Design of Equalized Maximum-Likelihood Receiver

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***Abstract*—A receiver structure referred to as equalized maximum likelihood (EML) is proposed to reduce the complexity of the maximum-likelihood sequence detector (MLSD). Performance of EML is evaluated through the bit-error rate (BER) measurement for a magnetic recording channel. EML exhibits 1-dB gain over the conventional partial response maximum-likelihood (PRML) for 10^{-5} BER.**

I. Introduction

MAXIMUM-LIKELIHOOD sequence detection (MLSD) is known to have optimum performance on an additive white Gaussian noise (AWGN) channel. However, as the length of a channel increases, the number of states in Viterbi detector grows exponentially as L^{ν}, where L is the number of input level and ν is the channel memory.

Many schemes have been proposed to reduce the complexity of the Viterbi detector. One common method is to use a linear equalizer to shape the channel to one having a shorter length [1]–[3]. In some of these approaches, the target response to which Viterbi detector is matched is chosen as a truncated version of the infinite length response. However this choice is not optimal for the finite length case.

In this paper, we have proposed the equalized maximum-likelihood (EML) system for a magnetic recording channel with a new criterion on the target response. The goal of EML is to jointly optimize the equalizer, $\mathbf{w}$, and the target response, $\mathbf{b}$, so that the performance of the receiver is maximized. Then, the Viterbi detector which is matched to the target response, $\mathbf{b}$, is employed for detection.

Several criteria can be applied for the target response. In [3], the mean-square error (MSE) between the equalizer output and the target channel response is minimized subject to a unit-energy constraint on the target response. In contrast, one tap of the target response in EML is confined to a unit value. Also, the position of this unit-tap is not restricted to the first tap. Therefore, EML has an added degree of freedom for choosing the position of the reference tap in the target response. The basic notion of changing a reference tap position in the target response was first introduced in [4] for multi-carrier systems. In this paper, the similar idea is applied to the magnetic recording channel and is shown to work better in the magnetic recording system. Since the target response in EML is not constrained to be minimum phase, we can equalize the channel more easily to the target response when the channel is not minimum phase as in a magnetic recording channel. This will be described in Section II. Later in the simulation section, EML is compared to the partial response maximum-likelihood (PRML) receiver, which is one of the popular choices in current magnetic recording read channel applications. In Section II, we explain the magnetic recording channel model and derive the optimal settings for EML. Superiority of EML is verified by bit-error rate (BER) simulations in Section III.

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II. EML Structure and Optimal Settings

We start with a channel model. Assuming the channel pulse response is $p(t)$, the received channel output may be expressed as

$$y(t) = \sum_k x_k p(t - kT) + n(t) \tag{1}$$

where x_k is the binary channel input data, T is the symbol period, and $n(t)$ denotes an additive white Gaussian noise with power spectral density $N_0/2$.

In a magnetic recording channel, the pulse response $p(t)$ in (1) is often obtained from a Lorentzian step response $s(t)$ by

$$p(t) = s(t) - s(t - T), \qquad \text{where } s(t) = \frac{1}{1 + (2t/pw_{50})^2}.$$

Here pw_{50} denotes the pulsewidth at 50% of its peak value.

In practice, a low-pass filter is employed as a front-end filter followed by a sampler. After sampling at time $t = kT$, the equivalent discrete time channel becomes $y_k = \sum_l p_l x_{k-l} + n_k$. Based on this sampled output, the vector form of the channel output can be found as

$$\mathbf{y}_k = P\mathbf{x}_k + \mathbf{n}_k$$

where

$$\begin{aligned} \mathbf{y}_k^T &= [y_k y_{k-1} \cdots y_{k-N}],\ \mathbf{x}_k^T \\ &= [x_k x_{k-1} \cdots x_{k-N-\nu}],\ \mathbf{n}_k^T \\ &= [n_k n_{k-1} \cdots n_{k-N}]. \end{aligned}$$

Here P is a Toeplitz matrix formed with the sampled pulse response sequence $\{p_l\}$ and N is the length of the equalizer $\mathbf{w}$.

