

# 다중 안테나 다중 사용자 하향 링크 환경에서 Regularized Channel Inversion 기법

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## Regularized Channel Inversion for Multiple-Antenna Users in Multiuser MIMO Downlink

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### ABSTRACT

Channel inversion is one of the simplest techniques for multiuser downlink systems with single-antenna users. In this paper, we extend the regularized channel inversion technique developed for the single-antenna user case to multiuser multiple-input multiple-output (MIMO) channels with multiple-antenna users. We first employ the multiuser preprocessing to project the multiuser signals near the null space of the unintended users based on the MMSE criterion, and then the single-user preprocessing is applied to the decomposed MIMO interference channels. In order to reduce the complexity, we focus on non-iterative solutions for the multiuser transmit beamforming and use a linear receiver based on an MMSE criterion. Simulation results show that the proposed scheme outperforms existing joint iterative algorithms in most multiuser configurations.

**Key Words** : Multiuser, MIMO, Downlink, Precoder Design, Regularized CI

### I. Introduction

Recently, the sum capacity of Gaussian vector broadcast channels has been studied, and in order to achieve the sum capacity, many advanced multiuser transmission schemes have been investigated for the downlink of multiuser multiple-input multiple-output (MIMO) wireless systems<sup>[1]</sup>. We consider a multiuser MIMO downlink system in which a base station (BS) equipped with multiple antennas transmits to several users simultaneously, each with multiple antennas. For the single-antenna user case, a channel inversion has been considered to suppress the cochannel interference<sup>[2]</sup>. In [3], as a generalization of the channel inversion, a joint channel

diagonalization was proposed for multiuser MIMO transmission systems where each user has multiple receive antennas. Similar algorithms based on a null space projection have been presented also in [4] and [5]. Compared to the work in [5], the method in [6] can achieve more diversity by iteratively placing the nulls at the combined signal outputs of the unintended users.

In these approaches, all the multiuser interference is eliminated by transmitting each user's data along the null space of the other users' channel matrix, which may limit the system performance due to noise enhancement. In order to get rid of this limitation, the optimum transmit precoding and receive combining matrices can be obtained by using

※ 본 연구는 서울시 산학연 협력사업 (ST090852 및 WR080951) 지원으로 수행되었습니다.

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논문번호 : KICS2009-09-389, 접수일자 : 2009년 9월 4일, 최종논문접수일자 : 2010년 3월 8일

a joint iterative optimization procedure based on a minimum mean-square error (MMSE) criterion, subject to total transmit power<sup>[7]</sup>.

In this paper, we propose a new transmit method for mitigating the cochannel interference in multiuser downlink systems. In contrast to the iterative algorithm in [7], we focus on non-iterative solutions for the transmit precoding matrix and then use a simple MMSE receiver to reduce the complexity. To this end, we extend the regularized channel inversion technique developed for the single-antenna users in [2], to the case of multiuser MIMO downlink in which each user has more than one receive antenna. The proposed precoding matrix consists of two parts: namely a multiuser preprocessing for decomposing a multiuser MIMO channel into a set of parallel single-user MIMO interference channels, and a single-user preprocessing for maximizing the sum rate under a sum power constraint. The multiuser preprocessing is designed by generalizing the earlier work in [2] to minimize the mean-square error (MSE) between the transmitted and received symbols, and the single-user preprocessing employs a combination of the channel diagonalization and power allocation matrix in order to maximize the sum rate subject to a total transmit power constraint.

## II. Multiuser MIMO System Model

In this section, we describe the system model of the multiuser MIMO downlink. The base station employs  $M$  transmit antennas and communicates with  $K$  users simultaneously. User  $j$ , ( $j = 1, 2, \dots, K$ ) has  $n_j$  receive antennas and we define  $N$  as  $N = \sum_{j=1}^K n_j$ . The channel model from the base to the  $j$ th user is represented by an  $n_j$  by  $M$  channel matrix  $\mathbf{H}_j$ , where the  $(p, q)$  entry of  $\mathbf{H}_j$  denotes the path gain from BS antenna  $q$  to antenna  $p$  of user  $j$ . The entries are independently and identically distributed (i.i.d.) according to  $\mathcal{N}_{\mathbb{C}}(0, 1)$ , i.e.,  $\mathbf{H}_j \in \mathbb{C}^{n_j \times M}$ .

