Bit Allocation and Pairing Methods for Distributed Antenna Systems with Limited Feedback

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***Abstract*—In this paper, we study bit allocation and pairing methods based on zero forcing beamforming for downlink multi-user distributed antenna systems with limited feedback. Before assigning the feedback bit for each distributed antenna (DA) port, we need to solve the pairing issue which determines the set of DA ports to support a user. To this end, we first analyze an upper bound of a mean rate loss between perfect channel state information systems and limited feedback systems. Since minimizing the obtained bound is a joint optimization problem with respect to the pairing and the bit allocation, it is difficult to identify a solution analytically. Instead, we propose a two-step algorithm which derives the pairing based on the bound of the rate loss, and then obtain the feedback bit allocation method independently. From simulation results, we confirm that the proposed algorithms offer about 135% performance gains over a conventional equal bit allocation scheme for 5 DA ports systems.**

I. Introduction

In recent years, distributed antenna systems (DAS) where multiple antenna units are separately located in a cell have received attentions owing to its potential for the extended cell coverage and the increased system capacity. Unlike conventional centralized antenna systems (CAS) where all antennas are co-located at the cell center, separated distributed antenna (DA) ports in the DAS reduce the access distance as well as the transmit power and co-channel interference, and thus can achieve improved cell-edge performance [1]. Several researches in DAS have been dedicated for analyzing the performance [2], [3] and determining the location of DA ports [4]. These studies were based on the assumption of perfect channel state information (CSI) at the transmitter.

In practical wireless communication systems, each receiver shares a codebook with a transmitter and sends back the index of a codeword to inform the CSI to the transmitter. For single-cell CAS, the work in [5] addressed a codebook design problem, and the performance of random vector quantization (RVQ) was analyzed in [6] where elements of the codebook are randomly generated.

In multi-cell CAS with limited feedback, systematic feedback bit allocation methods were proposed in [7], [8]. For DAS, [9] presented a feedback bit allocation problem where each user is paired only with the nearest DA port. Since any information exchange among transmitters is not allowed in [7]–[9], each user can only be supported by its own transmitter. For *centralized* joint processing (JP) systems where all transmitters share both the CSI and data through backhaul links, [10] and [11] considered per-cell based limited feedback systems and proposed bit allocation schemes to enhance the channel quantization quality. To reduce the backhaul signaling and overcome implementation issues, *decentralized* JP systems where cooperating transmitters share only users' data and do not require global CSI was considered in several papers [12], [13]. Although this decentralized JP systems may be viewed as a special case of the centralized JP systems, feedback bit allocation methods obtained in the centralized systems [10] [11] cannot be directly applied.

In this paper, we propose feedback bit allocation and DA port-user pairing techniques for multi-user multiple-input single-output (MISO) DAS with decentralized JP environments. In this configuration, each DA port employs zero forcing beamforming (ZFBF) in a distributed manner under per DA port power constraint. Unlike the multi-cell CAS where pairings between transmitters and users are mostly fixed, pairings in DAS can be adaptively selected to improve the overall system capacity.

To solve the pairing issues and the bit allocation, we first analyze an upper bound of a mean rate loss between full CSI and limited feedback systems. Since minimizing the obtained bound of the mean rate loss is a joint optimization problem with respect to the pairing and the bit allocation, it is difficult to identify a solution analytically. Instead, we provide an alternative approach, which derives a solution for the given pairing, and then obtains the feedback bit allocation method subsequently. In this way, the joint problem of pairing and bit allocation is fully decoupled into two sequential problems which determine the pairing and the bit allocation separately. Simulation results demonstrate that the proposed algorithms offer about a 135% performance gain over a conventional scheme for 5 DA ports systems.

Throughout this paper, bold lower case letters denote column vectors, and the superscript $(\cdot)^H$ represents the Hermitian operation. Also, $\mathbb{E}[\cdot]$ stands for the expectation and the set of all complex matrices of size m-by-n is defined by $\mathbb{C}^{m\times n}$.

