Real-Domain Decoder for Full-Rate Full-Diversity STBC with Multidimensional Constellations

Heunchul Lee, *Member, IEEE,* Jungho Cho, Jong-kyu Kim, and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—In this letter, we present a new maximum likelihood (ML) decoding algorithm for space time block codes (STBCs) that employ multidimensional constellations. We start with a lattice representation for STBCs which transforms complex channel models into real matrix equations. Based on the lattice representation, we propose a new decoding algorithm for quasi-orthogonal STBCs (QO-STBC) which allows simple ML decoding with performance identical to the conventional ML decoder. Multidimensional rotated constellations are constructed for the QO-STBCs to achieve full diversity. As a consequence, for quasi-orthogonal designs with an arbitrary number of transmit antennas N ($N \geq 4$), the proposed decoding scheme achieves full rate and full diversity while reducing the decoding complexity from $\mathcal{O}(M_c^{N/2})$ to $\mathcal{O}(M_c^{N/4})$ in a M_c-QAM constellation.**

***Index Terms*—Diversity methods, maximum likelihood decoding (MLD), multidimensional rotated constellations, space-time block codes (STBC).**

I. INTRODUCTION

THE use of multiple antennas with space-time code (STC) techniques has been studied since a simple transmit diversity concept was introduced by Alamouti [1]. The STC will play an important role in future wireless communication systems for high-quality multimedia services by offering additional capacity and/or diversity gain. Unfortunately, only for the two transmit antenna case, there exists a single-symbol decodable STC which achieves full rate and full diversity for complex constellations [2]. For the case with three or more transmit antennas, the transmission rate of space-time block codes (STBCs) from complex orthogonal designs is limited to 3/4 or lower [2]. The code rate can be made higher by sacrificing either full diversity or single-symbol decodability. A design of quasi-orthogonal STBC (QO-STBC) is one such case made at the expense of single-symbol decodability and the diversity order [3]. The performance and diversity property of the QO-STBC can be improved by rotating the constellation of the symbols [4] [5] [6].

For the maximum likelihood (ML) decoding for rate one QO-STBCs, the traditional ML decoding scheme divides the symbol set into two independent groups, resulting in $M_c^{N/2}$ candidates for the ML search [3]-[7] where N and M_c indicate the number of transmit antennas and the constellation size, respectively. Recently, QO-STBC with minimum decoding complexity have been proposed in [8] [9] [10] and [11]. However, these approaches suffers from a reduced code rate when more than four antennas are used. For example, the maximum achievable code rate of these codes is 3/4 for eight antennas. In this letter, we propose a QO-STBC that supports full rate for arbitrary number of transmit antennas with reduced decoding complexity.

Linear lattice-based decoders achieving the ML performance were proposed for multi-input multi-output (MIMO) systems in [12], which apply the sphere decoding (SD) algorithm based on a lattice representation. We will first introduce a new ML decoding algorithm based on the lattice real space signal models. Unlike the previous works in [8] [9] [10] and [11] where the ML decoding for QO-STBCs is considered in complex space, the real-domain decoding method presented provides a better understanding of the minimum decoding complexity in the quasi-orthogonal codes. We also present the multidimensional rotated constellations [13] to obtain full diversity in the proposed QO-STBC. As a consequence, the proposed scheme achieves full rate and full diversity while reducing the number of search candidates for the ML decoding from $M_c^{N/2}$ to $M_c^{N/4}$. A different STBC scheme with full rate and full diversity was presented based on the co-ordinate interleaved design in [14]. As mentioned in [14], this approach suffers from large peak-to-average power ratio problem due to zeros in the transmission matrix.

II. SYSTEM MODEL AND LATTICE REPRESENTATION

In this section, we present a lattice representation for a baseband space-time coded communication system with N transmit and M receive antennas. Throughout this letter, variables with a bar account for complex variables. For any complex notation $\bar{C}$, we denote the real and the imaginary part of $\bar{C}$ by $\Re[\bar{C}]$ and $\Im[\bar{C}]$, respectively. In particular, for a complex scalar $\bar{c}$, c_I and c_Q represent $\Re[\bar{c}]$ and $\Im[\bar{c}]$, respectively (i.e., $\bar{c} = c_I + jc_Q$).