Let  $l_j$  denote the number of data streams for user  $j$ . The base station desires to send the  $l_j \times 1$  vector  $\mathbf{x}_j$  of symbols to the  $j$ th user. Denoting  $x_{j,i}$  as the symbol transmitted on the  $i$ th spatial subchannel by the  $j$ th user, we can write  $\mathbf{x}_j = [x_{j,1}, x_{j,2}, \dots, x_{j,l_j}]^T$ . The user  $j$  employs a linear transmit precoding matrix  $\mathbf{T}_j = [\mathbf{t}_{j,1}, \mathbf{t}_{j,2}, \dots, \mathbf{t}_{j,l_j}]$  of size  $M \times l_j$ , which transforms the data vector  $\mathbf{x}_j$  to the  $M \times 1$  transmitted vector  $\mathbf{T}_j \mathbf{x}_j$ . Here,  $\mathbf{t}_{j,i}$  indicates the  $i$ th column of  $\mathbf{T}_j$ . Denoting the signal vector which is actually transmitted at the base by  $\sum_{j=1}^K \mathbf{T}_j \mathbf{x}_j$ , the received signal vector  $\mathbf{y}_j = [y_{j,1}, y_{j,2}, \dots, y_{j,n_j}]^T$  at the  $j$ th user can be written as

$$\mathbf{y}_j = \mathbf{H}_j \mathbf{T}_j \mathbf{x}_j + \mathbf{H}_j \sum_{i \neq j} \mathbf{T}_i \mathbf{x}_i + \mathbf{w}_j \quad (1)$$

where  $\mathbf{w}_j = [w_{j,1}, w_{j,2}, \dots, w_{j,n_j}]^T$  denotes the noise vector for user  $j$ . Here  $(\cdot)^T$  denotes the transpose of a matrix (or vector). The components  $w_{j,i}$  of the noise vector  $\mathbf{w}_j$  are i.i.d. with zero mean and variance  $\sigma_n^2$  for  $j = 1, 2, \dots, K$  and  $i = 1, \dots, n_j$ . Note that user  $j$  not only receives its desired signal  $\mathbf{T}_j \mathbf{x}_j$  through the channel  $\mathbf{H}_j$  but also the interference  $\sum_{i \neq j} \mathbf{T}_i \mathbf{x}_i$  from the signals destined for other users  $i \neq j$ .

Defining the network channel as

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \quad (2)$$

the corresponding signals at all the users can be arranged as

$$\mathbf{y}_s = \mathbf{H}_s \mathbf{T}_s \mathbf{x}_s + \mathbf{w}_s \quad (3)$$

where  $\mathbf{y}_s = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_K^T]^T$ ,  $\mathbf{T}_s = [\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_K]$ ,  $\mathbf{x}_s = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_K^T]^T$  and  $\mathbf{w}_s = [\mathbf{w}_1^T, \mathbf{w}_2^T, \dots, \mathbf{w}_K^T]^T$ .

We assume  $\mathbb{E}[\mathbf{w}_s \mathbf{w}_s^H] = \sigma_n^2 \mathbf{I}_N$ , where  $\mathbf{I}_d$  denotes an identity matrix of size  $d$ ,  $\mathbb{E}$  represents the expectation operator and  $(\cdot)^H$  represents the complex conjugate transpose of a vector (or matrix). We impose the power constraint  $\mathbb{E}[\text{tr}[\mathbf{x}_s \mathbf{x}_s^H]] = 1$  where  $\text{tr}[\cdot]$  denotes the trace operator of a matrix. The transmit precoding matrix  $\mathbf{T}_j$  satisfies the orthogonal condition such that  $\mathbf{T}_j \mathbf{T}_j^H$  is a diagonal matrix with nonnegative entries for  $(j = 1, 2, \dots, K)$ . Also it is assumed that  $\{\mathbf{T}_j\}_{j=1}^K$  are normalized as follows:  $\text{tr}[\mathbf{T}_s^H \mathbf{T}_s] = \sum_{j=1}^K \text{tr}[\mathbf{T}_j^H \mathbf{T}_j] = L$ , where  $L$

denotes the total number of data streams ( $L = \sum_{j=1}^K l_j$ ).

This implies that the actual transmitted vector  $\mathbf{T}_s \mathbf{x}_s$  has unit power, i.e.,  $\mathbb{E}[\|\mathbf{T}_s \mathbf{x}_s\|^2] = 1$ , where  $\|\cdot\|$  denotes the 2-norm of a vector. Then the average transmit signal-to-noise ratio (SNR) of the network is defined as  $\rho = 1/\sigma_n^2$ .

Let  $\mathbf{R}_j$  represent the receive combining matrix for the  $j$ th user. At the user  $j$ , the soft output data vector  $\hat{\mathbf{x}}_j$  can be expressed as  $\hat{\mathbf{x}}_j = \mathbf{R}_j \mathbf{y}_j$ . The mean squared errors (MSEs) between the transmitted and received signals are given as the diagonal elements in the error covariance matrix  $\varepsilon_s$ , which is defined by

$$\varepsilon_s = \mathbb{E}[(\mathbf{x}_s - \hat{\mathbf{x}}_s)(\mathbf{x}_s - \hat{\mathbf{x}}_s)^H] \quad (4)$$

where  $\hat{\mathbf{x}}_s = [\hat{\mathbf{x}}_1^T, \hat{\mathbf{x}}_2^T, \dots, \hat{\mathbf{x}}_K^T]^T$  is an estimate of  $\hat{\mathbf{x}}_s$ .

