

# MIMO 다운링크 채널에서 다중사용자 공간다중화를 위한 알고리즘

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## Triangulation Algorithm for Multi-user Spatial Multiplexing in MIMO Downlink Channels

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### 요 약

이 논문에서는 다중사용자 (multi-user) 다중안테나 (multiple-in multiple-out : MIMO) 시스템을 위한 전송기법을 제안한다. Costa가 증명한 더티페이퍼코딩 (dirty-paper coding: DPC)이론에 따르면 송신기가 수신기의 가우시안 분포를 갖는 간섭 신호를 알고 있는 경우 간섭 신호에 상관 없이 채널 용량을 얻을 수 있음이 알려져 있다. 단독사용자 채널의 경우 간단하며 효율적인 DPC기법들이 알려져 있으나 다중사용자 환경에는 실제 구현하는데 있어 복잡도와 관련해 많은 문제를 가지고 있다. 이 논문에서 우리는 다중사용자 환경에서 송신기에 알려진 간섭신호를 효율적으로 제어할 수 있는 네트워크 채널 행렬 삼각화 (network channel matrix triangulation: NMT) 기법을 제안하고자 한다. 제안하는 NMT 알고리즘은 다중사용자 MIMO 채널을 서로 독립된 병렬의 Single-input Single-output (SISO) 채널들로 변환하여 단독사용자 환경을 위해 제안되어 있는 기존 DPC기법들을 다중사용자 환경에서도 사용 가능케 한다. 시뮬레이션 결과를 통해 제안된 알고리즘이 다중사용자 환경의 채널 용량에 거의 접근함을 보일 것이다.

**Key Words** : Multi-input multi-output (MIMO) systems, Dirty paper coding (DPC), MIMO broadcast (MIMO-BC) channel, Sum rate capacity and Network Channel Matrix Triangulation (NMT)

### ABSTRACT

This paper studies the design of a multiuser multiple-input multiple-output (MIMO) system, where a base station (BS) transmits independent messages to multiple users. The remarkable "dirty paper coding (DPC)" result was first presented by Costa that the capacity does not change if the Gaussian interference is known at the transmitter noncausally. While several implementable DPC schemes have been proposed recently for single-user dirty-paper channels, DPC is still difficult to implement directly in practical multiuser MIMO channels. In this paper, we propose a network channel matrix triangulation (NMT) algorithm for utilizing interference known at the transmitter. The NMT algorithm decomposes a multiuser MIMO channel into a set of parallel, single-input single-output dirty-paper subchannels and then successively employs the DPC to each subchannel. This approach allows us to extend practical single-user DPC techniques to multiuser MIMO downlink cases. We present the sum rate analysis for the proposed scheme. Simulation results show that the proposed schemes approach the sum rate capacity of the multiuser MIMO downlink at moderate signal-to-noise ratio (SNR) values.

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## I. Introduction

Recently the sum capacity of multiple antenna Gaussian broadcast channels (BC) has been extensively studied by several authors<sup>[1]-[5]</sup>. While the maximization of the sum rate of the multi-input multi-output (MIMO) BC channels with a power constraint is a complex non-convex problem and a closed-form solution to the sum rate is not available yet, it is now well known that the dirty-paper coding (DPC) strategy<sup>[6]</sup> achieves the sum rate capacity of the MIMO BC. In this paper, we present a sum-capacity approaching transmit strategy for multiuser spatial multiplexing for MIMO downlink where each user has more than one antenna and the co-channel interference of other users is known at the base.

In a multiuser MIMO downlink, a base station uses multiple antennas to communicate with several co-channel users simultaneously. One of major challenges in such multiuser MIMO systems is to develop a capacity maximizing transmission scheme which considers the co-channel interference of other users. For the single antenna user case, channel inversion is one of the simplest transmission techniques for eliminating all multiuser interference<sup>[7]</sup>. A generalization of the channel inversion, called *block diagonalization (BD)*, was proposed in [8], where each user has multiple antennas. The key idea of the BD scheme is to eliminate all the multiuser interference by transmitting each user's data along the null space of the other users' channel matrix. However, when the interference is known at the transmitter noncausally, the BD algorithm based on the zero-interference condition is not the best choice for maximizing the achievable sum rate because the BD approach cannot utilize the known interference at the transmitter.