We assume that K symbols are transmitted in one STBC block. We define $\bar{\mathbf{x}} = [\bar{x}_1\ \bar{x}_2 \cdots \bar{x}_K]^t$ as the transmitted signal vectors, where $[\cdot]^t$ indicates the transpose of a vector or matrix. For an M_c-ary QAM modulation system, ηK bits are divided

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H. Lee was with the School of Electrical Engineering, Korea University, Seoul 136-702, Korea. He is now with the Department of Electrical Engineering, Stanford University, CA 94305, USA (e-mail: heunchul@gmail.com).

I. Lee is with the School of Electrical Engineering, Korea University, Seoul 136-701, Korea (e-mail: inkyu@korea.ac.kr).

J. Cho and J.-K. Kim are with the Agency for Defense Development, Daejeon, Korea (e-mail: {jungho, jongkyu}@add.re.kr).

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into K groups of η bits and independently mapped onto M_c-ary QAM constellation points, where $\eta = \log_2 M_c$ denotes the number of bits per symbol (two dimensions). Let $\bar{\mathbf{C}}$ denote the transmitted signal matrix of a $T \times N$ code design where T is the block size of the STBC. Therefore, a rate of the STBC use becomes

$$\zeta = K \log_2 M_c / T \quad \text{(bits/channel use).} \tag{1}$$

The (i, j) element $\bar{c}_j(i)$ of $\bar{\mathbf{C}}$ represents the signal transmitted from antenna j at time i. We assume that the transmitter is constrained in its total power to P, which means that the power consumed by each transmit antenna is equal to P/N. For simplicity we assume one receiver antenna ($M = 1$). Define $\bar{\mathbf{y}} = [\bar{y}_1\ \bar{y}_2 \ldots \bar{y}_T]^t$, $\bar{\mathbf{h}} = [\bar{h}_1\ \bar{h}_2 \ldots \bar{h}_N]^t$ and $\bar{\mathbf{n}} = [\bar{n}_1\ \bar{n}_2 \ldots \bar{n}_T]^t$, where $\bar{y}_i$ is the complex received signal at time i, $\bar{n}_i$ represents the independent and identically-distributed (i.i.d) complex Gaussian noise with variance σ_n^2 at time i, and $\bar{h}_i$ denotes the path gain from the ith transmit antenna to the receive antenna. The path gains $\bar{h}_i$ are modeled as samples of independent complex Gaussian random variables with variance 0.5 per dimension. Then the complex received signal vector $\bar{\mathbf{y}} \in \mathcal{C}^T$ can be written as

$$\bar{\mathbf{y}} = \bar{\mathbf{C}}\bar{\mathbf{h}} + \bar{\mathbf{n}}. \tag{2}$$

By properly transforming the received signal vector $\bar{\mathbf{y}}$ in (2) and decomposing the complex signals in $\bar{\mathbf{C}}$ into their real and imaginary parts, we can obtain an effective real channel model $\mathbf{H} \in R^{2T \times 2K}$ such that

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{3}$$

where $\mathbf{y} = [\ \Re[\bar{\mathbf{y}}^t]\ \Im[\bar{\mathbf{y}}^t]\ ]^t$, $\mathbf{x} = [\ \Re[\bar{\mathbf{x}}^t]\ \Im[\bar{\mathbf{x}}^t]\ ]^t$, and $\mathbf{n} = [\ \Re[\bar{\mathbf{n}}^t]\ \Im[\bar{\mathbf{n}}^t]\ ]^t$. Based on this lattice representation, the ML decoding requires the minimization of the metric

$$m(\mathbf{x}|\mathbf{y}) = ||\mathbf{y} - \mathbf{H}\mathbf{x}||^2$$

where $||\cdot||$ denotes the Euclidean norm.

