Orthogonalized Spatial Multiplexing for Rank-3 Transmission in Closed-Loop MIMO Systems

Changick Song · Inkyu Lee

Orthogonalized Spatial Multiplexing (OSM) scheme for Multiple-Input Multiple-Output (MIMO) channels has been originally developed for transmitting two data streams, which allows simple maximum ikelihood decoding at the receiver with small feedback information. In this paper, by extending the existing OSM scheme, we propose a new Spatial Multiplexing (SM) system which supports a larger number of data streams. To this end, we adopt three successive rotation precoding with four phase angles, so that we can decompose the MIMO channel into two identical subchannels. Moreover, it is shown that we can further improve the performance by applying multidimensional constellation rotation. Through the numerical results, we show that the proposed scheme outperforms the Singular Value Decomposition (SVD) based scheme while the computational complexity and the feedback overhead are reduced.

Keywords: Multiple-Input Multiple-Output (MIMO), Closed-loop, Spatial multiplexing

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless systems have been widely studied to increase the communication reliability and spectral efficiency. The capacity analysis of MIMO systems have shown significant gains over Single-Input Single-Output (SISO) systems [1],[2]. The expected benefits include a Spatial Multiplexing (SM) gain and a diversity gain. Especially, the SM gain enables extremely high spectral efficiency by transmitting independent data streams simultaneously through multiple transmit antennas.

In order to fully exploit the potential of multiple antennas and achieve the promised capacity, we can apply full Channel State Information (CSI) knowledge to the transmit side to optimize the transmission scheme according to current channel conditions. Many studies on such closed-loop MIMO systems have been based on Singular Value Decomposition (SVD) of the channel transfer matrix [3],[4]. The optimum linear precoder and decoder decouple the MIMO channel into several independent eigen subchannels, and allocate resources such as power and bits over these subchannels. In Time-Division Duplex (TDD) systems, such full CSI may be obtained using reciprocity, while in Frequency Division Duplex (FDD) systems, full CSI must be conveyed through a feedback channel. This is quite impractical, though, due to the large number of channel coefficients that need to be quantized and even under flat fading MIMO channels, the feedback requirements generally grow with the number of antennas and users. Another drawback of SVD based schemes is that the SVD operation requires high computational complexity and is numerically sensitive.

Recently, Orthogonalized Spatial Multiplexing (OSM) has been developed in [5] for closed-loop MIMO systems, which allows simple Maximum Likelihood (ML) detection at the receiver with small feedback information. This

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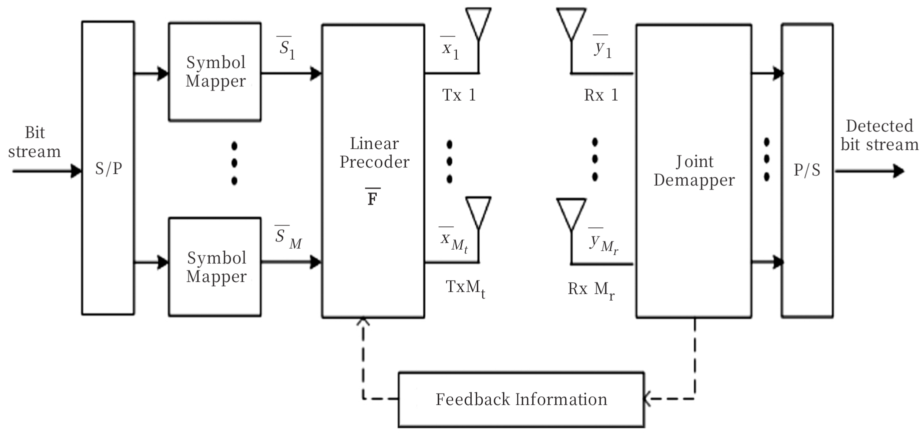

Figure 1. Schematic diagram of a closed-loop spatial multiplexing scheme with M_t transmit and M_r receive antennas

scheme achieves orthogonality of the channel by applying a simple rotation at the transmitter and as a result, the decomposed subchannels exhibit the same quality. Thus, compared to the conventional SVD-based transmission methods, the OSM scheme shows a good performance gain with reduced computational complexity and feedback overhead. Throughout this paper, we designate this OSM [5] scheme as an original OSM. However, the original OSM has a main limitation that it can support only two data streams simultaneously. In practical situations, this might be a problem for achieving high spectral efficiency.

In this paper, we propose an extended structure of the original OSM scheme which supports a larger number of data streams. We accomplish this goal by adopting three successive rotation precoding, so that the channel transfer matrix can be divide into two orthogonal subchannels with the same channel quality. The proposed scheme still exhibits better performance, and lower feedback overhead than SVD based schemes. Also the channel orthogonality of the proposed scheme significantly reduces the decoding complexity at the receiver from $O(M_c^3)$ (i.e., open-loop ML) which is prohibitable in the practical situation to $O(M_c^{3/2})$ at the cost of 4 phase angle feedback where M_c denotes the signal constellation size. Note that current development of integrated circuits and sphere decoding algorithms [6] allow the decoding complexity up to $O(M_c^2)$ without much difficulty. Next, we provide analytical work to determine the diversity order of proposed OSM. From our analysis, the diversity order of proposed OSM is derived as $D = M_r - 1$ for $M_t = M$ cases where M_r, M_t and M represent the number of receive antennas, transmit antennas and streams used, respectively.

