Optimization of Tap Spacings for the Tapped Delay Line Decision Feedback Equalizer

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***Abstract*—In this letter, we optimize tap spacings in a tapped delay line equalizer. We derive a set of nonlinear equations for the optimum tap spacings and tap weights for the decision feedback equalizer. A recursive numerical technique is used to obtain the optimum settings. With optimized tap spacings, the proposed solutions provide performance upper bound for the decision feedback equalizer for a given number of taps.**

***Index Terms*—Equalizer, intersymbol interference.**

I. Introduction

CHANNEL equalization has been widely used to compensate the Intersymbol Interference (ISI) in many communication systems. The decision feedback equalizer (DFE) is one popular equalization structure as a practical detection scheme [1], [2]. The feedforward filter in the DFE tries to concentrate the channel energy into the first sample, and then the feedback filter cancels the trailing intersymbol interference (ISI) using the previous decisions.

The optimum equalizer structure consists of a matched filter followed by a baud rate sampler and a tapped delay line equalizer [3]. When the equalizer is realized in a continuous time tap delay line structure, the position of the sampler and the equalizer can be switched. This is illustrated in Fig. 1 for the DFE structure. Here τ_i denotes the tap spacing for the feedforward filter w_i. Usually τ_i is set to the symbol period T or a fraction of T. Note that for the feedback filter b_i, a delay constant is always fixed to the symbol period T.

With an infinite number of taps, the uniformly spaced tapped delay line equalizer ($\tau_i = T$) achieves the optimum performance. In contrast, when the number of taps is limited to a finite number, we can improve the performance of the equalizer by allowing nonuniformly spaced taps.

Recently, some work has been reported on nonuniformly spaced tapped delay line equalizer structures [4], [5]. A method proposed in [4] first computes a sufficiently long T spaced or a fractionally spaced equalizer and then selects a finite number of taps using a search method based on the long equalizer taps to achieve the performance gain. Also, reference [5] provides suboptimum approaches to the tap selection problem without calculating coefficients of a long filter. However, neither of their approaches generates the optimum tap spacings.

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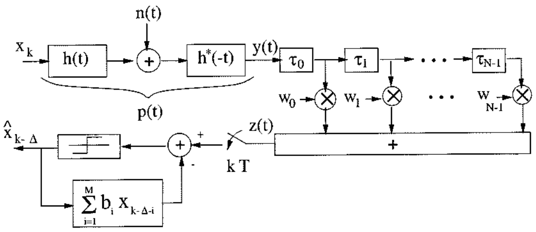

Fig. 1. Structure of the optimum DFE.

In this letter, we jointly optimize the tap weights w_i, b_i and the tap spacings τ_i. Then, the derived solutions provide the performance upper bound of the finite length DFE for a give number of taps.

In the following section, we explain our channel model and derive the optimum tap weights and tap spacings. In the simulation section, we compare the performance of the equalizer with the optimum tap spacings and that of the conventional uniformly spaced equalizer.

II. Nonuniformly Spaced Tap Equalizer

In Fig. 1, the channel output $y(t)$ is defined as

$$y(t) = \sum_m x_m p(t - mT) + n_h(t)$$

where x_m denotes the i.i.d. input sequence with power $\overline{\mathcal{E}}_x$, $p(t)$ is the cascade of the impulse response $h(t)$ and the matched filter $h^*(-t)$, and $n_h(t)$ is the noise signal at the end of the matched filter ($n_h(t) = n(t) * h^*(-t)$).[1] The noise $n(t)$ is assumed to be white and Gaussian with power density $N_0/2$. Here we assume that the overall channel response $p(t)$ is differentiable for all t.

The feedforward filter output $z(t)$ is written as $z(t) = \sum_{l=0}^{N-1} w_l \cdot y(t - \sum_{i=0}^{l} \tau_i)$ where N is the number of the feedforward filter taps. Assuming correct decisions and setting $b_0 = 1$, the error sequence at time kT at the slicer is defined as $e_k = \sum_{n=0}^{M} x_{k-\Delta-n} b_n - z(t)|_{kT}$ where M is the number of the feedback filter tap and Δ accounts for the decision delay. The effect of decision delay Δ in the decision feedback equalizer was analyzed in [6]. Note that Δ could be incorporated as the sampling phase into τ_0.

[1] $(\cdot)^*$ denotes complex conjugate.