Let us give a simple example to illustrate how the lattice representation for space-time codes works. The Alamouti code [1] provides full rate and full diversity in the complex orthogonal signal space, whose code matrix is given by

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 \\ \bar{x}_2^* & -\bar{x}_1^* \end{bmatrix}$$

where * represents the complex conjugate.

For this orthogonal design structure with $N = T = K = 2$, equation (3) can be written as

$$\mathbf{y} = \begin{bmatrix} y_{1,I} \\ y_{2,I} \\ y_{2,Q} \\ y_{2,Q} \end{bmatrix} = \mathbf{H}^o \begin{bmatrix} x_{1,I} \\ x_{2,I} \\ x_{1,Q} \\ x_{2,Q} \end{bmatrix} + \begin{bmatrix} n_{1,I} \\ n_{2,I} \\ n_{1,Q} \\ n_{2,Q} \end{bmatrix}$$

where $\mathbf{H}^o$ represents an orthogonal matrix

$$\mathbf{H}^o = [\mathbf{h}_1^o\ \mathbf{h}_2^o\ \mathbf{h}_3^o\ \mathbf{h}_4^o\ ] = \begin{bmatrix} h_{1,I} & h_{2,I} & -h_{1,Q} & -h_{2,Q} \\ -h_{2,I} & h_{1,I} & -h_{2,Q} & h_{1,Q} \\ h_{1,Q} & h_{2,Q} & h_{1,I} & h_{2,I} \\ -h_{2,Q} & h_{1,Q} & h_{2,I} & -h_{1,I} \end{bmatrix}. \tag{4}$$

Note that the column vectors $\{\mathbf{h}_k^o\}$ $(k = 1, 2, 3, 4)$ are orthogonal to each other. Then, the ML estimation of $x_{i,I}$ and $x_{i,Q}$ for $i = 1, 2$ can be decoupled for each in-phase/quadrature component. As an example, for $x_{1,I} = \Re[\bar{x}_1]$, the ML estimate $\hat{x}_{1,I}$ can be obtained by

$$\begin{aligned} \hat{x}_{1,I} &= \arg \min_{x_{1,I} \in \mathbf{u}} ||\mathbf{y} - \mathbf{h}_1^o x_{1,I}||^2 \\ &= \arg \min_{x_{1,I} \in \mathbf{u}} \left|\left| \bar{\mathbf{y}} - \begin{bmatrix} \bar{h}_1 \\ -\bar{h}_2 \end{bmatrix} x_{1,I} \right|\right|^2 \end{aligned}$$

where $\mathbf{u}$ denotes a PAM signal set with signals $\pm 1, \pm 3, \cdots, \pm(2^{\frac{\eta}{2}} - 1)$.

This simple ML detection result shows the single-symbol decodability of each in-phase/quadrature component in the case of orthogonal codes.

III. Maximum Likelihood Decoding Algorithm

In this section, we consider rate one quasi-orthogonal codes which employ multidimensional real-valued constellations. We first describe a detailed ML implementation of STBC based on the lattice representation described in the previous section, and then present a multidimensional rotation method to achieve full diversity.

Multidimensional real-valued (PAM) constellations are used to construct the complex-valued (QAM) transmitted signal vectors $\bar{\mathbf{x}}$. We first construct four $\frac{K}{2}$-dimensional real-valued PAM signal vectors, namely $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$, and employ multidimensional mappings of the four signals into $\mathbf{x}$ as $\mathbf{x} = [\ \mathbf{s}_{1,I}^t\ \mathbf{s}_{2,I}^t\ \mathbf{s}_{1,Q}^t\ \mathbf{s}_{2,Q}^t\ ]^t$. Note that $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$ are chosen from one of the $\frac{K}{2}$-dimensional constellation $\mathbf{u}^{\frac{K}{2}} = [u_1\ u_2 \cdots u_{\frac{K}{2}}]^t$ with $u_k = \pm 1, \pm 3, \cdots, \pm(2^{\frac{\eta}{2}} - 1)$. From (1), the rate is equal to $\zeta = \frac{\eta K}{T}$ bits/channel use. Then, the signal vectors $\bar{\mathbf{x}}$ can be represented by

$$\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{s}_{1,I} \\ \mathbf{s}_{2,I} \end{bmatrix} + j \begin{bmatrix} \mathbf{s}_{1,Q} \\ \mathbf{s}_{2,Q} \end{bmatrix}. \tag{5}$$

