A PDF-based Capacity Analysis of Diversity Reception Schemes over Composite Fading Channels using a Mixture Gamma Distribution

Jaehoon Jung, Sang-Rim Lee, Haewook Park, Hoon Lee and Inkyu Lee, *Senior Member, IEEE*
School of Electrical Eng., Korea University, Seoul, Korea
Email: {jhnjung, sangrim78, jetaime01, ihun1, inkyu}@korea.ac.kr

***Abstract*—In this paper, we analyze the ergodic capacity performance for diversity reception schemes over composite fading channels using a mixture gamma (MG) distribution. By adopting the MG distribution, various composite fading channel models can be approximated with mathematically tractable and highly accurate properties. In other words, signal-to-noise ratio statistics which contain complicated functions for each diversity reception scheme are formulated as a weighted sum of gamma distributions. With an aid of properties of the gamma distribution, we can derive closed-form expressions of an ergodic capacity for important diversity reception schemes such as maximal ratio combining and selection combining. We observe that our analysis can be expressed with the general number of receiver branches over various composite fading conditions. Simulation results verify that the derived ergodic capacity expressions match well with the empirical results.**

I. Introduction

The radio-wave propagation which includes shadowing and multipath fadings [1] has an impact on the performance of wireless communication systems. For realistic situations, the shadowing and the multipath fading appear simultaneously. These effects are modeled by composite fading channels such as Rayleigh-lognormal (RL) and Nakagami-lognormal (NL) distributions [2]. However, such fading models are not presented in a closed-form, which makes the performance analysis complicated [3].

In order to represent wireless propagation properties more tractably in comparison to the RL and NL distributions, various channel models have been proposed such as the generalized K (K_G) distribution [4] and the generalized gamma distribution [5]. Above all, many researches focused on the K_G distribution since a variety of fading conditions can be expressed by adjusting two shaping parameters with a closed-form [4]. Accordingly, the K_G distribution can be adopted as a system channel model in this paper.

Many researchers have endeavored to analyze the average symbol error probability (ASEP) of diversity reception schemes over the K_G fading channel [6]–[10]. Especially, the authors in [10] analyzed diversity and array gains by applying the mixture gamma (MG) distribution for maximal ratio combining (MRC) and selection combining (SC). Due to its mathematically tractable form, the results allow us to have meaningful insights for the system performance.

This research was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909).

From a perspective of the information theoretic analysis, capacity is an important measure of the system performance in wireless communication systems, as it provides a significant intuition on the theoretical transmission limit [11], [12]. For the L-branch diversity reception schemes over generalized fading channels, researchers in [13] and [14] analyzed the capacity with the moment generating function (MGF)-based approaches relying on complex functions such as the Meijer-G function [15] and the Fox-H function [16]. In this case, the probability density function (PDF) of the overall instantaneous signal-to-noise ratio (SNR) is in general not available in a simple form, and the ergodic capacity analysis becomes mathematically intractable, since it apparently involves the L-fold integral defined by the joint multivariate PDF of the instantaneous SNRs for the L branch case. For these reasons, the PDF-based approaches are generally difficult for the performance analysis [14].

To handle these problems, we adopt the MG distribution for the analysis. In [17], the MG distribution which is organized by a weighted sum of gamma distributions. Therefore, by adjusting its parameters, a variety of fading channels as well as the K_G distribution can be approximated with high accuracy. Note that the MGF and the cumulative distribution function (CDF) can also be given with expressions of the MG distribution, which allows simple performance analysis.

In this paper, we investigate the ergodic capacity analysis for diversity reception schemes over K_G fading channels using the MG distribution. We present expressions of the capacity for the MRC and the SC by applying the PDF-based approach in the independent and identically distributed (i.i.d.) case. Exploiting the properties of the MG distribution, the PDF of the MRC and the SC can also be expressed as the form of a weighted sum of the gamma distributions. As a result, the ergodic capacity of these diversity reception schemes can be obtained in an efficient way using the multinomial theorem in [18]. Simulations confirm that our derived analysis is well matched with empirical results.