In this paper we consider the normalized error covariance matrix  $\mathbf{E}_s$  which is equal to  $\varepsilon_s$  normalized with the variance of  $x_{j,i}$ ,  $1/L$ . Note that the normalization makes the diagonal entries of  $\mathbf{E}_s$  lie in the range from 0 to 1. From the system model in (3) and the definition of  $\varepsilon_s$  in (4), the normalized error covariance matrix  $\mathbf{E}_s$  can be rewritten as

$$\mathbf{E}_s = \frac{\varepsilon_s}{1/L} = (\mathbf{R}_s \mathbf{H}_s \mathbf{T}_s - \mathbf{I}_L)(\mathbf{R}_s \mathbf{H}_s \mathbf{T}_s - \mathbf{I}_L)^H - \frac{L}{\rho} \mathbf{R}_s \mathbf{R}_s^H \quad (5)$$

where  $\mathbf{R}_s = \text{diag}\{\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_K\}$ .

In this paper, we assume a linear MMSE receiver. Then for any transmit precoding matrices  $\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_K$ , the optimum solution for user  $j$  is given by [8]

$$\mathbf{R}_j = \mathbf{T}_j^H \mathbf{H}_j^H \left( \mathbf{H}_j \left( \sum_{k=1}^K \mathbf{T}_k \mathbf{T}_k^H \right) \mathbf{H}_j^H + \frac{L}{\rho} \mathbf{I}_{n_j} \right)^{-1} \quad (6)$$

Our goal is to develop a new transmit processing  $\mathbf{T}_s$  to minimize the total MSE  $\text{tr}[\mathbf{E}_s]$  under a total power constraint. No closed form solution is available to find the optimum global transmit matrix  $\mathbf{T}_s$  since the optimum precoding matrices  $\{\mathbf{T}_j\}_{j=1}^K$  are represented by functions of the optimum receive combining matrices  $\{\mathbf{R}_j\}_{j=1}^K$ , and the optimum receive matrices on the other hand are also functions of the transmit matrices  $\{\mathbf{T}_j\}_{j=1}^K$ . In the following sections, we derive a closed loop solution to the transmit precoding  $\mathbf{T}_s$  by utilizing the regularized channel inversion technique in [2].

### III. Channel Inversion for Users with Multiple antennas

In this section, we describe how the regularized channel inversion technique in [2] can be extended to multiuser MIMO downlink systems in which each user receives multiple data streams via multiple receive antennas. We assume that the base station provides full spatial multiplexing gain ( $l_j = n_j$ ) to every user. We note that this assumption is made to simplify the presentation. At the end of this section, the proposed transmit scheme will be extended to apply to the general case where  $1 \leq l_j \leq n_j$ .

Let  $\mathbf{h}_{i,j}^T$  denote the  $i$ th row of  $\mathbf{H}_j$ . Then, we have  $\mathbf{H}_j^T = [\mathbf{h}_{j,1}, \mathbf{h}_{j,2}, \dots, \mathbf{h}_{j,K}]$ . The analysis in [2] focused on the single-antenna user case where no receive combining processing is employed at the users. By applying the work of [2] directly to the general system model in (3), the network precoding matrix  $\mathbf{T}_s$  can be obtained in a form of

$$\begin{aligned} \mathbf{T}_s &= [\mathbf{t}_{1,1} \cdots \mathbf{t}_{j,1} \mathbf{t}_{2,1} \cdots \mathbf{t}_{j,2} \cdots \mathbf{t}_{K,1} \cdots \mathbf{t}_{j,K}] \\ &= \mathbf{H}_s^H (\mathbf{H}_s \mathbf{H}_s^H + \alpha \mathbf{I}_N), \end{aligned} \quad (7)$$

$$\begin{aligned} \mathbf{x}_s^j &= [\mathbf{x}_1^T \cdots \mathbf{x}_{j-1}^T \mathbf{x}_{j+1}^T \cdots \mathbf{x}_K^T]^T, \text{ and} \\ \mathbf{w}_s^j &= [\mathbf{w}_1^T \cdots \mathbf{w}_{j-1}^T \mathbf{w}_{j+1}^T \cdots \mathbf{w}_K^T]^T. \end{aligned}$$

where the optimal  $\alpha$  is determined according to the total transmit power and the noise variance.