In the following, we will show that the above mapping of $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$ allows a separate ML decoding for each symbol for quasi-orthogonal codes. Let us consider the following example of quasi-orthogonal designs [1] for N=T=K=4 as

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{x}_1 & \bar{x}_3 & \bar{x}_4 & \bar{x}_2 \\ \bar{x}_3^* & -\bar{x}_1^* & \bar{x}_2^* & -\bar{x}_4^* \\ \bar{x}_4^* & \bar{x}_2^* & -\bar{x}_1^* & -\bar{x}_3^* \\ \bar{x}_2 & -\bar{x}_4 & -\bar{x}_3 & \bar{x}_1 \end{bmatrix}. \tag{6}$$

In this case, $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$ are related to the symbols $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$, and $\bar{x}_4$ as

$$\mathbf{s}_{1,I} = \begin{bmatrix} x_{1,I} \\ x_{2,I} \end{bmatrix},\ \mathbf{s}_{2,I} = \begin{bmatrix} x_{3,I} \\ x_{4,I} \end{bmatrix},\ \mathbf{s}_{1,Q} = \begin{bmatrix} x_{1,Q} \\ x_{2,Q} \end{bmatrix},$$

and

$$\mathbf{s}_{2,Q} = \begin{bmatrix} x_{3,Q} \\ x_{4,Q} \end{bmatrix}.$$

[1]In this letter, we use a slightly different quasi-orthogonal code from the original design in [3] in order to make it simple to divide the transmitted symbol set into independent multidimensional constellations. It is worthy mentioning that any quasi-orthogonal code available in the literature achieves a ML decoding with a minimum decoding complexity by utilizing the proposed real-domain decoder.

Then, after some manipulations, we can obtain the corresponding effective channel $\mathbf{H}^q$ in (3) for the QO-STBC in (6) as

$$\mathbf{H}^q = [\mathbf{h}_1^q\ \mathbf{h}_2^q \cdots \mathbf{h}_8^q] = \begin{bmatrix} h_{1,I} & h_{4,I} & h_{2,I} & h_{3,I} & -h_{1,Q} & -h_{4,Q} & -h_{2,Q} & -h_{3,Q} \\ -h_{2,I} & h_{3,I} & h_{1,I} & -h_{4,I} & -h_{2,Q} & h_{3,Q} & h_{1,Q} & -h_{4,Q} \\ -h_{3,I} & h_{2,I} & -h_{4,I} & h_{1,I} & -h_{3,Q} & h_{2,Q} & -h_{4,Q} & h_{1,Q} \\ h_{4,I} & h_{1,I} & -h_{3,I} & -h_{2,I} & -h_{4,Q} & -h_{1,Q} & h_{3,Q} & h_{2,Q} \\ h_{1,Q} & h_{4,Q} & h_{2,Q} & h_{3,Q} & h_{1,I} & h_{4,I} & h_{2,I} & h_{3,I} \\ -h_{2,Q} & h_{3,Q} & h_{1,Q} & -h_{4,Q} & h_{2,I} & -h_{3,I} & -h_{1,I} & h_{4,I} \\ -h_{3,Q} & h_{2,Q} & -h_{4,Q} & h_{1,Q} & h_{3,I} & -h_{2,I} & h_{4,I} & -h_{1,I} \\ h_{4,Q} & h_{1,Q} & -h_{3,Q} & -h_{2,Q} & h_{4,I} & h_{1,I} & -h_{3,I} & -h_{2,I} \end{bmatrix}. \quad (7)$$