II. System Model

In this section, we describe a system model for the multi-user MISO DAS with finite-rate feedback. There are N DA ports equipped with M antennas and K users with a single antenna. It is assumed that DA ports share users' data but do not require global CSI, and each DA port supports its own set of users with ZFBF in a distributed manner. Let us define $\mathcal{U}_n \subset \{1,2,\cdots,K\}$ as a set of users supported by the n-th DA port, $\mathcal{D}_k \subset \{1,2,\cdots,N\}$ as a set of DA ports which serves the k-th user, and $\mathcal{D} = \bigcup_{j=1}^{K} \mathcal{D}_j$ as a set of active DA ports. Throughout this paper, we assume $|\mathcal{U}_n| \leq M$, $\forall n \in \mathcal{D}$

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and the composite channel model is employed which involves small scale fadings as well as path loss.

Then, the received signal at the k-th user is given by

$$y_k = \sum_{n\in\mathcal{D}} \sum_{j\in\mathcal{U}_n} \sqrt{P_{n,j}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \mathbf{x}_{n,j} + n_k \quad (1)$$

where $P_{n,j}$ represents the transmit power from the n-th DA port to the j-th user with per DA port power constraint P, i.e., $\sum_{j\in\mathcal{U}_n} P_{n,j} \le P$, $d_{n,k}$ indicates the distance between the n-th DA port and the k-th user, α denotes the path loss exponent, $\mathbf{h}_{n,k} \in \mathbb{C}^{M\times 1}$ stands for the channel vector from the n-th DA port to the k-th user, $\mathbf{x}_{n,j} \in \mathbb{C}^{M\times 1}$ equals the transmit signal vector from the n-th DA port to the j-th user with $\mathbb{E}[\|\mathbf{x}_{n,j}\|^2] = 1$, and n_k is the additive complex Gaussian noise with zero mean and unit variance. We assume that all elements in the channel vector are independent and identically distributed complex Gaussian random variables with zero mean and unit variance.

In this paper, we focus on the quantization procedure of channel direction information (CDI), which is defined as $\tilde{\mathbf{h}}_{n,k} = \mathbf{h}_{n,k}/\|\mathbf{h}_{n,k}\|$, and it is assumed that the n-th DA port perfectly knows channel quality information (CQI) of user $j \in \mathcal{U}_n$, i.e., $\|\mathbf{h}_{n,j}\|$, $\forall j \in \mathcal{U}_n$. In limited feedback systems, each user sends back indices of the quantized CDI to DA ports based on a codebook. Let us define the codebook for the n-th DA port and user $k \in \mathcal{U}_n$ as $\mathcal{W}_{n,k} = \{\mathbf{c}_{n,k,1}, \mathbf{c}_{n,k,2}, \cdots, \mathbf{c}_{n,k,2^{B_{n,k}}}\}$ where $\mathbf{c}_{n,k,i} \in \mathbb{C}^{M\times 1}$ denotes the i-th codeword of $\mathcal{W}_{n,k}$ with unit norm and $B_{n,k}$ indicates the allocated bits between the n-th DA port and the k-th user. Then, the quantized CDI from the n-th DA port to the k-th user is determined as $\hat{\mathbf{h}}_{n,k} = \arg\max_{\mathbf{c}\in\mathcal{W}_{n,k}} |\tilde{\mathbf{h}}_{n,k}^H \mathbf{c}|^2$. We assume that RVQ [6] is applied for the construction of the codebook and the total number of feedback bits is fixed as $\sum_{n\in\mathcal{D}_k} B_{n,k} = B^t$.

The transmit signal vector $\mathbf{x}_{n,k}$ from the n-th DA port to the k-th user is obtained as $\mathbf{x}_{n,k} = \hat{\mathbf{w}}_{n,k} s_k$ where $\hat{\mathbf{w}}_{n,k}$ equals the ZFBF vector from the n-th DA port to the k-th user which is chosen to satisfy $\hat{\mathbf{h}}_{n,j}^H \hat{\mathbf{w}}_{n,k} = 0$ with $\|\hat{\mathbf{w}}_{n,k}\| = 1$, $\forall j \in \mathcal{U}_n/\{k\}$ and s_k represents the scalar data symbol for the k-th user with $\mathbb{E}[|s_k|^2] = 1$. Then, (1) can be expressed as

$$\begin{aligned} y_k = & \sum_{n\in\mathcal{D}_k} \sqrt{P_{n,k}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,k} s_k \\ & + \sum_{j\ne k} \sum_{n\in\mathcal{D}_k \bigcap \mathcal{D}_j} \sqrt{P_{n,j}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,j} s_j \\ & + \sum_{l} \sum_{n\in\mathcal{D}_k^c \bigcap \mathcal{D}_l} \sqrt{P_{n,l}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,l} s_l + n_k \quad (2) \end{aligned}$$

where $\mathcal{D}_k^c$ is the complementary set of $\mathcal{D}_k$. Note that the first term in (2) accounts for the desired signal, while the second term indicates residual inter-user interference induced by the channel quantization error which becomes zero in full CSI systems. Also, the third term reflects the interference from a set of DA ports which does not serve the k-th user.

