Zero Forcing 수신기를 결합한 다중사용자 다중안테나 시스템의 성능 분석

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Performance Analysis of Multiuser MIMO Systems with Zero Forcing Receivers

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요 약

본 논문에서는, zero-forcing 수신기를 적용한 다중사용자 다중 입출력 시스템의 하향링크 채널을 고려한다. 공간분할다중접속 방식은 다중사용자 다이버시티 이득을 얻기 위하여 기지국의 송신안테나에 서로 다른 사용자를 할당하며, 그에 반하여 공간분할다중화 방식은 한 명의 사용자가 모든 안테나를 점유한다. 본 논문에서는 앞서 언급한 두 방식의 성능을 평가하기 위하여 두 시스템이 갖는 최대 성능에 대한 수학적인 분석을 수행한다. 이를 위하여 우리는 먼저 채널 용량에 대한 정확한 수식을 유도하고, 또한 채널 용량의 상한을 근사적으로 유도하여 더욱 간단한 형태의 수식을 제공한다. 마지막으로 시뮬레이션 결과를 통해 유도된 수식의 정확성을 검증하고 공간분할다중화 방식의 성능을 최적의 기법과 비교한다.

Key Words : multiuser diversity, multiple-input multiple-output (MIMO), zero-forcing (ZF), spatial division multiple access (SDMA)

ABSTRACT

In this paper, we consider multiuser multi-input/multi-output antenna systems with zero-forcing receivers in downlink. In this case, to exploit multiuser diversity, spatial-division multiple access (SDMA) system allows to assign different users to a part of transmit antennas at the base station whereas spatial-division multiplexing (SDM) system assigns all antennas to single user's data stream. In this paper, we present analytical frameworks to evaluate performance of these systems. We first analyze the performance of these two systems by deriving closed-form expressions of achievable throughput. Numerical results show that the derived expressions are very tight. In addition, we approximate the capacity expression of SDM and SDMA systems and compare the SDM with the optimal case.

Ⅰ. Introduction

Most wireless communication systems have been designed to support high-speed packet data trans-

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missions on a downlink channel. Those high-speed packet services can be provided by employing efficient usages of the transmission bandwidth. In such environments, much attention has been paid to multi-input multi-output (MIMO) antenna systems[1].

For single user communications, it has been shown in [2] that the MIMO channels exhibit significant capacity gains over single antenna systems as the number of antennas increases. When channel state information (CSI) is known at the transmitter, the mutual information can be maximized by adapting the transmit power to the channel via the water-filling technique after decomposing the MIMO channel into multiple parallel independent eigenmodes via singular value decompositions (SVD)[1].

In [3], the authors approximate the capacity expression of MIMO channels in multiuser scenarios by observing that the distribution of the mutual information in a Rayleigh fading channelobeys the Gaussian distribution. According to the result in [3], the most efficient architecture for the multiuser MIMO system is the system with an equal number of antennas at both the transmitter and receiver. Therefore, in this paper, we consider a system where the same number of antennas are equipped at both the transmitter and receiver.

In contrast to the optimal MIMO system with the eigen beamforming operation, we employ zero-forcing (ZF) receivers for MIMO systems to provide simpler transmit/receive operations. Instead of performing the SVD operation, the ZF equalizer is utilized to decompose the MIMO channel into multiple parallel streams. With the ZF receiver, we can consider two system models for multiuser scenarios. Those are the spatial-division multiple access (SDMA) system which allows to assign different users to a part of transmit antennas at the base station (BS) and the spatial-division multiplexing (SDM) system which assigns all antennas to single user's data stream. We analyze the performance of these two systems by comparing achievable throughput. To this end, we will derive closed form expressions for the achievable throughput of MIMO systems with multiple users and compare the performance among different system architectures. In addition, we approximate the throughput expressions of SDM and SDMA systems using the order statistics theory[4] to provide an insight into primary parameters which affect the system performance.

The numerical results show that the derived expressions are very tight compared with the results obtained by simulations and SDMA systems outperform SDM systems by around 3dB with four transmit antennas and 40 users. Also the achievable throughput approximation for the SDM describes well the real performance and it is shown that the performance comes within 2dB compared with the system employing the SVD as the number of users increases.

The paper is organized as follows: In section 2, the system model for the MIMO is described. The performance analysis of MIMO systems with the ZF receiver is presented in section 3. Finally, the numerical results and conclusion are presented in sections 4 and 5, respectively.