Unlike the effective channel of the orthogonal design in (4), only pairs of the column vectors $\{\mathbf{h}_1^q\ \mathbf{h}_2^q\}$, $\{\mathbf{h}_3^q\ \mathbf{h}_4^q\}$, $\{\mathbf{h}_5^q\ \mathbf{h}_6^q\}$ and $\{\mathbf{h}_7^q\ \mathbf{h}_8^q\}$ constitute orthogonal subsets. In this case, ML decoding should be performed jointly with pairs of in-phase/quadrature components: $\{x_{1,I}, x_{2,I}\}$, $\{x_{3,I}, x_{4,I}\}$, $\{x_{1,Q}, x_{2,Q}\}$ and $\{x_{3,Q}, x_{4,Q}\}$. In other words, ML decoding is achieved by decoding $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$ independently. For example, the ML estimate $\hat{\mathbf{s}}_{1,I} = [\hat{x}_{1,I}\ \hat{x}_{2,I}]^t$ can be obtained by

$$\begin{aligned} \hat{\mathbf{s}}_{1,I} &= \arg \min_{\mathbf{s}_{1,I} \in \mathbf{u}^2} \left\| \mathbf{y} - [\mathbf{h}_1^q\ \mathbf{h}_2^q] \begin{bmatrix} x_{1,I} \\ x_{2,I} \end{bmatrix} \right\|^2 \quad (8) \\ &= \arg \min_{\mathbf{s}_{1,I} \in \mathbf{u}^2} \left\| \bar{\mathbf{y}} - \begin{bmatrix} \bar{h}_1 & \bar{h}_4 \\ -\bar{h}_2 & \bar{h}_3 \\ -\bar{h}_3 & \bar{h}_2 \\ \bar{h}_4 & \bar{h}_1 \end{bmatrix} \mathbf{s}_{1,I} \right\|^2 . \end{aligned}$$

We notice that any complex-valued representation can be transformed into a real-valued representation as in (8), and the ML estimation metric in the real and complex representations requires the same amount of computation. Therefore, the QO-STBC with $N = 4$ can be decoded with linear complexity $\mathcal{O}(M_c)$ in a M_c-QAM constellation. It is also important to note that the traditional ML decoding scheme derives the ML decision metric for the pairs $(\mathbf{s}_{1,I}, \mathbf{s}_{1,Q})$ and $(\mathbf{s}_{2,I}, \mathbf{s}_{2,Q})$, respectively. For instance, the conventional ML metric for $(\mathbf{s}_{1,I}, \mathbf{s}_{1,Q})$ can be written as

$$[\hat{\mathbf{s}}_{1,I}\ \hat{\mathbf{s}}_{1,Q}] = \arg \min_{\mathbf{s}_{1,I}, \mathbf{s}_{1,Q} \in \mathbf{u}^2} \left\| \mathbf{y} - [\mathbf{h}_1^q\ \mathbf{h}_2^q\ \mathbf{h}_5^q\ \mathbf{h}_6^q] \begin{bmatrix} x_{1,I} \\ x_{2,I} \\ x_{1,Q} \\ x_{2,Q} \end{bmatrix} \right\|^2 . \quad (9)$$

The comparison between Equations (8) and (9) shows that compared to the conventional ML decoding methods, the proposed decoding algorithm reduces not only the number of metric computations but also the computational complexity per metric.

In general, for a rate one quasi-orthogonal code with $N = T = K = 2n$ ($n = 2, 4, 8, \cdots$), we construct four n-dimensional constellations, namely $\mathbf{s}_{1,I} = [x_{1,I}, \cdots, x_{n,I}]^t$, $\mathbf{s}_{2,I} = [x_{n+1,I}, \cdots, x_{2n,I}]^t$, $\mathbf{s}_{1,Q} = [x_{1,Q}, \cdots, x_{n,Q}]^t$ and $\mathbf{s}_{2,Q} = [x_{n+1,Q}, \cdots, x_{2n,Q}]^t$, and employ multidimensional mappings of the four signals as in Equation (5). Then we are now able to perform the ML estimation of $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$ and $\mathbf{s}_{2,Q}$ independently over the n-dimensional real-valued constellation $\mathbf{u}^n$. Note that the cardinality of the constellation $\mathbf{u}^n$ is $M_c^{N/4}$. Therefore, with our proposed ML decoding scheme, the decoding complexity reduces from $\mathcal{O}(M_c^{N/2})$ to $\mathcal{O}(M_c^{N/4})$ in a M_c-QAM system, where the complexity accounts for the number of candidates to search in the ML decoding.

