Sum Throughput Maximization for MIMO Underlay Cognitive Wireless Powered Communication Networks

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***Abstract*—This paper investigates multi-user multi-input multi-output cognitive wireless powered communication networks (WPCN), in which a secondary WPCN shares spectrum with a primary wireless information transfer system. A typical WPCN consists of two different phases. In the first downlink phase, a hybrid access point (H-AP) transfers energy to charge users, and then in the subsequent uplink phase, the users send information by using the harvested energy to the H-AP. We consider underlay cognitive WPCN without cooperation between the primary transmitter and the secondary H-AP. In this case, we formulate sum throughput maximization problem by taking the interference leakage to the primary network into consideration. The problem is generally non-convex due to coupled variables in the WPCN. To tackle this issue, we first convert the problem into equivalent convex form, and then identify the global optimal solution by applying the proposed iterative optimization algorithm. Finally, simulation results demonstrate that the proposed algorithm outperforms conventional schemes.**

I. Introduction

Over the past few years, energy harvesting which utilizes radio frequency (RF) signals has been studied as a promising solution for extending life time of traditional energy-constrained wireless devices without replacing their batteries [1]. Thanks to the development of such wireless energy transfer (WET) techniques, combining the WET with the wireless information transfer (WIT) has recently been an active research area to improve the system performance such as the simultaneous wireless information and power transfer [2]–[6] and the wireless powered communication networks (WPCN) [7]–[14]. While the former achieves both the WET and the WIT operation at the same time and frequency, the latter perform each operation separately over two sequential phases.

Specifically, in a downlink (DL) WET phase of the WPCN, a hybrid access point (H-AP) sends energy RF signals to users, and the energy is stored in users' rechargeable batteries. Then, in a subsequent uplink (UL) WIT phase, each user transmits its information signal to the H-AP using the harvested energy. Recently, the authors in [7] provided the optimal time allocation factor for each user in a single-input single-output (SISO) WPCN to maximize the sum throughput based on the time division multiple access approach. This work was extended to multi-input single-output scenarios in [9]–[11]. The work in [9] jointly optimized the time allocation and the DL-WET beamforming vectors to maximize the sum throughput. In [10] and [11], the H-AP beamforming solutions were also suggested for maximizing the minimum throughput among all users by utilizing space division multiple access techniques. To address the spectrum sharing issue of the WPCN with primary networks, a cognitive radio concept was applied in [15], where a secondary WPCN coexists with a primary WIT network in the same time and frequency band. However, the previous work for the cognitive WPCN (CWPCN) is limited to a single antenna case, and thus it is not easy to obtain insights on multiple antenna gains.

In this paper, we extend the previous SISO underlay CW-PCN (U-CWPCN) in [15] to a general multi-user multi-input multi-output (MU-MIMO) U-CWPCN where all secondary terminals are equipped with multiple antennas. The U-CWPCN assumes no cooperation between the primary transmitter (PT) and the H-AP, and thus the signals from the H-AP may interfere a primary receiver (PR). In this case, we need to jointly optimize the energy and information covariance matrices at the H-AP and the secondary users (SUs) as well as the time allocation factor between the UL and the DL phases of the WPCN. For U-CWPCN, the sum throughput maximization problem is generally non-convex due to coupled variables.

In the U-CWPCN, we propose a two-step approach to solve the sum throughput maximization problem. Specifically, after transforming the problem to an equivalent convex form, we obtain the energy and information covariance matrices using the Karush-Kuhn-Tucker (KKT) conditions and the subgradient ellipsoid methods for a given time allocation. Interestingly, it is shown that while the optimal information covariance matrix of each SU is determined by an iterative water-filling algorithm, the optimal energy covariance matrix is simply attained by a rank one matrix regardless of interference-temperature constraint (ITC) [16] for satisfying the minimum rate requirement of the primary network. Then, we find the Lagrange dual variables utilizing the ellipsoid process. Finally, the optimal time allocation is determined by a simple line search method. We note that our proposed algorithm for the U-CWPCN offers the global optimal solution for all power and interference constraints, and thus will serve as a meaningful reference for future research on the U-CWPCN. We also confirm from simulation results that our proposed algorithm outperforms conventional schemes.

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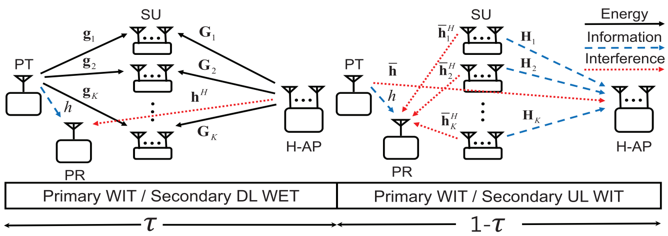

Fig. 1. Schematic diagrams of MU-MIMO U-CWPCN

Throughout the paper, we will use the following notations. The boldface uppercase, boldface lowercase and normal letters indicate matrices, vectors and scalars, respectively. The operator $(\cdot)^T$, $(\cdot)^H$ and $\mathbb{E}(\cdot)$ denote transpose, conjugate transpose and expectation, respectively. In addition, $|\mathbf{A}|$, tr$(\mathbf{A})$, rank$(\mathbf{A})$ and $\mathbf{I}_d$ represent determinant, trace, rank of a matrix $\mathbf{A}$ and an identity matrix of size d, respectively, and diag$(\mathbf{a})$ stands for a diagonal matrix whose diagonal elements consist of a vector $\mathbf{a}$. Also, $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is a positive semi-definite matrix.

