

# 높은 안테나 상관도를 갖는 다중입출력 공간 다중화 시스템을 위한 통계적 프리코딩 기법

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## Statistical Precoder Design for Spatial Multiplexing Systems in Correlated MIMO Fading Channels

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### 요 약

다중입출력 공간 다중화 시스템은 송수신 안테나 간 상관도가 있는 채널에서 심각한 성능 열화를 겪는다. 본 논문에서는 ML (maximum likelihood) 수신기를 결합한 다중입출력 무선통신 환경을 위해, 송신단에서 채널상관 행렬 정보만을 활용한 새로운 통계적 프리코딩 기법을 소개한다. 우리는 다차원 심볼 성상의 최소 유클리디언 거리를 최대화하는 두 가지 간단한 형식의 (closed-form solution) 프리코더, 회전 변환 및 파워 로딩 기법을 제안한다. 또한, 제안한 기법을 선형 zero-forcing (ZF) 수신기에 확장 적용하여 성능을 향상시키는 방법을 고안한다. 실험 결과를 통해 제안하는 기법은 ML 수신기 및 ZF 수신기에서 기존의 기법에 비하여 각각 2dB 및 8dB의 비트 에러율 성능 이득을 제공함을 확인할 수 있다.

**Key Words** : multiple-input multiple-output (MIMO), spatial multiplexing, antenna correlation, maximum-likelihood detection (MLD), statistical precoding

### ABSTRACT

It has been shown that the performance of multiple-input multiple-output (MIMO) spatial multiplexing systems is significantly degraded when spatial correlation exists between transmit and receive antenna pairs. In this paper, we investigate designs of a new statistical precoder for spatial multiplexing systems with maximum likelihood (ML) receiver which requires only correlation statistics at the transmitter. Two kinds of closed-form solution precoders based on rotation and power allocation are proposed by means of maximizing the minimum E tclidean distance of joint symbol constellations. In addition, we extend our results to linear receivers for correlated channels. We provide a method which yields the same profits from the proposed precoders based on a simple zero-forcing (ZF) receiver. The simulation shows that 2dB and 8dB gains are achieved for ML and ZF systems with two transmit antennas, respectively, compared to the conventional systems.

### I. Introduction

Considering the rapid increase in demand for

reliable communications and higher data rates in downlink cellular networks, the design of practical systems which can achieve huge capacity is of great

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importance. In recent years, multiple-input multiple-output (MIMO) systems have been intensively studied and proved that they can dramatically improve the system capacity<sup>[1][2]</sup>. However, this outstanding performance is realized only when the fading between transmit and receive antenna pairs is independently distributed. In a real propagation environment, the capacity degrades significantly in the presence of fading correlations<sup>[3,4]</sup>. The worst situation drastically deteriorates the MIMO capacity, which falls back to that of a single antenna system.

One of the most promising MIMO applications is a spatial multiplexing (SM) scheme<sup>[5]</sup> such as Bell Labs layered space-time (BLAST) systems<sup>[6]</sup>. Despite their high capacity gain, the performance of the SM systems is also shown to be quite sensitive to the correlation properties. To illustrate this, the bit error rate (BER) performance over correlated MIMO channels has been analyzed using a union bound approach with maximum-likelihood (ML) receiver<sup>[7]</sup> and zero-forcing (ZF) linear receiver<sup>[8,9]</sup>.

In future standards such as 3GPP-LTE<sup>[10]</sup> and IEEE 802.16m<sup>[11]</sup>, it is agreed that long-term channel statistics such as covariance matrix information can be acquired at the transmitter, from which more reliable transmission can be expected. Transmit antenna selection has been proposed as a means to enhance the error performance of correlated fading channels<sup>[12]</sup>. However, in this case, we may lose some SNR to enhance. On the other hand, a precoding method was introduced in [13] as a  $2 \times 2$  ML receiver based on transmit power allocation and symbol rotation, which minimizes the average mBER by finding the best setting by a numerical search. In a similar context, the authors in [14] proposed a power allocation scheme in a closed-form solution using their hybrid ZF and successive interference cancellation (SIC) receiver structure.