The mean squared error (MSE) is computed as

$$\begin{aligned}\sigma^2 =& E(|e_k|^2)\\ =& \overline{\mathcal{E}}_x \sum_{n=0}^{M} |b_n|^2 - 2\mathrm{Re}\left\{\sum_{l=0}^{N-1}\sum_{n=0}^{M} w_l b_n^* E\right.\\ &\left.\left(x_{k-\Delta-n}^* y\left(kT - \sum_{i=0}^{l}\tau_i\right)\right)\right\} + \sum_{l=0}^{N-1}\sum_{n=0}^{N-1} w_l w_n^* E\\ &\cdot\left(y\left(kT-\sum_{i=0}^{l}\tau_i\right)y^*\left(kT-\sum_{i=0}^{n}\tau_i\right)\right).\end{aligned}$$

Assuming the input sequence x_k is uncorrelated with the noise $n(t)$, it can be shown that

$$\begin{aligned}&E\left(x_{k-\Delta-n}^* y\left(kT-\sum_{i=0}^{l}\tau_i\right)\right)\\ &\quad= \overline{\mathcal{E}}_x p((\Delta+n)T - \sum_{i=0}^{l}\tau_i)\\ &\quad\cdot E\left(y\left(kT-\sum_{i=0}^{l}\tau_i\right)y^*\left(kT-\sum_{i=0}^{n}\tau_i\right)\right)\\ &\quad= \overline{\mathcal{E}}_x \sum_m p\left(mT-\sum_{i=0}^{l}\tau_i\right)p^*\left(mT-\sum_{i=0}^{n}\tau_i\right) + R_{l,n}\end{aligned}$$

where $R_{l,n}$ denotes the noise correlation as

$$R_{l,n} = \frac{N_0}{2}\int_{-\infty}^{\infty} h^*\left(\tau - \sum_{i=0}^{l}\tau_i\right)h\left(\tau-\sum_{i=0}^{n}\tau_i\right)d\tau.$$

For simplifying the notations, we define $p_{k,l}$ and $p'_{k,l}$ as $p_{k,l} = p(kT - \sum_{i=0}^{l}\tau_i)$ and $p'_{k,l} = p'(kT - \sum_{i=0}^{l}\tau_i)$. Here $(\cdot)'$ denotes the first derivative of a function. It is also useful to note that taking the partial derivatives of $p_{\Delta,l}$ and $p_{m,l}p_{m,n}$ with respect to τ_j yields

$$\frac{\partial}{\partial\tau_j}p_{\Delta,l} = \begin{cases}-p'_{\Delta,l}, & l \ge j\\ 0, & l<j\end{cases}$$

$$\frac{\partial}{\partial\tau_j}(p_{m,l}p_{m,n}^*) = \begin{cases}-p'_{m,l}p_{m,n}^* - p_{m,l}p_{m,n}^{*'}, & l,n\ge j\\ -p'_{m,l}p_{m,n}^*, & l\ge j>n\\ 0, & j>l,n\end{cases}$$

where we assume $l>n$, and $p_{m,n}^{*'}$ denotes the first derivative of $p^*(mT-\sum_{i=0}^{n}\tau_i)$. Here we neglect the derivative of the noise term by assuming that $R_{l,n}$ is independent of the tap spacing τ_i to simplify our derivation. Using the notations defined above, the MSE can be rewritten as

$$\begin{aligned}\sigma^2 =& \overline{\mathcal{E}}_x\sum_{n=0}^{M}|b_n|^2 - 2\overline{\mathcal{E}}_x\mathrm{Re}\left\{\sum_{l=0}^{N-1}\sum_{n=0}^{M} w_l b_n^* p_{\Delta+n,l}\right\}\\ &+\sum_{l=0}^{N-1}\sum_{n=0}^{N-1} w_l w_n^*\left(\overline{\mathcal{E}}_x\sum_m p_{m,l}p_{m,n}^* + R_{l,n}\right).\end{aligned}$$

To minimize the MSE, we take the derivative of σ^2 with respect to b_j, w_j and τ_j

$$\begin{aligned}\frac{\partial\sigma^2}{\partial b_j} =& 2\overline{\mathcal{E}}_x b_j - 2\overline{\mathcal{E}}_x\sum_{l=0}^{N-1} w_l p_{\Delta+j,l}, \quad \text{for } j=1,2\cdots M\\ \frac{\partial\sigma^2}{\partial w_j} =& -2\overline{\mathcal{E}}_x\left(\sum_{n=0}^{M} b_n p_{\Delta+n,j}^* - \sum_{l=0}^{N-1} w_l\right.\\ &\left.\cdot\left(\sum_m p_{m,l}p_{m,j}^* + \frac{R_{l,j}}{\overline{\mathcal{E}}_x}\right)\right),\\ &\text{for } j=0,1,\cdots N-1.\end{aligned} \tag{1}$$

By setting the above equations to zero and $\tau_i = T$, the classical Wiener filter solution for the T-spaced equalizer is obtained.