Now, we study a mean rate loss between full CSI and limited feedback systems for the DAS. First, the signal-to-noise-plus-interference ratio (SINR) with full CSI systems at the k-th user is expressed as

$$\begin{aligned} \mathrm{SINR}_k^{\mathrm{full}} &= \frac{\left|\sum_{n\in\mathcal{D}_k^{\mathrm{full}}} \sqrt{P_{n,k}^{\mathrm{full}}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \mathbf{w}_{n,k}\right|^2}{1+\sum_l \left|\sum_{n\in\left(\mathcal{D}_k^{\mathrm{full}}\right)^c \bigcap \mathcal{D}_l^{\mathrm{full}}} \sqrt{P_{n,l}^{\mathrm{full}}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \mathbf{w}_{n,l}\right|^2} \\ &\triangleq \frac{P_k^{\mathrm{full}}}{1+I_k^{\mathrm{full}}}, \end{aligned}$$

where $\mathcal{D}_k^{\mathrm{full}}$, $P_{n,k}^{\mathrm{full}}$ and $\mathbf{w}_{n,k} \in \mathbb{C}^{M\times 1}$ indicate a set of DA ports which support the k-th user, the transmit power from the n-th DA port to the k-th user and the beamforming vector from the n-th DA port to the k-th user with full CSI, respectively. Here, P_k^{full} and I_k^{full} represent the power of the desired signal and the interference with full CSI systems, respectively. Then, the average rate with full CSI systems at the k-th user is written by $R_k^{\mathrm{full}} = \mathbb{E}\left[\log_2\left(1+\mathrm{SINR}_k^{\mathrm{full}}\right)\right]$.

Also, the SINR with limited feedback for the k-th user is given by

$$\begin{aligned} \mathrm{SINR}_k^{\mathrm{LFB}} &= \frac{\left|\sum_{n\in\mathcal{D}_k} \sqrt{P_{n,k}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,k}\right|^2}{1+\sum_{j\ne k} \left|\sum_{n\in\mathcal{D}_j} \sqrt{P_{n,j}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,j}\right|^2} \\ &\triangleq \frac{P_k^{\mathrm{LFB}}}{1+I_k^{\mathrm{LFB}}}, \end{aligned}$$

where P_k^{LFB} and I_k^{LFB} indicate the power of the desired signal and the interference with limited feedback systems, respectively. The average rate at the k-th user for limited feedback systems can be obtained as $R_k^{\mathrm{LFB}} = \mathbb{E}\left[\log_2\left(1+\mathrm{SINR}_k^{\mathrm{LFB}}\right)\right]$.

Denoting $\Delta R_k \triangleq R_k^{\mathrm{full}} - R_k^{\mathrm{LFB}}$ as a mean rate loss for the k-th user, the average sum rate performance of limited feedback systems can be improved by minimizing $\sum_{j=1}^K \Delta R_j$ with respect to $\mathcal{D}_k$ and $B_{n,k}$ for all k and n. However, this optimization problem is highly complicated and requires global CSI. As an alternative way, we derive an upper bound of ΔR_k as

$$\begin{aligned} \Delta R_k &= \mathbb{E}\left[\log_2\left(1+\mathrm{SINR}_k^{\mathrm{full}}\right)\right] - \mathbb{E}\left[\log_2\left(1+\mathrm{SINR}_k^{\mathrm{LFB}}\right)\right] \\ &\le \mathbb{E}\left[\log_2\left(1+P_k^{\mathrm{full}}\right)\right] - \mathbb{E}\left[\log_2\left(1+P_k^{\mathrm{LFB}}\right)\right] \\ &\quad + \mathbb{E}\left[\log_2\left(1+I_k^{\mathrm{LFB}}\right)\right] \end{aligned}$$

where the inequality comes from the fact that the logarithm function is non-decreasing and the quantities I_k^{full} and I_k^{LFB} are non-negative.