In order to control both the inter-user interference that is caused by  $x_{j,i}$  onto the other  $K-1$  users through channel matrices  $\mathbf{H}_p$  for  $p \neq j$  and the intra-user interference that is seen by user  $j$  itself via the channel path  $\mathbf{h}_{j,p}^T$  for  $p \neq i$ , the resulting precoding vector  $\mathbf{t}_{j,i}$  in (7) is designed for projecting the symbol  $x_{j,i}$  near the null space (or on the null space for the zero-forcing (ZF) case) of  $\{\mathbf{H}_p\}_{p \neq j}$  and  $\{\mathbf{h}_{j,q}\}_{q \neq i}$  based on the MMSE criterion. In other words, the symbol  $x_{j,i}$  is multiplied by  $\mathbf{t}_{j,i}$  and received by antenna  $i$  at user  $j$  to minimize the MSE by compromising the maximization of the effective channel gain  $|\mathbf{h}_{j,i}^T \mathbf{t}_{j,i}|$  or  $x_{j,i}$  and the minimization of the interference  $\{\|\mathbf{H}_p \mathbf{t}_{j,i}\|\}$  and  $\{\|\mathbf{h}_{j,q}^T \mathbf{t}_{j,i}\|\}$  ( $p \neq j$  and  $q \neq i$ ). However, this direct application results in a performance loss compared to other joint transmit-receive optimization procedures since no receiver combining is performed at the users.

In order to fully exploit the multiple receive antenna diversity, when designing the precoding vector  $\mathbf{t}_{j,i}$  in the proposed scheme, we take into account only the inter-user interference seen by the other  $K-1$  users through channel matrices  $\{\mathbf{H}_p\}_{p \neq j}$  but not the intra-user interference seen by user  $j$  itself via the channel path  $\mathbf{h}_{j,p}^T$  for  $p \neq i$ . To this end, we define the complementary network channel model to the user  $j$ , excluding the  $j$ th user's channel model  $\mathbf{y}_j = \mathbf{H}_j \mathbf{x}_j + \mathbf{w}_j$  in (3), as

$$\mathbf{y}_s^j = \mathbf{H}_s^j \mathbf{x}_s^j + \mathbf{w}_s^j \quad (8)$$

where

$$\begin{aligned} \mathbf{H}_s^j &= [\mathbf{H}_1^T \cdots \mathbf{H}_{j-1}^T \mathbf{H}_{j+1}^T \cdots \mathbf{H}_K^T]^T, \\ \mathbf{y}_s^j &= [\mathbf{y}_1^T \cdots \mathbf{y}_{j-1}^T \mathbf{y}_{j+1}^T \cdots \mathbf{y}_K^T]^T, \\ \mathbf{T}_s^j &= [\mathbf{T}_1 \cdots \mathbf{T}_{j-1} \mathbf{T}_{j+1} \cdots \mathbf{T}_K]^T, \end{aligned} \quad (9)$$

We use the complementary channel model in (8) as the interference model for determining  $\mathbf{T}_j$ . The proposed solution consists of two parts in a form of  $\mathbf{T}_j = \mathbf{M}_j \mathbf{S}_j$ , where  $\mathbf{M}_j$  and  $\mathbf{S}_j$  represent a multiuser preprocessing and a singleuser preprocessing, respectively. We first find a projection vector for each data stream independently between streams belonging to the same user. The multiuser preprocessing is obtained as an orthonormal basis of the vector space of the projection vectors. Next, a single-user preprocessing is derived using existing single user algorithms, and the water-filling technique is applied for power allocation to all available spatial subchannels under a total transmit power constraint.

### 3.1 Multiuser Preprocessing $\mathbf{M}_j$

We first derive the projection matrix  $\mathbf{M}_j$  based on the MMSE criterion. To completely eliminate the interference to each user from the other  $K-1$  users, the projection matrix  $\mathbf{M}_j$  can be chosen to satisfy the null projection constraint  $\mathbf{H}_s^j \mathbf{M}_j = 0$ . In our scheme, we take into account the noise components by relaxing the null constraint.

Using the channel model in (8), we define the multiuser channel model as

$$\begin{bmatrix} y_{j,i} \\ \mathbf{y}_s^j \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{j,i}^T \\ \mathbf{H}_s^j \end{bmatrix} \begin{bmatrix} \mathbf{t}_{j,i} \mathbf{T}_s^j \\ \mathbf{x}_s^j \end{bmatrix} + \begin{bmatrix} w_{j,i} \\ \mathbf{w}_s^j \end{bmatrix}. \quad (10)$$

With this system model, it is possible to derive a closed form solution to the projection vector  $\mathbf{t}_{j,i}$  which projects the symbol  $x_{j,i}$  near the null space of  $\mathbf{H}_s^j$  so that the interference received by the rest  $K-1$  users is suppressed while minimizing the MSE for symbol  $x_{j,i}$ . Using the channel model in (10), we can formulate the regularized channel inversion matrix as

$$\mathbf{W}_{j,i} = (\mathbf{H}_{j,i}^H \mathbf{H}_{j,i} + \alpha \mathbf{I}_M)^{-1} \mathbf{H}_{j,i}^H, \quad (11)$$

where  $\mathbf{H}_{j,i} = [\mathbf{h}_{j,i} \mathbf{H}_j^T]^T$  and the optimal  $\alpha$  is given by  $\alpha = L/\rho$  to maximize the signal-to-interference plus noise ratio in a network<sup>[2]</sup>.

