Robust Resource Allocation Based Energy Harvesting in Distributed Antenna System

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***Abstract*—In this paper, we study a multiuser distributed antenna system with simultaneous wireless information and power transmission under the assumption of imperfect channel state information (CSI). Considering CSI errors, the minimum signal-to-interference-plus-noise ratio (SINR) is maximized subject to individual energy harvesting constraint for each mobile station by jointly designing beamforming and the power splitting (PS). First, an efficient algorithm is proposed to convert the original problem to a set of "dual" min-max power balancing problems. Then, an semi-definite programming based iterative algorithm is developed to achieve a local optimal rank-one solution. Simulation results are shown to validate the robustness and effectiveness of the proposed algorithm.**

I. Introduction

Recently, a distributed antenna system (DAS) has received more attentions as a new cellular communication structure to expand coverage and increase sum rates [1]. One of the limits in current cellular communication systems is the short lifetime of batteries. To combat the battery problem of mobile users, simultaneous wireless information and power transmission (SWIPT) has been studied in [2]-[5]. With the aid of the SWIPT, users can charge their devices based on the received signal [3]. By adopting the power splitting (PS) receiver, the SWIPT scheme for multiple-input single-output (MISO) downlink systems has been examined in [3] where perfect channel state information at the transmitter (CSIT) was assumed. In practice, however, due to channel estimation errors, it is not possible to obtain perfect CSIT [6].

On the other hand, some recent works have investigated SWIPT in DAS [7]-[9]. A tradeoff between the power transfer efficiency and the information transfer capacity has been introduced in [7]. The work in [8] examined a design of robust beamforming and PS for multiuser downlink DAS SWIPT. However, only one antenna was considered in each DA port. The authors in [9] investigated resource allocation for DAS SWIPT systems based on the worst-case model, where per-DA port power constraint was adopted. However, joint optimal design of transmit beamforming and the receive PS factor for SWIPT in DAS PS-based systems with multiple transmit antennas of each DA port, has not been considered in the literature yet.

Motivated by the existing literature [7]-[9], in this paper, we study a joint design of robust beamforming at the DA port and the receive PS factors at mobile stations (MSs) in DAS SWIPT systems with imperfect CSI. Our aim is to maximize the worst-case signal-to-interference-and-noise ratio (SINR) subject to EH constraint and per-DA port power constraint. For a given SINR target, the original problem is decomposed into a set of min-max per-DA port power balancing problems. The equivalent forms of all the constraints are derived. Then we prove that a solution of the relaxed semi-definite program (SDP) is always rank-two. Also, to recover a near-optimal rank-one solution, we employ a penalty function method instead of the conventional Gaussian randomization (GR) technique. Simulations results have been conducted to provide the robustness of the proposed algorithm, which has the superior performances compared with the conventional scheme.

II. System Model and Problem Formulation

We consider a multiuser downlink DAS SWIPT system model, which contains M DA ports and K single-antenna MSs. We assume that each DA port is equipped with N_T antennas, which have individual power constraint. All DA ports are physically connected to the main processing unit (MPU) via an exclusive radio frequency (RF) link. Furthermore, the DAS channel model consists of both small scale and large scale fading [1]. We denote the channel between the m-th DA port $(m = 1, ..., M)$ and the k-th MS $(k = 1, ..., K)$ as $\mathbf{h}_{m,k} = d_{m,k}^{-\gamma/2}\bar{\mathbf{h}}_{m,k}$, where $d_{m,k}$ indicates the distance between the m-th DA port and the k-th MS, γ equals the path loss exponent, and $\bar{\mathbf{h}}_{m,k} \in \mathbb{C}^{N_T\times 1}$ stands for the channel vector for small scale fading. For the k-th MS, the channel vector is given as $\mathbf{h}_k = [\mathbf{h}_{1,k}^T, ..., \mathbf{h}_{M,k}^T]^T$.

Due to channel estimation error, CSI is imperfect at each DA port. It is assumed that the uncertainty of the channel is determined by $\mathcal{H}_k$ as an Euclidean ball [6] as

$$\mathcal{H}_k = \left\{\hat{\mathbf{h}}_k + \Delta\mathbf{h}_k \left|\Delta\mathbf{h}_k^H\mathbf{\Phi}_k\Delta\mathbf{h}_k \le \varepsilon_k^2\right.\right\}, k = 1, 2, ..., K \quad (1)$$

where the ball is centered around the actual value of the estimated CSI vector $\hat{\mathbf{h}}_k$ from M DA ports to the k-th MS,

$\Delta\mathbf{h}_k \in \mathbb{C}^{MN_T\times 1}$ is the norm-bounded uncertainty vector, $\mathbf{\Phi}_k \in \mathbb{C}^{MN_T\times MN_T}$ represents the orientation of the region, and ε_k defines the radius of the ball.