For a Gaussian noise channel corrupted by an additive interference signal known to the transmitter, Costa studied the DPC with Gaussian interference in [6], which showed that the capacity does not change if the Gaussian interference is known at the transmitter noncausally. A simple application

of DPC may be implemented by presubtracting interference at the transmitter<sup>[9], [10]</sup>. Recently, quantizing to a lattice has been identified as an important tool for achieving the capacity<sup>[11], [12]</sup>. A complete end-to-end dirty paper transmission system which attains a significant portion of the channel capacity was proposed in [13]. Extending these results on single user cases, dirty paper coding has emerged as a building block in multiuser MIMO systems. As for MIMO BC channels, the DPC rate region is shown to coincide with the capacity region<sup>[2]</sup>. The optimal multiuser MIMO scheme based on the DPC technique can provide a substantial throughput gain over other traditional methods such as time-division multiple-access (TDMA)<sup>[14]</sup>. However, dirty-paper coding is a rather new and complicated scheme which has yet to be implemented in practical multiuser MIMO systems.

In this paper, we propose a divide-and-conquer strategy which decomposes the multiuser MIMO channel into parallel dirty-paper subchannels and then successively employs the DPC to each subchannel. Specifically, we first apply a linear preprocessing, called *network channel matrix triangulation (NMT) algorithm*, for generating dirty-paper subchannels, and then use successive dirty paper cancellation for each channel to achieve an interference-free channel. In consequence, the combination of the NMT algorithm and the single-user DPC allows us to realize the sum capacity of a multiuser MIMO channel using practical dirty paper coding techniques such as the lattice precoding and trellis shaping methods<sup>[12], [13]</sup>. In [1], a related work, called zero-forcing DPC, based on ordering and successive encoding was considered for the case where each user has a single antenna. In this paper, we extend and generalize this earlier work to the case where each user has more than one antenna.

We consider a single cell with  $M$  transmit antennas at the base station and  $K$  active users, each user  $j$  with  $n_j$  receive antennas. Let  $s_j$  denote the number of data streams for user  $j$ . In

order to maximize an achievable sum rate, at least  $M$  users should be served at the same time. To this end, we consider a NMT algorithm, where the base station sends a single stream ( $s_{j=1}$ ) to each user. Note that to achieve the maximum sum rate, the subset of active users, each user's number of data streams, and parameters on interference should be optimized according to the current channel condition. Although the proposed schemes are suboptimal due to the constraint of the fixed number of data streams for active users, the sum rate analysis and simulation results show that the achievable sum rate of the proposed NMT scheme is very close to the sum rate capacity.

This paper is organized as follows: In section II, we describe a general system model for the multiuser MIMO downlink. Section III presents a NMT algorithm to transmit a data stream for each user in the multiuser MIMO channel. In addition, we investigate the sum rate of the proposed NMT algorithm based on dirty-paper coding. In Section IV, simulation results are presented comparing the proposed method with other multiuser MIMO techniques. Finally, the paper is terminated with conclusions in Section V.

## II. System Model

In this section, we present the system model of a multiuser MIMO downlink system where the base station is transmitting to  $K$  independent users simultaneously and generating co-channel interference at all users. The base station employs  $M$  transmit antennas and user  $j$ , ( $j=1, \dots, K$ ) has  $n_j \geq 2$  receive antennas, referred to in the following as  $n_1, \dots, n_K \times M$ . For any general complex matrix  $M$ ,  $M^T$  denotes the transpose and  $M^H$  represents the complex conjugate transpose.  $M(c_1:c_2)$  stands for the submatrix obtained by taking columns  $c_1, c_{1+1}, \dots, c_2$  from  $M$ .  $Tr(M)$  indicates the trace and  $I_n$  is an identity matrix of size  $n$ .

In the discrete-time complex baseband MIMO case, the channel model from the base to the  $j$ th user is modeled by an  $n_j \times M$  channel matrix  $H_j$ , where the  $(p, q)$  entry of  $H_j$  represents the channel gain from base antenna  $q$  to antenna  $p$  of user  $j$ . We assume that  $H_j$  has full row rank and independent and identically distributed (i.i.d.) entries according to  $N_{\mathcal{C}}(0, 1)$ .