Since $\tilde{\mathbf{h}}_{m,j}$ and $\hat{\mathbf{h}}_{m,j}$ are identically distributed and CQI is perfectly known at DA ports, we have $\mathbb{E}\left[\log_2\left(1+P_k^{\mathrm{full}}\right)\right] = \mathbb{E}\left[\log_2\left(1+P_k^{\mathrm{LFB}}\right)\right]$ [6] [10]. Therefore, an upper bound of the mean rate loss can be further expressed as

$$\begin{aligned} &\Delta R_k \le \mathbb{E}[\log_2(1+I_k^{\mathrm{LFB}})] \\ &\le \log_2\left(1+\mathbb{E}\left[\sum_{j\ne k}\left|\sum_{n\in\mathcal{D}_j} \sqrt{P_{n,j}} d_{n,k}^{-\alpha/2} \mathbf{h}_{n,k}^H \hat{\mathbf{w}}_{n,j}\right|^2\right]\right) \quad (3) \end{aligned}$$

where (3) is given by the Jensen's inequality.

III. Proposed Pairing and Feedback Bit Allocation Methods

In this section, we present feedback bit allocation and DA port-user pairing algorithms which minimize the upper bound of the mean rate loss derived in the previous section. In general, paring and bit alloation are dependent on each other as shown in (3), and thus it is hard to obtain an analytic solution. Instead, we first propose a non-iterative bit allocation algorithm for given pairing. Then, based on the proposed bit allocation solution, it is shown that the pairing problem can be solved independent of the bit allocation.

A. Feedback Bit Allocation Scheme

In order to generate a solution for the minimization problem (3), we further derive a bound of (3) as a function of $\mathcal{D}_k$ and $B_{n,k}$ for $n \in \mathcal{D}_k$. The following theorem provides a bound of ΔR_k.

Theorem 1: Employing the distributed ZFBF to the multi-user MISO DAS, a mean rate loss ΔR_k is bounded by (4).

Proof: Applying the Cauchy-Schwarz inequality to (3), an upper bound of ΔR_k is expressed as

$$\begin{aligned}\Delta R_k &\leq \log_2\left(1+\mathbb{E}\left[\sum_{j\neq k}\left|\sum_{n\in\mathcal{D}_j}\sqrt{P_{n,j}}d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}\right|^2\right]\right)\\ &= \log_2\left(1+\mathbb{E}\left[\sum_{j\neq k}\left|\sum_{n\in\mathcal{D}_k\bigcap\mathcal{D}_j}\sqrt{P_{n,j}}d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}\right.\right.\right.\\ &\quad\left.\left.\left.+\sum_{n\in\mathcal{D}_k^c\bigcap\mathcal{D}_j}\sqrt{P_{n,j}}d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}\right|^2\right]\right)\\ &\leq \log_2\left(1+2\mathbb{E}\left[\sum_{j\neq k}\left|\sum_{n\in\mathcal{D}_k\bigcap\mathcal{D}_j}\sqrt{P_{n,j}}d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}\right|^2\right]\right.\\ &\quad\left.+2\mathbb{E}\left[\sum_{l}\left|\sum_{n\in\mathcal{D}_k^c\bigcap\mathcal{D}_l}\sqrt{P_{n,l}}d_{n,k}^{-\alpha/2}\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,l}\right|^2\right]\right)\\ &\triangleq \log_2\left(1+2U_k^B+2U_k^P\right),\end{aligned}$$

where U_k^B represents the power of residual inter-user interference due to the channel quantization error, and U_k^P reflects the interference from DA ports which are not paired with the k-th user. Here, a bound of U_k^B is given by

$$U_k^B \leq \sum_{j\neq k}\sum_{n\in\mathcal{D}_k\bigcap\mathcal{D}_j}\left|\mathcal{D}_k\bigcap\mathcal{D}_j\right|P_{n,j}d_{n,k}^{-\alpha}\mathbb{E}\left[\left|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}\right|^2\right], \quad (5)$$

where we use the Cauchy-Schwarz inequality. Note that $\mathbb{E}[|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}|^2]$ is bounded as [6]

$$\mathbb{E}\left[|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,j}|^2\right] = M\mathbb{E}\left[|\tilde{\mathbf{h}}_{n,k}^H\hat{\mathbf{w}}_{n,j}|^2\right] < \frac{M}{M-1}2^{-\frac{B_{n,k}}{M-1}}. \quad (6)$$