During one time slot, K independent signal streams are conveyed simultaneously to K MSs. Specifically, the transmit beamforming $\mathbf{v}_k^m \in \mathbb{C}^{N_T\times 1}$ is allocated for the k-th MS at the m-th DA port. Thus, we denote the joint transmit beamformer $\mathbf{v}_k \in \mathbb{C}^{MN_T\times 1}$ used by M DS ports for the k-th MS as $\mathbf{v}_k = vec\left(\begin{bmatrix} \mathbf{v}_{1,k} & \mathbf{v}_{2,k} & \ldots & \mathbf{v}_{M,k} \end{bmatrix}\right)$. Then, the transmitted signal to the k-th MS is $\mathbf{x}_k = \mathbf{v}_k s_k$, where $s_k \sim \mathcal{CN}(0,1)$ indicates the transmitted data symbol for the k-th MS.

The received signal at the k-th MS is expressed as

$$y_k = \mathbf{h}_k^H \sum\nolimits_{j=1}^{K} \mathbf{v}_j s_j + n_k,$$

where n_k represents the additive white gaussian noise (AWGN) with variance σ_k^2 at the k-th MS. We assume that each MS splits the received signal power into two parts using a power splitter [3]. The PS divides the $\rho_k \in (0,1]$ portion and the $1-\rho_k$ portion of the received signal power to the ID and the EH, respectively. Therefore, the split signal for the ID of the k-th MS is written as

$$y_k^{ID} = \sqrt{\rho_k}\big(\mathbf{h}_k^H \sum\nolimits_{j=1}^{K} \mathbf{v}_j s_j + n_k\big) + z_k,$$

where z_k stands for the AWGN with variance δ_k^2 during the ID process at the k-th MS. Then, the received SINR for the k-th MS is defined as

$$\mathrm{SINR}_k(\{\mathbf{v}_k\},\rho_k) = \frac{\rho_k |\mathbf{h}_k^H \mathbf{v}_k|^2}{\rho_k \sum_{j\neq k} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \rho_k \sigma_k^2 + \delta_k^2}. \quad (2)$$

Also, due to the broadcast nature of wireless channels, the energy carried by all signals, i.e., the $1-\rho_k$ portion of $\mathbf{v}_k$, can be harvested at the k-th MS, and the split signal for the EH of the k-th MS is thus given as

$$y_k^{EH} = \sqrt{1-\rho_k}\big(\mathbf{h}_k^H \sum\nolimits_{j=1}^{K} \mathbf{v}_j s_j + n_k\big).$$

Then, the harvested energy at the k-th MS is obtained as

$$E_k = \zeta_k (1-\rho_k) \left(\sum\nolimits_{j=1}^{K} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma_k^2\right)$$

where $\zeta_k \in (0 \;\; 1]$ is the energy conversion efficiency for the EH of the k-th MS.

In this paper, we assume that each DA port has its own power constraint $P_m\,(m = 1,...,M)$. Let us define an $MN_T \times MN_T$ square matrix $\mathbf{D}_m \triangleq \mathrm{diag}(\underbrace{0,...,0}_{(m-1)N_T}, \underbrace{1,...,1}_{N_T}, \underbrace{0,...,0}_{(M-m)N_T})$. Then, per-DA per power constraint is given as $\sum_{k=1}^{K} \mathrm{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H) \leq \alpha P_m, \forall m$. Our aim is to jointly optimize the transmit beamforming vector and the PS factor by maximizing the minimum SINR subject to EH constraint and per-DA power constraint. Then, the robust optimization problem is expressed as

$$\max_{\{\mathbf{v}_k\},\rho_k} \min_{\mathbf{h}_k\in\mathcal{H}_k} \quad \mathrm{SINR}_k(\{\mathbf{v}_k\},\rho_k) \quad (3a)$$

$$\text{s.t.} \quad \zeta_k (1-\rho_k) \big(\sum\nolimits_{j=1}^{K} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma_k^2\big) \geq e_k, \;\forall k, \quad (3b)$$

$$\sum\nolimits_{k=1}^{K} \mathrm{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H) \leq P_m, \;\forall m, \quad (3c)$$

$$0 < \rho_k \leq 1, \;\forall k, \quad (3d)$$

where e_k represents the required harvested power of the k-th MS. Problem (3) is non-convex and hard to solve efficiently.

III. Proposed Robust Joint Design

In this section, we propose a robust joint design algorithm for problem (3) to generates a local optimal rank-one solution.

To make problem (3) tractable, we decompose the problem into a set of the min-max per-DA port power balancing problems, one for each given SINR target $\Gamma > 0$ [8]. Using bisection search over Γ, the optimal solution to problem (3) can be obtained by solving the corresponding min-max per-DA port power balancing problem with different Γ. Then, for a given Γ, we focus on the following min-max per-DA port power balancing problem as

$$\min_{\{\mathbf{v}_k\},\rho_k} \max_{1\leq m\leq M} \frac{\sum_{k=1}^{K} \mathrm{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H)}{P_m} \quad (4a)$$

$$\text{s.t.} \quad \zeta_k (1-\rho_k) \big(\sum\nolimits_{j=1}^{K} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma_k^2\big) \geq e_k, \;\forall k, \quad (4b)$$

$$\mathrm{SINR}_k(\{\mathbf{v}_k\},\rho_k) \geq \Gamma, \;\forall k, \quad (4c)$$