II. System Model

Fig. 1 illustrates the MIMO communication systems with the ZF receiver comprising N_t transmit and N_r receive antennas. Denoting $\boldsymbol{H}$ as an $N_r \times N_t$ MIMO channel matrix whose (i,j)-th entry represents the channel response of the link between the jth transmit antenna and the ith receive antenna, the received signal model can be simply described as $\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n}$ where $\boldsymbol{n}$ denotes an independent and identically distributed (i.i.d) complex additive white Gaussian noise (AWGN) vector of length N_r with $E[\boldsymbol{n}^* \boldsymbol{n}] = \sigma^2 \boldsymbol{I}_{N_r}$ and $\boldsymbol{x}$ stands for the transmitted symbol vector of length N_t with $E[\boldsymbol{x}^* \boldsymbol{x}] = \sigma_x^2 \boldsymbol{I}_{N_t}$. Here $\boldsymbol{I}_a$ represents an identity matrix of size a and $()^*$ denotes the Hermitian transpose of the matrix. Also we assume that the user data $\boldsymbol{x}$ and the noise $\boldsymbol{n}$ are uncorrelated with each other.

With the ZF equalizer $\boldsymbol{W}_{ZF}$, the output of the ZF receiver can be written as $\hat{\boldsymbol{x}} = \boldsymbol{W}_{ZF}\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{W}_{ZF}\boldsymbol{n}$ where $\boldsymbol{W}_{ZF}$ is obtained from the pseudo-inverse

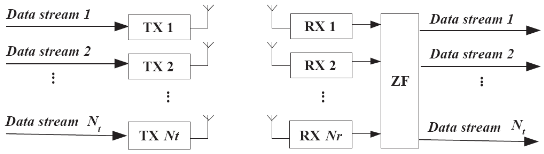

그림 1. ZF 수신기를 결합한 MIMO 시스템의 시스템 구조도
Fig. 1. System model for MIMO systems with the ZF receiver

of $\boldsymbol{H}$ defined as $\boldsymbol{W}_{ZF}=(\boldsymbol{H}^*\boldsymbol{H})^{-1}\boldsymbol{H}^*$. Then the output signal-to-noise ratio (SNR) at the nth stream after the ZF processing can be expressed by[5]

$$\theta_n=\frac{\rho}{N_t[(\boldsymbol{H}^*\boldsymbol{H})^{-1}]_{nn}}$$

where $[\boldsymbol{A}]_{ij}$ denotes the (i,j)-th entry of $\boldsymbol{A}$ and ρ represents the average received SNR. Assuming $N_r>N_t$, θ_n is a Chi-squared random variable with a degree of freedom equal to $2(N_r-N_t+1)$[5]. Here all θ_n's are i.i.d. Thus the probability density function (PDF) of θ_n can be written as

$$f_{\theta_n}(\theta)=\frac{N_t e^{N\theta/\rho}}{\rho(N_r-N_t)!}\left(\frac{N_t}{\rho}\theta\right)^{Nr-N_t}. \quad (1)$$

Since the MIMO channel is decomposed into N_t parallel channels with the ZF equalizer, the ergodic capacity can be obtained as the sum capacity of N_t parallel channels[5]

$$C_{ZF}=\sum_{n=1}^{N_t}E[\log_2(1+\theta_n)] \text{ (bps/Hz)}. \quad (2)$$

When the ZF receiver model in Fig. 1is extended to the multiuser case, we consider the following two architectures:

• SDM: Data streams of one user are transmitted through all transmit antennas at the BS. With the Max-Rate scheduling criterion[3], the BS can select the user with the largest $\sum\log_2(1+\theta_n)$ at each time slot.

• SDMA: The BS can select up to N_t different user streams at a given time which can best exploit each channel. Denoting θ_n^k as the output SNR at the nth spatial subchannel of the kth user, the BS allocates the nth spatial channel to the user with the highest $\log_2(1+\theta_n^k)$ for $n=1,\cdots,N_t$.

Ⅲ. Performance Analysis of Multiuser MIMO Systems with Linear Receivers

In this section, we analyze the performance of multiuser MIMO systems with ZF receivers. As mentioned earlier, an antenna configuration which exploits multiuser diversity most efficiently is the system employing an equal number of antennas at both the transmitter and receiver[3]. Therefore, from now on, we assume $N_t=N_r=N$ and all users experience statistically independent identical fading process. Applying the ZF equalizers at the receiver, N spatial channels are generated. We refer to the data stream transmitted via the nth spatial channel as the nth stream.