We assume that the base station desires to send the data symbol  $u_j$  to the  $j$ th user. We will consider NMT algorithm which employs a nonlinear mapping of the data symbol  $u_j$  to the  $M \times 1$  transmitted vector  $x_j$  destined for user  $j$ . Denoting the signal vector which is actually transmitted at the base by  $x_s = \sum_{j=1}^K x_j$ , the received signal at the  $j$ th user can be written as

$$y_j = H_j x_s + w_j = H_j x_j + \sum_{i \neq j} H_j x_i + w_j$$

where  $y_j \in C^{n_j}$  and  $w_j = [w_{j,1}, \dots, w_{j,n_j}]^T \in C^{n_j}$  denote the corresponding received signal and noise vectors, respectively. The components  $w_{j,i}$  of the noise vector  $w_j$  are i.i.d. with zero mean and unit variance for  $j=1, \dots, K$  and  $i=1, \dots, n_j$ . Note that user  $j$  not only receives its desired signal through the channel  $H_j$  but also the interference  $\sum_{i \neq j} H_j x_i$  from the signals destined for other users  $i \neq j$ .

Defining the network channel matrix as

$$H_s = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix},$$

the corresponding signals at all the users can be arranged as

$$y_s = H_s x_s + w_s,$$

where  $y_s = [y_1^T, y_2^T, \dots, y_K^T]^T$  and  $w_s = [w_1^T, w_2^T, \dots, w_K^T]^T$ .

In this paper, we construct a *transmit* precoding matrix  $\mathbf{P}_s$  of size  $M \times K$  and a *receive* combining matrix  $\mathbf{C}_s$  of size  $\sum_{j=1}^K n_j \times K$  such that the effective network channel model

$$\mathbf{H}_e = \mathbf{C}_s^H \mathbf{H}_s \mathbf{P}_s \quad (1)$$

is decomposed into dirty-paper subchannels, called *triangular form*, where  $\mathbf{P}_s$  and  $\mathbf{C}_s$  are restricted to column-orthonormal matrices. Then, in order to transmit the data vector  $\mathbf{u}_s = [u_1 u_2 \cdots u_K]^T$  via the triangular form channel  $\mathbf{H}_e$ , we apply successive dirty-paper encoding (from user  $j=1$  to  $K$ ) to determine the symbol  $s_j$  as a function of  $\mathbf{u}_s$  so that each user sees no interference from other users. As a result, the signal vector  $\mathbf{x}_s$  is given by

$$\mathbf{x}_s = \mathbf{P}_s \mathbf{s}_s$$

where  $\mathbf{s}_s = [s_1 s_2 \cdots s_K]^T$ . In this paper, we assume a transmit power constraint of  $P$ , i.e., the transmit signal vector must satisfy  $E[\|\mathbf{x}_s\|^2] = E[\|\mathbf{s}_s\|^2] \leq P$ .

### III. Network Channel Matrix Triangulation (NMT) Scheme

In this section, we describe the NMT scheme, where user  $j$  receives a single data stream ( $s_j=1$ ) by making full use of  $n_j$  receive antennas. The proposed NMT scheme can be applied to the MIMO downlinks where the number of active users is no larger than that of transmit antennas at the base station. In this paper, we focus on the case where the number of active users is equal to the number of transmit antennas of the base station ( $K=M$ ). This assumption is made to provide a fair comparison among different multiuser spatial multiplexing schemes as no scheduling is needed.

For the triangulation of network channel matrix  $\mathbf{H}_e$ , we consider the precoding and combining matrix as  $\mathbf{P}_s = \prod_{j=1}^K \mathbf{P}_j$  and  $\mathbf{C}_s = \prod_{j=1}^K \mathbf{C}_j$ , respectively.

Note that  $\mathbf{C}_s$  is a block diagonal matrix. By applying the precoding at the base and the combining matrix at the users, the network channel model can be written as

$$\begin{aligned} \mathbf{y}_e &= \mathbf{C}_s^H \mathbf{y}_s = \mathbf{C}_s^H \mathbf{H}_s \mathbf{P}_s \mathbf{s}_s + \mathbf{C}_s^H \mathbf{w}_s \\ &= \mathbf{H}_e \mathbf{s}_s + \mathbf{w}_e, \end{aligned} \quad (2)$$

where  $\mathbf{w}_e = \mathbf{C}_s^H \mathbf{w}_s$ .