$$0 < \rho_k \leq 1, \;\forall k. \quad (4d)$$

We represent $\alpha^*(\Gamma)$ as the optimal objective value of problem (4). Note that based on the equation $\alpha^*(\Gamma) = 1$ [8, Lemma 2], we can obtain the optimal beamforming solution for problem (3). Problem (4) is still non-convex in terms of the non-convex objective function (4a). First, we tackle the objective function (4a) by introducing an auxiliary variable α. Then, the min-max per-DA port power balancing problem (4) can be rewritten as

$$\min_{\{\mathbf{v}_k\},\,\rho_k,\,\alpha,\,\mathbf{h}_k\in\mathcal{H}_k} \quad \alpha \quad (5a)$$

$$\text{s.t.} \quad \sum\nolimits_{k=1}^{K} \mathrm{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H) \leq \alpha P_m, \forall m, \quad (5b)$$

$$(4b),\; (4c),\; (4d).$$

We can see that problem (5) has semi-infinite constraints (4b) and (4c), which are non-convex. To make the constraint (4b) tractable, the following lemma is introduced to convert (4b) into a quadratic matrix inequality (QMI).

Let us define an $MN_T \times MN_T$ square matrix $\mathbf{V}_k$ as $\mathbf{V}_k = \mathbf{v}_k \mathbf{v}_k^H$. By utilizing the Schur complement in [10], the constraint (4b) can be converted into

$$\begin{bmatrix} \zeta_k (1-\rho_k) & \sqrt{e_k} \\ \sqrt{e_k} & \big(\hat{\mathbf{h}}_k + \Delta\mathbf{h}_k\big)^H \mathbf{R} \big(\hat{\mathbf{h}}_k + \Delta\mathbf{h}_k\big) + \sigma_k^2 \end{bmatrix} \succeq \mathbf{0}, \quad (6)$$

where $\mathbf{R} \triangleq \sum_{k=1}^{K} \mathbf{V}_k$. Note that (6) is still non-convex. In order to remove the channel uncertainty in (6), the following lemma is required to convert the constraint (6) into linear matrix inequality (LMI).

Lemma 2: [13, Theorem 3.5] Let us denote $\mathbf{U}_k \in \mathbb{C}$, for $k \in [1,6]$. If $\mathbf{T}_i \succeq \mathbf{0}$ for $i = 1,2$, then the following QMI

$$\begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 + \mathbf{U}_3\mathbf{X} \\ (\mathbf{U}_2 + \mathbf{U}_3\mathbf{X})^H & \mathbf{U}_4 + \mathbf{X}^H\mathbf{U}_5 + \mathbf{U}_5^H\mathbf{X} + \mathbf{X}^H\mathbf{U}_6\mathbf{X} \end{bmatrix} \succeq \mathbf{0},$$

$$\mathbf{I} - \mathbf{X}^H\mathbf{T}_i\mathbf{X} \succeq \mathbf{0}, \text{ for } \forall \mathbf{X}$$

are equivalent to the LMI

$$\begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 & \mathbf{U}_3 \\ \mathbf{U}_2^H & \mathbf{U}_4 & \mathbf{U}_5^H \\ \mathbf{U}_3^H & \mathbf{U}_5 & \mathbf{U}_6 \end{bmatrix} + \lambda_1 \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_1 \end{bmatrix} + \lambda_2 \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_2 \end{bmatrix} \succeq \mathbf{0},$$

where $\lambda_i \geq 0$ $(i = 1,2)$. ■

To proceed, we set $\mathbf{X}=\Delta\mathbf{h}_k$, $\mathbf{T}_1=1/\varepsilon_k^2\mathbf{I}$, $\mathbf{T}_2=\mathbf{0}$, $\mathbf{U}_1=1-\rho_k$, $\mathbf{U}_2=\sqrt{e_k}$, $\mathbf{U}_3=\mathbf{0}_{1\times MN_T}$, $\mathbf{U}_4=\hat{\mathbf{h}}_k^H\mathbf{R}\hat{\mathbf{h}}_k+\sigma_k^2-t_k$, $\mathbf{U}_5=\hat{\mathbf{h}}_k^H\mathbf{R}$, $\mathbf{U}_6=\mathbf{R}$. Then, by exploiting Lemma 2, (6) can be equivalently modified as the following convex LMI

$$\mathbf{A}_k=\begin{bmatrix}\zeta_k(1-\rho_k) & \sqrt{e_k} & \mathbf{0}_{1\times MN_T}\\ \sqrt{e_k} & \hat{\mathbf{h}}_k^H\mathbf{R}\hat{\mathbf{h}}_k+\sigma_k^2-t_k & \hat{\mathbf{h}}_k^H\mathbf{R}\\ \mathbf{0}_{MN_T\times 1} & \mathbf{R}\hat{\mathbf{h}}_k & \mathbf{R}+\frac{t_k}{\varepsilon_k^2}\mathbf{I}\end{bmatrix}\succeq\mathbf{0}, \quad (7)$$

where $t_k\geq 0$ is a slack variable.