3.1 Closed Form Expression for Multiuser Capacity

In the case of $N_t=N_r=N$, the PDF of the output SNR θ_n in (1) reduces to an i.i.d. exponentially distribution which can be expressed as

$$f_{\theta_n}(\theta)=\frac{N}{\rho}e^{-N\theta/\rho}. \quad (3)$$

For the SDMA where the BS can select up to N users to exploit each antenna, N independent links with K users whose output SNR for each user can be modeled as (3). Defining the kth order statistics $\theta_{k:K}^n$ as the kth smallest of $\theta_n^1,\theta_n^2,\cdots,\theta_n^K$ [4], $\theta_{K:K}^n$ represents the largest output SNR for the nth stream among K users which is selected through the max-rate scheduler[3]. The max-rate scheduling scheme is an opportunistic scheduling scheme which maximizes the total throughput in the system.

According to the order statistics[4], the PDF of $\theta_{K:K}^n$ can be obtained as

$$f_{\theta_{K:K}^n}(\theta)=\frac{dF_{\theta_n}(\theta)^K}{d\theta}=KF_{\theta_n}(\theta)^{K-1}f_{\theta_n}(\theta). \quad (4)$$

By substituting (3) to (4), the capacity of the

SDMA with K users is expressed as

$$
\begin{aligned}
&C_{ZF}^{SDMA}(K) \\
&= \sum_{n=1}^{N}\int_0^{\infty}\log_2(1+\theta)K\frac{N}{\rho}e^{-N\theta/\rho}(1-e^{-N\theta/\rho})^{K-1}d\theta \\
&= \frac{KN^2}{\rho}\int_0^{\infty}\log_2(1+\theta)e^{-N\theta/\rho}(1-e^{-N\theta/\rho})^{K-1}d\theta. \quad (5)
\end{aligned}
$$

Using the binomial power series expansion and the integral equality[6, pp. 568]

$$
\int_0^{\infty}e^{-\mu x}\ln(1+\beta x)dx = \frac{1}{\mu}e^{\mu/\beta}E_1\left(\frac{\mu}{\beta}\right)
$$

where the exponential integral function $E_1(x)$ is defined as $\int_x^{\infty}e^{-t}t^{-1}dt$, (5) can be rewritten as

$$
\begin{aligned}
&C_{ZF}^{SDMA}(K) \\
&= \frac{KN^2}{\rho}\log_2(e) \\
&\times\int_0^{\infty}\ln(1+\theta)e^{-\frac{N}{\rho}\theta}\sum_{k=1}^{K-1}\binom{K-1}{k}(-1)^k e^{-\frac{N}{\rho}k\theta}d\theta \\
&= KN\log_2(e) \\
&\times\sum_{k=0}^{K-1}(-1)^k\frac{1}{k+1}\binom{K-1}{k}e^{\frac{N}{\rho}(k+1)}E_1\left(\frac{N}{\rho}(k+1)\right). \quad (6)
\end{aligned}
$$

Note that $E_1(x)$ can be efficiently computed by the approximation as in [7, pp. 231].

In contrast, for SDM systems, the BS selects the user with the highest sum rate, $\sum\log_2(1+\theta_n^k)$. However, it is a problem of great difficulty to describe an exact distribution of $\sum\log_2(1+\theta_n^k)$. To simplify the problem, we modify the SDM scheduling scheme as follows:

- The BS selects the kth user with the largest $\sum\theta_n^k$ instead of $\sum\log_2(1+\theta_n^k)$.
- Transmit power is distributed equally to each stream in order to make the received SNR equal for each stream.

Note that the user selection based on $\sum\theta_n^k$ instead of $\sum\log_2(1+\theta_n^k)$ does not degrade the performance since both metrics are monotonically increasing functions and the polynomial expansion of $\prod(1+\theta_n^k)$ includes the term $\sum\theta_n^k$.