The rest of the paper is as follows: The system model and the MG distribution are described in Section II. Then, we derive expressions of the ergodic capacity for the MRC and the SC by utilizing the MG distribution in Section III. From the simulation results in Section IV, we clarify the validity of our proposed analysis. Finally, this paper is terminated with conclusions in Section V.

Throughout the paper, we employ uppercase boldface letters

978-1-4799-4449-1/14/$31.00 ©2014 IEEE

for matrices and lowercase boldface letters for vectors. $\mathbb{E}[\cdot]$ denotes the expectation operator, $|\cdot|$ stands for the absolute value, and $(\cdot)^T$ means the transpose. Also, $\mathbf{I}_t$ indicates a $t \times t$ identity matrix and $n(\cdot)$ represents the number of elements in a set.

II. System Model

We consider single-input multiple-output systems where a diversity reception technique at a receiver is adopted over K_G fading channels. Assuming that the receiver is equipped with L antennas, the received signal $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

where $\mathbf{h} = [h_1, \cdots, h_L]^T \in \mathbb{C}^L$ indicates the K_G fading channel vector, s is the transmitted complex signal symbol with energy $E_s = \mathbb{E}[|s|^2]$, and $\mathbf{n} = [n_1, \cdots, n_L]^T \sim \mathcal{CN}(0, N_0\mathbf{I}_L)$.

In order to analyze the performance, we first investigate the PDF of SNR over K_G fading channels with $L = 1$. Then, the PDF of the SNR ρ can be obtained by [8]

$$f_\rho(x) = \frac{2\nu^{(k+m)/2} x^{(k+m-2)/2}}{\Gamma(m)\Gamma(k)} K_{k-m}\big(2\sqrt{\nu x}\big), \quad \text{for } x \geq 0 \quad (1)$$

where k and m stand for the shaping parameters of K_G fading channels which represent the multipath fading and shadowing effect, respectively, $\Gamma(\cdot)$ denotes the gamma function, $K_{k-m}(\cdot)$ is the modified Bessel function of order $k-m$, ν is defined as $\nu = km/\bar{\rho}$, and $\bar{\rho}$ equals the average SNR. From (1), important channel model statistics such as the CDF and the MGF can be derived [8]. However, these results contain some special functions such as a modified Bessel function, generalized hypergeometric functions, and the Whittaker function, which make the performance analysis difficult.

In order to tackle these problems, we adopt the MG distribution of the SNR in this model [10]. As mentioned before, the MG distribution has attractive properties such that an accurate approximation is possible for a variety of composite fadings by utilizing mathematically tractable expressions. The PDF of ρ in the form of the MG distribution is expressed by [17]

$$f_\rho(x) = \sum_{i=1}^{N} w_i f_i(x) = \sum_{i=1}^{N} \alpha_i x^{\beta_i - 1} e^{-\zeta_i x} \quad (2)$$

where $w_i = \alpha_i \Gamma(\beta_i)/\zeta_i^{\beta_i}$ means the normalization factor with $\sum_{i=1}^{N} w_i = 1$, $f_i(x) = \zeta_i^{\beta_i} x^{\beta_i - 1} e^{-\zeta_i x}/\Gamma(\beta_i)$ denotes the standard gamma distribution, and α_i, β_i, and ζ_i are the parameters of the i th mixture gamma component. Here, the number of terms N determines the accuracy measured by the mean square error (MSE) $\mathbb{E}\left[\left|f_{ext}(x) - f_{MG}(x)\right|^2\right]$ between the exact PDF $f_{ext}(x)$ and the MG distribution $f_{MG}(x)$, or the Kullback-Leibler divergence $\mathcal{D}_{KL} = \int_{-\infty}^{\infty} f_{ext}(x) \log \frac{f_{ext}(x)}{f_{MG}(x)} dx$. The effect of N can be verified in section IV.