To optimize the tap spacings, for $j=0,1,\ldots N-1$, we compute $\partial\sigma^2)/(\partial\tau_j$ as

$$\begin{aligned}\frac{\partial\sigma^2}{\partial\tau_j} =& \overline{\mathcal{E}}_x\left(2\mathrm{Re}\left\{\sum_{l=j}^{N-1}\sum_{n=0}^{M} w_l b_n^* p'_{\Delta+n,l}\right\}\right.\\ &- 2\mathrm{Re}\left\{\sum_{l=j}^{N-1}\sum_{n=0}^{j-1} w_l w_n^* \sum_m p'_{m,l}p_{m,n}^*\right\}\\ &\left.-\sum_{l=j}^{N-1}\sum_{n=j}^{N-1} w_l w_n^* \sum_m\left(p'_{m,l}p_{m,n}^* + p_{m,l}p_{m,n}^{*'}\right)\right).\end{aligned} \tag{2}$$

To further simplify these equations, after a careful derivation we can compute, for $j=0,1,\ldots N-2$,

$$\begin{aligned}\frac{\partial\sigma^2}{\partial\tau_j} - \partial\sigma^2 =& \overline{\mathcal{E}}_x\left(2Re\left\{w_j\sum_{n=0}^{M} b_n^* p'_{\Delta+n,j}\right\}\right.\\ &-2Re\left\{w_j\sum_{l=0}^{j-1} w_l^*\sum_m p'_{m,j}p_{m,l}^*\right.\\ &\left.+w_j\sum_{l=j+1}^{N-1} w_l^*\sum_m p'_{m,l}p_{m,j}^*\right\}\\ &-2Re\left\{w_j\sum_{l=j+1}^{N-1} w_l^*\sum_m\left(p'_{m,j}p_{m,l}^* + p_{m,j}p_{m,l}^{*'}\right)\right.\\ &\left.\left.+|w_j|^2\sum_m p'_{m,j}p_{m,j}^*\right\}\right)\\ =& 2\overline{\mathcal{E}}_x Re\left\{w_j\left(\sum_{n=0}^{M} b_n^* p'_{\Delta+n,j}\right.\right.\\ &\left.\left.-\sum_{l=0}^{N-1} w_l^*\sum_m p'_{m,j}p_{m,l}^*\right)\right\}.\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial\sigma^2}{\partial\tau_{N-1}} =& 2\overline{\mathcal{E}}_x Re\left\{w_{N-1}\left(\sum_{n=0}^{M} b_n^* p'_{\Delta+n,N-1}\right.\right.\\ &\left.\left.-\sum_{l=0}^{N-1} w_l^*\sum_m p'_{m,N-1}p_{m,l}^*\right)\right\}.\end{aligned} \tag{4}$$

In order to obtain w_i, b_i and τ_i which minimize the MSE, all the partial derivatives are set to zero. Since we are looking for nonzero tap values, we can eliminate the w_j term in (3). Combining (1), (3), and (4), we have finally a set of $2N+M$ nonlinear equations:

$$\sum_{n=0}^{M} b_n p^*_{\Delta+n,\,j} = \sum_{l=0}^{N-1} w_l \left(\sum_m p_{m,\,l} p^*_{m,\,j} + \frac{1}{\bar{\mathcal{E}}_x} R_{l,\,j}\right) \sum_{n=0}^{M} b_n p^{*'}_{\Delta+n,\,j}$$
$$= \sum_{l=0}^{N-1} w_l \sum_m p^{*'}_{m,\,j} p_{m,\,l}$$
$$b_i = \sum_{l=0}^{N-1} w_l p_{\Delta+i,\,l}$$

for $i = 1, 2, \ldots M$ and $j = 0, 1, \ldots N-1$.

The above equations can be transformed into a matrix form

$$\begin{bmatrix} \mathbf{Q} \\ \mathbf{dQ} \\ \mathbf{P} \end{bmatrix} \mathbf{w} = \begin{bmatrix} \mathbf{p} & \mathbf{P}^T \\ \mathbf{dp} & \mathbf{dP}^T \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix}$$

where column vectors $\mathbf{w}, \mathbf{b}, \mathbf{p}, \mathbf{dp}$ are defined as $\mathbf{w} = [w_0 \ \ w_1 \cdots w_{N-1}]^T$, $\mathbf{b} = [b_1 \ \ b_2 \cdots b_M]^T$, $\mathbf{p} = [p^*_{\Delta,\,0} p^*_{\Delta,\,1} \cdots p^*_{\Delta,\,N-1}]^T$, $\mathbf{dp} = [p^{*'}_{\Delta,\,0} p^{*'}_{\Delta,\,1} \cdots p^{*'}_{\Delta,\,N-1}]^T$ respectively and N by N matrices $\mathbf{Q} = [Q_{ij}]$, $\mathbf{dQ} = [dQ_{ij}]$ are given by $Q_{ij} = \sum_m p_{m,\,i} p^*_{m,\,j} + R_{i,\,j}/\bar{\mathcal{E}}_x$ and $dQ_{ij} = \sum_m p_{m,\,i} p^{*'}_{m,\,j}$. Also the M by N matrices $\mathbf{P} = [P_{ij}]$ and $\mathbf{dP} = [dP_{ij}]$ are given by $P_{ij} = p^*_{\Delta+1+i,\,j}$ and $dP_{ij} = p^{*'}_{\Delta+1+i,\,j}$. $\mathbf{0}$ and $\mathbf{I}$ are a zero column vector of length M and an identity matrix of size M, respectively. Here $[\cdot]^T$ denotes the transpose operation.