Define a new random variable Θ_k which stands for a sum of all θ_n's as $\Theta_k = \sum_{n=1}^{N}\theta_n^k$. Based on the above assumptions, the capacity formula of the K-user SDM can be rewritten as

$$
\begin{aligned}
C_{ZF}^{SDMA}(K) &= E\left[\max_k\left(\sum_{n=1}^{N}\log(1+\theta_n^k)\right)\right] \\
&\approx N\int_0^{\infty}\log_2\left(1+\frac{x}{N}\right)f_{\Theta_{K:K}}(x)dx \quad (7)
\end{aligned}
$$

where $f_{\Theta_{K:K}}(x)$ indicates the Kth order statistics of Θ_k. Since all θ_n^k's are i.i.d, the characteristic function of Θ_k[8] can be obtained by multiplying the characteristic functions of θ_n^k's. Therefore, the characteristic function $\psi_{\Theta_k}(\omega)$ is written as

$$
\psi_{\Theta_k}(\omega) = \prod_{n=1}^{N}\psi_{\theta_n^k}(\omega) = \left(1+j\frac{\rho}{N}\omega\right)^{-N}. \quad (8)
$$

Note that the right-hand side of (8) is the characteristic function of the gamma distribution[9] with the PDF

$$
f_{\Theta_k}(\theta) = \left(\frac{N}{\rho}\right)^N\frac{\theta^{N-1}e^{-N\theta/\rho}}{\Gamma(N)} \quad (9)
$$

where $\Gamma(N) = (N-1)!$ for integer N [7].
Since the cumulative density function (CDF) of (9) is given as[9]

$$
F_{\Theta_k}(x) = \frac{\gamma(N,(N/\rho)x)}{\Gamma(N)},
$$

where $\gamma(a,x)$ denotes an incomplete gamma function defined as $\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt$[6], the PDF of the Kth order statistics of Θ_k can be represented as

$$
f_{\Theta_{K:K}}(x) = K\left(\frac{\gamma(N,(N/\rho)x)}{\Gamma(N)}\right)^{K-1}\frac{(N/\rho)^N x^{N-1}e^{-Nx/\rho}}{\Gamma(N)}. \quad (10)
$$

To integrate (7) efficiently, the PDF in (10) is converted into[10]

$$
\begin{aligned}
f_{\Theta_{K:K}}(x) = \frac{K}{\Gamma(N)}&\sum_{k=1}^{K-1}\sum_{i=0}^{k(N-1)}(-1)^k\binom{K-1}{k}a_i^k\left(\frac{N}{\rho}\right)^{N+i} \\
&\times e^{-(k+1)(N/\rho)x}x^{N+i-1} \quad (11)
\end{aligned}
$$

where a_i^k for $0 \le i \le k(N-1)$ is recursively defined as

$$
\begin{aligned}
a_0^k &= 1,\ a_1^k = k \\
a_i^k &= \frac{1}{i}\sum_{n=1}^{\min(i,N-1)} \frac{n(k+1)-i}{n!} a_{i-n}^k,\ 2 \le i < k(N-1) \\
a_i^k &= \frac{1}{\Gamma(N)^k},\ i = k(N-1)
\end{aligned} \tag{12}
$$

By applying (11) to (7) and using the integral identity[11]

$$
\int_0^\infty \ln(1+t)e^{-\mu t}t^{n-1}dt = (n-1)!e^\mu \sum_{j=1}^{n} \frac{\Gamma(j-n,\mu)}{\mu^j}
$$

where $\Gamma(a,x)$ represents another incomplete gamma function which is defined as $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$ [6], the capacity of the K-user SDM system is finally computed by

$$
\begin{aligned}
& C_{ZF}^{SDM}(K) \\
&= \frac{K N \log_2(e)}{(N-1)!} \sum_{k=1}^{K-1} \sum_{i=1}^{k(N-1)} (-1)^k \binom{K-1}{k} a_i^k \left(\frac{N^2}{\rho}\right)^{N+i} \\
&\times \int_0^\infty \ln(1+t) e^{-(k+1)\frac{N^2}{\rho}t} t^{N+i-1} dt \\
&= \frac{K N \log_2(e)}{(N-1)!} \\
&\times \sum_{k=1}^{K-1} \sum_{i=1}^{k(N-1)} (-1)^k \binom{K-1}{k} a_i^k \left(\frac{N^2}{\rho}\right)^{N+i} (N+i-1)! \\
&\times e^{(k+1)\frac{N^2}{\rho}} \sum_{j=1}^{N+i} \frac{\Gamma\left(j-N-i,(k+1)\frac{N^2}{\rho}\right)}{\left[(k+1)\frac{N^2}{\rho}\right]^j}
\end{aligned} \tag{13}
$$

3.2 Capacity Bounds

Although the derived formulas for SDM and SDMA systems in (6) and (13) in the previous section are quite accurate, they do not provide much information about primary factors which affect the system performance. In this subsection, in order to gain an insight on multiuser diversity gains, we develop upper bounds for channel capacity for both the SDM and SDMA schemes with K users.

