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Zero-Forcing 기반의 BLAST 채널 용량

(Channel Capacity of BLAST based on the Zero-Forcing criterion)

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요 약

본 논문에서는 신호 대 잡음 비의 관점에서 Zero-Forcing 기반의 BLAST(Bell Labs Layered Space-Time) 구조의 채널 capacity를 점근적으로 분석하고자 한다. MIMO 채널 capacity에 관한 새로운 관계를 소개하고, ZF에 기반한 간섭 무효화를 수행할 때 DBLAST(Diagonal BLAST)에 의해 MIMO 채널의 하한값에 이름을 증명한다. 채널 capacity의 확률 밀도 함수의 정확한 최종식을 분석하고, 각 계층의 채널 capacity의 점근적 현상에 기반한 점근적 ergodic capacity의 최종식을 BLAST에서 유도한다. 본 논문에서 다뤄진 분석에 의해 MIMO 채널의 capacity 현상에 대한 통찰할 수 있다. 모의 실험의 결과를 통해서, 본 논문에서 다뤄진 광범위한 안테나 배열 사이즈에 대한 분석의 타당성과 정확성을 보여주고자 한다.

Abstract

In this paper, we present an asymptotical analysis of channel capacity of Bell labs layered space-time (BLAST) architectures based on a zero-forcing (ZF) criterion in the sense of signal-to-noise ratio (SNR). We begin by introducing a new relationship related to multi-input multi-output (MIMO) channel capacity. We prove that Diagonal Bell Labs Space-Time (DBLAST) attains the lower bound for MIMO channels when interference nulling is carried out based on the ZF-criterion. An exact closed-form expression for the probability density function of the channel capacity is analyzed. Based on the asymptotic behavior of the channel capacity of each layer, closed-form expressions for the asymptotic ergodic capacity are derived for BLAST. Based on the analysis presented in this paper, we gain an insight on the channel capacity behavior for a MIMO channel. Computer simulation results have verified the validity and accuracy of the proposed analysis for a wide range of antenna array sizes.

Keywords: Channel Capacity, multi-input multi-output, (MIMO) systems, Diagonal Bell Labs layered space-time (DBLAST), Zero-Forcing (ZF) criterion.

Ⅰ. Introduction

In recent years, the use of multiple antennas at both transmitter and receiver in wireless communication links has been shown to have the potential of achieving extraordinary bit rates[1~2]. The layered space-time architecture suggested in [3] provides extremely high spectral efficiencies without incurring any penalty in power or bandwidth. More recently, the optimality of the layered space-time concept has been proven in [4] and [5] in terms of channel capacity.

The channel capacity plays a central part in the design and analysis of multi-input multi-output (MIMO) communication systems. The ergodic capacity is the maximum mutual information averaged over all states of time-varying channels[2]. This ergodic capacity can be achieved using an adaptive transmission policy where the power and

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data rate vary depending on the instantaneous channel state. Other capacity definitions for time-varying channels with perfect transmitter and/or receiver channel state information (CSI) include outage capacity and minimum-rate capacity [6]. We consider a rich scattering flat fading MIMO environment where the complex valued propagation coefficients between all pairs of transmitter and receivers are statistically independent, and the channel matrix has full rank. To analytically evaluate these capacities, one needs to find the distribution of channel capacity. Without the distribution, these capacities are evaluated only by statistical simulations for specific channel models. In this case, it is difficult to analyze the effect of system parameters such as the number of transmit and receive antennas and/or signal-to-noise ratio (SNR).

In this paper we introduce an important equivalence related to the channel capacity of Bell Labs Space-Time (BLAST) based on a zero-forcing (ZF) criterion (BLAST-ZF). We prove that the lower bound presented in [1] is equal to the capacity of the Diagonal Bell Labs Space-Time (D-BLAST) architecture when interference nulling is carried out based on the ZF-criterion. This remarkable result allows a simple derivation of closed-form formulas for the ergodic capacity. We provide a closed-form expression for the ergodic capacity. This expression is then used to study the influence of system parameters such as the number of antennas and SNR on capacity. From this, we obtain the optimal number of antennas at the transmitter and receiver for a given total number of antennas in terms of channel capacity.

The remainder of this paper is organized as follows: In Section Ⅱ, we show that BLAST-ZF attains the lower bound for MIMO channels and present the exact pdf of the channel capacity of each layer in BLAST. In Section Ⅲ, we derive an asymptotic closed-form formula of the ergodic capacity of BLAST. Finally, the paper is terminated with conclusions in Section Ⅳ.

