Modeling and Verification of FEC Performance for Optical Transmission Systems Using a Proposed Uniformly Quantized Symbol Error Probability Model

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***Abstract*—We investigate the estimation of the bit-error rate (BER) performance of optical transmission systems with forward error correction (FEC) coding using a proposed uniformly quantized symbol error probability model. This model has been verified by the measurement of BER characteristics of coded and uncoded 10 Gb/s optical signals transmitted over 100 km. The measured results are very similar to the calculated results from the proposed model as well as Monte Carlo (MC) simulations. Our results suggest that the proposed uniformly quantized symbol error probability model using more than 8-decision levels can be applied to estimate BER performance for coded systems without degrading accuracy.**

***Index Terms*—Bit-error rate (BER) performance, coded systems, forward error correction (FEC), optical transmission systems, Reed–Solomon code, uniformly quantized symbol error probability model.**

I. Introduction

It has been shown that forward error correction (FEC) provides an additional system margin by increasing immunity to noise and pulse distortion in optical transmission systems [1]–[8], resulting in increased span length and transmission distance and system capacity in wavelength division multiplexed systems by increasing data rates of each channel or decreasing the channel spacing while keeping the same system performance. At present, a standard FEC code recommended by International Telecommunication Union (ITU) [9] is widely used in submarine systems and starting to be deployed in terrestrial systems.

As FEC code applications prevail in optical transmission systems, it becomes an important issue to estimate performance improvement due to FEC coding in order to find an appropriate FEC code before implementation. So far, several simulation techniques and theoretical evaluation methods have been suggested. Among them, Monte Carlo (MC) simulations which randomly generate values for uncertain variables over and over to simulate a model are the most exact approach that can be applied to virtually any systems [10]. However, optical transmission systems including FEC coding may have such low bit-error rates (BERs) that MC simulations require prohibitive simulation execution times. To speed up MC simulations of coded systems, importance sampling [11] and the simulation technique utilizing knowledge of channel noise statistics [12] were proposed. These methods require much less simulation times than MC simulations. Though these methods based on the binary symmetry channel (BSC) model are not suitable for estimating BER performance of optical transmission systems with FEC coding. Using a binary asymmetric channel (BAC) model, the performance limit of FEC coding for optical fiber channels was more elaborately evaluated [13]. This model still is not sufficient to represent characteristics of real optical fiber channels.

In this paper, we estimated the BER performance based on the proposed uniformly quantized symbol error probability model in optical transmission systems with FEC coding. The proposed uniformly quantized symbol error probability model can be applied any channels to estimate an accurate performance of coded systems. To verify the proposed uniformly quantized symbol error probability model, we estimated BER characteristics of strongly dispersion limited 10 Gb/s optical transmission systems using only the RS (255, 239) FEC code due to our experimental limitations. The simulation results were verified through the measurements of BER performance of the systems.

II. BER Performance of Optical Transmission Systems Using FEC Coding

A. Bit Error Probability With Hard Decision Decoding

Because the decoder for the RS codes using the Berlekamp-Massey algorithm or Euclidean algorithm is a t-error correcting bounded-distance decoder, symbol errors more than t give rise to either decoder error or decoder failure [14]. When a codeword $\mathbf{x}$ in $\boldsymbol{C}$ which is a q^m-*ary* $\mathrm{RS}(n, k)$ code with a minimum distance $d_{\min}$ is transmitted, the probability of the unsuccessful decoding $P_{U.D}(\mathbf{x})$ is given by

$$P_{U.D.}(\mathbf{x}) = P_E(\mathbf{x}) + P_F(\mathbf{x}) = 1 - P(\mathbf{x} \mid \mathbf{x}) \qquad (1)$$

Manuscript received June 30, 2003; revised August 17, 2004. This work was supported in part by the NRL program in South Korea.

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Digital Object Identifier 10.1109/JLT.2005.843452

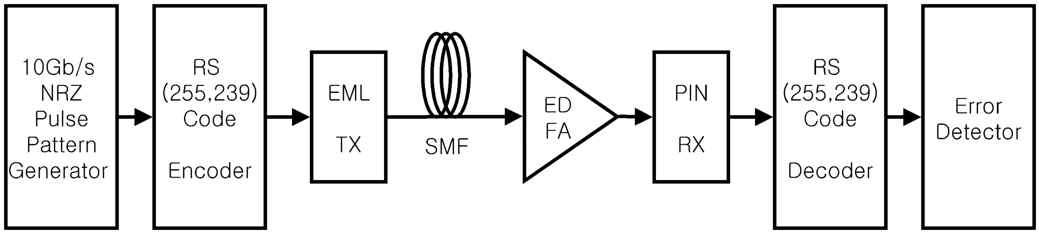

Fig. 1. Configuration of 10 Gb/s transmission systems for simulation and measurement of the improvement of BERs using FEC coding.

where $P_E(\mathbf{x})$ and $P_F(\mathbf{x})$ are the probability of decoder error and decoder failure, respectively, and $P(\mathbf{x} \mid \mathbf{x})$ is the probability of decoding $\mathbf{x}$ rightly when a codeword $\mathbf{x}$ is transmitted.