Denoting the mean and variance of the capacity of single user systems as μ and σ^2, respectively, the system throughput with K users is bounded as[4]

$$
C^K \le \mu + \frac{(K-1)\sigma}{\sqrt{2K-1}}. \tag{14}
$$

For the special case, if the capacity of each user obeys a Gaussian distribution, a tighter bound can be obtained from the weak law of large numbers[3] as

$$
C^K \approx \mu + \sqrt{2\sigma^2 \ln K}. \tag{15}
$$

As the Shannon capacity can be approximated by $\log_2\theta$ at high SNR ranges $(N \ll \rho)$[2], the average value of the achievable throughput of the nth stream for the SDMA is rewritten as

$$
\begin{aligned}
\mu_{SDMA}^n &= \int_0^\infty \log_2(1+\theta)\left(\frac{N}{\rho}\right)\exp\left(-\frac{N}{\rho}\theta\right)d\theta \\
&\approx \log_2 e \int_0^\infty \ln\theta \left(\frac{N}{\rho}\right)\exp\left(-\frac{N}{\rho}\theta\right)d\theta.
\end{aligned}
$$

Then, the mean of $C_{ZF}^{SDMA}(K)$ in (5) with N transmit antennas and $K=1$ can be expressed as $\mu_{ZF}^{SDMA} = N \times \mu_{SDMA}^n$.

Define μ_{SDMA} and σ_{SDMA}^2 as the mean and variance of $C_{ZF}^{SDMA}(1)$, respectively. Using the integral identity[6, pp.567]

$$
\int_0^\infty e^{-\mu x} \ln x\, dx = -\frac{1}{\mu}(\zeta + \ln\mu)
$$

where $\zeta = 0.5772\cdots$ denotes the Euler constant, μ_{SDMA} can be obtained as

$$
\mu_{SDMA} = N \log_2 e \left(\ln\frac{\rho}{N} - \zeta\right).
$$

Also σ_{SDMA}^2 can be computed as

$$
\begin{aligned}
\sigma_{SDMA}^2 &= E[(N\log_2(1+\theta))^2] - \mu_{SDMA}^2 \\
&= N^2\left((\log_2 e)^2\left[\frac{\pi^2}{6} + \left(\zeta + \ln\frac{N}{\rho}\right)^2\right] - (\mu_{SDMA}^i)^2\right) \\
&= N^2(\log_2 e)^2 \frac{\pi^2}{6}
\end{aligned} \tag{16}
$$

where we have used the integral identity[6, pp.567]

$$
\int_0^\infty e^{-\mu x}(\ln x)^2 dx = \frac{1}{\mu}\left[\frac{\pi^2}{6} + (\zeta + \ln\mu)^2\right].
$$

In contrast, for SDM systems, the average capacity is approximated as

$$\mu_{SDM} = N\int_0^\infty \log\left(1+\frac{\theta}{N}\right)\left(\frac{N}{\rho}\right)^N \frac{\theta^{N-1}e^{-N\theta/\rho}}{(N-1)!}d\theta$$
$$\approx (\log_2 e)\left(\frac{N}{\rho}\right)^N \frac{N}{(N-1)!}\int_0^\infty \ln\frac{\theta}{N}\theta^{N-1}e^{-N\theta/\rho}d\theta \quad (17)$$

for $N \ll \rho$. Using the Euler psi function $\psi(z)$ in [7] and the integral identity[6, pp.569]

$$\int_0^\infty x^{\nu-1}e^{-\mu x}\ln x\, dx = \mu^{-\nu}\Gamma(\nu)[\psi(\nu)-\ln\mu],$$

(17) can be computed as

$$\mu_{SDM} \approx \log_2 e\left(\frac{N^2}{\rho}\right)^N$$
$$\times \frac{N}{(N-1)!}\Gamma(N)\left(\frac{N^2}{\rho}\right)^{-N}\left[\psi(N)+\ln\frac{\rho}{N^2}\right]$$
$$= N\log_2 e\left[\psi(N)+\ln\frac{\rho}{N^2}\right]. \quad (18)$$