II. System Model and Problem Formulation

As illustrated in Fig. 1, we consider a multi-antenna U-CWPCN in which a primary WIT and a secondary WPCN operate in the same time and frequency resource blocks. In the primary system, there are a PT and a PR with a single antenna, and the secondary WPCN consists of a H-AP with M antennas and K SUs with N antennas. In this system, the PT and the H-AP have stable power supplies, whereas the SUs are not equipped with any embedded energy sources. Therefore, during the DL-WET phase, the SUs first harvest energy from the RF signals transmitted by the H-AP and the PT, and then in the subsequent UL-WIT phase, the SUs transmit their information to the H-AP using the harvested energy. We assume that the DL-WET and UL-WIT phases occupy τ and $1-\tau$ portion of the entire time resource block, respectively ($0 \leq \tau \leq 1$). It is also assumed that while the secondary terminals know global channel state information (CSI) of both primary and secondary networks, the primary terminals do not need to know CSI of the secondary network.

During the DL-WET phase of the U-CWPCN, the H-AP broadcasts the energy signal $\mathbf{s}_{\text{E}} \in \mathbb{C}^{M\times 1}$ with the covariance matrix $\mathbf{W} = \mathbb{E}[\mathbf{s}_{\text{E}}\mathbf{s}_{\text{E}}^H] \in \mathbb{C}^{M\times M}$. The transmit power at the H-AP is then expressed as tr$(\mathbf{W}) \leq P_{\text{H-AP}}$. Let us denote the baseband channels from the H-AP to SU i, $i = 1, \ldots, K$, and from the PT to SU i by $\mathbf{G}_i \in \mathbb{C}^{N\times M}$ and $\mathbf{g}_i \in \mathbb{C}^{N\times 1}$, respectively. Then, the received signal $\mathbf{r}_{\text{U},i} \in \mathbb{C}^{N\times 1}$ at SU i is expressed as

$$\mathbf{r}_{\text{U},i} = \mathbf{G}_i\mathbf{s}_{\text{E}} + \sqrt{P_{\text{P}}}\mathbf{g}_i s_{\text{P}} + \mathbf{n}_i,$$

where P_{P} is the transmit power at the PT, $s_{\text{P}} \sim \mathcal{CN}(0,1)$ stands for the information signal from the PT which is independent of $\mathbf{s}_{\text{E}}$, and $\mathbf{n}_i \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(0, \sigma_i^2\mathbf{I}_N)$ indicates the circularly symmetric complex Gaussian (CSCG) noise at SU i. Then, the harvested energy $Q_{\text{U},i}$ at SU i can be written by [2]

$$Q_{\text{U},i}(\mathbf{W}) = \tau\zeta_i\Big(\text{tr}(\mathbf{G}_i\mathbf{W}\mathbf{G}_i^H) + P_{\text{P}}\|\mathbf{g}_i\|^2\Big), \tag{1}$$

where $0 < \zeta_i \leq 1$ is the energy harvesting efficiency at SU i. For convenience, it is assumed that $\zeta_i = 1$, $\forall i$, throughout this paper.

In the UL-WIT phase, each SU transmits independent information signal vector $\mathbf{s}_i \in \mathbb{C}^{N\times 1}$ with the covariance matrix $\mathbf{S}_i = \mathbb{E}[\mathbf{s}_i\mathbf{s}_i^H] \in \mathbb{C}^{N\times N}$ to the H-AP. Due to the harvested energy constraint in (1), SU i cannot use the energy larger than $Q_{\text{U},i}(\mathbf{W})$, i.e., $(1-\tau)\text{tr}(\mathbf{S}_i) \leq Q_{\text{U},i}(\mathbf{W})$, $\forall i$. Let us define $\mathbf{H}_i \in \mathbb{C}^{M\times N}$ and $\bar{\mathbf{h}} \in \mathbb{C}^{M\times 1}$ as the UL channel from SU i to the H-AP and the interference channel from the PT to the H-AP, respectively. Then, the received signal $\mathbf{r}_{\text{U,H-AP}} \in \mathbb{C}^{M\times 1}$ at the H-AP can be given as

$$\mathbf{r}_{\text{U,H-AP}} = \sum_{i=1}^{K}\mathbf{H}_i\mathbf{s}_i + \bar{\mathbf{h}}s_{\text{P}} + \mathbf{n}_{\text{H-AP}}, \tag{2}$$

where $\mathbf{n}_{\text{H-AP}} \in \mathbb{C}^{M\times 1} \sim \mathcal{CN}(0, \sigma_{\text{H-AP}}^2\mathbf{I}_M)$ denotes the CSCG noise at the H-AP, and the second term in (2) represents the interference signal from the primary network. Since the UL-WIT occurs during $1-\tau$ portion of the time, the achievable sum throughput is obtained as [17]

$$R_{\text{U,H-AP}} = (1-\tau)\log_2\left|\mathbf{I}_M + \mathbf{R}_U^{-1}\sum_{i=1}^{K}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right|,$$

where $\mathbf{R}_U = \sigma_{\text{H-AP}}^2\mathbf{I}_M + P_{\text{P}}\bar{\mathbf{h}}\bar{\mathbf{h}}^H$ represents the interference-plus-noise covariance matrix at the H-AP.