We should consider how many bit errors occur when decoder error or decoder failure happens. We calculated the probability of bit errors due to the unsuccessful decoding $P_{(B.E|U.D)}(\mathbf{x})$ under these assumptions: 1) due to the highly imperfect nature of the RS codes, decoder failure $P_F(\mathbf{x})$ is dominant over decoder error $P_E(\mathbf{x})$; 2) the errors are passed through unchanged when decoder failure is declared [6]; 3) for low raw BER ($< 10^{-3}$), the possibility of the error pattern weights being larger than $d_{\min}$ is very low; 4) for low raw BER ($< 10^{-3}$), one bit error per symbol is likely to occur. The second, third and fourth assumptions mentioned above were validated through MC simulations. The first assumption allows us to approximate $P_{U.D}(\mathbf{x})$ to $P_F(\mathbf{x})$. When decoder failure occurs in a received word, the probability of symbol error can be bounded by $d_{\min}/n$ due to the second and third assumptions. Because we also found that one symbol error tends to bring about one bit error, $P_{B.E|U.D}(\mathbf{x})$ is approximately determined as the constant value $(d_{\min}/n) \cdot (1/m)$ regardless of the transmitted codeword $\mathbf{x}$. Consequently, bit error probability $P_{\text{bit error}}(\mathbf{x})$ can be evaluated as follows:

$$P_{\text{bit error}}(\mathbf{x}) \approx \frac{d_{\min}}{n} \cdot \frac{1}{m} \cdot [1 - P(\mathbf{x} \mid \mathbf{x})]. \tag{2}$$

B. Calculation of Bit Error Probability Using the Uniformly Quantized Symbol Error Probability Model

Fig. 1 shows the system configuration we considered. The first 1912 bits (239 symbols) of $2^{11} - 1$ 10 Gb/s nonreturn to zero (NRZ) pseudorandom binary sequences (PRBS) were input to the RS (255, 239) code encoder and encoded to a single RS codeword with 2040 bits (255 symbols) length, which results in a 10.66 Gb/s data rate. The reason why we selected RS (255, 239) code as FEC code is that this code is now recommended by ITU-T standard [9]. The transmitter was electrical absorption modulator integrated laser module (EML) with the negative chirp by large reverse bias. The input power into the fiber was set to -3.5 dBm. The transmitted optical signal over 100 km using the single mode fiber was amplified through an EDFA and then was detected by a PIN receiver. The system shown in Fig. 1 was modeled to calculate BER characteristics in real optical fiber channels. To consider both chromatic dispersion and nonlinear effects, optical signal propagation through single mode fiber was calculated by solving the nonlinear Schrödinger equation [16]. Because the ISI effect is an important factor to affect the transmission performance for high data rates, the ISI effect should be included to calculate BER characteristics. In [17], BER characteristics with the ISI effect for an ith bit can be expressed by

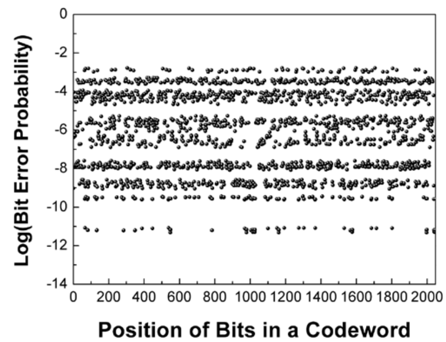

Fig. 2. Distribution plot of bit error probabilities when the RS (255, 239) codeword is transmitted over 100 km in Fig. 1.

$$\mathrm{BER}^i = \frac{1}{4} erfc \left[\sqrt{2} \left\{ \frac{\left(1 - c_{isi}^{+} - \tau\right) I_s^i}{\sigma_1} \right\} \right] + \frac{1}{4} erfc \left[\sqrt{2} \left\{ \frac{\left(\tau - c_{isi}^{-}\right) I_s^i}{\sigma_0} \right\} \right] \tag{3}$$

where I_s^i is the time-averaged signal photocurrent during the holding time at the sampling moment for an ith bit, τ is the decision threshold level setting relative to the rail-to-rail electrical pulse, and c_{isi}^{+} and c_{isi}^{-} are the normalized eye closure of electrical pulse at mark ("1") and space ("0"), respectively. σ_1 and σ_0 are the standard deviation of the total noise including shot noise, beating noises, and circuit noises for mark and space, respectively, [15]. The averaged BER can be calculated from adding BER for each bit and then divided by number of bits. The Fig. 2 shows the distribution of calculated bit error probabilities for each bit belonging to a single received RS encoded PRBS data pattern. X-axis and Y-axis represent the position of bits in the received word and bit error probabilities in the log scale, respectively. BER characteristics of the received word were recalculated with including error corrections of the decoder using (2).