In addition, the variance σ^2_{SDM} is obtained as

$$\sigma^2_{SDM} \approx N^2\left(E\left[\left(\ln\frac{\theta}{N}\right)^2\right] - \mu^2_{SDM}\right) \quad (19)$$

where $E[(\ln(\theta/N))^2]$ is computed as

$$E\left[\left(\ln\frac{\theta}{N}\right)^2\right] \approx \left(\frac{N}{\rho}\right)^N \frac{(\log_2 e)^2}{(N-1)!}\int_0^\infty \left(\ln\frac{\theta}{N}\right)^2 \theta^{N-1}e^{-N\theta/\rho}d\theta$$
$$= \frac{(\log_2 e)^2}{(N-1)!}\left(\frac{N^2}{\rho}\right)^N$$
$$\times \int_0^\infty (\ln x)^2 x^{N-1}e^{N^2x/\rho}dx. \quad (20)$$

Applying the integral identity[6, pp.572]

$$\int_0^\infty x^{\nu-1}e^{-\mu x}(\ln x)^2dx = \mu^{-\nu}\Gamma(\nu)\left([\psi(\nu)-\ln\mu]^2+\zeta(2,\nu)\right)$$

where $\zeta(z,q)$ represents the Riemann zeta function[7], (20) can be written as

$$E\left[\left(\ln\frac{\theta}{N}\right)^2\right] \approx (\log_2 e)^2\left(\left[\psi(N)-\ln\frac{N^2}{\rho}\right]^2+\zeta(2,N)\right) \quad (21)$$

After some manipulations, substituting (21) to (19) yields

$$\sigma^2_{SDM} \approx N^2(\log_2 e)^2\left(\frac{\pi^2}{6}-\sum_{n=1}^{N-1}\frac{1}{n^2}\right) \quad (22)$$

where we have used $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$[7]. As will be shown in section 4, the channel capacity of the SDM with K users can be closely expressed by the Gaussian approximation in (15) with (18) and (22).

The performance of the SDM and the SDMA can be compared by the following lemma.

Lemma 1.(Capacity Gain of the SDMA Systems): Define the capacity gain of the SDMA compared with the SDM as η. Then we have

$$\lim_{K\to\infty}\eta = \lim_{K\to\infty}\frac{C^K_{SDMA}}{C^K_{SDM}} > 1.$$

Proof : By substituting the results of (16) and (22) to (14), we have

$$\lim_{K\to\infty}\frac{C^K_{SDMA}}{C^K_{SDM}} = \frac{\sigma_{SDMA}}{\sigma_{SDM}}$$
$$= \sqrt{\frac{\pi^2}{\pi^2-6\sum_{n=1}^{N-1}(1/n^2)}} > 1. \quad (23)$$

Thus we can see that the capacity gain of SDMA systems over SDM systems increases as the number of antenna N grows.

Ⅳ. Numerical Results

In this section, we present numerical results to verify the performance analysis of MIMO systems with the linear equalizer at the receiver. To evaluate performance, we assume i.i.d Rayleigh fading channels which is supposed to be flat over transmission bandwith. Fig. 2 exhibits the capacity of MIMO systems with four transmit and four receive antennas and 40 users. To verify the accuracy of the derived closed form expressions, we compare the analysis results with the Monte Carlo simulations. As shown in the figure, the derived formulas match well with two MIMO system models. Also we see that the performance gain of the SDMA system overthe SDM is around 3dB.

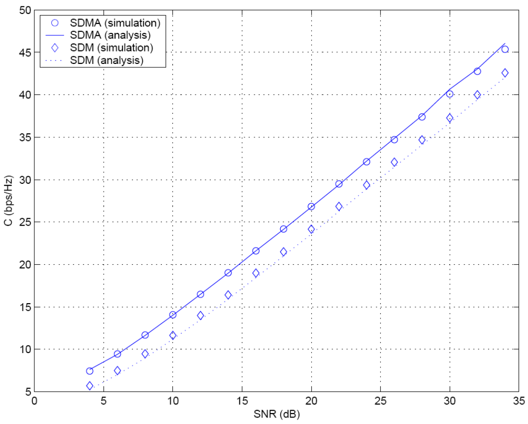

그림 2. ZF 수신기를 결합한 MIMO 시스템의 채널용량 (N=4, K=40)
Fig. 2. Capacity for the MIMO systems with ZF receivers (N=4, K=40)