Plugging (6) to (5) and exchanging the order of summations, it follows

$$\begin{aligned}U_k^B &\leq \frac{M}{M-1}\sum_{n\in\mathcal{D}_k}\sum_{j\in\mathcal{U}_n/\{k\}}\left|\mathcal{D}_k\bigcap\mathcal{D}_j\right|P_{n,j}d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}}\\ &\leq \frac{M}{M-1}|\mathcal{D}_k|\sum_{n\in\mathcal{D}_k}d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}}\sum_{j\in\mathcal{U}_n/\{k\}}P_{n,j} \quad (7)\\ &< \frac{PM}{M-1}|\mathcal{D}_k|\sum_{n\in\mathcal{D}_k}d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}}, \quad (8)\end{aligned}$$

where (7) follows from $|\mathcal{D}_k\bigcap\mathcal{D}_j| \leq |\mathcal{D}_k|$ and (8) is obtained from per DA port power constraint $\sum_{j\in\mathcal{U}_n/\{k\}}P_{n,j} < P$.

Now, we focus on U_k^P. Using a similar approach, U_k^P is bounded as

$$U_k^P \leq \sum_{l}\sum_{n\in\mathcal{D}_k^c\bigcap\mathcal{D}_l}\left|\mathcal{D}_k^c\bigcap\mathcal{D}_l\right|P_{n,l}d_{n,k}^{-\alpha}\mathbb{E}\left[\left|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,l}\right|^2\right].$$

Since the k-th user is not in the set $\mathcal{U}_n$ for $n \in \mathcal{D}_k^c\bigcap\mathcal{D}_l$, the random vector $\mathbf{h}_{n,k}$ is independent with $\hat{\mathbf{w}}_{n,l}$. Therefore, we have $\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,l} \sim \mathcal{CN}(0,1)$ and $|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,l}|^2$ is a chi-square random variable with degrees of freedom 2 with $\mathbb{E}[|\mathbf{h}_{n,k}^H\hat{\mathbf{w}}_{n,l}|^2] = 1$.

By applying this, a bound of U_k^P is given as

$$\begin{aligned}U_k^P &\leq \sum_{l}\sum_{n\in\mathcal{D}_k^c\bigcap\mathcal{D}_l}\left|\mathcal{D}_k^c\bigcap\mathcal{D}_l\right|P_{n,l}d_{n,k}^{-\alpha}\\ &\leq |\mathcal{D}_k^c|\sum_{n\in\mathcal{D}_k^c}d_{n,k}^{-\alpha}\sum_{l\in\mathcal{U}_n}P_{n,l}\\ &\leq P\left(N-|\mathcal{D}_k|\right)\sum_{n\in\mathcal{D}_k^c}d_{n,k}^{-\alpha} \quad (9)\end{aligned}$$

where the last inequality comes from the relations of $|\mathcal{D}_k^c\bigcap\mathcal{D}_l| \leq |\mathcal{D}_k^c| \leq N-|\mathcal{D}_k|$ and $\sum_{l\in\mathcal{U}_n}P_{n,l} \leq P$. Then, (4) is directly obtained from (5) and (9). ■

It is worth noting that $\mathcal{D}_k$ and $B_{n,k}$ for $n \in \mathcal{D}_k$ minimizing (4) are related to each other, and thus it is still hard to identify a solution analytically. To avoid this difficulty, we first determine the bit allocation for given pairing, i.e., $\mathcal{D}_k$ is fixed. Then, the bound of U_k^B in (5) can be optimized via bit allocation under the total number of bits constraint $\sum_{n\in\mathcal{D}_k}B_{n,k} = B^t$. Defining $\Psi_k = \sum_{n\in\mathcal{D}_k}d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}}$, the feedback bit allocation problem for the k-th user based on the bound (5) is formulated as

$$\min_{B_{n,k}\geq 0}\ \Psi_k,\ s.t.\ \sum_{n\in\mathcal{D}_k}B_{n,k} = B^t, \quad (10)$$