To obtain the value of $P(\mathbf{x} \mid \mathbf{x})$ in (2), we need to understand the characteristics of real optical fiber channels. As shown in Fig. 2, the error probability of each bit in the received cordword for RS (255, 239) codes is distributed from 10^{-11} to 10^{-3} when the average received optical power at the input of the PIN receiver is -18.3 dBm and the average BER value of 10^{-4}. Fig. 2 implies that the real optical fiber channels are very far from BSC. The error probability of each symbol calculated from the bit error probability distribution becomes to be very diverse as shown in Fig. 3(a) where X-axis and Y-axis represent the position of symbols in the received word and symbol

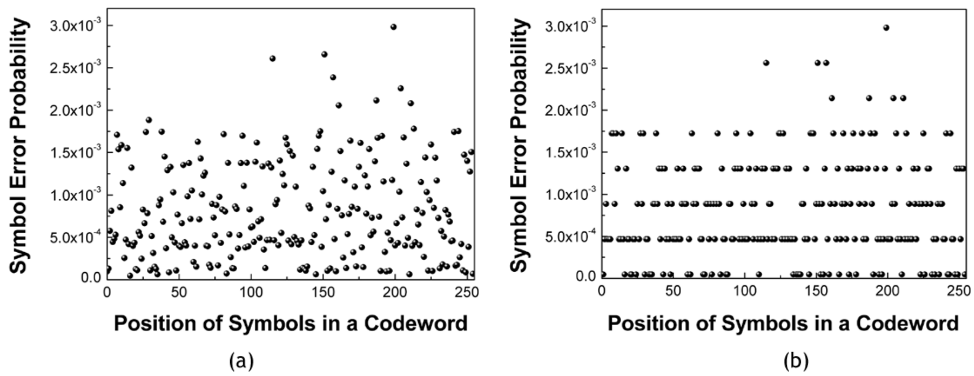

Fig. 3. Distribution plot of symbol error probabilities when the RS (255, 239) codeword is transmitted over 100 km. (a) Before quantization. (b) After 8-decision level quantization.

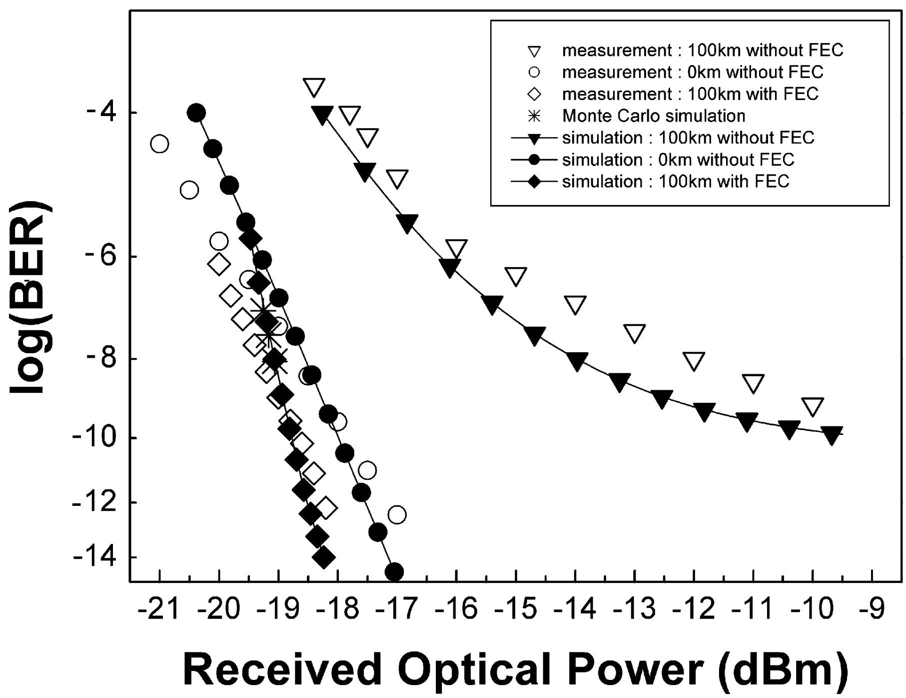

Fig. 4. BER characteristics as a function of received optical powers for 0- and 100-km transmission without FEC and 100-km transmission with the RS (255, 239) FEC coding.

error probabilities, respectively. If the error probability of each symbol belonging to a codeword were same (in other words, real optical channels had the BSC nature), the probability of occurrence for error patterns would be completely determined by their length and weight. Error patterns of the same length and weight occur with different probability. We have to evaluate the probabilities for all the cases of error patterns of weight $(d_{\min} - 1)/2$ or less and sum all of them. This often requires prohibitive simulation times. In the case that the RS (255, 239) code with the minimum distance 17 recommended by ITU-T is transmitted, the total number of correctable error patterns runs to $\sum_{i=0}^{8} {}_{255}C_i \approx 4.1 \times 10^{14}$.