In the meantime, during both phases, the PR receives the interference signal from the secondary WPCN. Thus, to ensure the primary network's performance, we impose the ITC on the secondary terminals so that the interference power is limited to a predefined interference level Γ. First, in the DL-WET phase, the received signal $r_{\text{U,PR}}^{\text{DL}}$ at the PR is given by

$$r_{\text{U,PR}}^{\text{DL}} = \sqrt{P_{\text{P}}}hs_{\text{P}} + \mathbf{h}^H\mathbf{s}_{\text{E}} + n_{\text{PR}}^{\text{DL}}, \tag{3}$$

where h and $\mathbf{h} \in \mathbb{C}^{M\times 1}$ stand for the DL channel from the PT to the PR and the interference channel from the H-AP to the PR, respectively, and $n_{\text{PR}}^{\text{DL}} \sim \mathcal{CN}(0, \sigma_{\text{PR,DL}}^2)$ denotes the CSCG noise at the PR in the DL-WET phase. It is worth noting that the received signal from the H-AP, i.e., $\mathbf{h}^H\mathbf{s}_{\text{E}}$ in (3), is interference at the PR with power $\mathbb{E}[(\mathbf{h}^H\mathbf{s}_{\text{E}})(\mathbf{h}^H\mathbf{s}_{\text{E}})^H] = \mathbf{h}^H\mathbf{W}\mathbf{h}$.[1] Therefore, the H-AP needs to determine the energy transmit covariance matrix $\mathbf{W}$ so that the ITC $\mathbf{h}^H\mathbf{W}\mathbf{h} \leq \Gamma$ is satisfied.

[1]Recently, it was investigated that deterministic multi-sine waveforms can be utilized as an energy carrying signal [18]. Thus, if the H-AP adopts those deterministic waveforms, the PR may remove the interference. However, a typical information receiving node may not be equipped with a multi-sine waveform canceller. Also, in order to accomplish perfect interference cancellation, the PR should know perfect CSI from the H-AP, which contradicts to the U-CWPCN strategy. Therefore, we assume that the energy signal from the H-AP is chaotic, and thus is not decodable by the PR.

Also, in the UL-WIT phase, the received signal $r_{\text{U,PR}}^{\text{UL}}$ at the PR can be expressed as

$$r_{\text{U,PR}}^{\text{UL}} = \sqrt{P_{\text{P}}}hs_{\text{P}} + \sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \mathbf{s}_i + n_{\text{PR}}^{\text{UL}}, \quad (4)$$

where $\bar{\mathbf{h}}_i \in \mathbb{C}^{N\times 1}$ and $n_{\text{PR}}^{\text{UL}} \sim \mathcal{CN}(0, \sigma_{\text{PR,UL}}^2)$ denote the channel from SU i to the PR and the CSCG noise at the PR, respectively, and the second term in (4) designates interference from the SUs with power $\sum_{i=1}^{K} \mathbb{E}[(\bar{\mathbf{h}}_i^H \mathbf{s}_i)(\bar{\mathbf{h}}_i^H \mathbf{s}_i)^H] = \sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \mathbf{S}_i \bar{\mathbf{h}}_i$. Thus, the information covariance matrices at the SUs must be designed to satisfy the ITC $\sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \mathbf{S}_i \bar{\mathbf{h}}_i \leq \Gamma$.

In this paper, it is of interest to maximize the sum throughput of the secondary WPCN under the ITCs. Thus, the problem in the U-CWPCN is formulated as

$$\begin{aligned}
\text{(P1)} \quad & \max_{\tau, \{\mathbf{S}_i \succeq 0\}, \mathbf{W} \succeq 0} (1-\tau) \log_2 \left| \mathbf{I}_M + \mathbf{R}_U^{-1} \sum_{i=1}^{K} \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right| \\
& \text{s.t. } (1-\tau)\text{tr}(\mathbf{S}_i) \leq Q_{\text{U},i}(\mathbf{W}), \forall i, && (5) \\
& \quad 0 \leq \tau \leq 1, \\
& \quad \text{tr}(\mathbf{W}) \leq P_{\text{H-AP}}, \\
& \quad \mathbf{h}^H \mathbf{W} \mathbf{h} \leq \Gamma, && (6) \\
& \quad \sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \mathbf{S}_i \bar{\mathbf{h}}_i \leq \Gamma. && (7)
\end{aligned}$$

In general, problem (P1) is non-convex due to the coupled variables in constraint (5), and thus is intractable in its current form. In Section III, we present an efficient method to find an optimal solution of (P1).