In this paper, we propose new statistical precoders to combat the effects of correlation for the ML receiver by using long-term average channel information. Unlike typical closed loop systems, the transmitter is not aware of all the fading coefficients. We present two precoding methods, i.e.,

rotation and power allocation, by means of maximizing the minimum squared Euclidean distance. Both schemes are derived in a closed-form as a function of the correlation coefficient. After that, we extend our results to simple linear receivers and provide a method which yields the same profits with a simple ZF receiver. From simulations, we show that the proposed schemes achieve about 2dB and 8dB gains over the conventional system with the ML and the ZF receiver, respectively.

The following notations are used throughout the paper. Normal letters represent scalar quantities, bold face letters indicate vectors, and boldface uppercase letters designate matrices. The superscripts  $(\cdot)^*$ ,  $(\cdot)^T$  and  $(\cdot)^H$  denote the complex conjugate, transpose and Hermitian transpose, respectively. For a matrix  $\mathbf{A}$ ,  $\det(\mathbf{A})$ ,  $r(\mathbf{A})$  and  $Tr(\mathbf{A})$  indicate the determinant, rank and trace of  $\mathbf{A}$ .  $C(\mathbf{A})$  and  $\mathcal{N}(\mathbf{A}^T)$  denote the column space and left nullspace of  $\mathbf{A}$ , respectively.  $\mathbf{I}_d$  and  $\mathbf{0}$  indicate an identity matrix of size  $d$  and a zero matrix, respectively. Lastly, we use a bar to account for real variables and denote the real and imaginary part of a complex vector  $c$  by  $Re[c]$  and  $Im[c]$ , respectively.

## II. System Description

### 2.1 System Model

We consider a single-user MIMO system with  $N_t$  transmit and  $N_r$  receive antenna arrays. Assuming flat-fading channels, the discrete-time baseband signal model is given by

$$\mathbf{y} = \sqrt{\frac{P}{N_t}} \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

where  $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$  is the vector of transmitted symbols,  $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$  is the received signal vector and  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  is the channel matrix which is known perfectly at the receiver through ideal channel estimation. The additive white Gaussian noise (AWGN) at the receiver is denoted by  $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$  with i.i.d. entries from  $CN(0, \sigma_n^2)$ . By assuming  $Tr(\mathbf{E}[\mathbf{x}\mathbf{x}^H]) \leq N_t$ , the total power available at the

transmitter becomes  $P$ . Then, the average received signal-to-noise ratio (SNR) is defined as  $\gamma = P/\sigma_n^2$ .

In ideal situation, the entries of  $\mathbf{H}$  follow i.i.d. Gaussian distribution. However, this assumption does not hold in real fading environments due to the lack of enough antenna spacing or rich scattering. One widely used MIMO correlation model is the ‘‘Kronecker model’’ of the form<sup>[3]</sup>

$$\mathbf{H} = \mathbf{R}_t^{1/2} \mathbf{H}_w \mathbf{R}_r^{1/2}$$

where  $\mathbf{R}_t$  and  $\mathbf{R}_r$  stand for the covariance matrices among the transmit and receive antennas, respectively, and  $\mathbf{H}_w \in C^{N_r \times N_t}$  consists of i.i.d. complex Gaussian entries with  $CN(0,1)$ . In this paper, we adopt an exponential correlation model for  $\mathbf{R}_t$  and  $\mathbf{R}_r$ <sup>[13]</sup>, where the  $(i,j)$ -th component of  $\mathbf{R}_t$  and  $\mathbf{R}_r$  is defined as  $\rho^{|i-j|}$  with  $\rho$  denoting the correlation coefficient. We will assume that  $\rho$  is known at the transmitter.

## 2.2 Union Bound of BER

A tight upper bound on the BER of the MIMO SM system was derived in [15] under the uncorrelated fading assumption. The pairwise-error probability (PEP) that the ML receiver decides  $\mathbf{x}_j$  erroneously when  $\mathbf{x}_i$  is transmitted is written as

$$PEP_{i,j} = E_{\mathbf{H}} \left\{ Q \left( \sqrt{\frac{d_{R,i,j}^2}{2\sigma_n^2}} \right) \right\}$$

where  $d_{R,i,j}^2 = \|\mathbf{H}(\mathbf{x}_i - \mathbf{x}_j)\|^2$  denotes the squared Euclidean distance between  $\mathbf{x}_i$  and  $\mathbf{x}_j$  at the receiver and  $Q(\cdot)$  indicates the Gaussian Q-function. Then, the BER  $P_b$  can be approximated by summing up all weighted PEPs as

$$P_b \leq \frac{1}{M^{N_t}} \sum_{i=1}^{M^{N_t}} \sum_{j=1, j \neq i}^{M^{N_t}} \frac{N_{b,i,j}}{N_t \log_2 M} PEP_{i,j} \quad (2)$$

where  $M$  represents the constellation size and  $N_{b,i,j}$  denotes the number of bit errors between  $\mathbf{x}_i$  and  $\mathbf{x}_j$  which depends on the mapping rule.