For the K_G fading channel model, the PDF of ρ in (1) can be approximated by the form of (2) with parameters α_i, β_i, and ζ_i as [17]

$$\alpha_i = \frac{\theta_i}{\sum_{j=1}^{N} \theta_j \Gamma(\beta_j) \zeta_j{}^{-\beta_j}}, \ \beta_i = m, \ \zeta_i = \frac{\nu}{t_i}, \ \theta_i = \frac{\nu^m y_i t_i{}^{\lambda - 1}}{\Gamma(m)\Gamma(k)}$$

where $\lambda = k - m$, and y_i and t_i are the weight factor and the abscissa for the Gaussian-Laguerre integration [19], respectively. Also, the CDF of ρ can be computed as

$$F_\rho(x) = \int_0^x f_\rho(t)\, dt = \sum_{i=1}^{N} \alpha_i \zeta_i{}^{-\beta_i} \gamma(\beta_i, \zeta_i x) \quad (3)$$

where $\gamma(\cdot,\cdot)$ indicates the lower incomplete gamma function defined as $\gamma(c, \sigma) \triangleq \int_0^\sigma t^{c-1} e^{-t} dt$.

Based on (2), the MGF is evaluated as [8]

$$\mathcal{M}_\rho(s) = \int_0^\infty e^{-sx} f_\rho(x) dx = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{(s + \zeta_i)^{\beta_i}}. \quad (4)$$

Compared to the exact MGF in [8], it is worth noting that (4) has a simple form, while the MGF in [8] includes a complicated Whittaker function. Using the PDF, the CDF, and the MGF obtained from the MG distribution, we will provide simple and clear solutions for a diversity and array gain with parameters α_i, β_i, and ζ_i in the following section.

III. Capacity Analysis of Diversity Schemes

In this section, we derive expressions of the ergodic capacity for MRC and SC using the MG distribution. First, the ergodic capacity C_{erg} is defined as

$$C_{erg} = \int_0^\infty \log_2(1 + x) f_\rho(x) dx. \quad (5)$$

To compute C_{erg} for diversity reception schemes, we need to identify the PDF of SNR in each scheme. Because of properties of the MG distribution as mentioned before, the PDF of the MRC and the SC can be expressed by the weighted sum of gamma distributions, which will be shown in the next subsections.

Therefore, the analysis of the capacity becomes manageable with a closed form of the derived PDFs. For simple calculation of the capacity, we can apply a useful identity in [1] as

$$\begin{aligned} \int_0^\infty \ln(1 + x) x^{n-1} e^{-\mu x} dx &= (n-1)! e^\mu \sum_{k=1}^{n} \frac{\Gamma(k - n, \mu)}{\mu^k} \\ &\triangleq I_n(\mu) \end{aligned} \quad (6)$$

where $\Gamma(\cdot,\cdot)$ indicates the upper incomplete gamma function defined as $\Gamma(a, b) = \int_b^\infty t^{a-1} e^{-t} dt$. By utilizing the function $I_n(\mu)$, the capacity analysis for the MRC and the SC is achieved in a simple manner. Now, we investigate the PDF of each diversity scheme and derive an expression of the capacity for positive integer shaping parameters k and m. For simple explanations, we consider the i.i.d. case. However, our result can be easily extended to the i.n.d. case.