In what follows, we construct  $\mathbf{P}_s$  and  $\mathbf{C}_s$  such that  $\mathbf{H}_e = \mathbf{C}_s^H \mathbf{H}_s \mathbf{P}_s$  has a triangular form. For an illustrative example, we consider the case of  $2, 2, 2, 2 \times 4$ , i.e., a base station with 4 transmit antennas and four users with 2 receive antennas, respectively. There are 4! possible transmission orders for the four user case. Without loss of generality, we consider the order is fixed as  $\pi = [1, 2, 3, 4]$ . Then, the corresponding network channel is given as  $\mathbf{H}_s^T = [\mathbf{H}_1^T \mathbf{H}_2^T \mathbf{H}_3^T \mathbf{H}_4^T]$ .

We define the singular-value decomposition (SVD) of the first user's channel as

$$\mathbf{H}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{V}_1^H = \mathbf{U}_1 \begin{bmatrix} \mathbf{\Lambda}_1^s & \mathbf{0} \end{bmatrix} \mathbf{V}_1^H \quad (3)$$

where  $\mathbf{\Lambda}_1^s$  denotes a diagonal square matrix whose  $i$ th diagonal element  $\lambda_{1,i}^s$  is equal to the  $i$ th non-zero singular value of the  $n_1$ -by- $M$  channel matrix  $\mathbf{H}_1$ , and  $\mathbf{V}_1$  consists of the  $n_1$  right singular vectors corresponding to non-zero singular values and the  $(M-n_1)$  vectors corresponding to zero singular values.

Using the SVD of the first user's channel in (3), we set the first precoding and combining matrix as  $\mathbf{P}_1 = \mathbf{V}_1$  and  $\mathbf{C}_1 = \text{diag} \mathbf{U}_1, \mathbf{I}_2, \mathbf{I}_2, \mathbf{I}_2$ , respectively. Then, the first effective channel is given by

$$\mathbf{H}_e^1 = \mathbf{C}_1^H \mathbf{H}_j \mathbf{P}_1 = \begin{bmatrix} \mathbf{U}_1^H \mathbf{U}_1 \Lambda_1 \mathbf{V}_1^H \mathbf{V}_1 \\ \mathbf{H}_2 \mathbf{V}_1 \\ \mathbf{H}_3 \mathbf{V}_1 \\ \mathbf{H}_4 \mathbf{V}_1 \end{bmatrix}. \quad (4)$$

Note that  $\mathbf{V}_1$  is independent of  $\mathbf{H}_j$  for  $j = 2, 3, 4$ , and each  $\mathbf{H}_j \mathbf{V}_1$  results in another random 2-by-4 complex matrix. The matrix in (4) can be written as

$$\mathbf{H}_e^1 = \begin{bmatrix} \lambda_{1,1}^s & 0 & 0 & 0 \\ 0 & \lambda_{1,2}^s & 0 & 0 \\ \times & \times & \times & \times \\ \vdots & \vdots & \vdots & \vdots \\ \times & \times & \times & \times \end{bmatrix},$$

where we denote  $\times$  as a non-zero element.

To help explain our approach, we depict the NMT algorithm in Figure 1. Let us define  $\hat{\mathbf{H}}_2$  as the submatrix obtained by taking the last three columns of the matrix  $\mathbf{H}_2 \mathbf{V}_1$  (see Figure 1). The SVD of  $\hat{\mathbf{H}}_2$  is given as

$$\hat{\mathbf{H}}_2 = \hat{\mathbf{U}}_2 \begin{bmatrix} \hat{\Lambda}_2^s & \mathbf{0} \end{bmatrix} \hat{\mathbf{V}}_2^H, \quad (5)$$

and the second precoding and combining matrix can be obtained as  $\mathbf{P}_2 = \text{diag} \mathbf{1}, \hat{\mathbf{V}}_2$  and  $\mathbf{C}_2 = \text{diag} \mathbf{I}_2, \hat{\mathbf{U}}_2, \mathbf{I}_2, \mathbf{I}_2$ . Then, the second effective channel model  $\mathbf{H}_e^2 = \mathbf{C}_2^H \mathbf{H}_e^1 \mathbf{P}_2$  is given as in Figure 1(c), where  $\lambda_{j,i}^s$  denotes the  $i$ th diagonal element of  $\Lambda_j^s$  (or  $\hat{\Lambda}_j^s$ ).