At high SNR regions, the PEP over joint transmit and receive correlated fading channels can be approximated by<sup>[7]</sup>

$$PEP_{i,j} \approx \left( \frac{1}{\det(\mathbf{R}_t)} \right) \left( \frac{2r(\mathbf{R}_t) - 1}{r(\mathbf{R}_t) - 1} \right) \left( \frac{1}{\gamma} \frac{N_t}{d_{T,i,j}^2} \right)^{r(\mathbf{R}_t)} \quad (3)$$

where we define  $d_{T,i,j}^2 = \|\mathbf{R}_t^{1/2}(\mathbf{x}_i - \mathbf{x}_j)\|^2$  as the Euclidean distance at the transmitter side before passing through the channel  $\mathbf{H}$ . The PEP expression (3) shows that the diversity order is determined by  $r(\mathbf{R}_t) \leq N_t$  and also an SNR loss is caused independently by  $\mathbf{R}_t$  and  $\mathbf{R}_r$ . It is obvious that the worst case of  $d_{T,i,j}^2$  should be maximized to minimize the PEP.

## III. Statistical Precoder Design

In this section, we will discuss how to design a precoding matrix for correlated MIMO systems under the assumption that only  $\mathbf{R}_t$  is available at the transmitter, which is a practical scenario in current system designs. In 802.16m standard, for example, the quantized information of  $\mathbf{R}_t$  is considered to be fed back regularly in order to perform codebook transformation<sup>[11]</sup>. Note that the effect of  $\mathbf{R}_r$  is irrelevant to the transmitter design.

One of the most desired design goal is to minimize the BER performance which is approximated in (2). However, this problem cannot be solved in a closed-form solution and numerical calculation is required<sup>[13]</sup>. To come up with an implementable solution, we note from the PEP analysis of (3) that the BER is inversely related to  $d_{T,i,j}^2$  over all possible  $(i,j)$  pairs. Since  $d_{T,i,j}^2$  is not a function of the whole channel matrix  $\mathbf{H}$  but only  $\mathbf{R}_t$ , we can handle  $d_{T,i,j}^2$  only in a statistical way using the precoder. We define a complex precoding matrix  $\mathbf{F} \in C^{N_t \times N_t}$  which precodes the data symbol vector  $\mathbf{s} \in C^{N_t \times 1}$ , i.e.,  $\mathbf{x} = \mathbf{F}\mathbf{s}$ .

Now our main purpose is to find a simple closed-form solution precoder  $\hat{\mathbf{F}}$  which satisfies

$$\hat{\mathbf{F}} = \arg \max_{\mathbf{F}} \min_{i \neq j} d_{T,ij}^2, \quad 1 < i, j < M^{N_t} \quad (4)$$

under the power constraint of  $\text{Tr}(\mathbf{F}\mathbf{F}^H) = N_t$  and the use of ML detection. We will use the notation  $d_{T,\min}^2$  to indicate  $\min_{(i,j)} d_{T,ij}^2$  for simplicity.

With no spatial correlation,  $d_{T,\min}^2$  is simply determined by the single symbol-error case. In this case, there is no need for the transmitter optimization and the conventional ML detection (i.e.,  $\mathbf{F} = \mathbf{I}_{N_t}$ ) is optimal. However, if  $\mathbf{R}_t$  becomes rank deficient, it makes the specific symbol vectors get closer. For example, for BPSK constellation of  $\{1, -1\}$  with  $N_t = 2$ ,  $d_{T,\min}^2 = 4$  is guaranteed when  $\mathbf{x}_i - \mathbf{x}_j = [2 \ 0]^T$  with no spatial correlation. However, under the high correlation of  $\rho = 0.9$ , the error vector  $[2 \ -2]^T$  becomes dominant and  $d_{T,\min}^2$  reduces to 0.8, which causes a severe performance loss.