A. Maximal Ratio Combining

The combined SNR for the MRC is given by $\rho_{MRC} = \sum_{i=1}^{L} \rho_i$ where ρ_i denote the SNR for the i th branch for

$$f_{\rho_{MRC}}(x) = (\Gamma(m))^L \sum_{k_1+\cdots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} {\alpha_i}^{k_i} \sum_{j=1}^{N}\sum_{l=0}^{k_j m} R_{jl}\frac{x^l}{l!}e^{-\zeta_j x} \tag{7}$$

$$\begin{aligned} C_{\rho_{MRC}} &= (\Gamma(m))^L \sum_{k_1+\cdots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} {\alpha_i}^{k_i} \sum_{j=1}^{N}\sum_{l=0}^{k_j m} \frac{R_{jl}}{l!}\int_0^\infty \log_2(1+x)x^{(l+1)-1}e^{-\zeta_j x}dx \\ &= \frac{(\Gamma(m))^L}{\ln 2} \sum_{k_1+\cdots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} {\alpha_i}^{k_i} \sum_{j=1}^{N}\sum_{l=0}^{k_j m} \frac{R_{jl}}{l!} I_{l+1}(\zeta_j). \end{aligned} \tag{8}$$

$i = 1, \cdots, L$. For independent ρ_i, the MGF of ρ_{MRC} can be expressed as

$$\mathcal{M}_{\rho_{MRC}}(s) = \mathbb{E}[e^{-s\rho_{MRC}}] = \prod_{i=1}^{L}\mathcal{M}_{\rho_i}(s). \tag{9}$$

From equation (9), the MGF of the MRC can be alternatively represented by using the multinomial theorem [18], which is given as

$$\left(\sum_{i=1}^{N} x_i\right)^L = \sum_{k_1+\cdots+k_N=L} \binom{L}{k_1,\cdots,k_N} \prod_{1\le i\le N} {x_i}^{k_i}$$

where $\binom{L}{k_1,\cdots,k_N} = \frac{L!}{k_1!\cdots k_N!}$. By applying this theorem, the MGF of the MRC is rewritten as

$$\mathcal{M}_{\rho_{MRC}}(s) = \left(\sum_{i=1}^{N}\frac{\alpha_i\Gamma(m)}{(s+\zeta_i)^m}\right)^L$$

From the relationship between the PDF and the MGF $\mathcal{L}^{-1}\{\mathcal{M}_\rho(s)\} = f_\rho(x)$, the PDF of the MRC can be obtained by the inverse Laplace transform of (9) as

$$f_{\rho_{MRC}}(x) = \mathcal{L}^{-1}\{\mathcal{M}_{\rho_{MRC}}(s)\} \tag{10}$$

Using the identity

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^m}\right\} = \frac{x^{m-1}}{(m-1)!}e^{-ax},$$

equation (10) follows as (7). where R_{jl} denotes the coefficient of the l th term for $a = \zeta_j$.

By utilizing the expression $I_n(\mu)$, the capacity of the MRC is calculated from (5) by (8). The key point which makes the PDF-based analysis possible is that by exploiting the MGF formulated with the MG distribution, the inverse Laplace transform can be calculated unlike the MGF containing other complex functions. Thus, with the derived expression of the PDF which consists of the form of gamma distributions, an efficient computation of the ergodic capacity of the MRC is made possible.

B. Selection Combining

For generalized L branches, the PDF of the SC is composed of the product of the PDF in (2) and the CDF in (3) as

$$f_{\rho_{SC}}(x) = LF_\rho(x)^{L-1}f_\rho(x). \tag{14}$$

Similar to the MRC case, the PDF of the SC is represented by the method of the multinomial theorem as (11). In order to make the equation (11) more tractable, we transform the product form of the lower incomplete gamma function $\prod_{1\le l\le N}\gamma(m,\zeta_l x)^{k_l}$ into a summation form. Applying the recurrence relation $\gamma(m,x) = (m-1)\gamma(m-1,x) - x^{m-1}e^{-x}$, $\gamma(m,\zeta_l x)$ is converted into

$$\gamma(m,\zeta_l x) = (m-1)!\left[1 - \sum_{k=0}^{m-1}\frac{{\zeta_l}^k}{k!}x^k e^{-\zeta_l x}\right]. \tag{15}$$