An example with eight transmit antennas (N=T=K=8) is given as

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{x}_1 & \bar{x}_5 & \bar{x}_6 & \bar{x}_2 & \bar{x}_7 & \bar{x}_3 & \bar{x}_4 & \bar{x}_8 \\ \bar{x}_5^* & -\bar{x}_1^* & \bar{x}_2^* & -\bar{x}_6^* & \bar{x}_3^* & -\bar{x}_7^* & \bar{x}_8^* & -\bar{x}_4^* \\ \bar{x}_6^* & \bar{x}_2^* & -\bar{x}_1^* & -\bar{x}_5^* & \bar{x}_4^* & \bar{x}_8^* & -\bar{x}_7^* & -\bar{x}_3^* \\ \bar{x}_2 & -\bar{x}_6 & -\bar{x}_5 & \bar{x}_1 & \bar{x}_8 & -\bar{x}_4 & -\bar{x}_3 & \bar{x}_7 \\ \bar{x}_7^* & \bar{x}_3^* & \bar{x}_4^* & \bar{x}_8^* & -\bar{x}_1^* & -\bar{x}_5^* & -\bar{x}_6^* & -\bar{x}_2^* \\ \bar{x}_3 & -\bar{x}_7 & \bar{x}_8 & -\bar{x}_4 & -\bar{x}_5 & \bar{x}_1 & -\bar{x}_2 & \bar{x}_6 \\ \bar{x}_4 & \bar{x}_8 & -\bar{x}_7 & -\bar{x}_3 & -\bar{x}_6 & -\bar{x}_2 & \bar{x}_1 & \bar{x}_5 \\ \bar{x}_8^* & -\bar{x}_4^* & -\bar{x}_3^* & \bar{x}_7^* & -\bar{x}_2^* & \bar{x}_6^* & \bar{x}_5^* & -\bar{x}_1^* \end{bmatrix}. \quad (10)$$

In this case, the ML decoding can be performed independently over each 4-dimensional real signal vector $\mathbf{s}_{1,I} = [x_{1,I}, \cdots, x_{4,I}]^t$, $\mathbf{s}_{2,I} = [x_{5,I}, \cdots, x_{8,I}]^t$, $\mathbf{s}_{1,Q} = [x_{1,Q}, \cdots, x_{4,Q}]^t$ and $\mathbf{s}_{2,Q} = [x_{5,Q}, \cdots, x_{8,Q}]^t$. For example, for the detection of $\mathbf{s}_{1,I} = [x_{1,I}, \cdots, x_{4,I}]^t$, the ML estimate $\hat{\mathbf{s}}_{1,I}$ is given by

$$\hat{\mathbf{s}}_{1,I} = \arg \min_{\mathbf{s}_{1,I} \in \mathbf{u}^4} \left\| \bar{\mathbf{y}} - \begin{bmatrix} \bar{h}_1 & \bar{h}_4 & \bar{h}_6 & \bar{h}_7 \\ -\bar{h}_2 & \bar{h}_3 & \bar{h}_5 & -\bar{h}_8 \\ -\bar{h}_3 & \bar{h}_2 & -\bar{h}_8 & \bar{h}_5 \\ \bar{h}_4 & \bar{h}_1 & -\bar{h}_7 & -\bar{h}_6 \\ -\bar{h}_5 & -\bar{h}_8 & \bar{h}_2 & \bar{h}_3 \\ \bar{h}_6 & -\bar{h}_7 & \bar{h}_1 & -\bar{h}_4 \\ \bar{h}_7 & -\bar{h}_6 & -\bar{h}_4 & \bar{h}_1 \\ -\bar{h}_8 & -\bar{h}_5 & -\bar{h}_3 & -\bar{h}_2 \end{bmatrix} \mathbf{s}_{1,I} \right\|^2 .$$

It is straightforward to see that the proposed ML decoding algorithm can be extended to the case of multiple receive antennas.