Solving this set of equations is a nonlinear optimization problem which does not have an analytical closed form solution. We solve this problem using the MATLAB optimization toolbox, which utilizes the Gauss–Newton method with a mixed quadratic and cubic line search procedure [7]. With a proper initial condition for w_i and τ_i, a locally optimum solution can be obtained recursively.

III. Simulations

We consider a low pass filter as a front end filter here. In the simulation, we use an exponential channel response

$$p(t) = \sum_{k=0}^{10} e^{-k/10} \mathrm{sinc}\left(\frac{t-kT}{T}\right).$$

We optimize the tap spacings τ_i by solving the nonlinear equations using an iterative numerical technique with all zero w_i and all one τ_i as an initial condition. Fig. 2 compares the output SNR of the uniformly spaced linear equalizer and that of the nonuniformly spaced equalizer with optimized tap spacings in various feedforward filter tap numbers at input SNR=10 dB with a 5 tap feedback filter. The optimum decision delays are determined in the uniformly spaced cases and the same delays are used for the nonuniformly spaced cases. Fig. 3 shows the bit error rate curves for the optimum spacing equalizer and T spaced equalizer with a 10 tap feedforward filter and a 5 tap feedback filter with actual decisions. From the simulations, it is clear that by optimizing the optimum tap spacings, we can achieve a better performance than the conventional uniformly spaced equalizer with the same number of taps.

In summary, we jointly optimize the tap spacings and the tap weights in the decision feedback equalizer. The nonlinear equations are derived for the optimum settings. Using a nonlinear optimization method, we compute the optimum tap spacings. Thus, the results with these optimized settings serve as a performance bound for equalizers with finite length. Finally it should be noted that the nonuniformly spaced tapped delay line structure increases the complexity in the sample and hold delay line block.

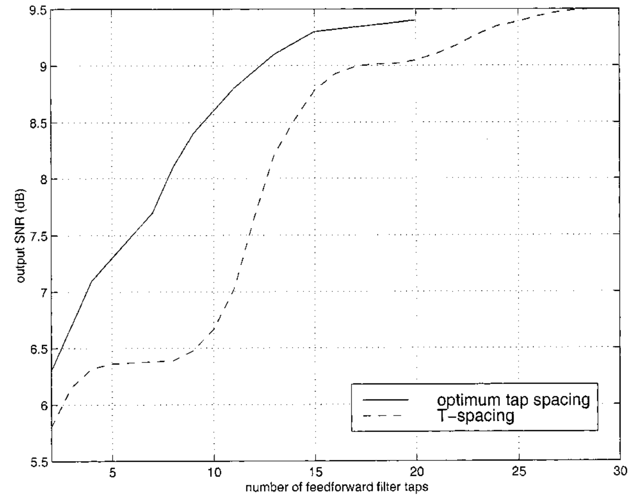

Fig. 2. SNR comparison with different feedforward filter lengths.

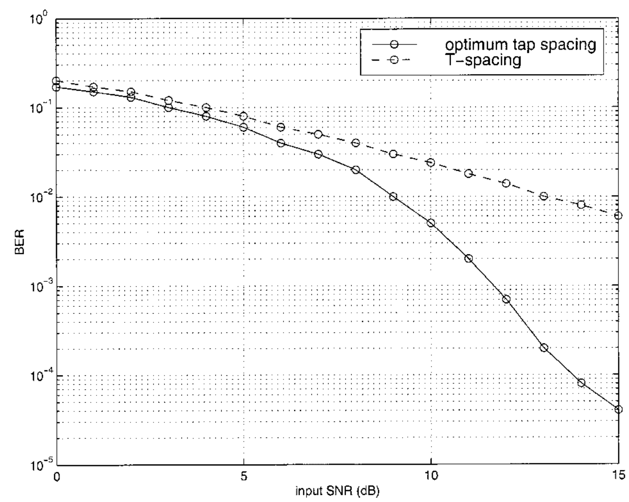

Fig. 3. Bit error rate curves ($N = 10$ and $M = 5$).

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