In what follows, we will show that after inserting equation (15) to (11), the PDF of the SC can be represented by the summation form which simplifies the PDF-based capacity analysis. For illustrative purposes, we begin by presenting the product form of $\gamma(m,\zeta_l x)$ for the case of $L = 3$ and $l = 1, 2$ as an example. In this case, the product form of $\gamma(m,\zeta_l x)$ is given as

$$\prod_{i=1}^{2}\gamma(m,\zeta_i x) = \{(m-1)!\}^2\left[1 - K(\zeta_1,\zeta_2;m) + T(\zeta_1,\zeta_2;m)\right]$$

where

$$\begin{aligned} K(\zeta_1,\zeta_2;m) &= \sum_{j=1}^{2}\sum_{k=0}^{m-1}\frac{{\zeta_j}^k}{k!}x^k e^{-\zeta_j x}, \\ T(\zeta_1,\zeta_2;m) &= \sum_{k=0}^{m-1}\frac{{\zeta_1}^k}{k!}x^k\sum_{p=0}^{m-1}\frac{{\zeta_2}^p}{p!}x^p e^{-(\zeta_1+\zeta_2)x}. \end{aligned}$$

After some manipulations, $T(\zeta_1,\zeta_2;m)$ can also be written by the weighted sum of gamma distributions as

$$T(\zeta_1,\zeta_2;m) = \sum_{s=0}^{2(m-1)}\sum_{\substack{i+j=s \\ 0\le i,j\le m-1}}\frac{{\zeta_1}^i{\zeta_2}^j}{i!j!}x^s e^{-(\zeta_1+\zeta_2)x}.$$

Now we consider the generalized L branch case for simplifying the PDF of the SC. By observing the characteristics of $K(\cdot)$ and $T(\cdot)$ for given L, we define $H(x,\mathcal{S},i)$ as (12) where $\mathcal{S}$ is a set with k_j repeated elements for each j ($j = 1,\cdots,N$), $\mathcal{P}_u$ means the u th tuple $\mathcal{P}_u = \{p_{u1},\cdots,p_{ur}\}$ which follows conditions $\sum_{k=1}^{r} p_{uk} = l, n(\mathcal{P}_u) = r$, $0 \le p_{uk} \le m-1$, B denotes ${}_{n(\mathcal{S})}C_r$, $\tilde{\mathcal{S}}$ represents a set which consists of subsets organized by choosing r elements from the given set $\mathcal{S}$, $\tilde{\mathcal{S}}_h$ stands for the h th subset of $\tilde{\mathcal{S}}$, and $\tilde{\mathcal{S}}_{hw}$ indicates the w th element of $\tilde{\mathcal{S}}_h$. For example, in case of $k_1 = 2$, $k_3 = 1$, and $r = 2$ for $L = 4$, $\mathcal{S}$ and $\tilde{\mathcal{S}}$ are given by $\mathcal{S} = \{1,1,3\}$ and $\tilde{\mathcal{S}} = \{(1,1),(1,3),(1,3)\}$, respectively.

Note that the PDF of the SC is represented by the weighted sum of $x^{m-1}e^{-\zeta_i x}$ and $x^{l+m-1}e^{-(\sum_{w=1}^{r}\zeta_{\tilde{\mathcal{S}}_{hw}}+\zeta_i)x}$ as (13).

$$f_{\rho_{SC}}(x) = L\sum_{i=1}^{N}\alpha_i \sum_{k_1+\cdots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j{\zeta_j}^{-m})^{k_j}\prod_{1\le l\le N}\gamma(m,\zeta_l x)^{k_l}x^{m-1}e^{-\zeta_i x}. \tag{11}$$

$$H(x,\mathcal{S},i) = \{(m-1)!\}^{L-1}\left[x^{m-1}e^{-\zeta_i x}+\sum_{r=1}^{L-1}\sum_{l=0}^{r(m-1)}\sum_{h=1}^{B}\sum_{u=1}^{U}\left\{(-1)^r\sum_{\substack{p_{uk}\in\mathcal{P}_u\\ \tilde{\mathcal{S}}_{hk}\in\tilde{\mathcal{S}}_h}}\prod_{w=1}^{r}\frac{\left(\zeta_{\tilde{\mathcal{S}}_{hw}}\right)^{p_w}}{p_w!}x^{l+m-1}e^{-(\sum_{w=1}^{r}\zeta_{\tilde{\mathcal{S}}_{hw}}+\zeta_i)x}\right\}\right] \tag{12}$$