We proposed the uniformly quantized symbol error probability model to reduce simulation time and obtain reasonable BERs. This model uniformly divides the range of the symbol error probabilities for a received codeword into L-decision levels and transforms each of them into the closest decision level. By transforming the error probabilities of the symbols for the received word into L values, we can reduce the total number of correctable error patterns with the different occurrence probabilities considerably. Fig. 3(b) shows the distribution of 8-decision level quantized symbol error probabilities we adopted in our simulation. Given that the error probabilities of 255 symbols for the received word are one of the eight values, the number of error patterns of weight l with the different occurrence probabilities can be determined as the case of casting l balls allowing overlapping among the different eight balls. Using the symbol H meaning the combination allowing overlapping, the total number of correctable error patterns occurring with different probabilities is bounded to $\sum_{i=0}^{8} {}_8H_i = 12\,780$, which requires reasonable simulation execution times. Even though the more quantization levels lead better performance, more than eight decision levels did not make a difference. We believe that eight decision levels can represent real optical channel. We did not consider ferwer than eight levels. So we calculated BER values using 8-decision level quantized symbol error probabilities.

As shown in Fig. 4, BER characteristics for 100-km transmission with the RS (255, 239) FEC coding were calculated by our proposed method and were also estimated by MC simulations for a few points which take relatively short time for comparison. The calculated results using the proposed model were almost identical with the MC simulation results. It implies that our proposed model can provide accurate estimation of BER characteristics using FEC coding.

C. Measurement of BERs With RS (255, 239) FEC Coding

In order to measure the performance improvement due to the RS (255, 239) FEC coding, we calculated BERs of the unencoded data transmitted at a 10.66 Gb/s data rate over the distance of 0 and 100 km. Fig. 4 shows the measured BER curves against averaged received optical power for 0- and 100-km transmissions without the FEC coding and 100 km transmission with the RS (255, 239) FEC coding. A variable attenuator was used after the EDFA to adjust the average received optical power at the input of the PIN receiver for BER measurements. Due to the RS (255, 239) FEC coding, BER performance was greatly improved and became to be an error-free operation for all the average received optical power. BER performance of 100-km transmission with the RS (255, 239) FEC coding was better than even that of 0-km transmission without the FEC coding for lower BER than 10^{-6}. The measured results were similar to the calculated results, which proves that the proposed uniformly quantized symbol error probability model can help predict BER performance well in optical transmission systems including the FEC coding.

III. Conclusion

Transmission performance of strongly dispersion limited 10 Gb/s optical transmission systems with the RS (255, 239) FEC coding have been analyzed and measured in this paper. In order to understand this transmission performance of FEC coded optical transmission systems, we firstly derived the BER equation available for RS coded optical transmission systems with hard decision. To calculate this BER equation, we proposed a uniformly quantized symbol error probability model. Our model is not only accurate because it is closely approximated to the characteristics of real optical channel, but also practical because it does not require long computation time. Finally, we verified our model through simulations and measurements. The measured BER characteristics using the RS (255, 239) FEC coding were similar to the simulated ones calculated from our proposed method as well as MC simulations. Our model can be applicable any systems other than optical transmission systems and the uniformly quantized symbol error probability model using more than 8-decision levels provides an effective way to calculate BER characteristics using the FEC coding within a reasonable time without degrading accuracy.

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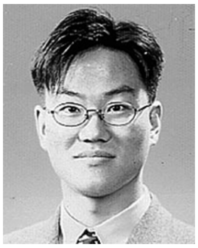

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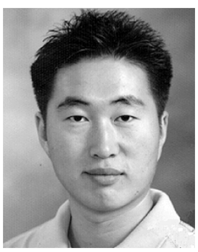

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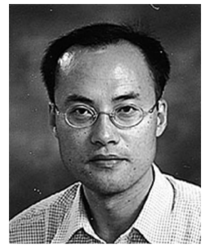

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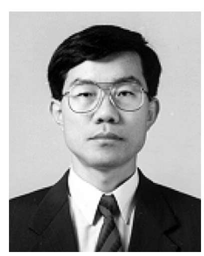

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