III. Optimal Solution

In this section, we solve problem (P1) and find a global optimal solution using a two-step procedure. First, we reformulate problem (P1) to an equivalent convex problem by introducing new variables $\bar{\mathbf{S}}_i = (1-\tau)\mathbf{S}_i$ for $i = 1, \ldots, K$ and $\bar{\mathbf{W}} = \tau \mathbf{W}$. Then, by the change of variables, we can rewrite problem (P1) equivalently as

(P1-1)

$$\begin{aligned}
& \max_{\tau, \{\bar{\mathbf{S}}_i \succeq 0\}, \bar{\mathbf{W}} \succeq 0} (1-\tau) \log_2 \left| \mathbf{I}_M + \frac{\mathbf{R}_U^{-1}}{1-\tau} \sum_{i=1}^{K} \mathbf{H}_i \bar{\mathbf{S}}_i \mathbf{H}_i^H \right| && (8) \\
& \text{s.t. } \text{tr}(\bar{\mathbf{S}}_i) \leq \text{tr}(\mathbf{G}_i \bar{\mathbf{W}} \mathbf{G}_i^H) + \tau P_{\text{P}} \|\mathbf{g}_i\|^2, \ \forall i, \\
& \quad 0 \leq \tau \leq 1, \\
& \quad \text{tr}(\bar{\mathbf{W}}) \leq \tau P_{\text{H-AP}}, \\
& \quad \mathbf{h}^H \bar{\mathbf{W}} \mathbf{h} \leq \tau \Gamma, \\
& \quad \sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \bar{\mathbf{S}}_i \bar{\mathbf{h}}_i \leq (1-\tau)\Gamma.
\end{aligned}$$

Since the objective function in (8) is concave on τ and $\{\bar{\mathbf{S}}_i\}$, and all the constraints of problem (P1-1) are affine functions, we can easily verify that (P1-1) is a jointly concave problem. To make the problem more tractable, we first fix the time allocation factor τ to $\bar{\tau}$, and obtain the optimal covariance matrices $\bar{\mathbf{W}}^\star$ and $\{\bar{\mathbf{S}}_i^\star\}$ for a given $\bar{\tau}$ in Section III-A. Then, in Section III-B, we will determine the optimal time allocation $\tau^\star$ based on the derived covariance matrices.

A. Optimal Transmit Covariance Matrices

For a given $\bar{\tau}$, (P1-1) is recast to the following problem:

(P1-2) $\mathcal{R}_U(\bar{\tau}) \triangleq$

$$\begin{aligned}
& \max_{\{\bar{\mathbf{S}}_i \succeq 0\}, \bar{\mathbf{W}} \succeq 0} (1-\bar{\tau}) \log_2 \left| \mathbf{I}_M + \frac{\mathbf{R}_U^{-1}}{1-\bar{\tau}} \sum_{i=1}^{K} \mathbf{H}_i \bar{\mathbf{S}}_i \mathbf{H}_i^H \right| \\
& \text{s.t. } \text{tr}(\bar{\mathbf{S}}_i) \leq \text{tr}(\mathbf{G}_i \bar{\mathbf{W}} \mathbf{G}_i^H) + \bar{\tau} P_{\text{P}} \|\mathbf{g}_i\|^2, \ \forall i, && (9) \\
& \quad \text{tr}(\bar{\mathbf{W}}) \leq \bar{\tau} P_{\text{H-AP}}, && (10) \\
& \quad \mathbf{h}^H \bar{\mathbf{W}} \mathbf{h} \leq \bar{\tau} \Gamma, && (11) \\
& \quad \sum_{i=1}^{K} \bar{\mathbf{h}}_i^H \bar{\mathbf{S}}_i \bar{\mathbf{h}}_i \leq (1-\bar{\tau})\Gamma, && (12)
\end{aligned}$$

where $\mathcal{R}_U(\bar{\tau})$ in (P1-2) indicates the maximum achievable rate. Since problem (P1-2) is convex and satisfies the Slater's condition, we can solve the problem by applying the Lagrange duality method [19].

First, let us express the Lagrangian of (P1-2) as

$$\begin{aligned}
& \mathcal{L}_U(\{\bar{\mathbf{S}}_i\}, \bar{\mathbf{W}}, \{\mu_i\}, \{\nu_j\}) = \\
& \quad (1-\bar{\tau}) \log_2 \left| \mathbf{I}_M + \frac{\mathbf{R}_U^{-1}}{1-\bar{\tau}} \sum_{i=1}^{K} \mathbf{H}_i \bar{\mathbf{S}}_i \mathbf{H}_i^H \right| - \text{tr}\left(\sum_{i=1}^{K} \mathbf{M}_i \bar{\mathbf{S}}_i \right) \\
& \quad + \text{tr}\left(\left(\sum_{i=1}^{K} \mu_i \mathbf{G}_i^H \mathbf{G}_i - \nu_2 \mathbf{h}\mathbf{h}^H - \nu_1 \mathbf{I}_M \right) \bar{\mathbf{W}} \right) \\
& \quad + \bar{\tau} P_{\text{P}} \sum_{i=1}^{K} \mu_i \|\mathbf{g}_i\|^2 + \nu_1 \bar{\tau} P_{\text{H-AP}} + \nu_2 \bar{\tau} \Gamma + \nu_3 (1-\bar{\tau})\Gamma,
\end{aligned}$$

where $\mathbf{M}_i$ is denoted as $\mathbf{M}_i = (\nu_3 \bar{\mathbf{h}}_i \bar{\mathbf{h}}_i^H + \mu_i \mathbf{I}_N)$ for $i = 1, \ldots, K$, and μ_i for $i = 1, \ldots, K$ and ν_j for $j = 1, 2, 3$ are the non-negative Lagrange dual variables associated with the constraints (9)-(12).

