Transmit Beamforming Method Based on Maximum-Norm Combining for MIMO Systems

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***Abstract*—In this paper, we present a low-complexity method to generate a transmit beamforming vector for multiple-input-multiple-output (MIMO) systems. We begin by introducing new definitions regarding orthogonality between two complex-valued vectors and then present new expressions of complex rotation matrices for the complex vector orthogonalization. The rotation matrices are utilized to derive the weight vector for the maximum-norm combining (MNC) process of two complex vectors, which provides a constructive basis for a new beamforming method. The proposed transmit beamforming method uses successive column combining of MIMO channel matrices based on MNC, and as a result, an approximate solution to the optimum beamforming vector is obtained. The proposed method offers a good tradeoff between complexity and performance. Simulation results demonstrate that the proposed beamforming method achieves the near-optimal performance with much reduced computational complexity, compared to the optimal beamforming scheme using singular-value decomposition (SVD) of the channel matrix.**

***Index Terms*—Multiple-input-multiple-output (MIMO) systems, transmit-beamforming technique, complex rotation matrix, maximum-norm combining (MNC).**

I. INTRODUCTION

THE use of multiple-input-multiple-output (MIMO) antennas, coupled with space-time processing, has been considered to be a promising means to increase the available capacity for high data rate wireless links [1] [2]. The expected benefits include higher system capacity and improved quality of service as a result of spatial multiplexing (SM) and diversity gain [3]. The fundamental tradeoff between the SM gain and the diversity gain has been characterized in [4], which reveals the relationship between two types of gain and provides insights to understand the overall resources provided by MIMO channels.

In open-loop systems where channel state information (CSI) is known only at the receiver, space-time code (STC) techniques exploit the transmit diversity, which yields an additional diversity and/or coding gain compared to a single-input single-output (SISO) system [5][6][7]. Specifically, the antenna diversity combining at the receiver has been studied to improve the performance of single-input multiple-output (SIMO) systems. A classical receiver technique for such systems is maximum-ratio combining (MRC) [8]. In contrast, closed-loop transmission strategies utilize knowledge of the channel at the transmitter to further enhance the capacity of MIMO systems [9][10]. In this case, the MRC algorithm was generalized for both transmit beamforming and receive combining in [11] where the concept of maximum ratio transmission (MRT) was presented. In order to maximize the received signal-to-noise ratio (SNR), the transmit beamforming and receive combining vectors need to be jointly designed by utilizing the singular-value decomposition (SVD) of the channel matrix [12][13], which will be referred to as *optimal beamforming scheme* for the rest of this paper.

There exist several approaches for computing the SVD of matrices. The Jacobi method is originally proposed for diagonalizing real symmetric matrices, and plays an important role in the computation of the SVD of matrices [14]. While the original *two-sided* Jacobi transformation requires both pre-multiplication and post-multiplication to cancel the largest off-diagonal elements, its *one-sided* version was introduced by discovering the equivalence between orthogonalizing two vectors and annihilating a matrix element [15] [16]. The most widely used algorithm for computing the full SVD is the Golub-Kahan algorithm [14]. However, in the beamforming transmission, since we do not need to compute all the singular vectors of the channel matrix, only the dominant singular vector can be efficiently obtained by using the iterative power method [17]. In this paper, we present an approximate method of generating transmit beamforming vectors, while its performance is very close to that of the optimal beamforming scheme. Note that all the SVD approaches, including the previously mentioned one- and two-sided Jacobi methods and the power method, are based on iterative algorithms. Instead, in this paper, we focus on non-iterative solutions to reduce the complexity while minimizing the performance loss.