Denoting to  $\mathbf{t}_{j,i}^o$  as the first column of  $\mathbf{W}_{j,i}$ ,  $\mathbf{t}_{j,i}^o$  can be obtained from (11) as

$$\mathbf{t}_{j,i}^o = \left( \sum_{K=1, K \neq j}^K \mathbf{H}_K^H \mathbf{H}_K + (\mathbf{h}_{j,i}^T)^H \mathbf{h}_{j,i} + \frac{L}{\rho} \mathbf{I}_M \right)^{-1} (\mathbf{h}_{j,i}^T)^H, \quad (12)$$

for  $i = 1, \dots, l_j$ . As shown in Equations (10) and (12), we find the projection vector  $\mathbf{t}_{j,i}^o$  for each data stream  $x_{j,i}$  independently between streams  $i = 1, \dots, l_j$  belonging to the same user  $j$ .

For  $\mathbf{T}_j = \mathbf{M}_j \mathbf{S}_j$  to be the projection matrix near the null space of  $\mathbf{H}_s^j$ , all the columns of  $\mathbf{M}_j$  are constrained to lie within the subspace spanned by vectors  $\{\mathbf{t}_{j,1}^o, \mathbf{t}_{j,2}^o, \dots, \mathbf{t}_{j,l_j}^o\}$ ,

Let  $\mathbf{m}_{j,i}$  denote the  $i$ th column of the projection matrix  $\mathbf{M}_j$ . Then, the column vectors  $\{\mathbf{m}_{j,1}, \mathbf{m}_{j,2}, \dots, \mathbf{m}_{j,l_j}\}$  of  $\mathbf{M}_j$  are obtained as an arbitrary orthonormal basis of the  $l_j$ -dimensional vector space spanned by vectors  $\{\mathbf{t}_{j,1}^o, \mathbf{t}_{j,2}^o, \dots, \mathbf{t}_{j,l_j}^o\}$  such that

$$\mathbf{m}_{j,i} \in R\left(\left[\mathbf{t}_{j,1}^o, \mathbf{t}_{j,2}^o, \dots, \mathbf{t}_{j,l_j}^o\right]\right) \quad (13)$$

and

$$\mathbf{m}_{j,p}^H \mathbf{m}_{j,q} = \delta_{pq}, \quad (14)$$

where  $R(\cdot)$  denotes the column space of a matrix and  $\delta_{pq}$  represents (discrete) Dirac delta function. For example, Gram-Schmidt orthogonalization can be utilized to obtain  $\mathbf{M}_j = [\mathbf{m}_{j,1}, \mathbf{m}_{j,2}, \dots, \mathbf{m}_{j,l_j}]$  by constructing an orthonormal basis of the column space of the matrix  $\mathbf{T}_j^o = [\mathbf{t}_{j,1}^o, \mathbf{t}_{j,2}^o, \dots, \mathbf{t}_{j,l_j}^o]$ .

In summary, the precoding matrix  $\mathbf{M}_j$  can be viewed as a projection matrix to the null space of

$\mathbf{H}_s^j$ , while regularized to minimize the MSE by taking into account the noise components. Compared to the regularized channel inversion given by (7), the proposed scheme provides a performance improvement as more degrees of freedom is provided in the design of  $\mathbf{t}_{j,i}$  by excluding the columns  $\mathbf{h}_{j,1}, \dots, \mathbf{h}_{j,i-1}, \mathbf{h}_{j,i+1}, \dots, \mathbf{h}_{j,n_j}$  in (12).

It is also important to note from (12) that mathematically speaking the proposed scheme does not require the dimensional condition on the transmit and receive antennas. Although it is possible to support  $L > M$  as in [2], in order to provide high sum rates, we assume that  $L > M$  and  $l_j \leq n_j$ .

### 3.2 Single-user Preprocessing $\mathbf{S}_j$

Once the multiuser preprocessing  $\{\mathbf{M}_j\}_{j=1}^K$  is applied for decomposing the multiuser MIMO channel into parallel single-user MIMO interference channels, any single-user MIMO schemes can be followed as a single-user preprocessing  $\{\mathbf{S}_j\}_{j=1}^K$  <sup>[4][5][6]</sup>. In other words, by applying the projection matrix  $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_K$ , the problem is now reduced to finding the optimal diagonalization processing  $\mathbf{S}_j$  for the single-user MIMO interference channels in (1). Let  $\mathbf{H}_j^e$  denote the effective channel matrix for user  $j$ , given by  $\mathbf{H}_j^e = \mathbf{H}_j \mathbf{M}_j$ . Single user algorithms based on the singularvalue decomposition (SVD) <sup>[9][10][11]</sup> can be applied to each user with the noise covariance term being replaced by the noise-plus-interference covariance matrix.