Ⅱ. Exact Probability Density Function Of Channel Capacity

In this section, we consider a MIMO system with N transmit and M receive antennas. Let us define the N-dimensional complex transmitted signal vector $\boldsymbol{x}$, and the M-dimensional complex received signal vector y. Then the received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where

$$\mathbf{H} = [\mathrm{h}_1 \mathrm{h}_2 \cdots \mathrm{h}_N] = \begin{pmatrix} \mathbf{h}_{11} \cdots \mathbf{h}_{\mathrm{LN}} \\ \vdots \ \ddots \ \vdots \\ \mathbf{h}_{\mathrm{N1}} \cdots \mathbf{h}_{\mathrm{NN}} \end{pmatrix}, \mathrm{n} = \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$$

Here h_i denotes the ith column of the channel matrix H. We assume that the additive noise terms in n are independent and identically-distributed complex Gaussian with variance σ_n^2. The channel coefficient h_{ij} of $\mathbf{H}$ represents the path gain from the transmit antenna i to the receive antenna j. The path gains are modeled as samples of independent complex Gaussian random variables with variance *0.5* per dimension. This choice models a Rayleigh fading environment. The channel matrix $\mathbf{H}$ is assumed to be known perfectly at the receiver only. In this case, in order to achieve the maximum capacity, the transmit power needs to be divided equally among all transmit antennas[2].

We will limit our attention to the case where $N \leq M$. Analysis of the channel capacity for D-BLAST has been presented in [5]. Each substream of D-BLAST experiences a periodically varying SNR. In this case, the capacity of a periodically varying channel with the channel state information known only to the receiver can be attained with a single code, resulting in the full capacity[5].

$$\sum_{n=1}^{N} \log_2(1 + SNR_n)$$

where $\mathrm{SNR_n}$ indicates the output SNR of the nth antenna's transmission when interference nulling is

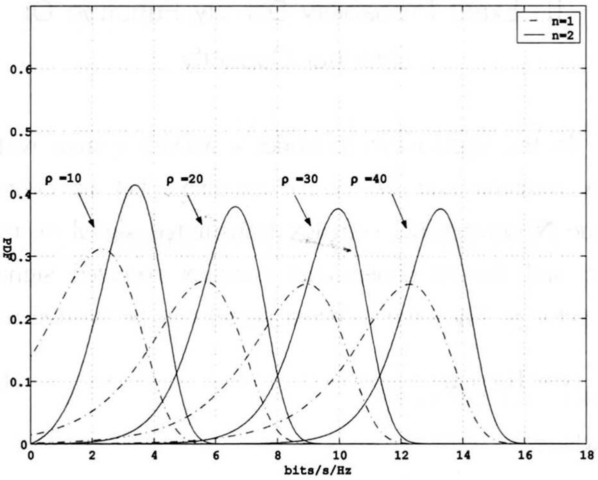

그림 1. 각 layer의 확률 밀도 함수 $f_{C_n(N,M,\rho)}$, $N = M = 4$

Fig. 1. The probability density function $f_{C_n(N,M,\rho)}$ of each layer with $N = M = 4$.

performed by combining the received signals based on the ZF-criterion.

We begin by introducing the following statistically equivalent equations

$$\sum_{n=1}^{N} \log_2(1+SNR_n) \quad \sum_{k=M-N+1}^{N} \log_2(1+\rho/N)\chi_{2k}^2 \tag{2}$$

where χ_{2k}^2 is a unit variance chi-square random variable with $2k$ degrees of freedom and ρ denotes the average received power to noise ratio at each receiver. Here we refer to two random variables x_1 and x_2 with the same statistics (i.e., $f_{x_1}(x) = f_{x_2}(x)$) as being statistically equivalent. The equivalence in (2) shows that BLAST-ZF attains the lower bound in [1]. The proof of the above equivalence is illustrated in Appendix. In this paper, we will derive closed-form expressions for the ergodic capacity of BLAST-ZF based on the equivalence.

We define the nth layer channel capacity as follows $C_n(N,M,\rho)$ as

$$\mathrm{C}_n(N,M,\rho) = \log_2(1+(\rho/N)\mathrm{X}_{2(M-N+n)}^2/2) \tag{3}$$

where X_{2k}^2 is a chi-squared random variable with $2k$ degrees of freedom. Note that $\mathrm{E}\left[\mathrm{X}_{2k}^2\right]$ is equal to $2k$.