Similarly, by denoting  $\hat{\mathbf{H}}_3$  as the submatrix obtained by taking the last two columns of the matrix  $\mathbf{H}_3 \mathbf{P}_1 \mathbf{P}_2$  and utilizing its SVD

$$\hat{\mathbf{H}}_3 = \hat{\mathbf{U}}_3 \hat{\Lambda}_3^s \hat{\mathbf{V}}_3^H, \quad (6)$$

we can obtain the third precoding matrix  $\mathbf{P}_3 = \text{diag} \mathbf{I}_2, \hat{\mathbf{V}}_3$  and combining matrix  $\mathbf{C}_3 = \text{diag} \mathbf{I}_2, \mathbf{I}_2, \hat{\mathbf{U}}_3, \mathbf{I}_2$ . Then, we have the resulting

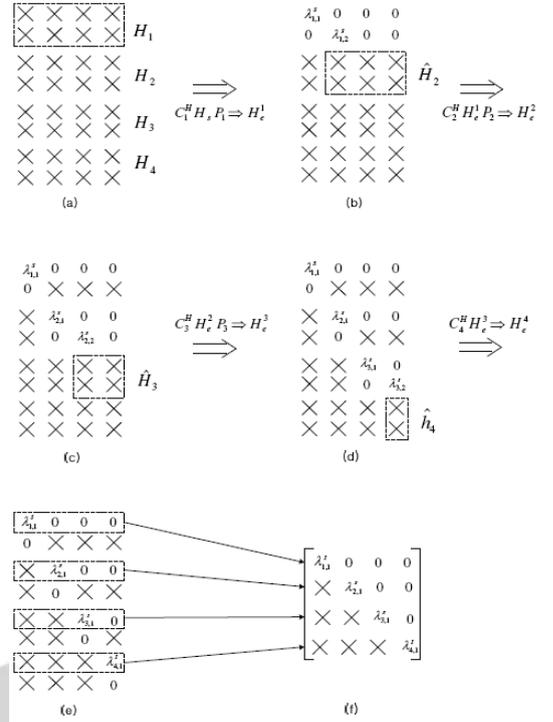


Fig. 1. NMT Scheme for  $\{2,2,2,2\} \times 4$  multiuser MIMO downlink channel.

effective channel matrix  $\mathbf{H}_e^3 = \mathbf{C}_3^H \mathbf{H}_e^2 \mathbf{P}_3$  as in Figure 1(d).

Finally, by denoting the last column of the matrix  $\mathbf{H}_4 \mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3$  as  $\hat{\mathbf{h}}_4$  and utilizing its SVD

$$\hat{\mathbf{h}}_4 = \hat{\mathbf{U}}_4 \begin{bmatrix} \lambda_{4,1}^s \\ 0 \end{bmatrix}, \quad (7)$$

we can determine the fourth precoding and combining matrix as  $\mathbf{P}_4 = \mathbf{I}_4$  and  $\mathbf{C}_4 = \text{diag} \mathbf{I}_2, \mathbf{I}_2, \mathbf{I}_2, \hat{\mathbf{U}}_4$ . Then, the corresponding effective channel matrix  $\mathbf{H}_e^4 = \mathbf{C}_4^H \mathbf{H}_e^3 \mathbf{P}_4$  is given as

$$\mathbf{H}_e^4 = \begin{bmatrix} \lambda_{1,1}^s & 0 & 0 & 0 \\ 0 & \times & \times & \times \\ \times & \lambda_{2,1}^s & 0 & 0 \\ \times & 0 & \times & \times \\ \times & \times & \lambda_{3,1}^s & 0 \\ \times & \times & 0 & \times \\ \times & \times & \times & \lambda_{4,1}^s \\ \times & \times & \times & 0 \end{bmatrix}. \quad (8)$$