Using the single user precoding technique in [10], we can write  $\mathbf{S}_j = \mathbf{Q}_j \sqrt{\mathbf{P}_j}$  where  $\mathbf{Q}_j$  is a unitary matrix which diagonalizes the matrix  $\mathbf{H}_j^e$  and  $\mathbf{P}_j$  denotes the power allocation matrix. The optimal unitary matrix  $\mathbf{Q}_j$  is given by  $\mathbf{Q}_j = \mathbf{V}_j^e$  <sup>[12]</sup>, where the  $l_j \times l_j$  matrix  $\mathbf{V}_j^e$  represents the right singular matrix of the  $n_j \times l_j$  matrix  $\mathbf{H}_j^e$  obtained from the SVD  $\mathbf{H}_j^e = \mathbf{U}_j^e \mathbf{A}_j^e \mathbf{V}_j^{eH}$ .

The power allocation matrix  $\mathbf{P}_j$  is a  $l_j \times l_j$  diagonal matrix with non-negative elements. We obtain the power allocation matrix  $\mathbf{P}_j$  which

maximizes the sum of information rates for all users subject to the sum power constraint. In general, the optimal power allocation to achieve the sum capacity of MIMO broadcast channels can be obtained by using an iterative waterfilling procedure<sup>[13]</sup>. Our algorithm based on the MMSE linear processing also needs to perform an iterative waterfilling method to compute the optimal  $\mathbf{P}_j$ . In this paper, we solve the power allocation problem by applying the waterfilling algorithm to the subchannels resulting from the intermediate solution  $\{\mathbf{T}_j = \mathbf{M}_j \mathbf{Q}_j\}_{j=1}^K$ .

The intermediate transmit processing  $\{\mathbf{T}_j = \mathbf{M}_j \mathbf{Q}_j\}_{j=1}^K$  decomposes the network channel matrix  $\mathbf{H}_s$  into parallel multiple MIMO interference channels. Substituting  $\{\mathbf{T}_j = \mathbf{M}_j \mathbf{Q}_j\}_{j=1}^K$  and the receive matrices  $\mathbf{R}_j$  of (6) into (5), we can obtain the normalized error covariance matrix  $\mathbf{E}_j$  for the  $j$ th user as

$$\mathbf{E}_j = \left( \mathbf{I}_j + \mathbf{Q}_j^H \mathbf{M}_j^H \mathbf{H}_j^H \left( \mathbf{H}_j \left( \sum_{k=1, k \neq j}^K \mathbf{M}_k \mathbf{Q}_k \mathbf{Q}_k^H \mathbf{M}_k^H \right) \mathbf{H}_j^H + \frac{L}{\rho} \mathbf{I}_{n_j} \right)^{-1} \mathbf{H}_j \mathbf{M}_j \mathbf{Q}_j \right)^{-1}. \quad (15)$$

Then, the effective channel gain of the  $i$ th stream of user  $j$  can be represented as the corresponding SINR, given by

$$\lambda_{j,i} = \frac{1}{\epsilon_{j,i}^2} - 1 \quad (16)$$

where  $\epsilon_{j,i}^2 = [\mathbf{E}_j]_{i,i}$ . Here  $[\cdot]_{i,i}$  denotes the  $i$ th diagonal entry of a matrix.

We perform power allocation under the assumption that the  $i$ th channel for user  $j$  has an equivalent channel gain of  $\lambda_{j,i}$  for  $j=1, \dots, K$  and  $i=1, \dots, l_j$ . In this case, by using the waterfilling algorithm, we can obtain the power allocation matrices  $\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K$  by the following maximization:

$$\{\mathbf{P}_j\}_{j=1}^K = \arg \max_{\sum_{j=1}^K \text{tr}\{\mathbf{P}_j\} \leq L} \sum_{j=1}^K \sum_{i=1}^{l_j} \log_2(1 + \lambda_{j,i} p_{j,i}), \quad (17)$$

where  $p_{j,i}$  denotes the  $i$ th diagonal element of  $\mathbf{P}_j$ .

We note again that the optimality of the waterfilling algorithm does not hold for the above solution because the effective channel gains  $\lambda_{j,i}$  used in the waterfilling technique are computed under the assumption that each data stream has equal transmit power (i.e.,  $\mathbf{P}_j = \mathbf{I}_j$ ), which is not true once the waterfilling solution  $\mathbf{P}_j$  ( $\mathbf{P}_j \neq \mathbf{I}_j$ ) is applied.

In conclusion, the transmit precoding matrix  $\mathbf{T}_j$  is given as  $\mathbf{T}_j = \mathbf{M}_j \mathbf{S}_j$ , where  $\mathbf{S}_j = \mathbf{Q}_j \sqrt{\mathbf{P}_j}$ , and the receive matrix