In what follows, we will present a multidimensional rotation method to achieve full diversity with reduced decoding complexity. In the conventional rotation schemes [4] [6], one half of the symbols in a quasi-orthogonal design is chosen from a signal constellation set $\mathcal{A}$, while the other half of them is selected from a rotated constellation $e^{j\phi}\mathcal{A}$, where ϕ denotes the rotation angle. This means that the proposed decoding scheme does not work with the conventional rotation approaches since the constellation rotation of half the transmitted symbols causes intersymbol interference among the data signals $\mathbf{s}_{1,I}$, $\mathbf{s}_{1,Q}$, $\mathbf{s}_{2,I}$ and $\mathbf{s}_{2,Q}$. Instead, we apply a rotation to the n-dimensional constellation $\mathbf{u}^n$ and use its rotated version $\mathbf{r}^n$ for all signals $\mathbf{s}_{1,I}$, $\mathbf{s}_{1,Q}$, $\mathbf{s}_{2,I}$ and $\mathbf{s}_{2,Q}$. In this case, the decoding algorithm described above works for rotated quasi-orthogonal codes (i.e., $\mathbf{s}_{1,I}$, $\mathbf{s}_{1,Q}$, $\mathbf{s}_{2,I}$ and $\mathbf{s}_{2,Q} \in \mathbf{r}^n$) with the same complexity as for the non-rotated case (i.e., $\mathbf{s}_{1,I}$, $\mathbf{s}_{1,Q}$, $\mathbf{s}_{2,I}$ and $\mathbf{s}_{2,Q} \in \mathbf{u}^n$).

For a rate one QO-STBC with $N = T = K = 2n$ ($n = 2, 4, 8, \cdots$), the rotated constellation $\mathbf{r}^n$ is obtained by applying the rotation matrix $\mathbf{G}_n$ of size n to $\mathbf{u}^n = [u_1\ u_2 \cdots u_n]^t$. The set of rotated points $\mathbf{r}^n = \mathbf{G}_n \mathbf{u}^n$ belongs to the n-dimensional lattice Z^n [13]. Based on the optimal rotation angles addressed in [4] and [6] for rate one QO-STBC, we can determine the rotation matrix $\mathbf{G}_n$ which achieves the maximum diversity order and coding gain. For example, in QO-STBC with $N = 4$, the two-dimensional constellation $\mathbf{r}^2$

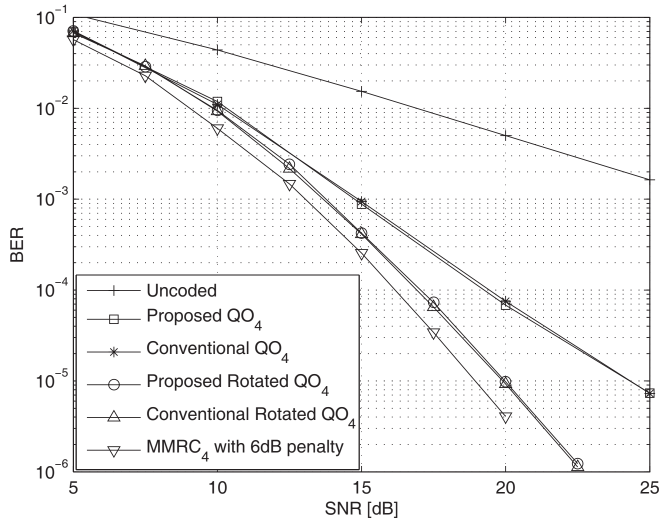

Fig. 1. Bit-error rates for ζ= 2 bits/channel use for four transmit antennas ($N = 4$).