The organization of this paper is as follows: In Section II, we present a system description and review the original OSM system. In Section III, we introduce a new extended OSM structure by successive rotation precoding and derive its diversity order. In Section IV, simulation results are presented comparing the proposed method with the conventional SVD based schemes and other alternative schemes. Finally, the paper is terminated with conclusions in Section V.

II. SYSTEM DESCRIPTION

In this section, we consider a spatial multiplexing scheme for closed-loop MIMO systems with M_t transmit and M_r receive antennas in a frequency flat fading channels. We basically assume that both transmitter and receiver have perfect CSI. Throughout this paper, normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. With a bar accounting for complex variables, for any complex notation $\bar{c}$, we denote the real and imaginary part of $\bar{c}$ by $Re[\bar{c}]$ and $Im[\bar{c}]$, respectively.

As shown in Figure 1, at the transmitter the information bit stream is mapped to symbols to yield the M-dimensional symbol vector $\bar{\mathbf{s}} = [\bar{s}_1\ \bar{s}_2\ \dots\ \bar{s}_M]^T$, where $[\,\cdot\,]^T$ indicates the transpose of a vector or matrix. Here M represents the number of streams to be transmitted. The $M_t \times M$ precoding matrix $\bar{\mathbf{F}}$ receives the symbol vector $\bar{\mathbf{s}}$ to generate the linearly precoded signal vector $\bar{\mathbf{x}}$ of length M_t as

$$\bar{\mathbf{x}} = \bar{\mathbf{F}}\,\bar{\mathbf{s}}$$

We assume that $Tr(\overline{F}^{\dagger}\overline{F}) = M$ where $Tr(\cdot)$ indicates the trace of a matrix, and $(\cdot)^{\dagger}$ denotes the complex conjugate transpose of a vector or matrix. The signal to noise ratio (SNR) ρ is defined as

$$\rho = \frac{E[\overline{s}^{\dagger}\overline{F}^{\dagger}\overline{F}\overline{s}]}{\sigma_n^2} = \frac{\sigma_s^2}{\sigma_n^2} Tr(\overline{F}^{\dagger}\overline{F}) = \frac{\sigma_s^2}{\sigma_n^2} M$$

where $E[\cdot]$ accounts for expectation and σ_s^2 and σ_n^2 stand for the symbol energy and noise variance, respectively.

At the receiver side, the complex received signal vector $\overline{y}$ is given by

$$\overline{y} = \overline{H}\overline{x} + \overline{n} \tag{1}$$

where $\overline{n}$ is a complex Gaussian noise vector with covariance matrix $\sigma_n^2 I_{M_r}$. Here the $M_r \times M_t$ channel response matrix can be written as

$$\overline{H} = \begin{bmatrix} \overline{h}_{1,1} & \cdots & \overline{h}_{1,M_t} \\ \cdots & \ddots & \cdots \\ \overline{h}_{M_r,1} & \cdots & \overline{h}_{M_r,M_t} \end{bmatrix} = [\overline{\boldsymbol{h}}_1 \cdots \overline{\boldsymbol{h}}_{M_t}]$$

where $\overline{h}_{ji}$ represents the channel response between the i-th transmit and the j-th receive antenna, and $\overline{h}_i$ denotes the i-th column vector of $\overline{H}$. The whole elements of the MIMO channel matrix $\overline{H}$ are obtained from an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance..

In what follows, we give a brief review of the original OSM scheme [5],[7]. The OSM orthogonalizes the channel by applying a rotation operation at the transmitter and transmits two independent data streams ($M = 2$). Then a single symbol decodable ML detection is employed at the receiver which greatly decreases the detection complexity. In this scheme, one can also attain substantial performance improvement by employing extra precoding in view of maximizing the minimum Euclidean distance $d_{\min}$ in the received signal vectors [7].

To orthogonalize the channel, the OSM adopts the following rotation as

$$\overline{x} = \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix} \begin{bmatrix} \overline{s}_1 \\ \overline{s}_2 \end{bmatrix} = \overline{F_{\theta} s}$$

where θ is the rotation phase angle applied to the second antenna, and can be written as $\theta = \tan^{-1}\left(\frac{K}{L}\right) \pm \frac{\pi}{2}$ with

$$L = \sum_{m=1}^{M_r} |\overline{h}_{m1}||\overline{h}_{m2}| \sin(\angle\overline{h}_{m2} - \angle\overline{h}_{m1})$$

and

$$K = \sum_{m=1}^{M_r} |\overline{h}_{m1}||\overline{h}_{m2}| \cos(\angle\overline{h}_{m2} - \angle\overline{h}_{m1}),$$

where $\angle$ indicates the phase of a complex number. This rotation angle θ allows the two groups of real components $(Re[\overline{s}_1], Re[\overline{s}_2])$ and $(Im[\overline{s}_1], Im[\overline{s}_2])$ to be detected independently. As a result, the ML detection complexity of the original OSM reduces from $O(M_c^2)$ to $O(M_c)$. Although, the OSM scheme has several advantages compared to the SVD based schemes, it has a limitation that only two streams can be supported. In the following, we derive a new OSM precoder for three stream transmission.