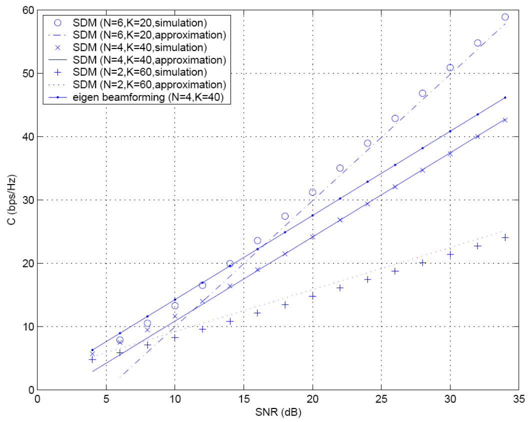

그림 3. 다양한 안테나 및 사용자수에 따른 SDM 시스템의 채널용량 성능
Fig. 3. Performance of SDM systems with various configurations

In Fig. 3, the performance of SDM systems with various configurations is illustrated. Fig. 3 shows that approximations with (15) describe well the SDM systems. In addition, we compare the SDM system with the optimal beamforming case with Gaussian approximation in [3]. Note that the optimal case can not be applied to the SDMA since orthogonality of unitary vectors cannot be satisfied if different users are assigned to each antenna. As shown in the figure, the performance of the SDM comes within 3dB compared with the optimal case. Also the performance of the SDMA in Fig. 2 approaches the optimal case. To perform the optimal eigen beamforming, channel information for all propagation paths should be fed back to the BS.

Thus the amount of feedback overhead increases as N_t, N_r and K grow. In comparison, for MIMO systems with ZF receivers, only scalar SNRs or acceptable data rates of each stream are sufficient for the feedback information and the performance gap decreases by combining with the multiuser diversity. Therefore, the MIMO systems employing the ZF receiver are better suited for systems where the bandwidth for feedback is limited as a reasonable performance can be achieved with reduced feedback overhead.

V. Conclusion

In this paper, we analyze the performance of multiuser MIMO systems with ZF receivers. We first derive the closed form expressions of capacity for MIMO systems with the ZF receiver in multiuser scenarios. As shown in simulation result section, the derived expressions accurately describe capacity for the multiuser MIMO system. Also, we compare the SDM and the SDMA by obtaining the capacity bounds and show that the performance gain decreases as thenumber of antenna increases.

References

〔1〕 I. E. Telatar, “Capacity of Multi-antenna Gaussian Channels”, *AT&T Bell Laboratories, Lucent Technologies*, Murray Hill, NJ, 1994

〔2〕 G. Foschini and M. Gans, “On limits of wireless communication in a fading environment when using multiple antennas”, *AT&T Bell Laboratories Tech. Memo,* September 1995

〔3〕 B. M. Hochwald, T. L. Marzetta, and V. Tarokh, “Multiple-Antenna Channel hardening and Its Implications for Rate-Feedback and Scheduling”, *IEEE Transaction on Information Theory*, vol. 50, pp. 1893-1909, September 2004

〔4〕 H. David and H. Nagaraja, *Order Statistics.* Wiley, 2003

〔5〕 D. A. Gore, J. Robert W. Heath, and A. J. Paulraj, "Transmit Selection in Spatial Multiplexing Systems", *IEEE Communications Letters*, vol. 6, pp. 491-493, November 2002

〔6〕 I. S. Gradshteyn and I. M. Razhik, *Table of Integrals, Series, and Products*. Academic Press, 6th ed., 2000

〔7〕 M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Dover, 1970

〔8〕 A. Papoulis and S. U. Pillai, Probability, Random Variables and Stochastic Processes. McGrawHill, 4th ed., 2002

〔9〕 J. G. Proakis, *Digital Communications*. McGrawHill, 4th ed., 2001

〔10〕 G. Fedele, "N-branch Diversity Reception of M-ary DPSK Signals in Slow Fading and Nonselective Nakagami Fading", *European Transactions on Telecommunications*, vol. 7, pp. 119-123, March-April 1996

〔11〕 M. S. Alouini and A. J. Goldsmith, "A Unified Approach for Calculation Error Rates of Linearly Modulated Signals over Generalized Fading Channels", *IEEE Transaction on Communications*, vol. 47, pp. 1324-1334, September 1999

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