We first discuss a method for orthogonalization of two complex-valued vectors. To this end, we introduce new definitions regarding orthogonality between two complex-valued vectors and present new expressions of complex rotation matrices for orthogonalizing an arbitrary pair of complex-valued vectors [18]. The orthogonalization of two real-valued vectors based on plane rotations was introduced in [16]. Also, several forms of general 2-by-2 complex-valued rotation matrices have been proposed for various purposes including the complex

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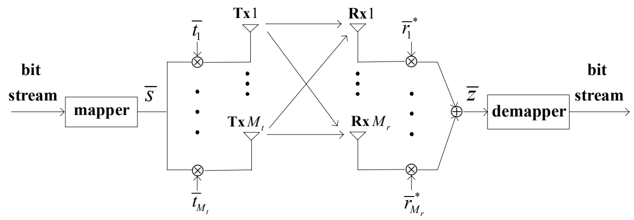

Fig. 1. Schematic diagram of a transmit beamforming and receive combining scheme with M_t transmit and M_r receive antennas.

vector orthogonalization in [19] and therein references. Unlike the previous works where the 2-by-2 rotation matrices were chosen from general representations without any theoretical basis, we propose a new expression of 2-by-2 complex rotation matrices with theoretical explanations in its derivation. As a result, we can deal with the complex vector orthogonalization problem more efficiently. The proposed rotation transformation is utilized to derive the unit-norm weight vector for maximum-norm combining (MNC) of two complex vectors. As a consequence, a new beamforming method is obtained which employs successive MNC to the MIMO channel matrix.

Simulation results show that this approach achieves a good trade-off between system performance and computational complexity. For example, for MIMO systems with four transmit and four receive antennas, the proposed beamforming scheme allows a computational complexity saving of about 60% for the same performance in comparison to the beamforming scheme based on the iterative power method [14] [17].

Throughout this paper, normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. With a bar accounting for complex variables, for any complex notation $\overline{c}$, we denote the real and imaginary part of $\overline{c}$ by $\Re[\overline{c}]$ and $\Im[\overline{c}]$, respectively.

The organization of the paper is as follows: Section II presents a general description of MIMO beamforming and combining systems and reviews conventional beamforming schemes. In Section III, we derive new expressions of complex rotation matrices for the complex vector orthogonalization. In Section IV, we propose a new beamforming method based on combining columns of a MIMO channel matrix successively. Section V shows the simulation results and compares the proposed method with the optimum beamforming scheme. Finally, the paper is terminated with conclusions in Section VI.

II. System Descriptions

In this section, we consider a MIMO system model with M_t transmit and M_r receive antennas, as shown in Figure 1. We assume that the elements of the MIMO channel matrix are obtained from an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance. The system consists of a beamforming processing at the transmitter and a combining processing at the receiver. At the transmitter, the data symbol $\overline{s}$ is multiplied by the unit-norm transmit weight vector $\overline{\mathbf{t}}$ $\big(||\overline{\mathbf{t}}|| = 1\big)$ to form the M_t-dimensional complex transmitted signal vector $\overline{\mathbf{x}}$, where $||\cdot||$ denotes the 2-norm of a vector.

Denoting $\overline{\mathbf{H}}$ and $\overline{\mathbf{y}}$ as the M_r-by-M_t channel matrix and the M_r-dimensional complex received signal vector, respectively, the complex received signal is given by

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\overline{\mathbf{x}} + \overline{\mathbf{n}}$$

where $\overline{\mathbf{x}} = \overline{\mathbf{t}}\,\overline{s}$ and $\overline{\mathbf{n}}$ is the complex Gaussian noise vector with the covariance matrix $\sigma_n^2 \mathbf{I}_{M_r}$. Here $\mathbf{I}_d$ indicates an identity matrix of size d.