To summarize, the precoding and combining matrices are given by

$$\mathbf{P}_s = \prod_{j=1}^4 \mathbf{P}_j = \mathbf{V}_1 \begin{bmatrix} 1 & 0 \\ 0 & \widehat{\mathbf{V}}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 & 0 \\ 0 & \widehat{\mathbf{V}}_3 \end{bmatrix}, \quad (9)$$

and

$$\mathbf{C}_s = \prod_{j=1}^4 \mathbf{C}_j = \text{diag} \mathbf{U}_1, \widehat{\mathbf{U}}_2, \widehat{\mathbf{U}}_3, \widehat{\mathbf{U}}_4,$$

respectively.

Recall that the NMT scheme assumes sending a single data stream to every user. To this end, as shown in Figure 1, each user needs to apply the first column of its combining matrix. Let the  $n_j \times 1$  vector  $\mathbf{u}_j^1$  ( $\widehat{\mathbf{u}}_j^1$ ) denote the first column of  $\mathbf{U}_j$  ( $\widehat{\mathbf{U}}_j$ ). Then with the precoding matrix  $\mathbf{P}_s$  in (9) and the new combining matrix

$$\mathbf{C}_s^\lambda = \begin{bmatrix} \mathbf{u}_1^1 & 0 & 0 & 0 \\ 0 & \widehat{\mathbf{u}}_2^1 & 0 & 0 \\ 0 & 0 & \widehat{\mathbf{u}}_3^1 & 0 \\ 0 & 0 & 0 & \widehat{\mathbf{u}}_4^1 \end{bmatrix},$$

the corresponding effective channel matrix (1) can be written as

$$\mathbf{H}_e = (\mathbf{C}_s^\lambda)^H \mathbf{H}_s \mathbf{P}_s = \begin{bmatrix} \lambda_{1,1}^s & 0 & 0 & 0 \\ \times & \lambda_{2,1}^s & 0 & 0 \\ \times & \times & \lambda_{3,1}^s & 0 \\ \times & \times & \times & \lambda_{4,1}^s \end{bmatrix}$$

which is equivalent to taking rows 1,3,5,7 of  $\mathbf{H}_e^A$  in (8).

For general cases, the effective network channel is given by

$$\mathbf{H}_e = \begin{bmatrix} \lambda_{1,1}^s & 0 & 0 & \cdots & 0 \\ \times & \lambda_{2,1}^s & 0 & \cdots & 0 \\ \times & \times & \lambda_{3,1}^s & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ \times & \times & \cdots & \times & \lambda_{K,1}^s \end{bmatrix}. \quad (10)$$

The triangular form channel matrix  $\mathbf{H}_e$  guarantees that for user  $j$ , no interference is caused by users

$j+1, j+2, \dots, K$ . Note that by the definitions in (3), (5), (6) and (7), the diagonal elements  $\lambda_{j,1}^s$  ( $j=1, \dots, K$ ) have the same distribution as the largest singular value of an  $n_j$ -by- $(M-j+1)$  complex random matrix<sup>[15]</sup>. It is also important to note that non-zero off-diagonal elements, denoted by  $\times$ , of  $\mathbf{H}_e$  have distribution  $N_{\mathcal{C}}(0,1)$ .

Now, we investigate the sum rate of the proposed NMT algorithm with dirty-paper coding. As seen in (10), the resulting multiuser MIMO channel matrix has a triangular form, thus we can apply successive dirty paper cancellation as in [1]. After the successive DPC application to Equation (10), the network channel is now given as

$$\mathbf{H}_e^{DPC} = \text{diag} \{ \lambda_{1,1}^s, \lambda_{2,1}^s, \dots, \lambda_{K,1}^s \}. \quad (11)$$

We obtain the sum rate for the channel  $\mathbf{H}_e^{DPC}$  by maximizing the sum of information rates for all users subject to the sum power constraint  $P$ . Let  $\sigma_j$  denote the power allocation for the  $j$ th user, ( $j=1, \dots, K$ ). Then, the sum rate  $R_{NMT-DPC}$  for the MNT channel in (11) can be written in terms of the following maximization:

$$R_{NMT-DPC} = \max_{\sum_{j=1}^K \sigma_j \leq P} \sum_{j=1}^K \log_2 (1 + (\lambda_{j,1}^s)^2 \sigma_j). \quad (12)$$