III. NEW OSM PRECODING SCHEME

In this section, we present an extended OSM scheme which can support three independent data streams. To simplify the presentation, we focus on $M_t = M = 3$. Note that the proposed scheme can be applied to all cases of $M_t > 3$. Let Q denote a QAM signal constellation of size M_c. Given the channel matrix $\overline{H}$ without precoding, the ML estimate of the transmitted vector $\overline{x}$ is given by

$$\hat{\overline{x}} = \operatorname{argmin}_{\overline{x} \in Q^3} \| \overline{y} - \overline{H}\overline{x} \|^2 \tag{2}$$

where $\|\cdot\|$ denotes the Euclidean norm. Note that the searching size for the ML estimate exponentially increases as the number of constellation points grow.

Equivalently, the real-valued representation of the system (1) can be written as [5]

$$\mathrm{y} = \begin{bmatrix} Re[\overline{\mathrm{y}}] \\ Im[\overline{\mathrm{y}}] \end{bmatrix} = \mathrm{Hx} + \mathrm{n}$$

where $\mathrm{x} = [Re(\overline{\mathrm{x}^T})\text{-}Im(\overline{\mathrm{x}^T})]^T$, $\mathrm{n} = [Re(\overline{\mathrm{n}^T})\text{-}Im(\overline{\mathrm{n}^T})]^T$ and H is represented by

$$\mathrm{H} = \begin{bmatrix} Re(\overline{\boldsymbol{H}})\text{-}Im(\overline{\boldsymbol{H}}) \\ Im(\overline{\boldsymbol{H}}) \quad Re(\overline{\boldsymbol{H}}) \end{bmatrix} = [\boldsymbol{h}_1\, \boldsymbol{h}_2\, \boldsymbol{h}_3\, \boldsymbol{h}_4\, \boldsymbol{h}_5\, \boldsymbol{h}_6]. \tag{3}$$

Here h_i denotes the i-th $2M_r$-dimensional column vector of the real-valued channel matrix H.

From the real-valued representation of the channel matrix in (3), it is easy to see that the column vectors h_1, h_2 and h_3 are orthogonal to h_4, h_5 and h_6, respectively (i.e., $h_1 \perp \mathrm{h}_4$, $\mathrm{h}_2 \perp \mathrm{h}_5$ and $\mathrm{h}_3 \perp \mathrm{h}_6$). We also note that column vectors h_1 h_2 and h_3 share the same property with h_4, h_5 and h_6. In other words, $\| \mathrm{h}_1 \| = \| \mathrm{h}_4 \|$, $\| \mathrm{h}_2 \| = \| \mathrm{h}_5 \|$, $\| \mathrm{h}_3 \| = \| \mathrm{h}_6 \|$ and the angles between each pair of vectors are the same as those between the corresponding vectors (i.e., $\overline{\mathrm{h}_i^T \mathrm{h}_j} = \overline{\mathrm{h}_{i+3}^T \mathrm{h}_{j+3}}$ for $i \neq j$). These properties do not change with the subsequent precoding process.

1. Orthogonalizing Precoding

Note that the ML detection in (2) requires the complexity $O(M_c^3)$. Now, we present a new OSM precoder which considerably simplifies the ML detection. To achieve this goal, we encode the three transmit data symbols as

$$\overline{\mathrm{x}} = \overline{\mathrm{F}_\theta}\, \overline{\mathrm{s}}$$

where $\overline{\mathrm{F}_\theta}$ indicates the orthogonalizing complex precoder. The system model in (1) then becomes

$$\overline{\mathrm{y}} = \overline{\mathrm{HF}_\theta}\, \overline{\mathrm{s}} + \overline{\mathrm{n}} = \overline{\mathrm{H}_\theta}\, \overline{\mathrm{s}} + \overline{\mathrm{n}} \tag{4}$$

Let us denote the effective channel by $\overline{\mathrm{H}_\theta} = \overline{\mathrm{HF}_\theta} = [\overline{\mathrm{h}_1^\theta\, \mathrm{h}_2^\theta\, \mathrm{h}_3^\theta}]$. Then the real valued representation of the effective channel matrix is equivalently denoted by $\overline{\mathrm{H}_\theta}$ as

$$\mathrm{H}_\theta = [\overline{\mathrm{h}_1^\theta \mathrm{h}_2^\theta \mathrm{h}_3^\theta \mathrm{h}_4^\theta \mathrm{h}_5^\theta \mathrm{h}_6^\theta}] = [\mathrm{H}_I \mathrm{H}_Q], \tag{5}$$

where the submatrices H_I and H_Q are defined as $\mathrm{H}_I = [\mathrm{h}_1^\theta\, \mathrm{h}_2^\theta \mathrm{h}_3^\theta]$ and $\mathrm{H}_Q[\mathrm{h}_4^\theta \mathrm{h}_5^\theta \mathrm{h}_6^\theta]$, respectively.