The basic structure of EML is shown in Fig. 1. In EML, the technique used to calculate the equalizer and the target channel response is similar to that used to compute the feedforward and feedback filters for the minimum MSE decision-feedback

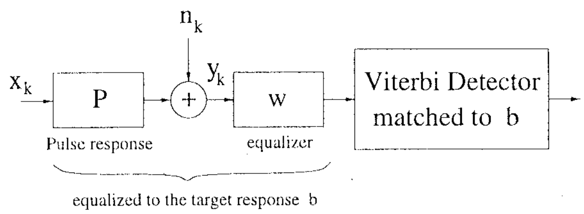

Fig. 1. The structure of EML.

equalizer (MMSE–DFE), except that the target channel response in EML is not restricted to being causal or monic as is the feedback filter in MMSE–DFE. Here we derive the equalizer setting which maximizes performance for the given complexity. The finite-impulse response (FIR) settings are derived in the same way as in [4].

The error signal between the equalizer output and the target response output is defined as $e_k = \mathbf{b}^T \mathbf{x}_{k-\Delta} - \mathbf{w}^T \mathbf{y}_k$ where $\mathbf{b}^T = [b_0 b_1 \cdots b_M]$ is the target response, $\mathbf{w}^T = [w_0 \quad w_1 \quad \cdots \quad w_N]$ is the equalizer and $\mathbf{x}_{k-\Delta}^T = [x_{k-\Delta}\, x_{k-\Delta-1}, \cdots, x_{k-\Delta-M}]$. Here Δ represents the decision delay.

Let $b_t = 1$ denote the reference tap ($t = 0, 1, \cdots, M$) where the position t determines to which tap the equalizer tries to concentrate the channel energy. Here the position t also represents the relative phasing of the two filters $\mathbf{b}$ and $\mathbf{w}$. For example, setting $t = M$ makes $\mathbf{b}$ anticausal.

Defining new augmented vectors $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{y}}_k$ as $\tilde{\mathbf{w}}^T = [w_0 \cdots w_N \quad -b_0 \cdots -b_{t-1} \quad -b_{t+1} \cdots -b_M]$ and $\tilde{\mathbf{y}}_k^T = [y_k \cdots y_{k-N}\, x_{k-\Delta} \cdots x_{k-\Delta-t+1}\, x_{k-\Delta-t-1} \cdots x_{k-\Delta-M}]$, we can rewrite the equation on the error signal as $e_k = x_{k-\Delta-t} - \tilde{\mathbf{w}}^T \tilde{\mathbf{y}}_k$. From the orthogonality principle, we have the following solution:

$$\tilde{\mathbf{w}} = R_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}^{-1} R_{\tilde{\mathbf{y}}x} \tag{2}$$

where $R_{\tilde{\mathbf{y}}_k x} = E(\tilde{\mathbf{y}} x_{k-\Delta-t})$, $R_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}} = E(\tilde{\mathbf{y}}_k \tilde{\mathbf{y}}_k^T)$. This optimized solution yields the corresponding MMSE as $\sigma_{EML}^2 = \overline{\mathcal{E}}_x - R_{\tilde{\mathbf{y}}x}^T R_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}^{-1} R_{\tilde{\mathbf{y}}x}$ where $\overline{\mathcal{E}}_x = E(|x_k|^2)$. In these expressions, the equalizer settings are functions of Δ and t.

If $t = 0$, (2) simply represents the familiar MMSE–DFE solution. The main difference from the DFE solution is that EML does not care about the position of the reference tap. When a channel is not minimum phase, we can improve the receiver performance by placing the reference tap of $\mathbf{b}$ at a location that is not necessarily the first tap. Thus, for a given finite filter, EML can shape the channel more effectively to the target channel. Noncausality of $\mathbf{b}$ can be simply implemented in the Viterbi algorithm with a delay.

Another advantage of EML is that the error sequence is close to a white sequence. The non-white error sequence deteriorates the performance of the MLSD designed for an AWGN channel. It can be shown that the error sequence of EML becomes white as the equalizer length goes to infinity [5]. Thus, the degradation due to the colored noise is not severe.