Combining (2) and (3), we obtain the channel capacity $C_z(N,M,\rho)$ of the layered space-time architecture with N transmit and M receive antenna as

$$C_z(N,M,\rho) = \sum_{n=1}^{N} C_n(N,M,\rho) = \sum_{k=M-N+1}^{M} \log_2(1+(\rho/N)X_{2k}^2/2)$$

Here $C_z(N,M,\rho)$ is a random variable dependent on $\mathbf{X}_{2k}^2$. The chi-squared distribution with $2k$ degrees of freedom is a result of evaluating the gamma distribution with $\alpha = k$ and $\lambda = 1/2$. It follows that the pdf of $\mathbf{X}_{2k}^2$ is given by [7]

$$f_{\mathrm{X}_{2k}^2}(x) = \frac{(\frac{1}{2})^k}{(k-1)!} x^{(k-1)} e^{(-x)}, x > 0 \tag{4}$$

From the dfinition $\mathrm{C}_n(N,M,\rho)$, the cumulative distribution function (cdf) of $\mathrm{C}_n(N,M,\rho)$ is obtained as

$$F_{\mathrm{C}_n(N,M,\rho)}(c) = P[\mathrm{C}_n(\mathrm{N},\mathrm{M},\rho) < \mathrm{c}] = P\left[\mathrm{X}^2_{2(M-N+n)} < \frac{1}{\bar{\rho}}(2^c-1)\right] \tag{5}$$

where $\bar{\rho} = \rho/2N$.

Differetiating (5) with respect to c, the pdf of $\mathrm{C_n}(\mathrm{N},\mathrm{M},\rho)$ can be derived as

$$F_{\mathrm{C}_n}(N,M,\rho)(c) = f_{\mathrm{X}^2_{2(M-N+n)}}\left(\frac{1}{\bar{\rho}}(2^c-1)\right)\frac{2^c \ln 2}{\bar{\rho}} = \frac{\ln 2}{(2\bar{\rho})^{M-N+n}(M-N+n-1)!} 2^c (2^c-1)^{M-N+n-1} e^{-\frac{1}{2\bar{\rho}}(2^c-1)} \tag{6}$$

Notice that the above pdf is derived for the channel capacity of the nth layer, $\mathrm{C}_n(N,M,\rho)$ in a BLAST system with N transmit and M receive antennas and $\bar{\rho} = \rho/2N$. Then, the ergodic capacity $E[\mathrm{C}_z(N,M,\rho)] = \sum_{n=1}^{N} E[\mathrm{C}_z(N,M,\rho)]$ can be obtained using $\mathrm{C}_n(N,M,\rho)$ for $n = 1,\cdots,N$.

However, deriving the true distribution of

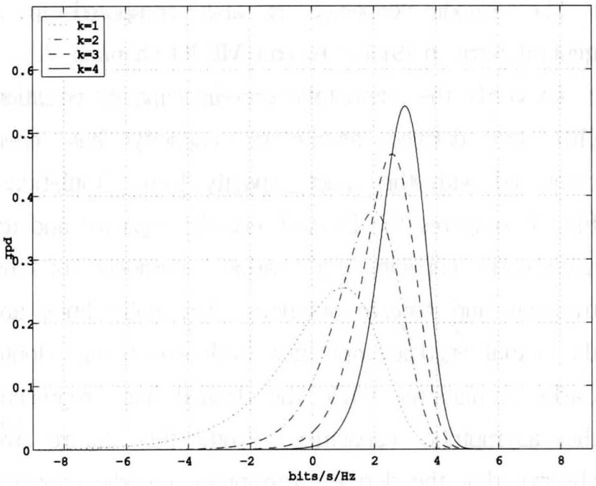

그림 2. Γ_k의 확률 밀도 함수의 확률 밀도 함수
Fig. 2. The probability density function of Γ_k.

$C_z(N, M, \rho)$ is difficult because of the complicated nature of $f_{c_n(N,M,\rho)}(c)$ in (6). Instead, we focus on the asymptotic statistical behavior of $C_z(N, M, \rho)$. Figure. 1. shows the true density function $f_{c_n(N,M,\rho)}(c)$ for N=M=4. We can see that as ρ increases, the shape of the density function remains unchanged. Based on this observation, we will present the asymptotic analysis of the channel capacity in the sense of SNR in the following sections.