Denoting $\overline{\boldsymbol{F}_d}$ as a d-th rotation matrix, the precoding matrix $\overline{\boldsymbol{F}_\theta}$ in (4) is composed of three successive processing as $\overline{\boldsymbol{F}_\theta} = \overline{\boldsymbol{F}_1 \boldsymbol{F}_2 \boldsymbol{F}_3}$ where

$$\overline{\mathrm{F}_1} = \begin{bmatrix} e^{j\theta_3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overline{\mathrm{F}_2} = \begin{bmatrix} \cos\theta_2 & \text{-}\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\overline{\mathrm{F}_3} = \begin{bmatrix} e^{j\theta_3} & 0 & 0 \\ 0 & e^{j\theta_4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Consequently, we can obtain $\overline{\boldsymbol{F}_\theta}$ as

$$\overline{\boldsymbol{F}_\theta} = \begin{bmatrix} \cos\theta_2 e^{j(\theta_1+\theta_3)} & \text{-}\sin\theta_2 e^{j(\theta_1+\theta_4)} & 0 \\ \sin\theta_2 e^{j\theta_3} & \cos\theta_2 e^{j\theta_4} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Here we define $\overline{\mathrm{H}_{\theta_1}} = \overline{\mathrm{HF}_1}$ and $\overline{\mathrm{H}_{\theta_2}} = \overline{\mathrm{HF}_1\mathrm{F}_2}$ Recall that the column vectors h_1, h_2 and h_3 are orthogonal to h_4, h_5, and h_6, respectively. First, let us consider four vectors h_1, h_2, h_4 and h_5. Denoting span(u,v) as the subspace spanned by u and v, we can allocate these four vectors in two orthogonal subspaces span($\mathrm{h}_1^{\theta_1}$, $\mathrm{h}_2^{\theta_1}$) and span($\mathrm{h}_4^{\theta_1}$, $\mathrm{h}_5^{\theta_1}$) only if we have $\mathrm{h}_1^{\theta_1} \perp \mathrm{h}_5^{\theta_1}$ and $\mathrm{h}_2^{\theta_1} \perp \mathrm{h}_4^{\theta_1}$. Since $\mathrm{h}_2^\theta \cdot \mathrm{h}_4^\theta = -\mathrm{h}_1^\theta \cdot \mathrm{h}_5^\theta$ where $\mathrm{h}_i \cdot \mathrm{h}_j$ denotes the inner (dot) product between vectors h_i and h_j, the orthogonality can be achieved when $\mathrm{h}_1^{\theta_1}, \mathrm{h}_5^{\theta_1} = (\mathrm{h}_1\cos\theta_1 + \mathrm{h}_5\cos\theta_1) \cdot \mathrm{h}_5^{\theta_1}$ becomes zero. After some manipulations, θ_1 can be computed as [5]

$$\theta_1 = \tan^{-1}\frac{w_B}{w_A}$$

where $w_A = \boldsymbol{h}_4^T \boldsymbol{h}_5$ and $w_B = -\boldsymbol{h}_1^T \boldsymbol{h}_5$.

Next, the orthogonality between $\mathrm{h}_1^{\theta_1}$ and $\mathrm{h}_2^{\theta_1}$ is easily achieved by a precoder $\overline{F}_2$ if the inner product of $h_1^{\theta_2}$ and $h_2^{\theta_2}$ becomes zero as

$$(h_1^{\theta_2})^T h_2^{\theta_2} = (\| h_2^{\theta_1} \|^2 - \| h_1^{\theta_1} \|^2)\cos\theta_2 \sin\theta_2 + (\cos^2\theta_2 - \sin^2\theta_2)(h_1^{\theta_1})^T h_2^{\theta_1} = 0 \quad (6)$$

Note that the rotation processing $\overline{F}_2$ does not have any effects on the orthogonality between two subspaces. Solving equation (6) with respect to θ_2 yields the rotation angle

$$\theta_2 = \tan^{-1}\left(\frac{w_C \pm \sqrt{w_C^2 + 4w_D^2}}{2w_D}\right)$$

where $w_C = \| h_2^{\theta_1} \|^2 - \| h_1^{\theta_1} \|^2$ and $w_D (h_1^{\theta_1})^T h_2^{\theta_1}$. Here we note that $\| h_1^{\theta_2} \|^2$ is maximized for $\theta_2 = \tan^{-1}\left(\frac{w_C \pm \sqrt{w_C^2 + 4w_D^2}}{2w_D}\right)$ and $\| h_2^{\theta_2} \|^2$ is maximized for $\theta_2 = \tan^{-1}\left(\frac{w_C \pm \sqrt{w_C^2 + 4w_D^2}}{2w_D}\right)$. Throughout this paper, we consider only the first case ($\| h_1^{\theta_2} \| \geq \| h_2^{\theta_2} \|$).