In general, the precoding matrix  $\mathbf{F}$  can be any  $N_t \times N_t$  complex matrix. However, with no constraint, too many variables make it difficult to solve the problem of (4). In this paper, we consider two well-known shapes of precoder, i.e., rotation and power allocation under the assumption of  $N_t = 2$ .

### 3.1 Rotational Precoding

We consider the Jacobi rotation matrix  $\mathbf{F}(\theta)$  defined by<sup>[16]</sup>

$$\mathbf{F}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where  $\theta$  denotes a rotation angle. By applying  $\mathbf{F}(\theta)$  at the transmitter,  $d_{T,ij}^2(\theta)$  can be computed as

$$\begin{aligned} d_{T,ij}^2(\theta) &= \|\mathbf{R}_t^{1/2} \mathbf{F}(\theta) \mathbf{e}_{ij}\|^2 \\ &= \left\| \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e_{1,ij} \\ e_{2,ij} \end{bmatrix} \right\|^2 \\ &= \|\mathbf{e}_{ij}\|^2 + \rho A \cos 2\theta + \rho B \sin 2\theta \end{aligned} \quad (5)$$

where the error vector  $\mathbf{e}_{ij} = [e_{1,ij} \ e_{2,ij}]^T = \mathbf{s}_i - \mathbf{s}_j$  is defined between two different codewords  $\mathbf{s}_i$  and  $\mathbf{s}_j$  and we have  $A = e_{1,ij}e_{2,ij}^* + e_{1,ij}^*e_{2,ij}$  and  $B = |e_{1,ij}|^2 - |e_{2,ij}|^2$ .

Solving the problem (4) seems difficult even with

this  $2 \times 2$  rotation matrix case of (5), because exhaustive calculations are required for  $d_{T,ij}^2(\theta)$  for

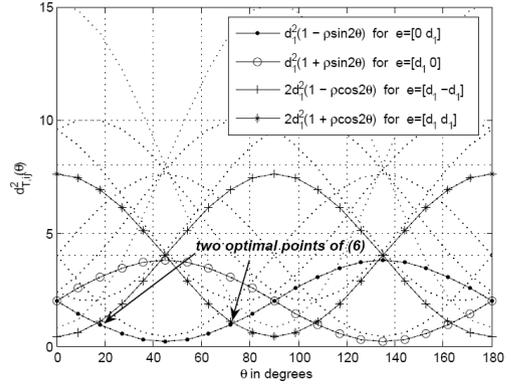


그림 1. 4-QAM 상에서 각 에러 벡터 후보에 대한  $d_{T,ij}^2(\theta)$  ( $\rho = 0.9$ )

Fig. 1.  $d_{T,ij}^2(\theta)$  for all candidate error vectors over 4-QAM ( $\rho = 0.9$ )

every  $i, j$  and  $\theta$  combinations. However, the problem becomes simpler by utilizing the symmetry property in its objective function. Note that  $d_{T,ij}^2(\theta)$  is a sinusoid with a period  $\pi$  and there always exists a symmetric pair of two sinusoids whose phase difference is  $\pi/2$ . Therefore, the search range of  $\theta$  can be reduced to  $0 \leq \theta \leq \pi/2$ .

For the case of  $N_t = 2$ , there are  $M^2(M^2 - 1)$  possible error vector candidates for  $M$ -QAM modulation. Utilizing symmetric properties of QAM modulations, among all candidate error vectors, we can consider only four error vectors  $\mathbf{e}_{ij}$  as

$$S = [0 \ d_1]^T, [d_1 \ 0]^T, [d_1 \ -d_1]^T, [d_1 \ d_1]^T$$

where  $d_1 = \sqrt{6\epsilon_x/(M-1)}$  denotes the minimum Euclidean distance of the  $M$ -QAM constellation with average symbol energy  $\epsilon_x$ <sup>[17]</sup>.