At the receiver, the transmitted data symbol can be recovered by combining M_r received symbols by the receive weight vector $\overline{\mathbf{r}}$ $(||\overline{\mathbf{r}}|| = 1)$. In Figure 1, $\overline{t}_i$ and $\overline{r}_i$ indicate the i-th element of $\overline{\mathbf{t}}$ and $\overline{\mathbf{r}}$, respectively, and $(\cdot)^*$ denotes the complex conjugate. Then, the corresponding estimated symbol $\overline{z}$ can be written as

$$\overline{z} = \overline{\mathbf{r}}^\dagger\, \overline{\mathbf{H}}\, \overline{\mathbf{t}}\, \overline{s} + \overline{\mathbf{r}}^\dagger \overline{\mathbf{n}} \tag{1}$$

where $(\cdot)^\dagger$ denotes the complex conjugate transpose of a vector or matrix.

Note that $\overline{\mathbf{r}}^\dagger\, \overline{\mathbf{H}}\, \overline{\mathbf{t}}$ in (1) corresponds to the effective channel gain. In order to maximize the output SNR, we need to properly choose the transmit weight vector $\overline{\mathbf{t}}$ and the receive weight vector $\overline{\mathbf{r}}$. We assume an MRC receiver given by

$$\overline{\mathbf{r}} = \frac{\overline{\mathbf{H}}\,\overline{\mathbf{t}}}{||\overline{\mathbf{H}}\,\overline{\mathbf{t}}||}, \tag{2}$$

which is optimal in terms of the output SNR when the noise is white [8].

Without any constraint on the beamforming vector, it can be shown that the channel gain $\overline{\mathbf{r}}^\dagger\, \overline{\mathbf{H}}\, \overline{\mathbf{t}}$ is maximized when the dominant right singular vector of the channel matrix $\overline{\mathbf{H}}$ corresponding to the largest singular value λ_{max} of $\overline{\mathbf{H}}$ is employed as $\overline{\mathbf{t}}$ [13]. In this case, the maximum channel gain is obtained as

$$|\overline{\mathbf{r}}^\dagger\, \overline{\mathbf{H}}\, \overline{\mathbf{t}}| = ||\overline{\mathbf{H}}\, \overline{\mathbf{t}}|| = \lambda_{max}. \tag{3}$$

This beamforming technique based on the dominant singular vector of the channel matrix is optimal in terms of the SNR performance. Therefore, the optimal solution of the channel gain maximization in Equation (3) requires the computation of the dominant singular vector of the M_r-by-M_t channel matrix, and the power method is known to be particularly suited for the computation. The iterative power method determines the transmit vector $\overline{\mathbf{t}}$ by performing the following iteration

$$\overline{\mathbf{r}}^{(i+1)} = \frac{\overline{\mathbf{H}}\,\overline{\mathbf{t}}^{(i)}}{||\overline{\mathbf{H}}\,\overline{\mathbf{t}}^{(i)}||}$$

and

$$\overline{\mathbf{t}}^{(i+1)} = \frac{\overline{\mathbf{H}}^\dagger\,\overline{\mathbf{r}}^{(i+1)}}{||\overline{\mathbf{H}}^\dagger\,\overline{\mathbf{r}}^{(i+1)}||},$$

where we start with an initial vector $\overline{\mathbf{t}}^{(0)}$and then, after the iteration k, the channel gain of $||\overline{\mathbf{H}}\overline{\mathbf{t}}^{(k)}||$ is achieved. Note that $||\overline{\mathbf{H}}\overline{\mathbf{t}}^{(k)}||$ approaches λ_{max} as the iteration process converges.

The power method described above involves $(8M_tM_r + 4M_t{+}4M_r)N_{it}$ real multiplications for the M_r-by-M_t channel matrix, where N_{it} indicates the number of iterations. In this paper, we propose a new transmit beamforming method which achieves a reduction of up to $\frac{1}{3}$ in the complexity compared to the power method while maintaining the near optimum performance.

III. ROTATION TRANSFORMATIONS FOR COMPLEX VECTOR ORTHOGONALIZATION

In this section, we present the mathematical background for the proposed beamforming method. We first discuss an orthogonalization problem of two complex-valued vectors and derive new expressions of complex rotation matrices for the complex vector orthogonalization. Meanwhile, we provide many useful insights on the connection between the vector orthogonalization process and the vector-norm maximization.