Next, we transform (4c) to the convex one. Due to the definition of SINR_k and $\mathcal{H}_k$, (4c) can be recast as

$$\rho_k\left|\left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)^H\mathbf{v}_k\right|^2\geq\Gamma\big(\rho_k\sum\nolimits_{j\neq k}\left|\left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)^H\mathbf{v}_j\right|^2+\rho_k\sigma_k^2+\delta_k^2\big),$$

and thus, it follows

$$\rho_k\big(\left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)^H\mathbf{M}_k\left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)+\sigma_k^2\big)\geq\delta_k^2, \quad (8)$$

where $\mathbf{M}_k=\frac{1}{\Gamma}\mathbf{V}_k-\sum_{j\neq k}\mathbf{V}_j$.

Also, we utilize a similar method for (8) as follows. By applying Lemma 1, (8) can be changed into

$$\begin{bmatrix}\rho_k & \delta_k\\ \delta_k & \left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)^H\mathbf{M}_k\left(\hat{\mathbf{h}}_k+\Delta\mathbf{h}_k\right)+\sigma_k^2\end{bmatrix}\succeq\mathbf{0}. \quad (9)$$

To remove the channel uncertainty $\Delta\mathbf{h}_k$ in (9), Lemma 2 is adopted, and the constraint (9) is equivalently modified as

$$\mathbf{B}_k=\begin{bmatrix}\rho_k & \delta_k & \mathbf{0}_{1\times MN_T}\\ \delta_k & \hat{\mathbf{h}}_k^H\mathbf{M}_k\hat{\mathbf{h}}_k+\sigma_k^2-r_k & \hat{\mathbf{h}}_k^H\mathbf{M}_k\\ \mathbf{0}_{MN_T\times 1} & \mathbf{M}_k\hat{\mathbf{h}}_k & \mathbf{M}_k+\frac{r_k}{\varepsilon_k^2}\mathbf{I}\end{bmatrix}\succeq\mathbf{0}, \quad (10)$$

where $r_k\geq 0$ is a slack variable. Defining $\hat{\mathbf{V}}_{m,k}$ as $\hat{\mathbf{V}}_{m,k}=\mathbf{D}_m\mathbf{V}_k$, problem (5) is thus reformulated as

$$\begin{aligned}\min_{\{\mathbf{V}_k\},\rho_k,\alpha,t_k,r_k}\quad & \alpha\\ \text{s.t.}\quad & \sum\nolimits_{k=1}^{K}\mathrm{tr}(\hat{\mathbf{V}}_{m,k})\leq\alpha P_m,\ \forall m,\\ & \mathbf{A}_k\succeq\mathbf{0},\ \mathbf{B}_k\succeq\mathbf{0},\ \mathbf{V}_k\succeq\mathbf{0},\ (4\mathrm{d}),\\ & t_k\geq 0,\ r_k\geq 0,\ \mathrm{rank}(\mathbf{V}_k)=1,\forall k.\end{aligned} \quad (11)$$

Problem (11) is difficult to solve in general due to the rank-one constraint. Therefore, we employ the semi-definite relaxation (SDR) technique [11] which simply drops the constraints $\mathrm{rank}(\mathbf{V}_k)=1$ for all $\mathbf{V}_k$'s. Then, problem (11) becomes a convex problem which can be solved efficiently by CVX tool [12]. In the following theorem, we show that a solution $\mathbf{V}_k^*$ to problem (11) satisfies $\mathrm{rank}(\mathbf{V}_k^*)\leq 2$.

Theorem 1: If problem (11) is feasible, the rank of a solution $\mathbf{V}_k^*$ to problem (11) is less than or equal to 2.

Proof: See Appendix A. ■

After $\mathbf{V}_k^*$ is obtained, if $\mathrm{rank}(\mathbf{V}_k^*)=1$, we can compute an optimal solution $\mathbf{v}_k$ by eigenvalue decomposition (EVD) of $\mathbf{V}_k^*$. If $\mathrm{rank}(\mathbf{V}_k^*)=2$, we use the conventional Gaussian randomization (GR) technique [11] to find $\mathbf{v}_k$. In particular, the GR technique generates a suboptimal solution. Hence, when $\mathrm{rank}(\mathbf{V}_k^*)=2$, we will propose an iterative algorithm to recover the optimal rank-one solution by following the approach in [15].

First, since $\hat{\mathbf{V}}_{m,k}$ is always semi-positive definite, we have $\mathrm{tr}(\hat{\mathbf{V}}_{m,k})\geq\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$. Thus, we can prove that $\mathrm{rank}(\hat{\mathbf{V}}_{m,k})=1$ if $\mathrm{tr}(\hat{\mathbf{V}}_{m,k})\leq\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$. Then, we can transform the constraint $\mathrm{rank}(\hat{\mathbf{V}}_{m,k})=1$ into the single reverse convex constraint as

$$\sum\nolimits_{k=1}^{K}(\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k}))\leq 0.$$

Note that the function $\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$ on the set of Hermitian matrices is convex. When $\sum_{k=1}^{K}(\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k}))$ is small enough, $\hat{\mathbf{V}}_{m,k}$ will approach $\lambda_{\max}(\hat{\mathbf{V}}_{m,k})\hat{\mathbf{v}}_{m,k}^{\max}(\hat{\mathbf{v}}_{m,k}^{\max})^H$, where $\hat{\mathbf{v}}_{m,k}^{\max}$ represents the eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$ with $\|\hat{\mathbf{v}}_{m,k}^{\max}\|=1$. Then the optimal transmit beamformer vector can be expressed by

$$\mathbf{v}_{m,k}=\sqrt{\lambda_{\max}(\hat{\mathbf{V}}_{m,k})}\hat{\mathbf{v}}_{m,k}^{\max}, \quad (12)$$

which satisfies the rank-one constraint.