Following the waterfilling solution, we can solve (12) to get the sum rate

$$R_{NMT-DPC} = \sum_{j=1}^K \log_2 (1 + (\lambda_{j,1}^s)^2 \sigma_j) \quad (13)$$

with the waterfilling solution  $\sigma_j = \left[ \chi_p - \frac{1}{(\lambda_{j,1}^s)^2} \right]_+$ ,

where  $\chi_p$  is chosen such that  $P = \sum_{j=1}^K \sigma_j$ .

As addressed above,  $\lambda_{j,1}^s$  has the same probability density function (pdf) as the dominant singular value corresponding to the  $n_j$ -by- $(M-j+1)$  random MIMO channel. This aspect can be shown in Figure 2, where we present the complementary cumulative distribution functions (CCDFs) of sum rate for the 2,2,2,2x4 downlink at a signal-to-noise ratio (SNR) of 10 dB. We generate 10,000 random complex matrices  $\mathbf{H}_g$  and employ the NMT scheme to yield the corresponding DPC channels  $\mathbf{H}_e^{DPC}$  in (11). Then we evaluate the sum rate  $R_{NMT-DPC}$  by taking expectation with respect to  $\mathbf{H}_g$ . For the sake of comparison, we also include the equivalent rate for the NMT scheme, denoted by  $R_{NMT-eq}$ , which indicates the sum rate based on the channel  $\mathbf{H}_e^{DPC}$  where the diagonal elements  $\lambda_{j,1}^s$  are now obtained by using the SVD of random complex  $n_j$ -by- $(M-j+1)$  random MIMO channels for  $j=1, \dots, 4$ . In this particular case, we use four matrices of size 2-by-4, 2-by-3, 2-by-2, and 2-by-1 per each realization. For the channel inversion case, one of receive antennas at each user is selected for the transmission of one data stream. Note that the transmit power at the base should be normalized to satisfy the power constraint. Therefore, to obtain the maximum achievable sum rate of the channel inversion,

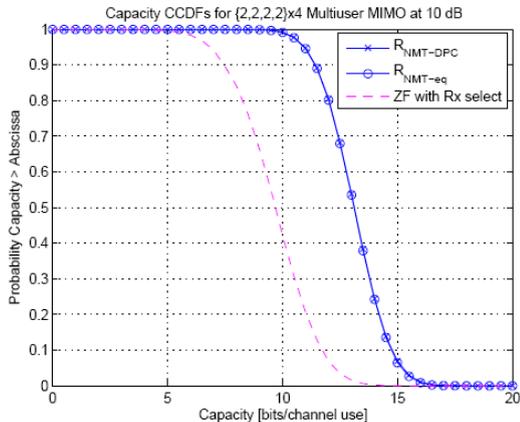


Fig. 2. CCDFs of sum rate for {2,2,2,2}x4 multiuser MIMO downlink.

denoted by *ZF with Rx select*, the user's receive antenna selection is performed at the base to minimize the normalization factor of the transmit signal. In this simulation, an exhaustive search is employed to find the optimal antenna subset by computing the normalization factor over  $\prod_{j=1}^K n_j$  receive antenna subsets. In the figure, it is clear that  $R_{NMT-DPC}$  is the same as  $R_{NMT-eq}$ , which demonstrates that our claim on the distribution of  $\lambda_{j,1}^s$  in (10) is correct. The figure also shows that in terms of the sum rate performance the proposed NMT scheme is far superior to the ZF scheme with receive antenna selection.