Now, the search range of the problem (4) can be simplified and we have

$$\begin{aligned} \hat{\theta} &= \arg \max_{0 \leq \theta \leq \pi/2} \min d_{T,ij}^2(\theta) \\ &= \arg \max_{0 \leq \theta \leq \pi/2} \min [d_1^2(1 - \rho \sin 2\theta), d_1^2(1 + \rho \sin 2\theta), \\ &\quad 2d_1^2(1 - \rho \cos 2\theta), 2d_1^2(1 + \rho \cos 2\theta)]. \end{aligned} \quad (6)$$

In Figure 1,  $d_{T,ij}^{\rho}$  for all possible error vectors are depicted for 4-QAM modulation with  $\rho = 0.9$ , where dotted lines represent the cases of error vectors which do not belong to  $S$ . Here we can confirm that it is sufficient to consider only four error vectors in  $S$  to maximize the minimum value of  $d_{T,ij}^{\rho}$ . We also learn from simulations that the same relations hold for higher modulation levels such as  $M=16$  and 64.

Moreover, Figure 1 shows that one of the last two candidates  $2d_1^{\rho}(1-\rho\cos 2\theta)$  and  $2d_1^{\rho}(1+\rho\cos 2\theta)$  in (6) can be further removed because we need only one single value for  $\theta$ . Then, the optimum  $\hat{\theta}$  which maximizes the rest three sinusoidal functions is determined at their intersections in  $0 \leq \theta \leq \pi/4$  as shown in Figure 1. Accordingly, by solving the following two equations

$$\begin{cases} d_1^{\rho}(1-\rho\sin 2\hat{\theta}) = 2d_1^{\rho}(1-\rho\cos 2\hat{\theta}), & \text{for } \rho \geq 0.5 \\ d_1^{\rho}(1-\rho\sin 2\hat{\theta}) = d_1^{\rho}(1+\rho\sin 2\hat{\theta}), & \text{for } \rho < 0.5, \end{cases}$$

we obtain  $\hat{\theta}$  for the optimal rotational precoding as

$$\hat{\theta} = \begin{cases} \frac{1}{2} \cos^{-1} \left( \frac{2 + \sqrt{5\rho^2 - 1}}{5\rho} \right), & \text{for } \rho \geq 0.5 \\ 0, & \text{for } \rho < 0.5. \end{cases} \quad (7)$$

This indicates that if the correlation is weaker than  $\rho = 0.5$ ,  $d_{T,\min}^{\rho}$  is still determined by the single error pattern, and the optimal precoding reduces to the conventional ML detection, i.e.,  $\mathbf{F}(0) = \mathbf{I}_{N_t}$ . In fact, it is shown that the BER performance of the ML receiver is not much influenced until the correlation exceed a certain level<sup>[7]</sup>. Note that (7) is a function of  $\rho$  only, not the constellation size  $M$ .

### 3.2 Power Allocation

In this section, we employ another precoder in the form of power allocation defined as

$$\mathbf{F}(p) = \begin{bmatrix} \sqrt{2p} & 0 \\ 0 & \sqrt{2(1-p)} \end{bmatrix}$$

where the parameter  $p$  ( $0 \leq p \leq 1$ ) determines the power ratio between two data symbols. Then, we have

$$\begin{aligned} d_{T,ij}^{\rho}(p) &= \|\mathbf{R}_t^{1/2} \mathbf{F}(p) \mathbf{e}_{i,j}\|^2 \\ &= 2p|e_{1,i,j}|^2 + 2(1-p)|e_{2,i,j}|^2 + \rho A \sqrt{4p(1-p)}. \end{aligned} \quad (8)$$

Assuming  $p \geq 0.5$ , we can express the problem by substituting the same candidates in  $S$  into (8) as

$$\begin{aligned} \hat{p} &= \arg \max_{p \geq 0.5} \min_{\mathbf{e}_q \in S} d_{T,ij}^{\rho}(p) \\ &= \arg \max_{p \geq 0.5} \min_{\mathbf{e}_q \in S} [2d_1^{\rho}p, 2d_1^{\rho}(1-p), 2d_1^{\rho}(1-\rho\sqrt{4p(1-p)}), \\ &\quad 2d_1^{\rho}(1+\rho\sqrt{4p(1-p)})]. \end{aligned} \quad (9)$$

Clearly, the fourth term can be eliminated again. By solving the equations, the optimal factor  $\hat{p}$  is derived as

$$\hat{p} = \begin{cases} \frac{4\rho^2}{4\rho^2 + 1}, & \text{for } \rho \geq 0.5 \\ 0.5, & \text{for } \rho < 0.5. \end{cases} \quad (10)$$

Likewise,  $\mathbf{F}(\hat{p})$  is valid for  $\rho \geq 0.5$ . We have noticed that the proposed power weight  $\hat{p}$  is analogous to the solution obtained in [14]. However, the precoding approach of [14] is mainly based on the analysis of their hybrid ZF and SIC detection, while the proposed scheme optimizes the  $N_t$ -dimension signal constellations which is directly connected to the mechanism of the ML detection.