Then, the dual function $\mathcal{G}_U(\{\mu_i\}, \{\nu_i\})$ of (P1-2) is defined by

$$\mathcal{G}_U(\{\mu_i\}, \{\nu_j\}) = \max_{\{\bar{\mathbf{S}}_i \succeq 0\}, \bar{\mathbf{W}} \succeq 0} \mathcal{L}(\{\bar{\mathbf{S}}_i\}, \bar{\mathbf{W}}, \{\mu_i\}, \{\nu_j\}). \quad (13)$$

Due to strong duality, the following equality also holds for all $\bar{\tau}$ as

$$\mathcal{R}_U(\bar{\tau}) = \min_{\{\mu_i\}, \{\nu_j\}} \mathcal{G}(\{\mu_i\}, \{\nu_j\}). \quad (14)$$

Therefore, we can find a solution of $\{\bar{\mathbf{S}}_i\}$ and $\bar{\mathbf{W}}$ by solving (13) and (14) alternatively. First, we determine the optimal energy covariance matrix $\bar{\mathbf{W}}^\star$ in the following lemma.

Lemma 1: Let us define a matrix $\mathbf{B} \triangleq \sum_{i=1}^{K} \mu_i^\star \mathbf{G}_i^H \mathbf{G}_i - \nu_2^\star \mathbf{h}\mathbf{h}^H$. Then, for given $\tau > 0$, $\mu_i > 0 \ \forall i$, and $\nu_2 \geq 0$, the optimal energy transmit covariance matrix $\bar{\mathbf{W}}^\star$ of (13) and the optimal dual variable $\nu_1^\star$ are respectively written by

$$\begin{aligned}
\bar{\mathbf{W}}^\star &= \tau P_{\text{H-AP}} \mathbf{u}_{\mathbf{B},1} \mathbf{u}_{\mathbf{B},1}^H, && (15) \\
\nu_1^\star &= \lambda_{\mathbf{B},1}
\end{aligned}$$

where $\lambda_{\mathbf{B},1}$ and $\mathbf{u}_{\mathbf{B},1}$ represent the largest eigenvalue and its associated eigenvector of $\mathbf{B}$, respectively.

Proof: See Appendix A. ■

From Lemma 1, it is clear that the optimal energy covariance matrix $\bar{\mathbf{W}}^\star$ is always rank one. Therefore, without loss of generality, one can set the energy carrying signal at the H-AP as $\mathbf{s}_\text{E} = \sqrt{P_\text{H-AP}}\mathbf{u}_{\mathbf{B},1}s$, where s stands for an arbitrary complex random signal with zero mean and unit variance.

Now, we examine the information covariance matrices $\{\bar{\mathbf{S}}_i\}$. Let us define $\hat{\mathbf{S}}_i = \frac{\mathbf{M}_i^{\frac{1}{2}}\bar{\mathbf{S}}_i(\mathbf{M}_i^{\frac{1}{2}})^H}{1-\bar{\tau}}$ and ignore the constant terms with respect to $\hat{\mathbf{S}}_i$ in (13). Then, problem (13) is rephrased by a function of $\hat{\mathbf{S}}_i$ as

$$\max_{\hat{\mathbf{S}}_i\succeq 0} \log_2 |\mathbf{I}_M + \mathbf{C}_i\hat{\mathbf{S}}_i\mathbf{C}_i^H| - \text{tr}(\hat{\mathbf{S}}_i) \tag{16}$$

where we have $\mathbf{C}_i = (\mathbf{I}_M + \sum_{i\neq j}\mathbf{E}_j\hat{\mathbf{S}}_j\mathbf{E}_j^H)^{-\frac{1}{2}}\mathbf{E}_i$ and $\mathbf{E}_i = \mathbf{R}_U^{-\frac{1}{2}}\mathbf{H}_i\mathbf{M}_i^{-\frac{1}{2}}$. We denote singular value decomposition $\mathbf{C}_i = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^H$, where $\mathbf{U}_i \in \mathbb{C}^{M\times r_i}$ and $\mathbf{V}_i \in \mathbb{C}^{M\times r_i}$ represent the left and right singular vectors of $\mathbf{C}_i$, respectively, with $r_i = \text{rank}(\mathbf{C}_i)$, and $\mathbf{\Sigma}_i = \text{diag}(\sigma_{i,1}, \dots, \sigma_{i,r_i})$ is a diagonal matrix consisting of singular values $\sigma_{i,k}$ for $k = 1, \dots, r_i$ with $\sigma_{i,1} \geq \cdots \geq \sigma_{i,r_i}$.