Thus, in order to make $\sum_{k=1}^{K}(\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k}))$ as small as possible, we adopt the exact penalty method [10]. First, introducing a sufficiently large penalty ratio $\theta>0$, the alternative formulation is considered as

$$\min_{\{\mathbf{V}_k\},\rho_k,\alpha,t_k,r_k}\quad\alpha \quad (13a)$$

$$\text{s.t.}\ \mathbf{A}_k\succeq\mathbf{0},\ \mathbf{B}_k\succeq\mathbf{0},\ \mathbf{V}_k\succeq\mathbf{0},\ (4\mathrm{d}), \quad (13b)$$

$$\sum\nolimits_{k=1}^{K}(\mathrm{tr}(\hat{\mathbf{V}}_{m,k})+\theta(\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k})))\leq\alpha P_m \quad (13c)$$

$$t_k\geq 0, r_k\geq 0,\ \forall k. \quad (13d)$$

For (13c), the difference $\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$ will be minimized when θ is large enough. Clearly, (13c) is set to minimize $\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k})$. Note that (13c) is non-convex due to the coupled θ and $\hat{\mathbf{V}}_{m,k}$. To eliminate the coupling between θ and $\hat{\mathbf{V}}_{m,k}$, we apply the following lemma to provide an effective approximation of (13c).

Lemma 3: Let us define $\mathbf{C}\in\mathbb{H}_+$ and $\mathbf{E}\in\mathbb{H}_+$. Then, it always follows $\lambda_{\max}(\mathbf{C})-\lambda_{\max}(\mathbf{E})\geq\boldsymbol{e}_{\max}^H(\mathbf{C}-\mathbf{E})\boldsymbol{e}_{\max}$, where $\boldsymbol{e}_{\max}$ denotes the eigenvector corresponding to the maximum eigenvalue of $\mathbf{E}$. ■

According to Lemma 3, we propose an iterative algorithm to recover a local optimal solution. For given some feasible $\{\hat{\mathbf{V}}_{m,k}^{(n)}\}$ to problem (13), we get

$$\begin{aligned}&\mathrm{tr}(\hat{\mathbf{V}}_{m,k}^{(n+1)})+\theta\Big[\mathrm{tr}(\hat{\mathbf{V}}_{m,k}^{(n+1)})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k}^{(n)})\\ &\quad-(\hat{\mathbf{v}}_{m,k}^{\max,(n)})^H\big(\hat{\mathbf{V}}_{m,k}^{(n+1)}-\hat{\mathbf{V}}_{m,k}^{(n)}\big)\hat{\mathbf{v}}_{m,k}^{\max,(n)}\Big]\\ &\leq\mathrm{tr}(\hat{\mathbf{V}}_{m,k}^{(n)})+\theta\big(\mathrm{tr}(\hat{\mathbf{V}}_{m,k}^{(n)})-\lambda_{\max}(\hat{\mathbf{V}}_{m,k}^{(n)})\big),\end{aligned} \quad (14)$$

where the superscript n represents the n-th iteration.

Hence, the following SDP problem generates an optimal solution $\mathbf{V}_{m,k}^{(n+1)}$ that is better than $\mathbf{V}_{m,k}^{(n)}$ to problem (13) as

$$\min_{\{\mathbf{V}_k\},\rho_k,\alpha,t_k,r_k}\quad\alpha \quad (15a)$$

$$\text{s.t.}\ (13\mathrm{b}),\ (13\mathrm{d}), \quad (15b)$$

$$\sum_{k=1}^{K}\Big\{\mathrm{tr}(\hat{\mathbf{V}}_{m,k})+\theta\big[\mathrm{tr}(\hat{\mathbf{V}}_{m,k})-(\hat{\mathbf{v}}_{m,k}^{\max,(n)})^H\hat{\mathbf{V}}_{m,k}\hat{\mathbf{v}}_{m,k}^{\max,(n)}\big]\Big\}\leq\alpha P_m,\forall m. \quad (15c)$$

To summarize, we can solve problem (3) with a given Γ, and a bisection search algorithm is applied to update Γ for the objective value $\alpha^*=1$. Then, this process is repeated until convergence. For the bisection method, we need to determine an upper bound $\Gamma_{\max}$ as $0 < \Gamma < \Gamma_{\max}$. Then, it follows

$$\mathrm{SINR}_k\left(\{\mathbf{v}_k\}, \rho_k\right) \leq \frac{\rho_k \left|\mathbf{h}_k^H \mathbf{v}_k\right|^2}{\rho_k \sigma_k^2 + \delta_k^2} \leq \frac{\left\|\mathbf{h}_k\right\|^2 \sum_{j=1}^{M} P_m}{\sigma_k^2 + \delta_k^2}.$$

From this, we can set $\Gamma_{\max}$ as $\max\limits_{k} \left\{ \frac{\|\mathbf{h}_k\|^2 \sum_{j=1}^{M} P_m}{\sigma_k^2 + \delta_k^2} \right\}$. Due to monotonicity of α, the bisection search algorithm needs $\mathcal{O}\left(\log_2 \frac{\Gamma_{\max}}{\eta}\right)$ iterations, where η is a small positive constant which controls the accuracy of the bisection search algorithm. It is noted that this bisection search algorithm converges to the optimal solution $\mathbf{v}_k^*$ for problem (3).