## IV. Extension to Linear Receiver

In the previous section, we have presented two effective ways of precoding based on the ML receiver. Although, numerous practical algorithms for the ML detection have been proposed recently<sup>[18,19]</sup>, they still bear the burden of high complexity especially with strong channel codes. To alleviate the complexity issue, we extend our results to the case where a simple ZF linear receiver is applied. In conventional ZF receiver systems, there exists no known statistical precoding way which would overcome the correlation effect. In this section, we propose a modified ZF receive filter, which can directly adopt the precoding schemes shown in the previous section. The description will be based on  $N_t = 2$  case for simplicity.

The real-valued equivalent representation of the system model (1) is given as<sup>[18]</sup>

$$\bar{\mathbf{y}} = \begin{bmatrix} Re[\mathbf{y}] \\ Im[\mathbf{y}] \end{bmatrix} = \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}} \quad (11)$$

where  $\bar{\mathbf{x}} = [Re[\mathbf{x}^T] \ Im[\mathbf{x}^T]]^T$ ,  $\bar{\mathbf{n}} = [Re[\mathbf{n}^T] \ Im[\mathbf{n}^T]]^T$ , and

$$\bar{\mathbf{H}} = \begin{bmatrix} Re[\mathbf{H}] & -Im[\mathbf{H}] \\ Im[\mathbf{H}] & Re[\mathbf{H}] \end{bmatrix} = [\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2 \ \bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4].$$

Here  $\bar{\mathbf{n}}$  represents the real Gaussian noise vector with zero mean and covariance matrix  $(\sigma_n^2/2)\mathbf{I}_{2N_r}$ .

Denoting a  $4 \times 2N_r$  real matrix  $\bar{\mathbf{B}}$  as the receive filter, the output signal vector  $\bar{\mathbf{z}}$  is denoted by

$$\bar{\mathbf{z}} = \bar{\mathbf{B}}^T \bar{\mathbf{y}} = \bar{\mathbf{B}}^T \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{v}}$$

where  $\bar{\mathbf{v}} = \bar{\mathbf{B}}^T \bar{\mathbf{n}}$  is the output noise vector with the covariance matrix  $(\sigma_n^2/2)\bar{\mathbf{B}}\bar{\mathbf{B}}^T$ .

Then, the ZF problem can now be generalized to find  $\bar{\mathbf{B}}$  which forces the interference between the two effective symbols  $Re[\mathbf{x}]$  and  $Im[\mathbf{x}]$  to zero. The necessary and sufficient condition is that the subspace spanned by the  $\bar{\mathbf{h}}_1$  and  $\bar{\mathbf{h}}_2$  pair is orthogonal to that spanned by  $\bar{\mathbf{h}}_3$  and  $\bar{\mathbf{h}}_4$ , which can be achieved by projection. This was originally introduced in a group detection (GD) problem<sup>[20]</sup> which aims to perform subgroup ML detection. Here we want to make separate detection for two effective symbols  $Re[\mathbf{x}]$  and  $Im[\mathbf{x}]$ .

We apply the QR decomposition to  $2N_r \times 2$  effective channel matrices  $[\bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4]$  and  $[\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2]$  represented by

$$\begin{aligned} [\bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4] &= \begin{bmatrix} \mathbf{Q}_{Re}^{(1)} & \mathbf{Q}_{Re}^{(0)} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{Re}^{(1)} \\ 0 \end{bmatrix} \\ [\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2] &= \begin{bmatrix} \mathbf{Q}_{Im}^{(1)} & \mathbf{Q}_{Im}^{(0)} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{Im}^{(1)} \\ 0 \end{bmatrix} \end{aligned}$$

where  $\mathbf{Q}_{Re}^{(1)} \in R^{2N_r \times 2}$  and  $\mathbf{Q}_{Re}^{(0)} \in R^{2N_r \times (2N_r - 2)}$  denote the matrices whose columns form an orthonormal basis for  $C([\bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4])$  and  $N([\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2])$ , respectively, and

$\mathbf{R}_{Re}^{(1)}$  is an upper-triangular matrix. The same definition follows for  $\mathbf{Q}_{Im}^{(1)}$ ,  $\mathbf{Q}_{Im}^{(0)}$  and  $\mathbf{R}_{Im}^{(1)}$ . Here, the notations  $Re$  and  $Im$  are employed to emphasize that the matrices are intended for  $Re[\mathbf{x}]$  and  $Im[\mathbf{x}]$ , respectively.