$\mathbf{R}_j$  is obtained by Equation (6). Given  $\{\mathbf{T}_j\}_{j=1}^K$  and  $\{\mathbf{R}_j\}_{j=1}^K$  the SINR of the  $i$ th stream of user  $j$  is equal to

$$\text{SINR}_{j,i} = \frac{1}{e_{j,i}^2} - 1 \quad (18)$$

where  $e_{j,i} =$

$$\left[ \left( \mathbf{I}_j + \mathbf{T}_j^H \mathbf{H}_j^H \left( \mathbf{H}_j \left( \sum_{k=1, k \neq j}^K \mathbf{T}_k \mathbf{T}_k^H \right) \mathbf{H}_j^H + \mathbf{H}_j^H + \frac{1}{\rho} \mathbf{I}_{n_j} \right)^{-1} \mathbf{H}_j \mathbf{T}_j \right)^{-1} \right]_{i,i}.$$

Therefore, the maximum achievable sum rate of the proposed algorithm is given by

$$R_{new} = \sum_{j=1}^K \sum_{i=1}^{l_j} \log(1 + \text{SINR}_{j,i}). \quad (19)$$

So far we have assumed  $n_j = l_j$ . For the case of  $n_j > l_j$ , the proposed algorithm can be applied as follows. Let the SVD of  $\mathbf{H}_j$  be  $\mathbf{H}_j = \mathbf{U}_j \mathbf{\Lambda}_j \mathbf{V}_j^H$ . Then the  $n_j \times M$  channel matrix  $\mathbf{H}_j$  can be reduced to an  $l_j \times M$  matrix  $\mathbf{H}_j^r$  by premultiplying  $\mathbf{H}_j$  by  $\mathbf{U}_j^r$  as  $\mathbf{H}_j^r = \mathbf{U}_j^{rH} \mathbf{H}_j$ , where the  $n_j \times l_j$  matrix  $\mathbf{U}_j^r$  is obtained by taking the first  $l_j$  column vectors of  $\mathbf{U}_j$  which correspond to the  $l_j$  largest singular values. We consider the reduced-sized version  $\mathbf{H}_j^r$  as a new channel matrix  $\mathbf{H}_j^r$  for user  $j$ . The resulting  $L \times M$

matrix  $\mathbf{H}_s^r = [\mathbf{H}_1^{rT} \mathbf{H}_2^{rT} \dots \mathbf{H}_K^{rT}]^T$  can be viewed as a multiuser MIMO channel, and thus the proposed scheme can be now applied to  $\mathbf{H}_s^r$  to find the transmit processing  $\mathbf{T}_s = [\mathbf{T}_1 \dots \mathbf{T}_K]$ . Finally, given  $\mathbf{T}_s$  and the original channel matrices  $\mathbf{H}_1, \dots, \mathbf{H}_K$ , we can compute  $\mathbf{R}_j$  for  $j = 1, \dots, K$  from Equation (6).

#### IV. Simulation Results

In this section, we provide simulation results and comparisons to demonstrate the efficacy of our proposed scheme in terms of the sum rate. The simulation results for the sum rate are obtained through a Monte Carlo simulation. In our simulations, we denote  $[M, n(l), K]$  for a  $K$ -user MIMO downlink scenario where the base station employs  $M$  transmit antennas and each user receives  $l$  data streams via  $n$  receive antennas.

Figure 1 shows the sum rate performance of the proposed scheme and the regularized channel inversion, denoted by Reg-CI, with respect to SNR in dB. Note that the average SNR is given by  $\rho = 1/\sigma_n^2$ . In this figure, we set  $M=8$  and  $L=N=8$ . While the Reg-CI based on Equation (7) does not require any receiver processings, the proposed scheme employs the linear MMSE receiver for each user. Therefore, as can be seen from the simulation results depicted in Figure 1, no

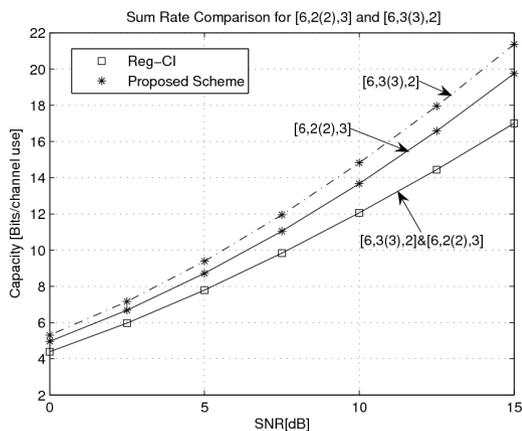


그림 1. 제안하는 기법과 Reg-CI간의 Sum Rate 비교  
Fig 1. Sum rate comparison between the proposed scheme and Reg-CI

performance gain is achieved for the case of the Reg-CI even when the number of receive antennas per user increases since no receiver coordination is available. Figure 1 shows that the proposed method can achieve much higher channel capacity than the Reg-CI. In particular, the performance gap of the proposed scheme over the Reg-CI increases with the number of receive antennas per user since a larger diversity gain is achieved from the receiver coordination at the users.

In the following simulations, the proposed algorithm is compared with two iterative algorithms: Nu-SVD represents a scheme based on the null-space criterion developed in [6] and T-MMSE indicates a scheme based on a minimum total MSE criterion under a total transmit power constraint presented in [7].