IV. Computational Complexity

In this section, we evaluate the computational complexity of the proposed robust design method. Now, we will present the complexity comparison by adopting the analysis in [14]. The complexities of the proposed algorithm are shown in Table I. Here, we denote n, $L^{\max} = \log_2 \frac{\Gamma_{\max}}{\eta}$ and $D^{\max}$ as the number of decision variables, the bisection search number and the local search number in [8], respectively.

1) Algorithm 1 in problem (15) involves $2K$ LMI constraints of size $MN_T + 2$, K LMI constraints of size MN_T, and $4K + M$ linear constraints.

2) Conventional scheme in [8] consists of $2K$ LMI constraints of size $MN_T + 1$, K LMI constraints of size MN_T, and $2K + M$ linear constraints.

For example, for a system with $M = 3, K = 2, N_T = 3$, $L^{\max} = Q^{\max} = 6$, and $D^{\max} = 100$, the complexities of the proposed algorithm and the conventional scheme [8] are $\mathcal{O}(1.96 \times 10^9)$ and $\mathcal{O}(4.31 \times 10^{10})$, respectively. Thus the complexity of the proposed algorithm is only 4.5% of that of the conventional scheme in [8].

V. Simulation Results

In this section, we numerically compare the performance of the proposed algorithms with a circular antenna layout and set $M = 3, K = 3$, and $N_T = 4$. The power of each DA port is set to $P_1 = \frac{P}{6}$, $P_2 = \frac{P}{3}$, and $P_3 = \frac{P}{2}$ as in [8]. Three DA ports form an equilateral triangle while all MSs are uniformly distributed inside a disc with the cell radius $R = \sqrt{112/3}$ m centered at the centroid of the triangle. The j-th DA port is located at $\left(r\cos\frac{2\pi(j-1)}{M},\ r\sin\frac{2\pi(j-1)}{M}\right)$ for $j = 1, ..., M$ with $r = \sqrt{3/7}R$ as in [1]. The pathloss exponent γ is set to be 3. According to this setting, a received SNR loss of 23.5 dB is observed at cell edge users compared to cell center users. All channel coefficients $\bar{\mathbf{h}}_{m,k} \in \mathbb{C}^{N_T \times 1}$ are modelled as Rician fading. The channel vector $\bar{\mathbf{h}}_{m,k}$ is given as $\bar{\mathbf{h}}_{m,k} = \sqrt{\frac{K_R}{1+K_R}} \bar{\mathbf{h}}_{m,k}^{LOS} + \sqrt{\frac{1}{1+K_R}} \bar{\mathbf{h}}_{m,k}^{NLOS}$, where $\bar{\mathbf{h}}_{m,k}^{LOS}$ indicates the line-of-sight (LOS) component with $\|\bar{\mathbf{h}}_{m,k}^{LOS}\|^2 = d_{m,k}^{-\gamma/2}$, $\bar{\mathbf{h}}_{m,k}^{NLOS}$ represents the Rayleigh fading component as $\bar{\mathbf{h}}_{m,k}^{NLOS} \sim \mathcal{CN}(0, d_{m,k}^{-\gamma/2}\mathbf{I})$, and K_R is the Rician factor equal to 3. For simplicity, it is assumed that all MSs have the same set of parameters, i.e., $\zeta_k = \zeta, \delta_k^2 = \delta^2, \sigma_k^2 = \sigma^2$, and $e_k = e$ for $k = 1, ..., K$. In addition, we set $\sigma^2 = -50$ dBm, $\delta^2 = -30$ dBm, and $\zeta = 0.3$. Also, all the channel uncertainties are chosen to be the same as $\varepsilon_k = \varepsilon, \forall k$.

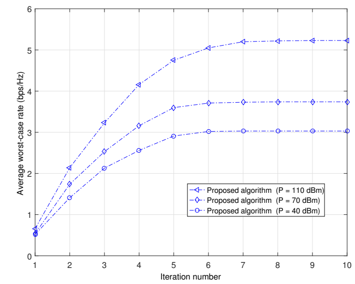

Fig. 1. Convergence performance for various P

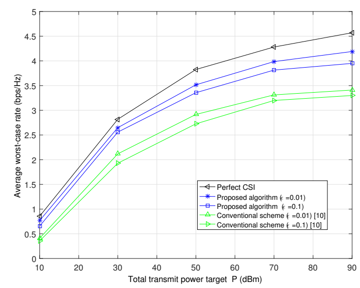

Fig. 2. Average worst-case rate versus P for various ε

Fig. 1 investigates the convergence performance with e = 3 dBm and $\varepsilon = 0.01$. It is clear that the proposed algorithm indeed converge in all cases. After 7 iterations, the steady average worst-case rate is achieved for all P.