It is obvious that the above problem is convex with respect to $B_{n,k}$ for $n \in \mathcal{D}_k$. Then, the Lagrangian function of Ψ_k is constructed as

$$\mathcal{L} = \sum_{n\in\mathcal{D}_k}d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}} + \nu\left(\sum_{n\in\mathcal{D}_k}B_{n,k}-B^t\right), \quad (11)$$

where ν stands for the Lagrange multipliers. Using (11), the Karush-Kuhn-Tucker (KKT) conditions for problem (10) are obtained as

$$\frac{\partial\mathcal{L}}{\partial B_{n,k}} = 0,\ \text{and}\ \sum_{n\in\mathcal{D}_k}B_{n,k}-B^t = 0. \quad (12)$$

From the zero-gradient condition of (12), we have

$$\frac{\partial\mathcal{L}}{\partial B_{n,k}} = -\frac{\ln 2}{M-1}\ d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}} + \nu = 0. \quad (13)$$

Then, it is easily verified that from (13), the bit allocation solution can be written as

$$B_{n,k} = \left(\mu + (M-1)\log_2 d_{n,k}^{-\alpha}\right)^+ \quad (14)$$

where $(x)^+ = \max(x,0)$ and μ is chosen to satisfy $\sum_{n\in\mathcal{D}_k}B_{n,k} = B^t$. Thus, we can determine the number of feedback bits by applying a simple water-filling algorithm (14). Note that if B^t is not large enough, then some of DA ports in $\mathcal{D}_k$ may not get assigned any bits. Denoting $\tilde{\mathcal{D}}_k$ as

$$\Delta R_k < \log_2\left(1+2P\left(\frac{M}{M-1}|\mathcal{D}_k|\sum_{n\in\mathcal{D}_k} d_{n,k}^{-\alpha}2^{-\frac{B_{n,k}}{M-1}} + (N-|\mathcal{D}_k|)\sum_{n\in\mathcal{D}_k^c} d_{n,k}^{-\alpha}\right)\right) \quad (4)$$

a set of DA ports where non-zero feedback bits are assigned from the k-th user, we can set $\mathcal{D}_k$ as $\mathcal{D}_k = \tilde{\mathcal{D}}_k$ since DA ports in $\mathcal{D}_k - \tilde{\mathcal{D}}_k$ do not have any meaningful information for the k-th user.

B. DA port and User Pairing Scheme

In this subsection, we propose DA port-user pairing algorithm with the bit allocation computed in Section III-A. Since the bit allocation solution (14) is obtained for a given $\mathcal{D}_k$, pairing should be determined before a user calculates the number of feeback bits. To this end, we first derive an bound of ΔR_k based on the proposed bit allocation solution in (14). We start with the lemma for investigating the bound of a mean rate loss.

Lemma 1: When the proposed bit allocation solution (14) is employed, the mean rate loss is bounded as

$$\Delta R_k < \log_2\left(1+2Pf\left(\mathcal{D}_k\right)\right) \quad (15)$$

where $f(\mathcal{D}_k)$ is defined as (16), at the top of the next page.

Proof: To fulfill the total number of feedback bits constraint, μ should satisfy

$$\sum_{n\in\mathcal{D}_k} B_{n,k} = \sum_{n\in\mathcal{D}_k}\left(\mu+(M-1)\log_2 d_{n,k}^{-\alpha}\right)^+ = \sum_{n\in\tilde{\mathcal{D}}_k}\left(\mu+(M-1)\log_2 d_{n,k}^{-\alpha}\right) = B^t. \quad (17)$$

From (17), μ is calculated as $\mu = \left(B^t - (M-1)\sum_{n\in\tilde{\mathcal{D}}_k}\log_2 d_{n,k}^{-\alpha}\right)/|\tilde{\mathcal{D}}_k|$. By setting $\mathcal{D}_k$ as $\mathcal{D}_k = \tilde{\mathcal{D}}_k$, the number of allocated bits to the n-th DA port is given as

$$B_{n,k} = \frac{B^t}{|\mathcal{D}_k|} + (M-1)\log_2\frac{d_{n,k}^{-\alpha}}{\left(\prod_{m\in\mathcal{D}_k} d_{m,k}^{-\alpha}\right)^{1/|\mathcal{D}_k|}}.$$