Then, by the KKT optimality conditions, a solution of (16) is expressed as

$$\bar{\mathbf{S}}_i^\star = (1-\tau)\mathbf{M}_i^{-\frac{1}{2}}\mathbf{V}_i\mathbf{D}_i\mathbf{V}_i^H(\mathbf{M}_i^{-\frac{1}{2}})^H, \tag{17}$$

where $\mathbf{D}_i = \text{diag}(d_{i,1}, \dots, d_{i,r_i})$ with $d_{i,k} = \max(0, \log_2 e - \frac{1}{\sigma_{i,k}^2})$ for $k = 1, \dots, r_i$. Note that the derived solution $\bar{\mathbf{S}}_i^\star$ in (17) is related to the input covariance matrices $\{\bar{\mathbf{S}}_j\}_{j\neq i}$ of other SUs. This can be resolved by an iterative water-filling procedure [17]. Specifically, first we set $\bar{\mathbf{S}}_i^{(0)} = \mathbf{0}\ \forall i$, where $\bar{\mathbf{S}}_i^{(n)}$ indicates a solution of SU i at the n-th iteration. At each iteration, we update $\bar{\mathbf{S}}_i^{(n)}$ from (17) with $\bar{\mathbf{S}}_j = \bar{\mathbf{S}}_j^{(n)}$ for $j \leq i-1$ and $\bar{\mathbf{S}}_j = \bar{\mathbf{S}}_j^{(n-1)}$ for $j \geq i+1$. We repeat this procedure until $\{\bar{\mathbf{S}}_i^{(n)}\}$ converges. Since the problem is jointly concave and each iteration maximizes the cost function (13), this process indeed converges to the optimal solution $\{\bar{\mathbf{S}}_i^\star\}$.

Now, we turn our focus to the dual variables of problem (14). As shown in [2], this is effectively solved by the subgradient ellipsoid methods. The subgradient γ_i of the dual function with respect to μ_i is given by $\gamma_i = \text{tr}(\mathbf{G}_i\bar{\mathbf{W}}\mathbf{G}_i^H + \bar{\tau}P_\text{P}\mathbf{g}_i\mathbf{g}_i^H - \bar{\mathbf{S}}_i)$ for $i = 1, \dots, K$, while the subgradients δ_2 and δ_3 with respect to ν_2 and ν_3 are attained by $\delta_2 = \bar{\tau}\Gamma - \mathbf{h}^H\bar{\mathbf{W}}\mathbf{h}$ and $\delta_3 = (1-\bar{\tau})\Gamma - \sum_{i=1}^K \bar{\mathbf{h}}_i^H\bar{\mathbf{S}}_i\bar{\mathbf{h}}_i$, respectively. Utilizing these parameters, one can apply the ellipsoid method to find the optimal dual variables. The entire process of identifying the optimal solution of (P1-2) is summarized in Algorithm 1.

Algorithm 1: Optimal solution for problem (P1-2)

Initialize μ_i for $i = 1, \dots, K$ and ν_j for $j = 2, 3$.
Repeat
  Compute $\bar{\mathbf{W}}^\star$ from (15).
  Set $n = 0$ and initialize $\bar{\mathbf{S}}_i^{(0)} = \mathbf{0}$ for $i = 1, \dots, K$.
  Repeat
    Set $n = n + 1$.
    Update $\bar{\mathbf{S}}_i^{(n)}$ from (17) for $i = 1, \dots, K$.
  Until $\bar{\mathbf{S}}_i^{(n)}$ for $i = 1, \dots, K$ converge.
  Update $\{\mu_i\}$ and $\{\nu_j\}$ using the ellipsoid method.
Until $\{\mu_i\}$ and $\{\nu_j\}$ converge.

B. Optimal Time Allocation

Having obtained the optimal covariance matrices $\bar{\mathbf{W}}^\star$ and $\{\bar{\mathbf{S}}_i^\star\}$, the remaining work is to determine the time allocation factor τ by computing $\tau^\star = \arg\max_{0<\tau<1} \mathcal{R}_U(\tau)$. Note that for the given optimal covariance matrices $\bar{\mathbf{W}}^\star$ and $\{\bar{\mathbf{S}}_i^\star\}$, $\mathcal{R}_U(\tau)$ in (P1-2) is a concave function with respect to τ. Therefore, the optimal $\tau^\star$ can be efficiently found by using the line search methods such as golden section search. The overall algorithm for calculating the optimal solution of the U-CWPCN is summarized in Algorithm 2.

Algorithm 2: Optimal solution for U-CWPCN

Initialize $a = 0$, $b = 1$, and $\omega = \frac{-1+\sqrt{5}}{2}$.
Repeat
  Update $\tau_1 = a + (1-\omega)b$ and $\tau_2 = b + (1-\omega)a$.
  Obtain $\mathcal{R}_U(\tau_1)$ and $\mathcal{R}_U(\tau_2)$ from Algorithm 1.
  If $\mathcal{R}_U(\tau_1) > \mathcal{R}_U(\tau_2)$, set $a = \tau_1$.
  Else set $b = \tau_2$.
Until $|a - b|$ converges.
Obtain $\tau^\star = (a+b)/2$.