It was shown in [16] that given two real-valued vectors the norm of one column is maximized and the norm of the other is minimized when the two columns become orthogonal to each other by a rotation transformation. In order to generalize the observation to the complex case, we first define three different kinds of orthogonality between two complex-valued vectors: *inner* orthogonality, *outer* orthogonality, and *complex* orthogonality [18].

Definition 1: Given two complex vectors $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$, consider the Hermitian product of two (complex-valued) vectors as

$$\begin{aligned}\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle &= \overline{\mathbf{h}}_1^{\dagger} \overline{\mathbf{h}}_2 \\ &= \langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R + j \langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_I ,\end{aligned}$$

where $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R \triangleq \Re\left[\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle\right]$ and $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_I \triangleq \Im\left[\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle\right]$. We refer to two complex-valued vectors $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$ as being *inner* orthogonal or *outer* orthogonal in $\mathcal{C}^{M_r}$ if $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R = 0$ or $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_I = 0$ is satisfied, respectively. Furthermore, $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$ are referred to as *complex* orthogonal if $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R = 0$ and $\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_I = 0$ at the same time. Therefore, *complex* orthogonality means a stronger condition.

For any complex vector $\overline{\mathbf{v}}$, we define two real-valued vectors as

$$\mathbf{v} = \begin{bmatrix} \Re[\overline{\mathbf{v}}] \\ \Im[\overline{\mathbf{v}}] \end{bmatrix}, \quad \text{and} \quad \dot{\mathbf{v}} = \begin{bmatrix} -\Im[\overline{\mathbf{v}}] \\ \Re[\overline{\mathbf{v}}] \end{bmatrix}. \tag{4}$$

It is easy to show the following properties:

$$||\mathbf{v}|| = ||\dot{\mathbf{v}}|| \quad \text{and} \quad \mathbf{v} \cdot \dot{\mathbf{v}} = 0 \tag{5}$$

where $(\cdot)$ denotes the inner (dot) product between two real vectors.

Also, from *Definition* 1 and Equation (4), we have the relations

$$\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R = \mathbf{h}_1 \cdot \mathbf{h}_2 = \dot{\mathbf{h}}_1 \cdot \dot{\mathbf{h}}_2 \tag{6}$$

and

$$\langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_I = -\mathbf{h}_1 \cdot \dot{\mathbf{h}}_2 = \mathbf{h}_2 \cdot \dot{\mathbf{h}}_1 . \tag{7}$$

In what follows, we will utilize the properties (5), (6), and (7) to derive a new rotation matrix for achieving three different kinds of orthogonality between two complex vectors. Given two complex vectors $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$, we consider a rotation matrix $\overline{\mathbf{F}}$ such that two vectors $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ in

$$\left[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'\right] = \left[\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2\right] \overline{\mathbf{F}} \tag{8}$$

become either inner, outer or complex orthogonal. Here, we should emphasize that we are trying to establish orthogonality between columns $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ so that the norm of $\overline{\mathbf{h}}_1'$ is maximized, which is based on the connection between the vector orthogonalization process and the vector-norm maximization [16]. In the following section, the transformation $\overline{\mathbf{F}}$ will be used to find the MNC weight vector, which gives a constructive basis for a new beamforming strategy on how to construct the transmit weight vector $\overline{\mathbf{t}}$.