Until now, we have shown that by adopting two rotation matrices $\overline{F}_1$ and $\overline{F}_2$, we can obtain fully orthogonalized column vectors $h_1^{\theta_2}$, $h_2^{\theta_2}$, $h_4^{\theta_2}$ and $h_5^{\theta_2}$ (i.e., $h_1^{\theta_2} \perp h_2^{\theta_2} \perp h_4^{\theta_2} \perp h_5^{\theta_2}$). Note that we have already orthogonalized all four vectors, thus additional rotation on $h_1^{\theta_2}$ and $h_4^{\theta_2}$, or on $h_2^{\theta_2}$ and $h_5^{\theta_2}$ does not destroy the orthogonality. Therefore, we can employ an additional precoder F_3 satisfying

$$\text{span}(h_1^{\theta}, h_3^{\theta}) \perp \text{span}(h_4^{\theta}, h_6^{\theta})$$

$$\text{and span}(h_2^{\theta}, h_3^{\theta}) \perp \text{span}(h_5^{\theta}, h_6^{\theta})$$

to achieve the groupwise orthogonality which is our final goal, i.e.,

$$\text{span}(h_1^{\theta}, h_2^{\theta}, h_3^{\theta}) \perp \text{span}(h_4^{\theta}, h_5^{\theta}, h_6^{\theta}).$$

This can be achieved by setting $(h_1^{\theta})^T h_6^{\theta} = 0$ and $(h_2^{\theta})^T h_6^{\theta} = 0$, and we respectively obtain θ_3 and θ_4 in $\overline{F}_3$ as

$$\theta_3 = \tan^{-1}\left(\frac{-(h_1^{\theta_2})^T h_6^{\theta_2}}{(h_4^{\theta_2})^T h_6^{\theta_2}}\right)$$

$$\text{and } \theta_4 = \tan^{-1}\left(\frac{-(h_2^{\theta_2})^T h_6^{\theta_2}}{(h_5^{\theta_2})^T h_6^{\theta_2}}\right).$$

Consequently, using three successive rotation precodings, we can allocate the vectors in H_I and H_Q into the two orthogonal subspaces, i.e., $\text{span}(H_I) \perp \text{span}(H_Q)$, and thus the ML solution in (2) can be rewritten as

$$\hat{\overline{s}}_I = \operatorname{argmin}_{\overline{s} \in Re[Q^3]} \| y - H_I s_I \|^2 \quad (7)$$

$$\text{and } \hat{\overline{s}}_Q = \operatorname{argmin}_{\overline{s} \in Im[Q^3]} \| y - H_Q s_Q \|^2 \quad (8)$$

where $s_I = Re[\overline{s}_I]$ and $s_Q = Im[\overline{s}_I]$ stand for real-valued 3-dimensional hyper constellation symbols with dimension of $\sqrt{M_c}$. Note that here H_I and H_Q are stochastically equivalent, since we have $\| h_1^{\theta} \| = \| h_4^{\theta} \|$, $\| h_2^{\theta} \| = \| h_5^{\theta} \|$ and $\| h_3^{\theta} \| = \| h_6^{\theta} \|$ in the aspect of magnitude, and $h_1^{\theta} \cdot h_2^{\theta} = -h_4^{\theta} \cdot h_5^{\theta}$, $h_1^{\theta} \cdot h_3^{\theta} = -h_4^{\theta} \cdot h_6^{\theta}$ and $h_2^{\theta} \cdot h_3^{\theta} = -h_5^{\theta} \cdot h_6^{\theta}$ in view of the phase.

Now, we briefly address the complexity issue. The computational complexity for the proposed OSM precoding matrix is much simpler than the SVD based schemes [3],[4],[9], since the right singular vectors should be iteratively calculated for $M \geq 3$. In case of $M_t = M_r = 3$, the Jacobi SVD in [14] which exhibits a high degree of potential parallelism requires at least **456** floating-point multiplications [15], whereas only 94 multiplications are enough for the proposed OSM. Moreover, in the case of ARITH-MSE or ARITH-BER, we additionally need to compute the power allocation matrix for determining the power distribution among spatial modes, which is not required for the proposed OSM. Also, considering the feedback overhead of the proposed scheme, the OSM has an advantage for systems with a band limited feedback channel. More specifically, the SVD based transmissions require full CSI to determine the precoders at the transmitter, while the proposed OSM precoder is attainable with only four phase angles feedback.

We now look at the complexity issue in the receiver side. Through the simple ML equations in (7) and (8), the channel orthogonalizing process guarantees considerable reduction in the detection complexity from $O(M_c^M)$ to $O(M_c^{M/2})$ compared to the conventional open-loop ML. For example, in the case of **16**-QAM, the number of searching candidates reduces from **4096** to **128**, and from **262144** to **1024** in the **64**-QAM. It appears that the proposed OSM shows still slightly high decoding complexity compared to the SVD based designs. However, SVD based designs require much higher complexity in the precoder design which is based on the iterative algorithm. Furthermore, considering highly developed integrated circuits and many effective algorithms [6],[8] which allows the complexity up to $O(M_c^2)$, it is more desirable to improve the error performance even allowing for a slight increase of decoding complexity in view of the high speed data transmission.

2. Diversity Analysis

In the case of *Optimal Unitary Precoding* (OUP) introdeced in [9], the channel gain associated with each subchannel is directly proportional to their singular values. This means that the subchannel with the lowest gain dominates the error performance. In order to prevent this problem, ARITH-MSE [3] and ARITH-BER [4] schemes allocate more power to weaker modes for the improved system performance based on the Minimum Mean Square Error (MMSE) and minimum Bit Error Rate (BER) criteria, respectively. The diversity order of the SVD was derived in [10] and [11] as M_r-M_t + 1 for the case of full stream transmission ($M = M_t$), which shows that no diversity gain is obtained when $M_r = M_t$. Also it has been shown in [12] that the SVD based power loading designs such as ARITH-MSE and ARITH-BER cannot increase the diversity order.