Ⅲ. Closed-Form Formula For Asymptotic Ergodic Capacity

In this section, we proceed to estimate the channel capacity $C_z(N, M, \rho)$. For high SNRs, the capacity expression $C_n(N, M, \rho) = \log_2(1+\rho/N)X_{2k}^2/2)$ can be approximated by

$$\begin{aligned} \widetilde{\mathbf{C}}_n(N, M, \rho) &= \log_2(1+(\rho/N)\mathbf{X}^2_{2(M-N+n)}/2) \\ &= \log_2(\overline{\rho}) + \log_2\mathbf{X}^2_{2(M-N+n)} \\ &= \log_2(\overline{\rho}) + \Gamma_{M-N+n} \end{aligned} \quad (7)$$

where $\Gamma_k = \log_2\mathbf{X}^2_{2\mathrm{k}}$. Here Γ_{M-N+n} accounts for the nth layer channel capacity obtained by subtracting the effect of ρ. This approximate value is lower than the actual capacity. Note that $\widetilde{\mathbf{C}}_{\mathrm{n}}(\mathrm{N}, \mathrm{M}, \rho)$converges to $\mathbf{C}_n(N, M, \rho)$ for high ρ. After some straightforward algebraic manipulations in Equations (4)-(6), the pdf of Γ_k can be obtained as

$$f_{\Gamma_k}(c) = \frac{\ln 2}{2^k(k-1)!}2^{ck}e^{-\frac{1}{2}2^c} \text{for} -\infty \le c \le \infty \quad (8)$$

The pdf in (8) is plotted in Fig. 2. Note that the curves in Fig. 1 can be asymptotically obtained by shifting the plots in Fig. 2 by $\log_2(\overline{\rho})$, as can be seen in (7).

The mean of $\widetilde{\mathbf{C}}_n(N, M, \rho)$ in (7) can be completely evaluated from (8). In what follows, we present a closed-form expression of the ergodic capacity by computing the characteristic function of equation (8).

By definition, the characteristic function of (8) is computed as [7]

$$\begin{aligned} \Phi_{\Gamma_k}(w) &= \int_{-\infty}^{\infty} f_{\Gamma_k}(x)e^{jwx}dx \\ &= \int_{-\infty}^{\infty} \frac{\ln 2}{2^k(k-1)!}2^{xk}e^{-\frac{1}{2}2^x}e^{jwx}dx \\ &= (1+\frac{jw}{(k-1)\ln 2})\Phi_{\Gamma_{k-1}}(w) \end{aligned}$$

Therefore, by induction, we have

$$\Phi_{\Gamma_k}(w) = \left(1+\frac{jw}{(k-1)\ln 2}\right)\cdots\left(1+\frac{jw}{\ln 2}\right)\Phi_{\Gamma_1}(w)$$

where

$$\Phi_{\Gamma_1}(w) = \int_{-\infty}^{\infty}\frac{\ln 2}{2}2^x e^{-\frac{1}{2}2^x}e^{jwx}dx$$

Also, the moment theorem states that the moments of Γ_k are given by [7]

$$E[\Gamma_k^m] = \frac{1}{j^m}\frac{d^m\Phi_{\Gamma_k}(w)}{dw^m}|_{w=0}$$

From the above, the mean $E[\Gamma_1]$ can be represented as

$$\begin{aligned} E[\Gamma_k] &= \frac{1}{j}\frac{d^m\Phi_{\Gamma_k}(w)}{dw^m}|_{w=0} \\ &= E[\Gamma_1] + (\frac{1}{(k-1)\ln 2} + \cdots + \frac{1}{\ln 2})\Phi_{\Gamma_k}(w)|_{w=0} \quad (9) \\ &= E[\Gamma_1] + \frac{1}{\ln 2}\sum_{m=2}^{k-1}\frac{1}{m} \end{aligned}$$

where $E[\Gamma_1]$ denotes $\frac{1}{j}\frac{d\Phi_{\Gamma_1}(w)}{dw}|w=0$. Here we have used the fact that $\Phi_{\Gamma_k}(0)=1$.

$E[\Gamma_1]$ can be expressed as [8]

$$\begin{aligned} E[\Gamma_1] &= 1-\frac{\gamma}{\ln 2} \\ &= \lim_{l\to\infty}\left\{\log_2(2l)-\frac{1}{\ln 2}\sum_{k=1}^{l}\frac{1}{k}\right\} \end{aligned}$$

where $\gamma = 0.57721566\cdots$ is called the Euler-Mascheroni constant[8].