Plugging this into (4) yields the result in (15). ■

Since $f(\mathcal{D}_k)$ in (16) is independent of $B_{n,k}$, $\mathcal{D}_k$ can be determined prior to allocating feedback bits. After the pairing is obtained, we apply the proposed bit allocation solution in (14). The problem for minimizing (15) with respect to $\mathcal{D}_k$ can be equivalently formulated as

$$\min_{\mathcal{D}_k} \; f(\mathcal{D}_k), \; s.t. \; \mathcal{D}_k \subset \{1,2,\cdots,N\}. \quad (18)$$

To calculate the optimal solution for (18), we need exhaustive search over all subsets of $\{1,2,\cdots,N\}$, and thus the search size equals 2^N. Hence, the search complexity of the pairing method (18) increases exponentially as N grows. However, the number of DA ports N is not large in practical systems, and thus it would be reasonable to calculate $f(\mathcal{D}_k)$ for 2^N candidates.

After $\mathcal{D}_k$ is obtained from the proposed pairing method, we apply the proposed bit allocation solution in (14). The overall proposed two-step algorithm is summarized as below.

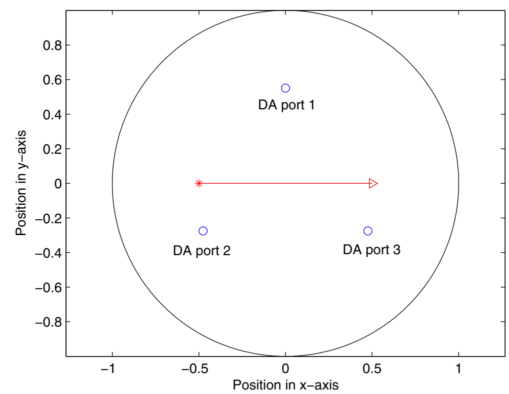

Fig. 1. The trajectory of a moving user with $N = 3$

Algorithm1. Proposed pairing and bit allocation algorithm
Obtain $\mathcal{D}_k$ from (18).
Sort $\{d_{n,k} \mid n\in\mathcal{D}_k\}$ in descending order, and compute μ.
For $n\in\mathcal{D}_k$
Calculate $B_{n,k} = \mu + (M-1)\log_2 d_{n,k}^{-\alpha}$.
If $B_{n,k}\le 0$, set $B_{n,k}=0$ and update μ.
End

In general, the number of feedback bits from the proposed algorithm is a real number. Thus, to make integer bits, we simply apply a ceiling or floor operation for all n and k.

IV. Simulation Results

In this section, simulation results are provided to demonstrate the efficacy of our proposed algorithms. Throughout simulations, the cell radius and the path loss exponent are set to $R = 1$ and $\alpha = 3.75$, respectively. Also, we assume that the locations of DA ports are determined by the results in [4] and define the signal-to-noise ratio (SNR) as P. First, we consider a user configuration as illustrated in Figure 1. Here, the user moves horizontally from the asterisk point $(-0.5, 0)$ to the triangular point $(0.5, 0)$. According to the user location depicted in Figure 1, the number of assigned feedback bits is computed and plotted as a function of the user position in Figure 2 for $M = 3$, $N = 2$ and $B^t = 15$. In the plot, we employ the proposed bit allocation algorithm with the proposed pairing method (18). It is shown that the user selects $\mathcal{D}_k = \{1, 2\}$ as the user moves from $(-0.5, 0)$ to $(-0.125, 0)$. As the user is getting closer to the point $(-0.125, 0)$, the number of bits allocated to the DA port 1 slightly increases while $B_{2,k}$ decreases. When the user moves from $(-0.125, 0)$ to $(0.125, 0)$, the pairing solution is obtained as $\mathcal{D}_k = \{1, 2, 3\}$, i.e., the user is supported by all DA ports. We can see that DA port 2 no longer serves the user when the user moves away from $(0.125, 0)$, and $B_{3,k}$ increases as the user gets closer to DA port 3.