In Fig. 2, we exhibit the average worst-case rate versus P for various ε with $e = 3$ dBm. It is observed that the proposed algorithm outperforms the conventional scheme. Also, as ε increases, the performance gap between our proposed algorithm and the conventional scheme becomes larger.

VI. Conclusion

In this paper, we have studied a robust jointly design of beamforming and PS in multiuser DAS SWIPT systems in present of imperfect CSIT. To solve the worst-case SINR maximization problem, we have proposed an iterative algorithm based on penalty function method and SDP. Simulation results have demonstrated the validity of the proposed algorithm.

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Appendix A
Proof of Theorem 1

If the rank-one constraint is ignored, problem (11) becomes convex and satisfies the Slater's condition. Thus, its

TABLE I
COMPLEXITY ANALYSIS OF DIFFERENT ALGORITHMS

Algorithms	Complexity Order
Algorithm 1	$\mathcal{O}\big(nL^{\max}Q^{\max}\sqrt{2K(MN_T{+}2){+}KMN_T{+}4K{+}M}\big\{2K(MN_T{+}2)^3{+}K(MN_T)^3{+}n[2K(MN_T{+}2)^2{+}K(MN_T)^2]{+}4K{+}M{+}n^2\big\}\big)$ where $n=\mathcal{O}(M^2N_T^2{+}3K{+}1)$
Conventional scheme [8]	$\mathcal{O}\big(nD^{max}\sqrt{K(3MN_T{+}2){+}2K{+}M}[2K(MN_T{+}1)^3{+}KM^3N_T^3{+}n(2K(MN_T{+}1)^2{+}KM^2N_T^2{+}2K{+}M){+}n^2]\big)$ where $n=\mathcal{O}(M^2N_T^2{+}2K{+}1)$

duality gap is zero [10]. Assume that the dual variables $\{\mathbf{C}_k\}\in\mathbb{H}_+,\{\mathbf{Q}_k\}\in\mathbb{H}_+,\{\mathbf{S}_k\}\in\mathbb{H}_+$ and $\{\mu_m\}\geq 0$ correspond to the constraint $\mathbf{A}_k\succeq\mathbf{0}$, $\mathbf{B}_k\succeq\mathbf{0},\mathbf{V}_k\succeq\mathbf{0}$ and $\sum_{k=1}^{K}\mathrm{tr}(\mathbf{D}_m\mathbf{V}_k)\leq\alpha P_m$ in (11), respectively. Then, the Lagrangian dual function of the problem (11) is given by

$$\mathcal{L}=\alpha-\sum\nolimits_{k=1}^{K}\left(\mathrm{tr}\left(\mathbf{C}_k\mathbf{A}_k\right)+\mathrm{tr}\left(\mathbf{Q}_k\mathbf{B}_k\right)+\mathrm{tr}\left(\mathbf{S}_k\mathbf{V}_k\right)\right)+\sum\nolimits_{m=1}^{M}\mu_m\Big(\sum\nolimits_{k=1}^{K}\mathrm{tr}(\mathbf{D}_m\mathbf{V}_k)-\alpha P_m\Big). \tag{16}$$

Since $\mathbf{C}_k$ and $\mathbf{T}_k$ are Hermitian matrices, we have

$$\mathrm{tr}(\mathbf{C}_k\mathbf{A}_k)=\mathrm{tr}(\mathbf{C}_k\mathbf{G}_k^H\mathbf{T}\mathbf{G}_k)+\mathrm{tr}(\mathbf{C}_k\mathbf{F}_k),$$
$$\mathrm{tr}(\mathbf{Q}_k\mathbf{B}_k)=\mathrm{tr}(\mathbf{Q}_k\mathbf{G}_k^H\mathbf{M}_k\mathbf{G}_k)+\mathrm{tr}(\mathbf{Q}_k\mathbf{E}_k),$$

where $\mathbf{G}_k=[\ \mathbf{0}\quad\hat{\mathbf{h}}_k\quad\mathbf{I}\ ]$,

$$\mathbf{E}_k=\begin{bmatrix}\rho_k & \delta_k & \mathbf{0}_{1\times MN_T}\\ \delta_k & \sigma_k^2{-}r_k & \mathbf{0}_{1\times MN_T}\\ \mathbf{0}_{MN_T\times 1} & \mathbf{0}_{MN_T\times 1} & \frac{r_k}{\varepsilon_k^2}\mathbf{I}\end{bmatrix},$$

$$\mathbf{F}_k=\begin{bmatrix}\zeta_k(1{-}\rho_k) & \sqrt{e_k} & \mathbf{0}_{1\times MN_T}\\ \sqrt{e_k} & \sigma_k^2{-}t_k & \mathbf{0}_{1\times MN_T}\\ \mathbf{0}_{MN_T\times 1} & \mathbf{0}_{MN_T\times 1} & \frac{t_k}{\varepsilon_k^2}\mathbf{I}\end{bmatrix}.$$