Combining (7) and (9), we obtain the asymptotic ergodic capacity of the nth layer as

$$\begin{aligned} E\left[\tilde{C}_n(N,M,\rho)\right] &= \log_2(\bar{\rho}) + E[\Gamma_{M-N+n}] \\ &= \log_2(\bar{\rho}) + E[\Gamma_1] + \frac{1}{\ln 2}\sum_{m=1}^{M-N+n-1}\frac{1}{m} \end{aligned} \tag{10}$$

Consequently, for a general MIMO system with N transmit and M receive antennas ($N \le M$), the asymptotic ergodic capacity of BLAST is

$$\begin{aligned} E\left[\tilde{C}_z(N,M,\rho)\right] &= \sum_{n=1}^{N} E\left[\tilde{C}_z(N,M,\rho)\right] \\ &= N\log_2(\rho/2N) + NE[\Gamma_1] + \frac{1}{\ln 2}\sum_{n=1}^{N}\sum_{m=1}^{M-N+n-1}\frac{1}{m} \end{aligned} \tag{10}$$

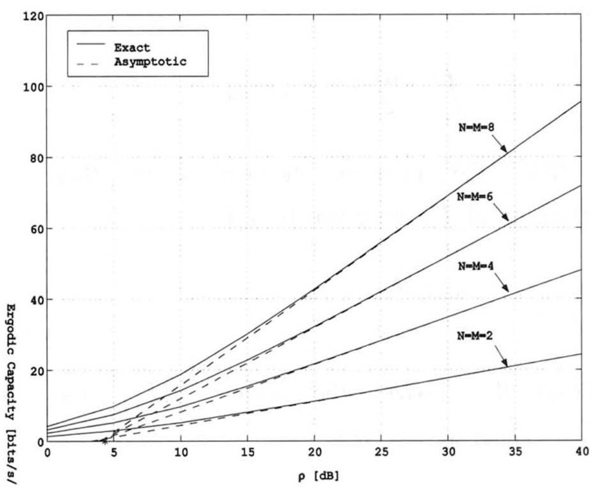

그림 3. 에르고딕적 용량 $E[C_z(N,M,\rho)]$과 점근적 용량 $E[\widetilde{C}_z(N,M,\rho)]$의 비교

Fig. 3. Comparison between exact ergodic channel capacity $E[C_z(N,M,\rho)]$ and its asymptotic capacity $E[\widetilde{C}_z(N,M,\rho)]$.

The ergodic capacity is also computed in a general form in [9] for Ricean MIMO channel.

To verify the asymptotic ergodic capacity equation (10), the derived asymptotic capacity has been compared with the exact capacity from simulations. Fig. 3 compares the BLAST ergodic capacity and its asymptotic capacity for various numbers of the transmit and receive antennas. The solid lines are the actual ergodic capacities evaluated using Monte Carlo simulations, while the dashed lines represent the asymptotic capacities. From this figure we observe that the derived asymptotic ergodic capacity results are indistinguishable from the true BLAST ergodic capacity for SNR higher than 20 dB.

Table 1 contains the incremental ergodic channel capacity corresponding to additional transmit and receive antennas. The table shows how the capacity increases as the number of antennas at either the transmit or receive side grows. Thus, the gross channel capacity for the N by M BLAST can be obtained by summing all elements within the upper-left M by N block. From this table, we can gain an insight on the capacity contribution incurred

표 1. Tx/Rx 안테나 수에 따른 에르고딕적 용량 영향

Table 1. Contribution to ergodic capacity with respect to the number of transmit (column) and receive antennas (row).