Now, the orthogonality conditions become  $C([\bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4]) \perp C[\mathbf{Q}_{Re}^{(0)}]$  and  $C([\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2]) \perp C[\mathbf{Q}_{Im}^{(0)}]$ . By choosing arbitrary two column vectors of  $\mathbf{Q}_{Re}^{(0)}$  as the receive filter for  $Re[\mathbf{x}]$ , we can eliminate the interference from  $Im[\mathbf{x}]$  completely and vice versa. Without loss of generality, we have finally

$$\bar{\mathbf{B}} = [\mathbf{Q}_{Re}^{(0)}(1:2) \ \mathbf{Q}_{Im}^{(0)}(1:2)] \quad (12)$$

where  $(\mathbf{A})_{(1:2)}$  stands for a matrix composed of the first and second column of  $\mathbf{A}$ .

As the effective channels combined with  $\bar{\mathbf{B}}$  is constituted by two  $2 \times 2$  real submatrices as

$$\bar{\mathbf{B}}^T \bar{\mathbf{H}} = \begin{bmatrix} \mathbf{Q}_{Re}^{(0)T}(1:2) [\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2] & 0 \\ 0 & \mathbf{Q}_{Im}^{(0)T}(1:2) [\bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4] \end{bmatrix},$$

we can achieve the precoding gain in the same way as the ML receiver by applying  $\bar{\mathbf{F}}$ . Note that the conventional ZF receiver  $\bar{\mathbf{H}}^+ = (\bar{\mathbf{H}}^T \bar{\mathbf{H}})^{-1} \bar{\mathbf{H}}^T$  tries to eliminate the whole interference among all the components to achieve  $\bar{\mathbf{H}}^+ \bar{\mathbf{H}} = \mathbf{I}_4$ .

We mention that although our method is described only for  $N_t = 2$ , we can extend the results to an arbitrary number of  $N_t$  with a proper choice of pairing. The corresponding precoder  $\bar{\mathbf{F}}$  in real domain is simply a  $2N_t \times 2N_t$  block diagonal matrix whose all  $N_t$  submatrices are equal to  $\mathbf{F}$ . Also we note that a single fixed precoder  $\mathbf{F}(\theta)$  (or  $\mathbf{F}(p)$ ) can be applied for both the ML and the proposed ZF receiver structure.

## IV. Simulation Results

This section demonstrates the efficacy of the proposed precoding schemes through Monte-Carlo simulations. We consider a  $2 \times 2$  SM system with  $\rho = 0.9$  at the transmitter. In Figure 2, the BER

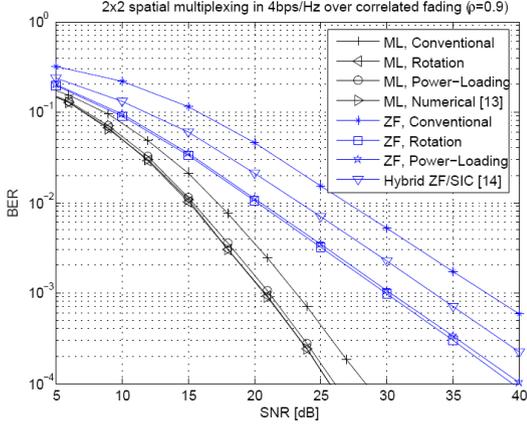


그림 2.  $2 \times 2$  공간 다중화 시스템의 비트에러율 성능 ( $\rho = 0.9$ )  
Fig. 2. Bit-error probability of  $2 \times 2$  spatial multiplexing systems with  $\rho = 0.9$

performance is plotted over a flat Rayleigh fading using 4-QAM constellations for both the ML and the proposed ZF receivers. By applying our proposed rotation and power allocation, we achieve about 2.5dB and 8dB gains over the conventional ML and ZF receiver, respectively, at a BER of  $10^{-4}$ . The proposed rotation slightly performs better than the power allocation. Also we draw the BER performance of the precoder found by exhaustive search in (2)<sup>[13]</sup>. The curves show that the proposed rotation acquires the best performance by simply optimizing  $\theta$ . Moreover, the proposed ZF receiver with rotation performs 4dB better than the system in [14].