$$f_{\rho_{SC}}(x) = L\sum_{i=1}^{N}\alpha_i \sum_{k_1+\cdots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j{\zeta_j}^{-m})^{k_j}H(x,\mathcal{S},i). \tag{13}$$

$$\begin{aligned} C_{\rho_{SC}} &= \int_0^\infty \log_2(1+x)L\sum_{i=1}^{N}\alpha_i \sum_{k_1+\cdots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j{\zeta_j}^{-m})^{k_j}H(x,\mathcal{S},i)dx \\ &= \frac{L}{\ln 2}\sum_{i=1}^{N}\alpha_i \sum_{k_1+\cdots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j{\zeta_j}^{-m})^{k_j}\int_0^\infty \ln(1+x)H(x,\mathcal{S},i)dx. \end{aligned} \tag{16}$$

$$\int_0^\infty \ln(1+x)H(x,\mathcal{S},i)dx = \{(m-1)!\}^{L-1}\left(I_m(\zeta_i)+\sum_{r=1}^{L-1}\sum_{l=0}^{r(m-1)}\sum_{h=1}^{B}\sum_{u=1}^{U}(-1)^r\sum_{\substack{p_{uk}\in\mathcal{P}_u\\ \tilde{\mathcal{S}}_{hk}\in\tilde{\mathcal{S}}_h}}\prod_{w=1}^{r}\frac{\left(\zeta_{\tilde{\mathcal{S}}_{hw}}\right)^{p_w}}{p_w!}I_{l+m}\left(\sum_{w=1}^{r}\zeta_{\tilde{\mathcal{S}}_{hw}}+\zeta_i\right)\right). \tag{17}$$

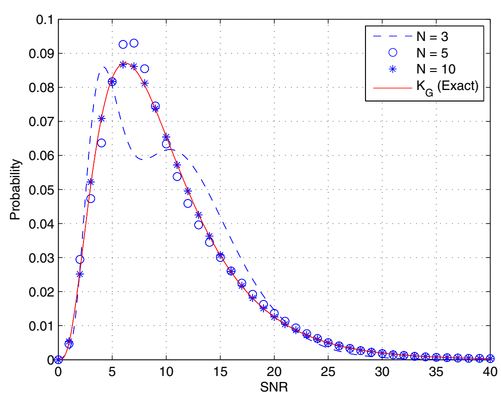

Fig. 1. Effect of N between the K_G distribution and MG distribution ($k = 5$ and $m = 7$)

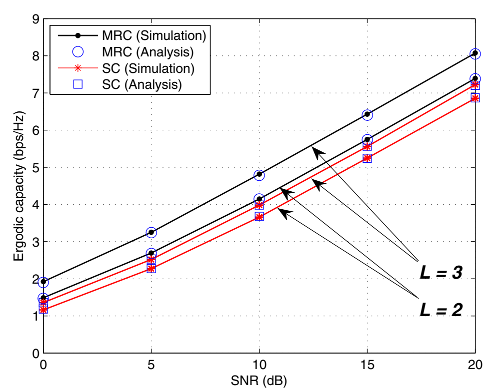

Fig. 2. Ergodic capacity of MRC and SC versus the average output SNR ($L = 2$ and 3)

Therefore, the ergodic capacity of the SC is computed by using (13) as (16). In (16), the integral term is simply calculated with $I_n(\mu)$ as (17).

It can be seen that the PDF of the SC in (14) is directly derived by the PDF and the CDF of a single branch using the MG distribution. This is due to the fact that the product form of the lower incomplete gamma function can be modified by the weighted sum of gamma distributions. Then, after applying some mathematical manipulations, the capacity analysis of the SC follows a similar way to that of the MRC.