M	$N=1$	2	3	4	5
1	$\log_2(\rho/2N)$	$\log_2(\rho/2N)$	$\log_2(\rho/2N)$	$\log_2(\rho/2N)$	$\log_2(\rho/2N)$
	$1-\frac{\gamma}{\ln 2}$	$1-\frac{\gamma}{\ln 2}$	$1-\frac{\gamma}{\ln 2}$	$1-\frac{\gamma}{\ln 2}$	$1-\frac{\gamma}{\ln 2}$
2	$1/\ln(2)$	·	·	·	·
3	$1/(2\ln(2))$	$1/\ln(2)$	·	·	·
4	$1/(3\ln(2))$	$1/(2\ln(2))$	$1/\ln(2)$	·	·
5	$1/(4\ln(2))$	$1/(3\ln(2))$	$1/(2\ln(2))$	$1/\ln(2)$	·
6	$1/(5\ln(2))$	$1/(4\ln(2))$	$1/(3\ln(2))$	$1/(2\ln(2))$	$1/\ln(2)$
7	$1/(6\ln(2))$	$1/(5\ln(2))$	$1/(4\ln(2))$	$1/(3\ln(2))$	$1/(2\ln(2))$

표 2. $G_{n_r}(N,M)$에서의 다양한 안테나 size에 대한 $G_{n_t}(N,M)$의 한계 이득

Table 2. Marginal gain of $G_{n_r}(N,M)$ over $G_{n_t}(N,M)$ for various antenna array sizes in bits/s/Hz.

	$N=1$	2	3	4	5
$M=1$	·	·	·	·	·
2	1.4288	·	·	·	·
3	3.1120	0.1931	·	·	·
4	3.9536	1.9964	-0.6578	·	·
5	4.5066	2.9101	1.2177	-1.3109	·
6	4.9154	3.5113	2.1795	0.6127	-1.8419
7	5.2382	3.9544	2.8149	1.6088	0.1161

by each additional antenna. Denote G_{nt}(N,M) and $G_{nr}(N,M)$ as the transmit and receive capacity gain resulting from increasing the number of antennas form (N,M) to $(N+n_t,M)$ and (N,M) to $(N,M+n_r)$, respectively. Then, we have

$$\mathrm{G}_{n_t}(\mathrm{N,M}) = \mathrm{n_t} \times \left(\log_2\left(\frac{\rho}{2(N+n_t}\right) + E[\Gamma_1]\right) - N\log_2\left(1+\frac{n_t}{N}\right) + \frac{1}{\ln 2}\sum_{n=1}^{n_t}\sum_{m=1}^{M-N-n}\frac{1}{m}$$

and

$$\mathrm{G}_{n_r}(N,M) = \frac{1}{\ln 2}\sum_{n=1}^{n_r}\sum_{m=M-N+n}^{M+n-1}\frac{1}{m}$$

The marginal gain of $G_{N_t=1}(N,M)$ over $G_{N_r=1}(N,M)$ is given in Table 2 where the marginal gain is defined as the capacity increase difference $(G_{N_t=1}(N,M) - G_{N_r=1}(N,M))$ For example, in the case of (4,5) system, one additional antenna at the transmitter and receiver side leads to (5,5) and (4,6), respectively. In this case, putting one more antenna at the receive side provides the capacity increase of 1.3109 bits/s/Hz compared to the case when one more antenna is employed to the transmit side. Consequently, based on the analysis listed in this table, the optimal antenna allocation for transmit and receive sides can be determined.

Ⅳ. Conclusion

In this paper, we have shown that Diagonal Bell Labs Space-Time attains the lower bound for MIMO channels when interference nulling is carried out based on the ZF criterion. Using this remarkable result, we have obtained a closed-form expression for the asymptotic ergodic capacity for layered space-time architectures. Based on the derived capacity expressions, we are able to acquire better insight into the capacity analysis. Especially, the optimal antenna allocation between the transmitter and the receiver has been presented as a result of our analysis.

APPENDIX

PROOF OF THE EQUIVALENCE IN (2)

In this section, we will show that the equation (2) is statistically equivalent as follows:

$$\sum_{n=1}^{N}\log_2(1+SNR_n) \quad \sum_{k=M-N+1}^{M}\log_2(1+(\rho/N)\chi^2_{2k}) \tag{11}$$

We will limit our attention to a practical case (N≤ M) for the ZF equalization.

For analytical conveniences, we assume that the BLAST detection operation proceeds from layer N to *1*. Therefore, the ith layer has *i-1* interferences, and then, from (1), the modified received signal can be described as

$$\mathbf{y}_i = \widetilde{\mathbf{H}}_i + \mathbf{n} = [\ \mathbf{h}_1\mathbf{h}_2\cdots\mathbf{h}_i]\mathbf{x}_i + \mathbf{n}$$

where x_i is the *i-1* dimensional undetected signal vector and $\widetilde{\mathbf{H}}_i$ denotes the corresponding M by i channel matrix.