The real-valued representation of (8) can be written by [14] [20]

$$\begin{aligned}&\begin{bmatrix} \Re\left[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'\right] & -\Im\left[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'\right] \\ \Im\left[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'\right] & \Re\left[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'\right] \end{bmatrix} \\ &\quad = \begin{bmatrix} \Re\left[\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2\right] & -\Im\left[\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2\right] \\ \Im\left[\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2\right] & \Re\left[\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2\right] \end{bmatrix} \begin{bmatrix} \Re[\overline{\mathbf{F}}] & -\Im[\overline{\mathbf{F}}] \\ \Im[\overline{\mathbf{F}}] & \Re[\overline{\mathbf{F}}] \end{bmatrix}.\end{aligned} \tag{9}$$

Using the notations in (4), Equation (9) is transformed to

$$\left[\mathbf{h}_1' \; \mathbf{h}_2' \; \dot{\mathbf{h}}_1' \; \dot{\mathbf{h}}_2'\right] = \left[\mathbf{h}_1 \; \mathbf{h}_2 \; \dot{\mathbf{h}}_1 \; \dot{\mathbf{h}}_2\right] \begin{bmatrix} \Re[\overline{\mathbf{F}}] & -\Im[\overline{\mathbf{F}}] \\ \Im[\overline{\mathbf{F}}] & \Re[\overline{\mathbf{F}}] \end{bmatrix}. \tag{10}$$

We first derive the rotation transformation $\overline{\mathbf{F}}$ for obtaining the inner orthogonality between $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ in (8) $\left(\text{i.e., } \langle \overline{\mathbf{h}}_1', \overline{\mathbf{h}}_2' \rangle_R = 0\right)$. From (6), it is obvious that in order to achieve the inner orthogonality, we need to make the real-valued vectors $\mathbf{h}_1'$ and $\dot{\mathbf{h}}_1'$ perpendicular to $\mathbf{h}_2'$ and $\dot{\mathbf{h}}_2'$, respectively, in (10) (i.e., $\mathbf{h}_1' \perp \mathbf{h}_2'$ and $\dot{\mathbf{h}}_1' \perp \dot{\mathbf{h}}_2'$). In this case we can arrive at new orthogonal pairs of real-valued vectors $\{\mathbf{h}_1', \mathbf{h}_2'\}$ and $\{\dot{\mathbf{h}}_1', \dot{\mathbf{h}}_2'\}$ by applying the following rotation transformation to a pair of $\{\mathbf{h}_1, \mathbf{h}_2\}$ and $\{\dot{\mathbf{h}}_1, \dot{\mathbf{h}}_2\}$ as

$$\left[\mathbf{h}_1' \; \mathbf{h}_2' \; \dot{\mathbf{h}}_1' \; \dot{\mathbf{h}}_2'\right] = \left[\; [\mathbf{h}_1 \; \mathbf{h}_2]\, \mathbf{R}(\theta) \quad \left[\dot{\mathbf{h}}_1 \; \dot{\mathbf{h}}_2\right] \mathbf{R}(\theta) \;\right] \tag{11}$$

where $\mathbf{R}(\theta)$ is the well-known two-dimensional Givens rotation matrix [14] defined as

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

In a complex domain, Equation (11) is equivalent to

$$[\overline{\mathbf{h}}_1' \; \overline{\mathbf{h}}_2'] = [\overline{\mathbf{h}}_1 \; \overline{\mathbf{h}}_2] \overline{\mathbf{I}}(\theta) \tag{12}$$

where the *inner* rotation matrix $\overline{\mathbf{I}}(\theta)$ is given as

$$\overline{\mathbf{I}}(\theta) = \mathbf{R}(\theta). \tag{13}$$

Then, from (6), we can show that the *inner* orthogonality between $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ is achieved ($\langle \overline{\mathbf{h}}_1', \overline{\mathbf{h}}_2' \rangle_R = 0$) if the following rotation angle is employed [18]

$$\theta_I = \tan^{-1}\left(\frac{A - \sqrt{A^2 + 4B^2}}{2B}\right) \tag{14}$$

where $A = ||\overline{\mathbf{h}}_1||^2 - ||\overline{\mathbf{h}}_2||^2$ and $B = \langle \overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2 \rangle_R$.