IV. Simulation Results

In this section, we confirm the validity of our ergodic capacity analysis through Monte Carlo simulations. First, we observe the accuracy of the MG distribution according to the number of terms N. Figure 1 presents the comparison of the PDF between the K_G distribution and the MG distribution with various N for $k = 5$, $m = 7$, and $\bar{\rho} = 10$ dB. From this plot, it is verified that the MG distribution becomes more accurate as N grows. In particular, when N equals 10, the MG distribution nearly corresponds to the exact PDF. As a result, we can conclude that the PDF of the K_G fading channel is well approximated by utilizing the MG distribution with larger N. Throughout the simulations, the number of terms N is set to 10.

Then, we provide numerical results for the capacity of diversity reception schemes. As expected, the capacity of the MRC is higher than that of the SC. In Figure 2, the ergodic capacity of the MRC and the SC is depicted with respect to SNR for the different number of branches with the fading parameters $k = 5$ and $m = 2$. As L increases, the performance

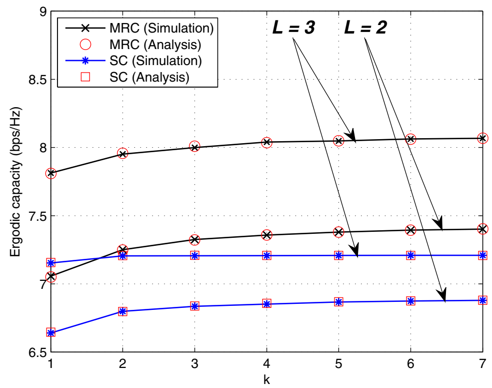

Fig. 3. Ergodic capacity of MRC and SC for different multipath fading factor k ($L = 2$ and 3)

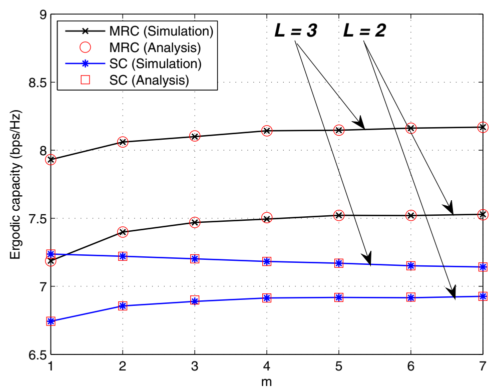

Fig. 4. Ergodic capacity of MRC and SC for different shadowing factor m ($L = 2$ and 3)

enhancement of the MRC is larger than that of the SC. As seen in this figure, the analysis results from equations in (8) and (16) are accurately matched with the actual results over all SNR ranges.

Next, we present simulation results of the capacity for various shaping parameters k and m in the generalized fading channel. We set the average SNR as 20 dB. In Figure 3, the ergodic capacity is plotted with respect to different multipath fading factors k for $m = 2$. Also, Figure 4 illustrates the ergodic capacity with various shadowing factors m for $k = 5$. From these two figures, it is shown that the capacity of each diversity reception scheme has a tendency to converge when the shaping parameter becomes large. For the SC, it can be seen that as the shaping parameter increases, a gain of the capacity with respect to L becomes less, while the gain for the MRC is invariant. Since the variance of the output SNR for the K_G fading channel becomes smaller as the shaping parameter k or m increases, the selective gain for the PDF decreases. Simulation results reveal that this effect becomes more pronounced for the shadowing parameter m. Again, the proposed analysis exactly matches with the simulation results.

V. Conclusions

In this paper, we have provided the ergodic capacity analysis of diversity reception schemes over K_G fading channel models by employing the MG distribution. Due to properties of the MG distribution, PDFs of SNR for both MRC and SC are represented by the form of the weighted sum of gamma distributions. Therefore, compared to the MGF-based analysis, our PDF-based approach allows us to perform the capacity analysis for the MRC and the SC with a simple manner. Also, our proposed capacity analysis generates the same result as the simulations for different scenarios of the fading environment.