In this case, $\mathrm{SNR_i}$ is obtained as

$$SNR_i = (\rho/N)\frac{1}{E\left[\ \|\mathrm{g}_i\|^2\right]} \tag{12}$$

where g_i denotes the ith row of the ZF equalizer matrix

$$\mathrm{G} = (\widetilde{\mathrm{H}}_i^H\widetilde{\mathrm{H}}_i)^{-1}\widetilde{\mathrm{H}}_i^H$$

Equivalently, we have

$$\begin{aligned}\|\mathbf{g}_i\|^2 &= \left[\mathbf{G}\mathbf{G}^H\right]_{ii} \\ &= \left[\left(\widetilde{\mathbf{H}}_i^H\widetilde{\mathbf{H}}_i\right)^{-1}\right]_{ii}\end{aligned} \tag{13}$$

As introduced in [5], $\widetilde{\mathbf{H}}_i$ is unitarily equivalent to an M by i matrix

$$\begin{bmatrix} \gamma_{2M} & k_{2(i-1)} & 0 & \cdots & 0 \\ 0 & \gamma_{2(M-1)} & k_{2(i-2)} & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \gamma_{2(M-i+2)} & k_2 \\ \vdots & \vdots & 0 & 0 & \gamma_{2(M-i+1)} \\ 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where both γ_{2j}^2 and k_{2j}^2 are chi-squared random variables with *2j* degrees of freedom.

Let us define the matrix $\mathbf{D}_i = (\widetilde{\mathbf{H}}_i{}^H \widetilde{\mathbf{H}}_i)$ Then $\mathbf{D}_i$ is obtained as

$$\begin{bmatrix} \gamma_{2M}^2 & \gamma_{2M}k_{2(i-1)} & 0 & \cdots & 0 \\ k_{2(i-1)}\gamma_{2M} & \gamma_{2(M-1)}^2 + k_{2(i-1)}^2 & \ddots & \cdots & 0 \\ 0 & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \gamma_{2(M-i+2)}^2 + k_4^2 & \gamma_{2(M-i+2)}k_2 \\ 0 & \cdots & 0 & k_2\gamma_{2(M-i+2)} & \gamma_{2(M-i+1)}^2 + k_2^2 \end{bmatrix}$$

which shows hat $\mathrm{Det}(\mathrm{D}_i) = \Pi_{\mathrm{k}=(\mathrm{M}-\mathrm{i}+1)}^{\mathrm{M}} \gamma_{2\mathrm{k}}^2$.

Also, the inverse of the square matrix $\mathbf{D}_i$ can be written as [10]

$$\mathbf{D}_i^{-1} = \frac{1}{\mathrm{Det}(\mathrm{D}_t)} \mathrm{adj}(\mathbf{D}_i)$$

where the adjoint matrix *adj*$(\mathbf{D}_i)$ represents the transpose of the cofactor matrix.

Then, (13) can be expressed as

$$\| \mathrm{g}_i \|^2 = \left[\mathrm{D}_t^{-1}\right]_{ii} = \frac{\mathrm{C}_{ii}}{\mathrm{Det}(\mathrm{D}_i)}$$

where the cofactor C_{ii} denotes the determinant of the matrix $\mathbf{D}_i$ with the ith column and the *i*th row discarded, i.e., $C_{ii} = \mathrm{Det}(\mathrm{D}_{t-1})$. It follows

$$\| \mathrm{g}_i \|^2 = \frac{\mathrm{Det}(\mathrm{D}_{i-1})}{\mathrm{Det}(\mathrm{D}_i)} = \frac{1}{\gamma_{2(M-i+1)}^2}$$

where we use the fact that $C_{ii} = Det(\mathrm{D}_i) = \Pi_{k=(M-i+1)}^{M} \gamma_{2k}^2$. Substituting (14) into (12), we obtain

$$SNR_i = (\rho/N)\gamma_{2(M-i+1)}^2$$

Consequently, we can rewrite the left-hand side of (11) as

$$\begin{aligned} \sum_{n=1}^{N} \log_2(1+\mathrm{SNR}_n) &= \sum_{\mathrm{n}=1}^{\mathrm{N}} \log_2(1+\rho/N)\gamma_{2(M-n+1)}^2) \\ &= \sum_{n=M-N+1}^{M} \log_2 1 + (\rho/N)\gamma_{2n}^2 \end{aligned}$$

which shows that (11) is *statistically* true.

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