In contrast, the proposed OSM can achieve the higher diversity order than the SVD based schemes even in the case of $M_r = M_t$. In the following theorem, we derive the diversity order of the proposed OSM.

- Theorem 1: The diversity order of the proposed OSM is given by M_r-1.
- Proof: Let $\| h_1^{\theta} \|^2 = \lambda_1$ and $\| h_2^{\theta} \|^2 = \lambda_2$. Using the ML criterion, the Pairwise Error Probability (PEP) of $\overline{s}$ and $\overline{\hat{s}}$ given CSI, can be written as

$$P(\overline{s} \rightarrow \overline{\hat{s}} \mid H) = P(\|\overline{y} \rightarrow \overline{H_{\theta} s} \|^2 \geq \|\overline{y} - \overline{H_{\theta} \hat{s}} \|^2)$$

$$\leq Q\left(\sqrt{\frac{\|\overline{H_{\theta} e}\|^2}{2\sigma_n^2}}\right)$$

$$= Q\left(\sqrt{\frac{\|H_I e_I + H_Q e_Q \|^2}{2\sigma_n^2}}\right)$$

where $\overline{e} = \overline{\hat{s}} - \overline{s}$, and e_I and e_Q are defined as $Re(\overline{e})$ and $Im(\overline{e})$, respectively. Here H_I and H_Q are orthogonal to each other and share the same channel quality. Moreover under the QAM constellation, either e_I or e_Q can be assumed to be zero in the worst case. Therefore, we can rewrite (9) as

$$P(\overline{s} \rightarrow \overline{\hat{s}} \mid H) = Q\left(\sqrt{\frac{\|H_I e_I \|^2}{2\sigma_n^2}}\right).$$

Note that $E[s_I s_I^T] = (\sigma_s^2/2) I_M$. Using the Chernoff's bound for the Q-function the conditional PEP given H can be bounded as

$$P(\overline{s} \rightarrow \overline{\hat{s}} \mid H) \leq \exp(\|H_1^{\theta} d^1(s_I, \hat{s}_I) + H_2^{\theta} d^2(s_I, \hat{s}_I)$$
$$+ H_3^{\theta} d^3(s_I, \hat{s}_I) \|^2 \sigma_s^2 / 8\sigma_n^2)$$

where $d^i(s_I, \hat{s}_I)$ is defined as the normalized squared Euclidean distances computed on the i-th element of all the multidimensional symbols s_I. In the worst case, $d^i(s_I, \hat{s}_I)$ becomes zero except for $i = 2$, since we have the maximum average PEP among all other possible combinations. Thus, the PEP can be rewritten as

$$P(\overline{s} \rightarrow \overline{\hat{s}}) \leq E\left[\frac{1}{2} \exp\left(- \frac{\sigma_s^2 \|d^2(s_I, \hat{s}_I)\|^2}{8\sigma_n^2} \lambda_2\right)\right.$$

$$= \int_0^{\infty} \exp\left(- \frac{\sigma_s^2 \|d^2(s_I, \hat{s}_I)\|^2}{8\sigma_n^2} \lambda_2\right) f_{\Lambda_2}(\lambda_2) d\lambda_2$$

$$= G(\infty) - G(0) \tag{10}$$

where $f_{\Lambda_2}(\lambda_2)$ denotes the marginal Probability Density Function (PDF) of λ_2 and $G(x)$ is defined as the indefinite

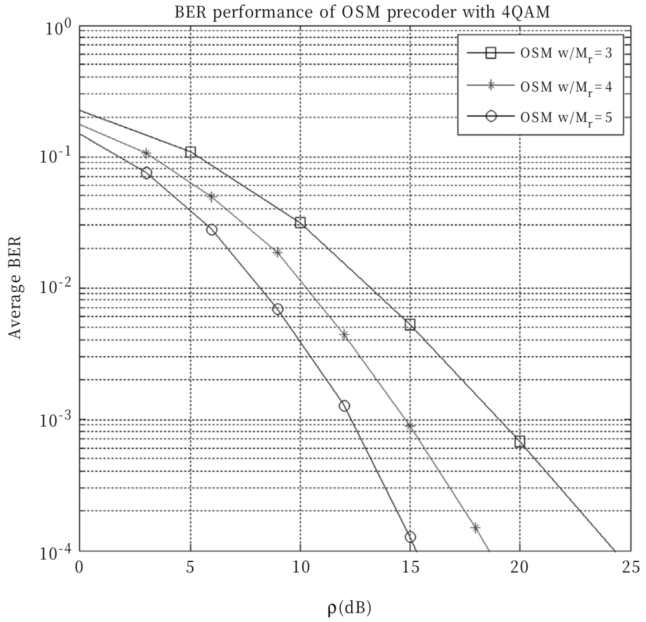

Figure 2. BER performance of the proposed OSM with $M_t=3$ and various M_r

integral

$$\int \exp\left(-\frac{\sigma_s^2 \|d^2(\mathrm{s}_I,\hat{\mathrm{s}}_I)\|^2}{8\sigma_n^2}\lambda_2\right) f_{\Lambda_2}(x)dx_2.$$