IV. Simulation Results

In this section, we evaluate the average sum throughput of the MU-MIMO U-CWPCN by numerical simulations. For simplicity, we set the noise variance at the H-AP to be one, i.e., $\sigma_\text{H-AP}^2 = 1$. In addition, we assume that the PT uses unit transmit power $P_\text{P} = 1$ unless specified otherwise. All channel coefficients follow an independent complex Gaussian distribution with zero mean and unit variance. Throughout the section, a notation (M, N, K) is adopted to denote a WPCN with K SUs each having N antennas and the H-AP with M antennas. The signal-to-noise ratio (SNR) of the primary and secondary networks are defined as $\text{SNR}_\text{P} = P_\text{P}$ and $\text{SNR}_\text{S} = P_\text{H-AP}$, respectively. Also, we fix Γ in (6) and (7) as $\Gamma = 5$ unless otherwise indicated. To the best of our knowledge, there is no existing schemes in the MU-MIMO U-CWPCN under consideration. In the simulation, we compare our proposed method with equal power (EP) scheme. For instance, one may adopt a simple scheme with $\mathbf{W}_\text{EP,U} = \min(\frac{\Gamma}{\|\mathbf{h}\|^2}, \frac{P_\text{H-AP}}{M})\mathbf{I}_M$. In this case, $\{\mathbf{S}_i\}$ are obtained from Algorithm 1 with $\mathbf{W} = \mathbf{W}_\text{EP,U}$. The time allocation τ is determined from Algorithm 2 or is fixed to $\tau = \frac{1}{2}$.

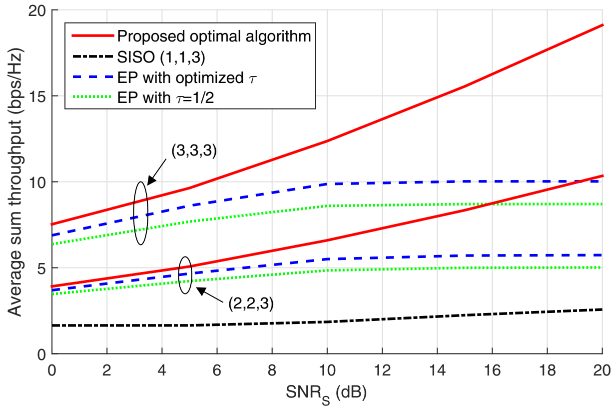

Fig. 2. Average sum throughput performance as a function of $\mathrm{SNR_S}$ for $(1, 1, 3)$, $(2, 2, 3)$ and $(3, 3, 3)$

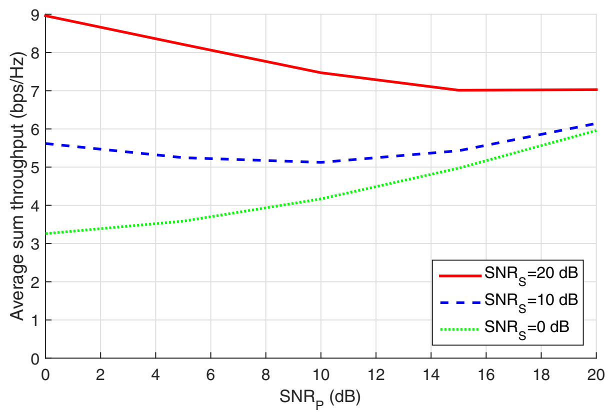

Fig. 3. Average sum throughput performance as a function of $\mathrm{SNR_P}$ for $(2, 2, 2)$ systems

Fig. 2 depicts the average sum throughput performance of the proposed algorithm and the EP scheme for the U-CWPCN of $(2, 2, 3)$ and $(3, 3, 3)$ systems with $\Gamma = 5$. As expected, we can check that the proposed optimal algorithm outperforms the conventional EP scheme for all cases. One interesting observation is that while the proposed scheme achieves a sum throughput performance gain as the SNR of the secondary network $\mathrm{SNR_S}$ range. This is because the transmit power at the H-AP is constrained by the ITC, and thus the harvested energy at the SUs no longer increases even if $\mathrm{SNR_S}$ becomes high. Therefore, a gain of the proposed method is more pronounced at high $\mathrm{SNR_S}$. It is shown that at $\mathrm{SNR_S} = 10$dB, our method obtains about 36 % and 44 % gains over the EP scheme with $\tau = \frac{1}{2}$ in $(2, 2, 3)$ and $(3, 3, 3)$ systems, respectively. We also plot that the performance curve with $(1, 1, 3)$ correspond to the conventional single antenna U-CWPCN [15] with 3 SUs. Due to the ITC, it is shown that the sum throughput of the $(1, 1, 3)$ U-CWPCN system becomes saturated at large $\mathrm{SNR_S}$.

In Fig. 3, we draw the average sum throughput of the U-CWPCN with respect to the primary network's SNR $\mathrm{SNR_P}$. In the U-CWPCN, as $\mathrm{SNR_P}$ grows, the interference power from the PT to the H-AP in the UL-WIT phase increases. As a result, the throughput performance of the secondary WPCN may be reduced. In particular, when $\mathrm{SNR_S}$ is sufficiently low, each SU relies more on the harvested energy from the PT's signal, and thus the gains from the increased $\mathrm{SNR_P}$ will be more pronounced. Throughout the simulations, we confirm that our algorithm outperforms conventional schemes.

V. Conclusions

This paper has investigated the MU-MIMO CWPCN where a secondary WPCN with multiple-antenna terminals shares the same spectrum with a primary WIT network. In this configuration, we have provided the optimal time allocation and transmit covariance matrices to maximize the sum throughput while protecting the primary network's performance in the U-CWPCN scenario. The optimal transmit energy and information covariance matrices have been determined for a given time allocation. Then, the optimal time allocation has been found by a simple line search method. Simulation results have confirmed that the proposed algorithm outperforms the EP method.