The filter settings $\mathbf{w}$ and $\mathbf{b}$ are computed with respect to Δ and t in (2). By carrying out an exhaustive search over all

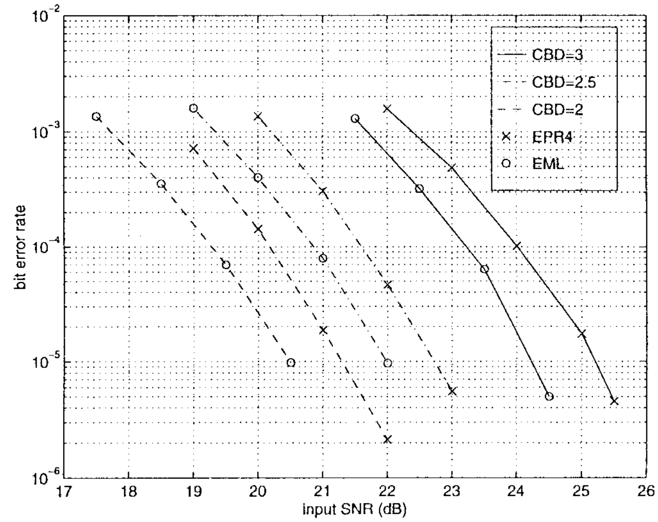

Fig. 2. Comparison of BER.

TABLE I
TARGET RESPONSE $\mathbf{b}$ OF EML

$\frac{pw_{50}}{T}$	2.0	2.5	3.0
$\mathbf{b}$	$[1\ 0.039\ -0.543\ -0.365]$	$[-0.484\ -0.638\ 0.177\ 1]$	$[-0.500\ -0.707\ 0.302\ 1]$

possible Δ and t, we can choose the filter settings $\mathbf{w}$ and $\mathbf{b}$ which maximize the performance of EML. For comparing the performance of different EML settings, the MSE could be used as a simple criterion [4], [5]. However, it turned out to generate suboptimal solutions in some simulations. Also, the actual bit-error rate (BER) measurement may take too long. In order to avoid long Monte Carlo simulations, a simple but accurate semi-analytic error rate computation technique developed in [6] can be employed as a performance measure. Similar expressions for the performance of a sequence detector, taking the colored noise sequence and misequalization into account, can be also found in [7], [8]. After choosing the optimal EML settings with the semi-analytic technique, the performance is evaluated by longer computer simulations in the following section.

III. SIMULATIONS

In this section, we evaluate the performance of EML and compare to that of the partial-response maximum likelihood (PRML) in a magnetic recording channel modeled in Section II.

EPR4 target response introduced by Thapar [9] has been a popular choice of the PRML receiver for a recording density from 1.5 to 3 in a current magnetic recording industry. EPR4 detector presets the target response as $\mathbf{b} = [1\ 1\ -1\ -1]$.

In all simulations, the target response length M is set to 3 requiring 8-state Viterbi detector in EML. Also, a ten-tap feedforward filter, $\mathbf{w}$, is used ($N+1 = 10$). Table I shows the optimized target response $\mathbf{b}$ of EML for different recording densities. The noise power is adjusted for each densities so that the overall BER is around 10^{-5}. A tap value equal to one indicates the reference tap. Simulations show that the optimal reference tap is either the first one $(t = 0)$ or the last one $(t = M)$ in most cases. Note that the optimal target response will change depending on the noise power level.

Fig. 2 plots the simulated BER of EML with eight states and EPR4. For each BER simulation point, 500 errors are counted up by Monte Carlo simulation. The x-axis indicates the input SNR and CBD represents the channel bit density.

This plot shows that EML outperforms EPR4 at all densities. Specifically, EML has 1-dB gain over EPR4 for 10^{-5} BER. It is expected that the gain increases as the recording density grows. It should be noted that the complexity of Viterbi detector in EML will increase because the target response is not an integer number. EML also generates more output levels in comparison with EPR4. However, the increased complexity becomes more manageable with the advancement of the VLSI technology.

In summary, the EML jointly optimizes the equalizer setting and the target response for a given complexity. The EML takes advantage of the fact that the target response $\mathbf{b}$ can be chosen to be noncausal (or mixed phase). We conclude that the complexity for MLSD can be significantly reduced while achieving the performance gain over the conventional PRML detector.

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