scheme in a Multicell Environment," *IEEE Transactions on Wireless Communication*, vol. 8, pp. 4981–4987, October 2009.

[10] N. Jindal, "MIMO Broadcast Channels with Finite-Rate Feedback," *IEEE Transactions on Information Theory*, vol. 52, pp. 5045–5060, November 2006.

[11] W. Santipach and M. Honig, "Asymptotic capacity of beamforming with limited feedback," in *Proc. IEEE ISIT*, p. 290, June 2004.

[12] W. Santipach and M. Honig, "Signature Optimization for CDMA with Limited Feedback," *IEEE Transactions on Information Theory*, vol. 51, pp. 3475–3492, October 2005.

[13] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Third Edition, The Johns Hopkins University Press, Baltimore and London, 1996.