Figure 2 provides the sum rate comparison for the  $[4,2(2),2]$  case. In low SNR ranges, the Nu-SVD shows the poor performance in comparison with the T-MMSE due to noise enhancement. On the other hand, as the SNR increases, the Nu-SVD outperforms the T-MMSE since the influence of noise becomes negligible at high SNRs and the channel diagonalization based on the SVD is more effective in terms of sum capacity. More importantly, the proposed scheme which takes the advantages of both approaches is superior to both

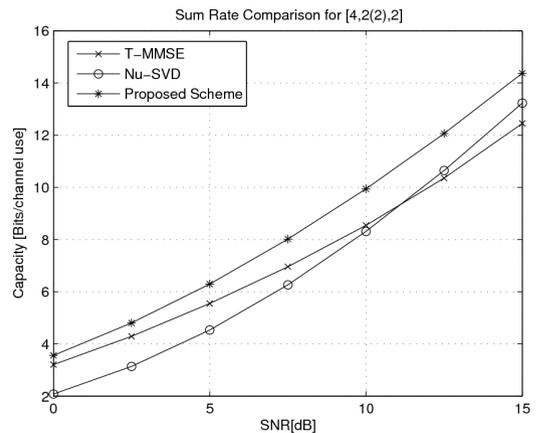


그림 2. 제안하는 기법과 반복적 알고리즘 기법 간의 Sum Rate 비교  
Fig 2. Sum rate comparison between the proposed scheme and iterative algorithms

the T-MMSE and Nu-SVD at all SNR values.

## V. Conclusion

In this paper, we have successfully generalized the regularized channel inversion technique which is originally developed for the single-antenna users into multiuser MIMO downlink systems where each user receives multiple data streams via multiple receive antennas. Compared to the original channel inversion technique, the proposed scheme improves the system performance by allowing receiver antenna coordination at each user. The proposed scheme maintains a low complexity by using non-iterative solutions for both the transmit and receive processing. Simulation results demonstrate that the proposed algorithm is a promising strategy in multiuser MIMO downlink systems with the base station transmitting multiple data streams per each user.

## References

- [1] G. Caire and S. Shamai, "On the Achievable Throughput of a Multiantenna Gaussian Broadcast Channel," *IEEE Transactions on Information Theory*, Vol.49, pp.1691-1706, July 2003.
- [2] C. B. Peel, B. Hochwald, and A. L. Swindlehurst, "A Vector-Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication-Part I: Channel Inversion and Regularization," *IEEE Transactions on Communications*, Vol.53, pp.195 - 202, January 2005.
- [3] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-Forcing methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels," *IEEE Transactions on Signal Processing*, Vol.52, pp.461-471, February 2004.
- [4] A. Bourdoux and N. Khaled, "Joint TX-RX optimisation for MIMOSDMA based on a null-space constraint," in Proc. *IEEE VTC '02*, pp.171-174, September 2002.
- [5] L.-U. Choi and R. D. Murch, "A Transmit Preprocessing Technique for Multiuser MIMO Systems Using a Decomposition," *IEEE Transactions on Wireless Communications*, Vol.3, pp.20-24, January 2004.
- [6] Z. Pan, K.-K. Wong, and T.-S. Ng, "Generalized Multiuser Orthogonal Space-Division Multiplexing," *IEEE Transactions on Wireless Communications*, Vol.6, pp.1969-1973, November 2004.
- [7] J. Zhang, Y. Wu, S. Zhou, and J. Wang, "Joint Linear Transmitter and Receiver Design for the Downlink of Multiuser MIMO Systems," *IEEE Communications Letters*, Vol.9, pp.991-993, November 2005.
- [8] A. J. Tenenbaum and R. S. Adve, "Joint Multiuser Transmit-Receive Optimization Using Linear Processing," in Proc. *IEEE ICC '04*, Vol.1, pp.588 - 592, June 2004. 6
- [9] G. G. Raleigh and J. M. Cioffi, "Spatio-Temporal Coding for Wireless Communication," *IEEE Transactions on Communications*, Vol.46, pp.357 - 366, March 1998.
- [10] H. Sampath, P. Stoica, and A. Paulraj, "Generalized Linear Precoder and Decoder Design for MIMO Channels Using the Weighted MMSE Criterion," *IEEE Transactions on Communications*, Vol.49, pp.2198-2206, December 2001.
- [11] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx Beamforming Design for Multi-carrier MIMO Channels: A Unified Framework for Convex Optimization," *IEEE Transactions on Signal Processing*, Vol.51, pp.2381-2401, September 2003.
- [12] J. B. Andersen, "Array Gain and Capacity for Known Random Channels with Multiple Element Arrays at Both Ends," *IEEE Journal on Selected Areas in Communications*, Vol.18, pp.2172-2178, November 2000.
- [13] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative Water-filling for Gaussian Vector Multiple Access Channels," *IEEE Transactions on Information Theory*, Vol.50, pp.145-152, January 2004.

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