It is important to point out that in Equation (12) the column $\overline{\mathbf{h}}_1'$ has its maximum norm value when the inner rotation $\overline{\mathbf{I}}(\theta)$ $\left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$ is applied with the rotation angle $\theta = \theta_I$ [1]

[1]Note that when $\theta_I = \tan^{-1}\left(\frac{A+\sqrt{A^2+4B^2}}{2B}\right)$, $\left|\left|\overline{\mathbf{h}}_2'\right|\right|$ is maximized while $\left|\left|\overline{\mathbf{h}}_1'\right|\right|$ is minimized.

so that the inner orthogonality between two vectors $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ is obtained. In other words, $\left\|\overline{\mathbf{h}}_1'\right\|$ is maximized over all possible θ by substituting the above angle θ_I into Equation (12), and this results in

$$\left\|\overline{\mathbf{h}}_1'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} + \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_R^2}. \quad (15)$$

In this case, the norm value of $\overline{\mathbf{h}}_2'$ is minimized as

$$\left\|\overline{\mathbf{h}}_2'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} - \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_R^2},$$

since the Frobenius norm remains unchanged for any unitary matrix $\overline{\mathbf{F}}$ in (8), i.e., $\left\|\overline{\mathbf{h}}_1'\right\|^2+\left\|\overline{\mathbf{h}}_2'\right\|^2=\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2$.

Next, we derive a rotation matrix for achieving the outer orthogonality $\left(\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle_I=0\right)$. From (7), we can see that the outer orthogonality is obtained by applying a rotation transformation onto a pair of columns $\{\mathbf{h}_1,\dot{\mathbf{h}}_2\}$ and $\{\mathbf{h}_2,\dot{\mathbf{h}}_1\}$ so that $\mathbf{h}_1'\perp\dot{\mathbf{h}}_2'$ and $\mathbf{h}_2'\perp\dot{\mathbf{h}}_1'$. In this case, the corresponding rotation operation is expressed as

$$\left[\mathbf{h}_1'\ \dot{\mathbf{h}}_2'\ \mathbf{h}_2'\ \dot{\mathbf{h}}_1'\right]=\left[\ \left[\mathbf{h}_1\ \dot{\mathbf{h}}_2\right]\mathbf{R}(\theta)\quad\left[\mathbf{h}_2\ \dot{\mathbf{h}}_1\right]\mathbf{R}(\theta)\ \right] \quad (16)$$

which becomes, in a complex notation,

$$[\overline{\mathbf{h}}_1'\ \overline{\mathbf{h}}_2']=[\overline{\mathbf{h}}_1\ \overline{\mathbf{h}}_2]\overline{\mathbf{O}}(\theta) \quad (17)$$

where the *outer* rotation matrix $\overline{\mathbf{O}}(\theta)$ is given as

$$\overline{\mathbf{O}}(\theta)=\begin{bmatrix}\cos\theta & j\sin\theta\\ j\sin\theta & \cos\theta\end{bmatrix}. \quad (18)$$

After some mathematical manipulations, we can obtain the rotation angle for the outer orthogonality between $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$ as [18]

$$\theta_o=\tan^{-1}\left(\frac{A-\sqrt{A^2+4C^2}}{2C}\right) \quad (19)$$

where $C=\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_I$.

We notice that when the outer rotation $\overline{\mathbf{O}}(\theta)$ is applied to (8) as in (17), the norm value of $\overline{\mathbf{h}}_1'$ is maximized by applying the above rotation angle θ_o, and this results in

$$\left\|\overline{\mathbf{h}}_1'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} + \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_I^2}, \quad (20)$$

while minimizing the norm value of $\overline{\mathbf{h}}_2'$ as

$$\left\|\overline{\mathbf{h}}_2'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} - \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_I^2}. \quad (21)$$