Next, we consider a case where users are uniformly located within a cell. It is assumed that each DA port allocates the transmit power evenly to paired users, i.e., $P_{n,j} = P/|\mathcal{U}_n|$ for $j \in \mathcal{U}_n$. Figure 3 exhibits the average sum rate curves of the

$$f(\mathcal{D}_k) = \frac{M}{M-1} |\mathcal{D}_k|^2 \, 2^{-\frac{B^t}{|\mathcal{D}_k|(M-1)}} \left(\prod_{n \in \mathcal{D}_k} d_{n,k}^{-\alpha} \right)^{1/|\mathcal{D}_k|} + (N - |\mathcal{D}_k|) \sum_{n \in \mathcal{D}_k^c} d_{n,k}^{-\alpha} \tag{16}$$

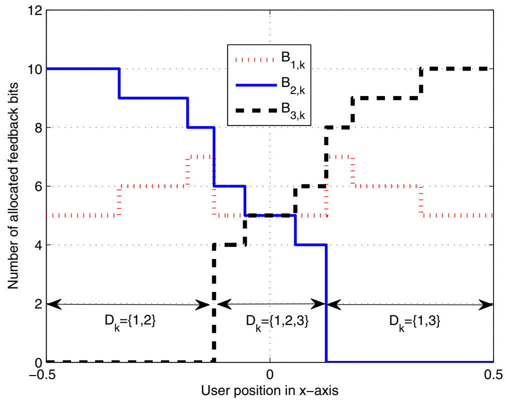

Fig. 2. The number of allocated bits with respect to the user location with $N = 3$, $M = 2$ and $B^t = 15$

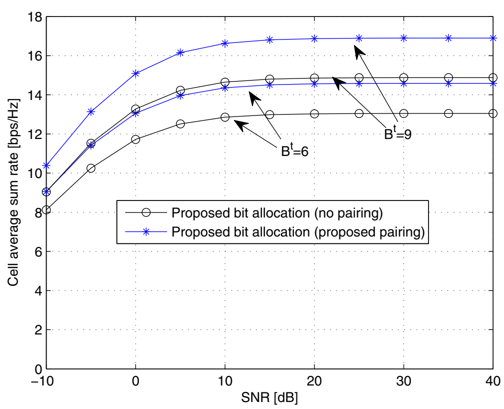

Fig. 3. Performance of the proposed non-iterative algorithm with different pairing methods for $N = 3$ and $M = K = 5$

proposed bit allocation algorithm with the proposed pairing method for $N = 3$ and $M = K = 5$. We also plot the sum rate performance of the systems with no pairing where all DA ports support all users, i.e., $\mathcal{D}_k = \{1, \cdots, N\}$, $\forall k$. From the plot, we figure out that the proposed pairing scheme provides about 13% gain at SNR = 20 dB over the no pairing case with $B^t = 9$.

Figure 4 presents the average sum rates of the proposed bit allocation algorithm and the equal bit allocation scheme with the proposed pairing method for $N = 5$ and $M = K = 3$. We can see that at SNR = 20 dB, the proposed scheme offers about 135% and 50% performance gains over the equal bit allocation for $B^t = 10$ and 15, respectively.

V. Conclusion

In this paper, we have studied bit allocation and pairing methods for multi-user MISO DAS with limited feedback. We have first introduced upper bounds of a mean rate loss between the full CSI and the limited feedback systems. To minimize the obtained bound of the mean rate loss, we have proposed a two-step algorithm which determines the pairing first and then

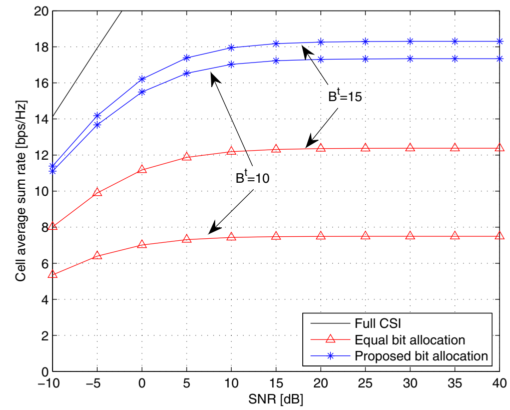

Fig. 4. The average sum rate performance comparison with $N = 5$ and $M = K = 3$

computes the number of allocated bits. The numerical results have demonstrated that the proposed algorithms offer a large performance gains over the equal bit allocation scheme.

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