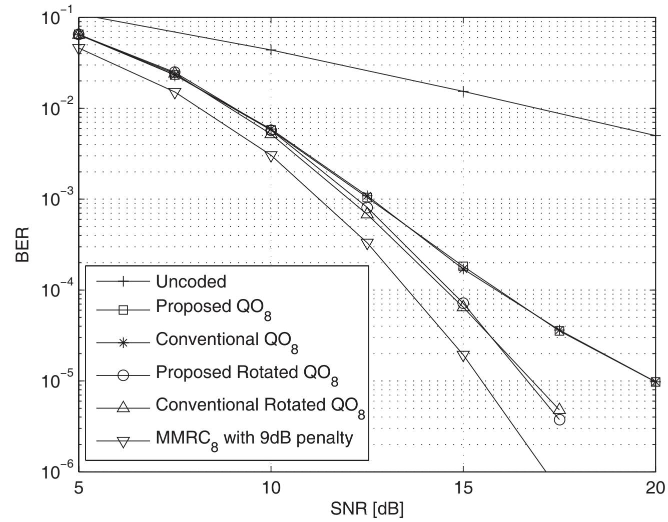

Fig. 2. Bit-error rates for ζ= 2 bits/channel use for eight transmit antennas ($N = 8$).

is obtained from $\mathbf{u}^2$ with the rotation matrix

$$\mathbf{G}_2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where the optimal angle is given as $\theta = \frac{1}{2}\text{atan}(\frac{1}{2}) = 13.2825°$, regardless of η.

We note that the conventional rotation schemes increase the number of signals in the constellation as one half of the symbols in a quasi-orthogonal design are rotated. The proposed scheme also exhibits such constellation expansion since $\mathbf{s}_{1,I}$, $\mathbf{s}_{2,I}$, $\mathbf{s}_{1,Q}$, and $\mathbf{s}_{2,Q}$ in Equation (5) are chosen from one of the $\frac{K}{2}$-dimensional rotated constellation $\mathbf{r}^n$. For instance, in the case of $N = 4$ and $\eta = 2$, the proposed mapping results in 16-QAM transmission constellation.

IV. Simulation Results

In this section, we present simulation results that demonstrate the efficacy of our proposed methods. Denote QO_N as a rate one QO-STBC with N transmit antennas. We compare the bit-error rate (BER) performance of the proposed ML decoding (denoted by *Proposed* QO_N) and the conventional ML decoding (denoted by *Conventional* QO_N) for QO_N [4] [6]. We will also present performance comparisons of the proposed rotation method (denoted by *proposed rotated* QO_N) with other existing rotation schemes (denoted by *conventional rotated* QO_N) for QO_N [6]. Figures 1 and 2 depict the BER performance of QO_4 and QO_8 with or without rotation for ζ= 2 bits/channel use corresponding to the conventional 4-QAM modulation scheme. For the proposed rotated QO_8, we use the rotation matrix $\mathbf{G}_4$ presented in [15]. These simulation results reflect that the proposed ML decoding algorithm provides the performance identical to the original ML decoding detection [4] [6] with and without rotation. In these two figures, we also plot BER curves of maximum ratio receive combining (MRRC) for comparison purposes. We denote an MRRC scheme with one transmit and M receive antennas by MRRC_M. Power penalties of 6 dB and 9 dB in MRRC for $M = 4$ and 8, respectively, are incurred for a fair comparison with other transmit diversity schemes. We can see that for both QO_4 and QO_8 there is no difference in the diversity order between MRRC_M and the proposed QO_N with multidimensional rotated constellation (for $M = N$). Here the performance gap between MRRC_N and the rotated QO_N can be explained in terms of the minimum determinant (i.e., coding gain) for which transmit diversity schemes are inferior to receive diversity techniques.

V. Conclusion

In this letter, we have used the real space signal models for rate one QO-STBCs in order to provide a better understanding and a useful insight on the minimum decoding complexity. We have also utilized multidimensional rotated constellations to achieve full diversity for the quasi-orthogonal codes. Especially, it is shown that in order to achieve both the full diversity and minimum decoding complexity for QO-STBCs, we should employ the multidimensional rotated constellations instead of the conventional full-diversity methods using rotated QAM constellations. Simulation results have demonstrated that for systems with more than or equal to four transmit antennas, the proposed STBC scheme achieves full rate and full diversity with reduced decoding complexity. The same idea can be extended to general space-time codes with different rate and size, and is also applied to other MIMO systems, such as layered space-time architectures [16].

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