Taking partial derivative of (16) with respect to $\mathbf{V}_k$ and applying the KKT conditions [10], it follows

$$\sum\nolimits_{m=1}^{M}\mu_m\mathbf{D}_m-\Big(\mathbf{G}_k\mathbf{C}_k\mathbf{G}_k^H+\tfrac{1}{\Gamma}\mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^H+\mathbf{S}_k\Big)=0. \tag{17}$$

Let $\{\mathbf{C}_k^*\},\{\mathbf{Q}_k^*\},\{\mathbf{S}_k^*\}$ and $\{\mu_m^*\}$ be the optimal dual solution to problem (11). Note that $\mathbf{Q}_k^*\mathbf{B}_k^*=\mathbf{0}$ from the complementary slackness conditions of problem (11). Since the size of $\mathbf{Q}_k^*$ and $\mathbf{B}_k^*$ is $(MN_T+2)\times(MN_T+2)$, we have $\mathrm{rank}(\mathbf{Q}_k^*)+\mathrm{rank}(\mathbf{B}_k^*)\leq MN_T+2$. Denoting r_k^* as the optimal solution to problem (11), r_k^* in $\mathbf{B}_k^*$ in (11) is non-negative. If $r_k^*>0$, $r_k^*\mathbf{I}{+}\mathbf{M}_k^*$ has full rank. We will prove that $r_k^*\neq 0$ by contradiction.

If $r_k^*=0$, the constraint $\|\Delta\mathbf{h}_k\|^2\leq\varepsilon_k^2$ does not hold since r_k^* is the dual variable for (10). Note that the condition $\|\Delta\mathbf{h}_k\|^2\leq\varepsilon_k^2$ is the only constraint on $\Delta\mathbf{h}_k$. If $\Delta\mathbf{h}_k$ is the worst channel uncertainty which minimizes $q\triangleq\rho_k|\mathbf{h}_k^H\mathbf{v}_k|^2\Big/\Big(\rho_k\sum_{j\neq k}|\mathbf{h}_k^H\mathbf{v}_j|^2+\rho_k\sigma_k^2+\delta_k^2\Big)$, we can always find a scalar $\omega>1$ which satisfies $\|\Delta\mathbf{h}_k\|^2=\varepsilon_k^2$. Substituting the channel uncertainty $\omega\Delta\mathbf{h}_k$ in q, we can find a SINR lower than that obtained by $\Delta\mathbf{h}_k^*$. This is contradictory to the assumption that $\Delta\mathbf{h}_k^*$ minimizes the SINR. Thus, it follows $r_k^*\neq 0$, which leads to $r_k^*>0$. As a result, $r_k^*\mathbf{I}{+}\mathbf{M}_k^*$ becomes full rank, and we have $\mathrm{rank}(\mathbf{B}_k^*)\geq N$. Furthermore, since $\mathrm{rank}(\mathbf{Q}_k^*)$ is non-zero. Thus, the rank of $\mathbf{Q}_k^*$ equals 1. Similarly, we can show that $\mathrm{rank}\left(\mathbf{C}_k^*\right)=1$. Then, it follows $\mathrm{rank}\big(\mathbf{G}_k^H(\mathbf{C}_k^*+\frac{1}{\Gamma}\mathbf{Q}_k^*)\mathbf{G}_k\big)\leq\mathrm{rank}\big(\mathbf{G}_k^H\mathbf{C}_k^*\mathbf{G}_k\big)+\frac{1}{\Gamma}\mathrm{rank}\big(\mathbf{G}_k^H\mathbf{Q}_k^*\mathbf{G}_k\big)=2$.

Thus, multiplying both sides of (17) with $\mathbf{V}_k^*$ yields

$$\big(\sum\nolimits_{m=1}^{M}\mu_m^*\mathbf{D}_m\big)\mathbf{V}_k^*=\big(\mathbf{G}_k\left(\mathbf{C}_k^*+\tfrac{1}{\Gamma}\mathbf{Q}_k^*\right)\mathbf{G}_k^H{+}\mathbf{S}_k^*\big)\mathbf{V}_k^*,$$

where it is noted that $\mathbf{S}_k^*\mathbf{V}_k^*=\mathbf{0}$. Since $\sum_{m=1}^{M}\mu_m^*\mathbf{D}_m$ has full rank, following the rank inequality $\mathrm{rank}(\mathbf{AB})\leq\min(\mathrm{rank}(\mathbf{A}),\mathrm{rank}(\mathbf{B}))$, we can finally prove that

$$\mathrm{rank}\big\{\big(\sum\nolimits_{m=1}^{M}\mu_m^*\mathbf{D}_m\big)\mathbf{V}_k^*\big\}=\mathrm{rank}\left(\mathbf{V}_k^*\right)\leq\mathrm{rank}\Big(\mathbf{G}_k\big(\mathbf{C}_k^*+\frac{1}{\lambda}\mathbf{Q}_k^*\big)\mathbf{G}_k^H\Big)\leq 2.$$

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