As shown in [10],[13], $G(x)$ approaches zero as $x \to \infty$ since it has an exponential factor with a negaive exponent. Thus the PEP is bounded by $G(0)$ at high SNR. Also λ_2 is equivalent to the squared second largest singular value of the matrix $[\overline{\mathrm{h}_1 \mathrm{h}_2}]$. The first-order expansion of the marginal pdf $f_{\Lambda_2}(\lambda_2)$ can be approximated to [10],[13]

$$f_{\Lambda_2}(\lambda_2) \approx \mu \lambda_2^{(M_r-1)-1} \quad \text{for} \quad \lambda_2 \to 0$$

where μ is a constant. Hence, the PEP bound at high SNR is approximately given as

$$P(\overline{\mathrm{s}} \to \overline{\hat{\mathrm{s}}}) \approx -\int \exp\left(-\frac{\sigma_s^2 \|d^2(\mathrm{s}_I,\hat{\mathrm{s}}_I)\|^2}{8\sigma_n^2}\lambda_2\right) \mu \lambda_2^{(M_r-1)-1} d\lambda_2$$

$$= \nu \left(\frac{\sigma_s^2}{8\sigma_n^2} \|d^2(s_I,\hat{s}_I)\|^2\right)^{-(M_r-1)}$$

where ν is a constant. Now we can see that the proposed OSM achieves the diversity order of M_r-1. ■

As we described previously, the error performance of the proposed OSM mainly depends on the minimum channel gain λ_2 in (10).

Hence, we can further improve the performance by adopting multidimensional rotation matrix $\overline{\mathrm{P}}$ on the transmit signal vector $\overline{\mathrm{s}}$ as

$$\overline{\mathrm{P}} = \begin{bmatrix} \cos\Theta_1 & -\sin\Theta_1 & 0 \\ \sin\Theta_1 & \cos\Theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta_2 & 0 & -\sin\Theta_2 \\ 0 & 1 & 0 \\ \sin\Theta_2 & 0 & \cos\Theta_2 \end{bmatrix} \tag{11}$$

We determine this matrix through an exhaustive computer search over $0 \le \Theta_i \le \frac{\pi}{4}$ $(i=1,2)$ with respect to the average BER. The resulting $\overline{\mathrm{P}}$ is obtained with $\Theta_1 = \Theta_2 = 24.637°$ in (11). With the rotation matrix $\overline{\mathrm{P}}$, we can construct the multidimensional rotated constellation $\overline{\mathrm{r}} = \overline{\mathrm{P}}\,\overline{\mathrm{s}}$ and equivalently $\mathrm{r}_I = \overline{\mathrm{P}}\,\mathrm{s}_I$ and $\mathrm{r}_Q = \overline{\mathrm{P}}\,\mathrm{s}_Q$. We refer to the OSM rotated by $\overline{\mathrm{P}}$ as Constellation Rotated OSM (OSM-CR)

3. OSM with rank-4 transmission

In this section, we briefly discuss the case of 4-stream

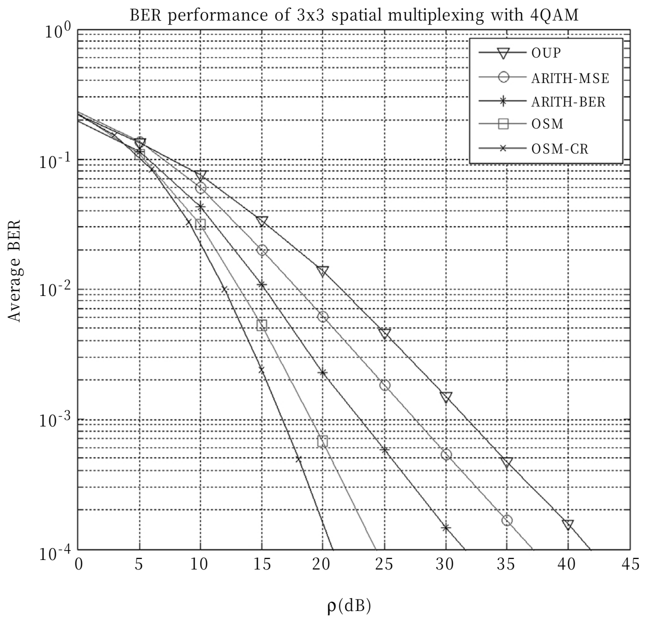

Figure 3. BER performance comparison between the proposed OSM and the SVD based schemes

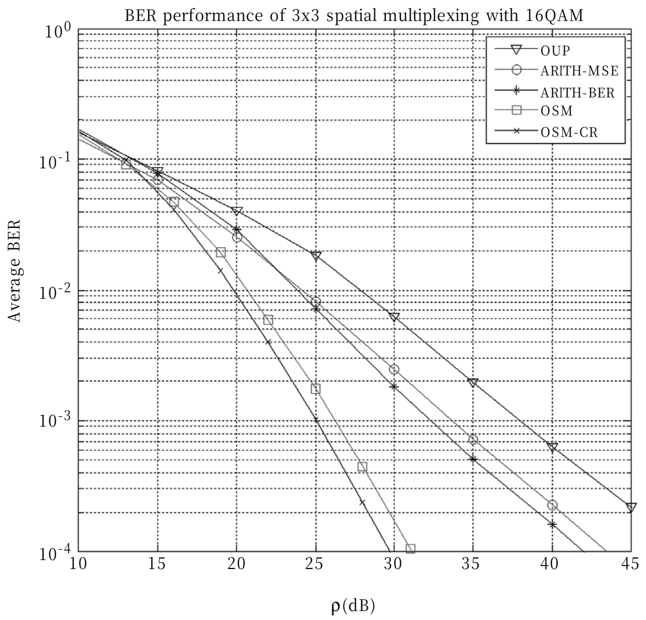

Figure 4. BER performance comparison between the proposed OSM and the SVD based schemes