Appendix A
Proof of Lemma 1

We first prove that $\mu_i^\star > 0$ for $i = 1, \ldots, K$. If $\mu_i^\star$ equals 0 for some i, the dual function in (13) becomes unbounded, i.e., $\mathcal{G}_U(\{\mu_i\}, \{\nu_j\}) \rightarrow \infty$, since the optimal $\bar{\mathbf{S}}_i^\star$ can be written by

$$\bar{\mathbf{S}}_i^\star = c\mathbf{q}\mathbf{q}^H,$$

with $c \rightarrow \infty$, where $\mathbf{q}$ is a vector in the nullspace of $\bar{\mathbf{h}}_i$. Therefore, to obtain a bounded optimal solution, we must have $\mu_i^\star > 0, \ \forall i$.

Next, we show that $\mathbf{A} \triangleq \sum_{i=1}^K \mu_i^\star \mathbf{G}_i^H \mathbf{G}_i - \nu_2^\star \mathbf{h}\mathbf{h}^H - \nu_1^\star \mathbf{I}_M$ is a negative semi-definite matrix. The KKT conditions of problem (P1-2) with respect to $\bar{\mathbf{W}}^\star$ are expressed as [19]

$$\mathbf{A}\bar{\mathbf{W}}^\star = \mathbf{0}, \tag{18}$$

$$\nu_1^\star(\bar{\tau} P_{\text{H-AP}} - \mathrm{tr}(\bar{\mathbf{W}}^\star)) = 0, \tag{19}$$

$$\nu_2^\star(\bar{\tau}\Gamma - \mathbf{h}^H \bar{\mathbf{W}}^\star \mathbf{h}) = 0, \tag{20}$$

$$\bar{\mathbf{W}}^\star \succeq \mathbf{0}.$$

We will prove by contradiction. Let us suppose that the matrix $\mathbf{A}$ is negative definite. In this case, it is easy to show that $\bar{\mathbf{W}}^\star$ in (18) is a zero matrix, which results in $\nu_1^\star = \nu_2^\star = 0$ due to the complementary slackness conditions in (19) and (20). Then, we have $\mathbf{A} = \sum_{i=1}^K \mu_i^\star \mathbf{G}_i^H \mathbf{G}_i$, which is a positive semi-definite matrix since $\mu_i^\star > 0, \forall i$. Note that this contradicts the assumption $\mathbf{A} \prec 0$, and thus $\mathbf{A}$ cannot be a negative definite matrix.

Second, suppose that $\mathbf{A}$ has at least one positive eigenvalue. Then, the optimal energy covariance matrix becomes $\bar{\mathbf{W}}^\star = c\mathbf{u}_{\mathbf{A},1}\mathbf{u}_{\mathbf{A},1}^H$, where c is a positive number and $\mathbf{u}_{\mathbf{A},1}$ indicates an eigenvector of $\mathbf{A}$ corresponding to the positive eigenvalue $\lambda_{\mathbf{A},1}$. As mentioned before, the dual function is unbounded by setting $c \rightarrow \infty$, and all eigenvalues of $\mathbf{A}$ should be non-positive. As a result, we can conclude that the matrix $\mathbf{A}$ is negative semi-definite.

Based on the derived results, we now prove that $\bar{\mathbf{W}}^\star$ is given by (15). Let us define the eigenvalue decomposition of $\mathbf{B} \triangleq \sum_{i=1}^{K} \mu_i^\star \mathbf{G}_i^H \mathbf{G}_i - \nu_2^\star \mathbf{h}\mathbf{h}^H$ as $\mathbf{B} = \mathbf{U_B}\mathbf{\Lambda_B}\mathbf{U_B}^H$, where $\mathbf{U_B} \in \mathbb{C}^{M\times M}$ and $\mathbf{\Lambda_B} = \mathrm{diag}(\lambda_{\mathbf{B},1}, \ldots, \lambda_{\mathbf{B},M})$ with $\lambda_{\mathbf{B},1} \geq \ldots \geq \lambda_{\mathbf{B},M}$ represent the eigenvector matrix and the eigenvalue matrix of $\mathbf{B}$, respectively. To guarantee the negative semi-definite property of $\mathbf{A} = \mathbf{U_B}(\mathbf{\Lambda_B} - \nu_1^\star \mathbf{I})\mathbf{U_B}^H$, the optimal dual variable $\nu_1^\star$ should be equal to the maximum eigenvalue of $\mathbf{B}$, i.e., $\nu_1^\star = \lambda_{\mathbf{B},1} > 0$. Then, from (18), the optimal energy covariance matrix can be expressed as $\bar{\mathbf{W}}^\star = \alpha \mathbf{u}_{\mathbf{B},1}\mathbf{u}_{\mathbf{B},1}^H$ for any non-negative number α, since we have $\mathbf{A}\mathbf{u}_{\mathbf{B},1} = 0$. From the complementary slackness condition (19), it follows $\mathrm{tr}(\bar{\mathbf{W}}^\star) = \alpha = \tau P_{\text{H-AP}}$, which implies that the optimal energy covariance matrix $\bar{\mathbf{W}}^\star$ is written by $\bar{\mathbf{W}}^\star = \tau P_{\text{H-AP}} \mathbf{u}_{\mathbf{B},1}\mathbf{u}_{\mathbf{B},1}^H$. This completes the proof.

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