References

[1] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. John Wiley & Sons, 2nd ed., 2005.

[2] G. L. Stüber, *Principles of Mobile Communications*. Kluwer Academic Publishers, 2nd ed., 2001.

[3] C. Tellambura, A. J. Mueller, and V. K. Bhargava, "Analysis of M-ary phase-shift keying with diversity reception for land mobile satellite channels," *IEEE Transactions on Vehicular Technology*, vol. 46, pp. 910–922, November 1997.

[4] P. M. Shankar, "Error rates in generalized shadowed fading channels," *Wireless Personal Communications*, vol. 28, pp. 233–238, February 2004.

[5] A. Laourine, M.-S. Alouini, S. Affes, and A. Stéphenne, "On the performance analysis of composite multipath/shadowing channels using the $\mathcal{G}$-distribution," *IEEE Transactions on Wireless Communications*, vol. 57, pp. 1162–1170, February 2009.

[6] P. M. Shankar, "Performance analysis of diversity combining algorithms in shadowed fading channels," *Wireless Personal Communications*, vol. 37, pp. 61–72, April 2006.

[7] I. M. Kostić, "Analytical approach to performance analysis for channel subject to shadowing and fading," *IEE Proceedings*, vol. 152, pp. 821–827, December 2005.

[8] P. S. Bithas, N. C. Sagias, P. T. Mathiopoulos, G. K. Karagiannidis, and A. A. Rontogiannis, "On the Performance Analysis of Digital Communications over Generalized-K Fading Channels," *IEEE Communications Letters*, vol. 10, pp. 353–355, May 2006.

[9] P. S. Bithas, P. T. Mathiopoulos, and S. A. Kotsopoulos, "Diversity Reception over Generalized-K Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 6, pp. 1–6, October 2007.

[10] J. Jung, S.-R. Lee, H. Park, and I. Lee, "Diversity Analysis over Composite Fading Channels using a Mixture Gamma Distribution," *in Proc. IEEE ICC 2013*, June 2013.

[11] S.-R. Lee, S.-H. Moon, J.-S. Kim, and I. Lee, "Capacity Analysis of Distributed Antenna Systems in a Composite Fading Channel," *IEEE Transactions on Wireless Communications*, vol. 11, pp. 1076–1086, March 2012.

[12] S.-R. Lee, S.-H. Moon, H.-B. Kong, and I. Lee, "Optimal Beamforming Schemes and Its Capacity Behavior for Downlink Distributed Antenna Systems," *IEEE Transactions on Wireless Communications*, vol. 12, pp. 2578–2587, June 2013.

[13] M. Renzo, F. Graziosi, and F. Santucci, "Channel Capacity over Generalized Fading Channels:a Novel MGF-based Approach for Performance Analysis and Design of Wireless Communication Systems," *IEEE Transactions on Vehicular Technology*, vol. 59, pp. 127–149, January 2010.

[14] F. Yilmaz and M.-S. Alouini, "A Unified MGF-Based Capacity Analysis of Diversity Combiners over Generalized Fading Channels," *IEEE Transactions on Communications*, vol. 60, pp. 862–875, March 2012.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. Academic Press, 6th ed., 2000.

[16] A. Kilbas and M. Saigo, *H-Transforms: Theory and Applications*. CRC Press, 2004.

[17] S. Atapattu, C. Tellambura, and H. Jiang, "A Mixture Gamma Distribution to Model the SNR of Wireless Channels," *IEEE Transactions on Wireless Communications*, vol. 10, pp. 4193–4203, December 2011.

[18] W. Feller, *An Introduction to Probability Theory and its Applications*. John Wiley & Sons, 3rd ed., 1968.

[19] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. Dover Publications, 1965.