Now we are ready to establish the *complex* orthogonality between $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ ($\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle=0$) by means of the above two rotation transformations in (13) and (18). The underlying process for this problem is equivalent to an orthogonalization of the columns $\mathbf{h}_1'$, $\mathbf{h}_2'$, $\dot{\mathbf{h}}_1'$, and $\dot{\mathbf{h}}_2'$. We note that the inner rotation operation in (12) forces the real part of Hermitian product $\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle_R$ to zero and does not affect the imaginary part $\left(\text{i.e., }\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle_I=\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_I\right)$. Similarly the outer rotation in (17) makes $\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle_I$ become zero and leaves the real part unchanged $\left(\text{i.e., }\left\langle\overline{\mathbf{h}}_1',\overline{\mathbf{h}}_2'\right\rangle_R=\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_R\right)$. This means that the inner orthogonality between columns established in (12) remains unaffected by subsequent outer operations of (17).

Considering this fact, we can arrive at *complex* orthogonal vectors $\overline{\mathbf{h}}_1'$ and $\overline{\mathbf{h}}_2'$ by means of the following transformations

$$\begin{aligned}[\overline{\mathbf{h}}_1'\ \overline{\mathbf{h}}_2'] &= [\overline{\mathbf{h}}_1\ \overline{\mathbf{h}}_2]\overline{\mathbf{I}}(\theta_I)\overline{\mathbf{O}}(\theta_c)\\ &= [\overline{\mathbf{h}}_1\ \overline{\mathbf{h}}_2]\overline{\mathbf{C}}(\theta_I,\theta_c)\end{aligned}$$

where θ_I is given by (14) and

$$\theta_c=\tan^{-1}\left(\frac{D-\sqrt{D^2+4C^2}}{2C}\right) \quad (22)$$

where $D=\sqrt{\left(\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2\right)^2+4\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_R^2}$. Here the *complex rotation matrix* $\overline{\mathbf{C}}(\theta_1,\theta_2)$ is given by

$$\overline{\mathbf{C}}(\theta_1,\theta_2)=\begin{bmatrix}\cos(\theta_1,\theta_2) & \sin(\theta_1,\theta_2)\\ -\sin(\theta_1,\theta_2)^* & \cos(\theta_1,\theta_2)^*\end{bmatrix} \quad (23)$$

where we define *complex cosine* $\cos(\theta_1,\theta_2)$ and *complex sine* $\sin(\theta_1,\theta_2)$ as

$$\cos(\theta_1,\theta_2)=\cos\theta_1\cos\theta_2+j\sin\theta_1\sin\theta_2$$

and

$$\sin(\theta_1,\theta_2)=\sin\theta_1\cos\theta_2+j\cos\theta_1\sin\theta_2.$$

We notice that the angle θ_c given by (22) is nothing but the rotation angle θ_O for the outer rotation operation to the channel matrix $[\overline{\mathbf{h}}_1'\ \overline{\mathbf{h}}_2']=[\overline{\mathbf{h}}_1\ \overline{\mathbf{h}}_2]\overline{\mathbf{I}}(\theta_I)$, where we have used the fact from (20) and (21) that $\left\|\overline{\mathbf{h}}_1'\right\|^2-\left\|\overline{\mathbf{h}}_2'\right\|^2=\sqrt{\left(\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2\right)^2+4\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle_R^2}$.

In this case, the norm of $\overline{\mathbf{h}}_1'$ is maximized as

$$\left\|\overline{\mathbf{h}}_1'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} + \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left|\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle\right|^2}, \quad (24)$$

while the norm of $\overline{\mathbf{h}}_2'$ is minimized as

$$\left\|\overline{\mathbf{h}}_2'\right\|^2 = \frac{\|\overline{\mathbf{h}}_1\|^2+\|\overline{\mathbf{h}}_2\|^2}{2} - \sqrt{\left(\frac{\|\overline{\mathbf{h}}_1\|^2-\|\overline{\mathbf{h}}_2\|^2}{2}\right)^2+\left|\left\langle\overline{\mathbf{h}}_1,\overline{\mathbf{h}}_2\right\rangle\right|^2}.$$

Comparing (24) with (15) and (20), we can conclude that the complex orthogonalization results in larger $\left|\left|\overline{\mathbf{h}}_1'\right|\right|^2$ than the inner and outer orthogonalization as $\left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle\right|^2$ is always greater than $\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_R^2$ and $\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_I^2$.