#### IV. Numerical Results

In this section, we compare the performance of the proposed scheme in terms of the sum rate through Monte Carlo simulation. The sum capacity denotes the maximum sum rate which can be achieved using DPC. The simulation results for the sum rate capacity is obtained by using the iterative water-filling algorithm proposed in [16]. Notice that the sum rate  $R_{NMT-DPC}$  in (13) depends on the ordering of the users. There are  $K!$  possible orderings of  $K$  users. While in the previous sections we computed the sum rate using a fixed order  $\pi = [1, 2, \dots, K]$  for the NMT schemes (denoted by  $R_{NMT-DPC}$ ), we can improve the performance by choosing the order which maximizes the sum rate for each channel realization (denoted by  $R_{NMT-DPC}$  w/ ordering). In this paper, we use a limited set of the user ordering in circularly shifted form, i.e.,  $\pi \in [1, 2, 3, 4], [4, 1, 2, 3], [3, 4, 1, 2], [2, 3, 4, 1]$  for the four user case to reduce simulation complexity. For the sake of comparison, we also consider the achievable sum rates of other methods including the greedy TDMA. For the greedy TDMA, we use a much simpler technique of TDMA in which the base transmits to only a single user with the best channel quality at each time.

We will first illustrate the sum rate

performance of the proposed schemes for multiuser MIMO downlinks with a four transmit antenna base station ( $M=4$ ). Figure 3 compares the achievable sum rate of the proposed NMT scheme with other schemes for the  $2,2,2,2 \times 4$  case. It is seen from the figure that the sum rate performance of the proposed scheme is improved by allowing different orders. The proposed scheme provides much higher sum rate than the ZF with Rx select and greedy TDMA. More interestingly, the proposed NMT scheme with ordering approaches close to the sum capacity. We believe that the performance gap between the sum capacity and the proposed NMT schemes in Figure 3 mainly results from the constraint of the fixed number of data streams for all the active

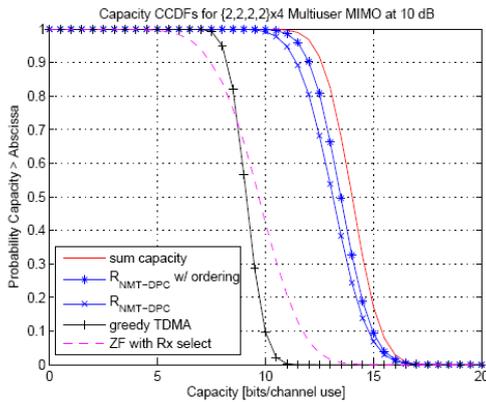


Fig. 3. CCDFs of sum rate for  $\{2,2,2,2\} \times 4$  multiuser MIMO downlink.

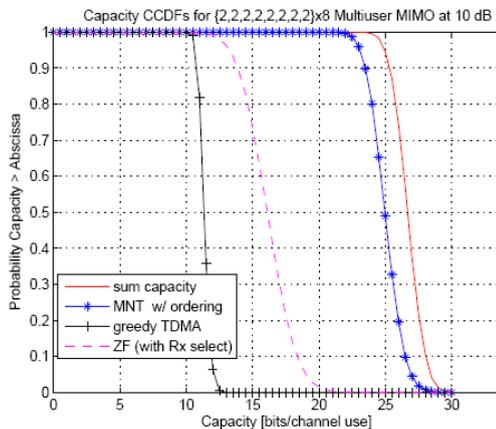


Fig. 4. CCDFs of sum rate for  $\{2,2,2,2,2,2,2,2\} \times 8$  multiuser MIMO downlink.

users regardless of current channel qualities. Note that the optimal sum rate can be achieved by optimizing the number of data streams for each active user according to the current channel condition.

In Figure 4, we plot the sum rate performance for the  $\{2,2,2,2,2,2,2,2\} \times 8$  case. The figure again shows that the sum rate achieved by the proposed NMT scheme is very close to the sum rate capacity.

## V. Conclusion

In this paper, we have proposed a new linear preprocessing, called *network channel matrix triangulation (NMT) algorithm*, for achieving dirty-paper subchannels over which we can use successive dirty paper cancellation to generate an interference-free channel. The combination of the NMT algorithm and the single-user DPC allows us to realize the sum capacity of the multiuser MIMO channel using practical dirty paper coding techniques such as the lattice precoding and trellis shaping methods combined with channel codes. We have analyzed the achievable sum rate of the proposed NMT algorithms based DPC and compared it with the sum rate capacity of MIMO Gaussian broadcast channels. Our simulation results show that the achievable sum rate of the proposed NMT scheme is very close to the optimal sum rate.

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