data transmissions (i.e., $M_t = 4$). In this case, the two group decodable approach is also possible. One simple method is as in the following. Let us consider a complex channel matrix $\overline{\mathrm{H}}$ consisting of four vectors. First, we can fully orthogonalize the first three columns of $\overline{\mathrm{H}}$ using SVD. Then, following the similar approach in section III-1, we can easily allocate the subspaces spanned by the real part and the imaginary part of $\overline{\mathrm{H}}$ into two orthogonal subspaces. Certainly, this method have an advantage in terms of both the precoding complexity and error

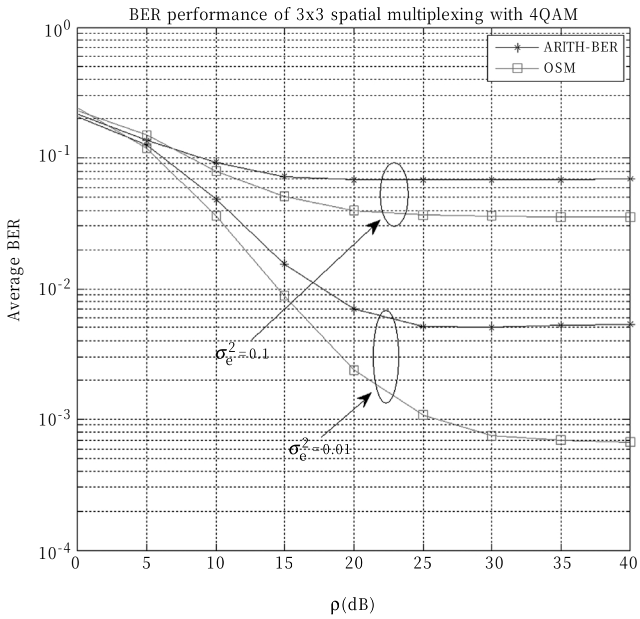

Figure 5. BER performance comparison of the proposed OSM and the ARITH-BER with channel estimation error σ_e^2.

performance than the case where the SVD is applied to the entire $\overline{\mathrm{H}}$. Note that the aforementioned approach still requires a SVD operation based on the iterative process. The closed-form solution for rank-4 cases is still unknown and we defer this problem for the future work.

IV. SIMULATION RESULTS

In this section, Monte Carlo simulations were performed to illustrate the performance of the proposed OSM precoders as a function of SNR (ρ) in dB scale. In all simulations, we depict the BER comparison of the proposed OSM schemes, and SVD based schemes considering three independent data streams.

First, in Figure 2, we present the Monte Carlo simulations for the BER performance of the OSM precoder with $M_t = 3$ and various M_r. From this plot, we see that the derived diversity expression M_r-1 precisely reflects the slope of the BER curves.

In Figure 3, we compare the BER performance of the proposed OSM with the existing SVD based schemes in 4 QAM constellation. We observe that the proposed OSM-CR provides approximately 10 dB and 15 dB gains over the ARITH-BER and ARITH-MSE, respectively, at a BER of 10^{-4} with much reduced complexity in the feedback overhead and the computation at the transmitter.

We also provide a simulation result for the case of 16 QAM in Figure 4. The similar trends are shown but it is remarkable that the performance gap between proposed schemes and SVD schemes becomes larger as the modulation level grows. Note that extensive computer search shows the optimum values of $\Theta_1 = \Theta_2 = 24.637°$ for OSM-CR do not depend on the modulation level.

In practical scenarios, the mismatch between the true channel $\overline{\mathrm{H}}$ and the estimated channel $\hat{\mathrm{H}}$ is inevitable due to an estimation error. Finally, in Figure 5, we further discuss the performance of the proposed scheme in the case where the perfect channel estimation has been failed. We assume that $\hat{\mathrm{H}}$ is related to $\overline{\mathrm{H}}$ as $\hat{\mathrm{H}} = \overline{\mathrm{H}} + \mathrm{E}$ where the elements of E are i.i.d. complex Gaussian random variables with variance σ_e^2. Figure 5 shows that the proposed OSM outperforms the SVD based precoding schemes even in that case of the channel estimation error.

V. CONCLUSION

In this paper, we proposed a new efficient precoder design for closed-loop MIMO systems. The main objective of this work is the extension of the original OSM scheme to multiple data stream transmission. The essential advantage of the proposed OSM is to maximize the system performance with low complexity in the precoding computation as well as the detection. Moreover, the proposed scheme considerably reduces the feedback amount compared to the conventional SVD based schemes. Finally, the simulation results confirm the efficiency of the proposed OSM schemes in practical situations.

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