Next, we present the results for frequency-selective channels with orthogonal frequency division multiplexing (OFDM) techniques<sup>[21]</sup>. A convolutional code with polynomials (133,171) in octal notation is utilized with a pseudorandom bit-interleaver. Packet size is set to 256 bits with 4-QAM modulation. For a channel model, a five-tap exponentially decaying profile is used. Figures 3 and 4 illustrate the packet error rate (PER) for the code rate of 1/2 and 3/4, respectively. We can confirm in both results that the tendency is similar to the flat-fading channel case. The proposed rotation achieves about 2dB and 7dB gains at a PER of  $10^{-2}$  for the ML and the ZF receiver, respectively, in

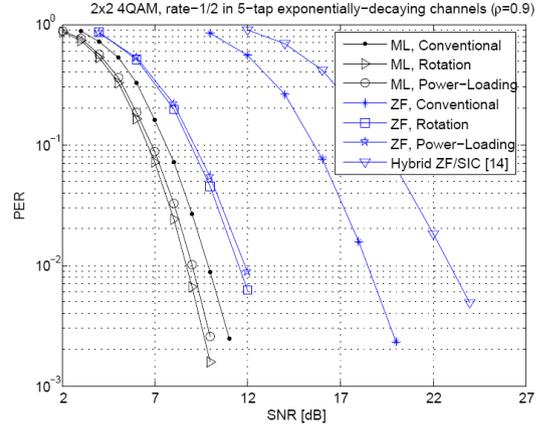


그림 3. 1/2 코드율의 길쌈 부호를 결합한  $2 \times 2$  공간 다중화 시스템의 패킷에러율 성능 ( $\rho = 0.9$ )  
Fig. 3. Packet-error probability of  $2 \times 2$  rate-1/2 convolutional coded spatial multiplexing systems with  $\rho = 0.9$

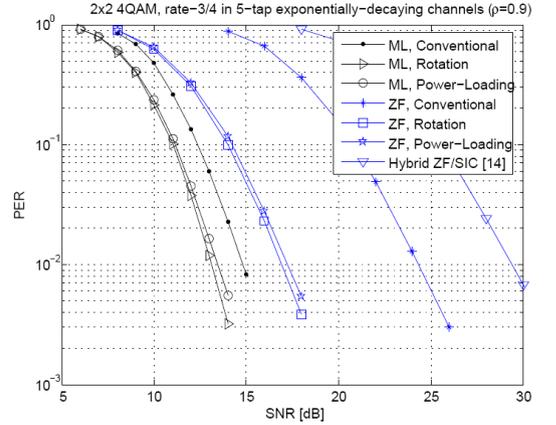


그림 4. 3/4 코드율의 길쌈 부호를 결합한  $2 \times 2$  공간 다중화 시스템의 패킷에러율 성능 ( $\rho = 0.9$ )  
Fig. 4. Packet-error probability of  $2 \times 2$  rate-3/4 convolutional coded spatial multiplexing systems with  $\rho = 0.9$

Figure 4. Note that the hybrid ZF and SIC detection scheme in [14] performs even worse than the conventional ZF scheme here due to error propagation. One can see that the performance gap of 4dB between the ML and the ZF receiver still exists.

## V. Conclusion

In this paper, we have presented new precoder design schemes for correlated MIMO spatial

multiplexing systems by making use of the channel correlation information at the transmitter. Two precoding strategies have been proposed in order to minimize the BER performance of the ML receiver. Instead of dealing with the BER equation directly, we have derived the closed-form solutions which maximize the minimum squared Euclidean distance. In addition, we have extended our results to the case of a linear ZF receiver. The simulation results are presented to illustrate the validity of the proposed precoding schemes in correlated channels with both the ML and the proposed ZF receiver. We note that the proposed scheme for ZF receiver can be extended to the better criteria which give a good balance between the desired signal power and the noise power such as signal-to-interference-plus-noise ratio (SINR) maximization<sup>[22]</sup>.

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