In the following section, we will utilize the connection of the vector orthogonalization with the vector-norm maximization to describe our new beamforming technique. In particular, we will see that the orthogonalization transformations in Equations (13), (18), and (23) provide solutions to maximum-norm combining of two complex-valued vectors.

IV. New Transmit Beamforming Method

In this section, we present a new beamforming method which can be realized with low complexity while maintaining the performance comparable to the optimal beamforming. We exploit the aspect of the norm maximization achieved by the orthogonalization transformations to specify the combining vector for two complex column vectors. The proposed beamforming consists of $M_t - 1$ successive column combining of the MIMO channel matrix based on MNC.

First we study an MNC technique which combines two vectors $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$ by the unit-norm weight vector $\overline{\mathbf{w}}$ of length 2 ($||\overline{\mathbf{w}}|| = 1$) in the form

$$\overline{\mathbf{h}}_1' = \left[\overline{\mathbf{h}}_1 \ \overline{\mathbf{h}}_2\right] \overline{\mathbf{w}}. \tag{25}$$

The goal is to choose $\overline{\mathbf{w}}$ which maximizes the output norm $||\overline{\mathbf{h}}_1'||$. As noted in the previous section, the optimal solution of maximizing $||\overline{\mathbf{h}}_1'||$ in the above equation is directly related to the orthogonalization problem in Equation (8). Specifically, the norm maximization of $\overline{\mathbf{h}}_1'$ can be achieved by employing the first column of $\overline{\mathbf{C}}(\theta_I, \theta_c)$ in Equation (23), denoted by

$$\overline{\mathbf{w}}_c = \begin{bmatrix} \cos(\theta_I, \theta_c) \\ -\sin(\theta_I, \theta_c)^* \end{bmatrix}. \tag{26}$$

We notice that the *complex weight vector* $\overline{\mathbf{w}}_c$ achieves the maximum value (24), which is identical to the larger singular value λ_{max} of the $M_r \times 2$ matrix $\left[\overline{\mathbf{h}}_1 \ \overline{\mathbf{h}}_2\right]$.

In a similar way, by taking the first column of $\overline{\mathbf{I}}(\theta_I)$ and $\overline{\mathbf{O}}(\theta_o)$, the *inner and outer weight vectors* are defined as, respectively,

$$\overline{\mathbf{w}}_I = \begin{bmatrix} \cos\theta_I \\ -\sin\theta_I \end{bmatrix} \text{ and } \overline{\mathbf{w}}_o = \begin{bmatrix} \cos\theta_o \\ j\sin\theta_o \end{bmatrix}. \tag{27}$$

The corresponding output norm values $||\overline{\mathbf{h}}_1'||$ are given by Equations (15) and (20), respectively.

From the equations (15), (20) and (24), we can see that compared to the complex weight vector, the inner or outer weight vector leads to a small decrease in the norm. Comparison of Equations (15) and (20) indicates that a larger norm value can be achieved by selecting one of two weight vectors $\overline{\mathbf{w}}_I$ and $\overline{\mathbf{w}}_o$ according to the following criterion: Choose the inner weight vector $\overline{\mathbf{w}}_I$ if $\left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_R\right| \geq \left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_I\right|$, otherwise choose the outer weight vector $\overline{\mathbf{w}}_o$.

Figure 2 depicts the cumulative distribution functions (CDFs) of the squared norm value achieved by the MNC process in (25) for 2-by-1 vectors $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$. Here, we assume that the elements of $\overline{\mathbf{h}}_1$ and $\overline{\mathbf{h}}_2$ are drawn from an i.i.d. complex Gaussian distribution with zero mean and unit variance. It is demonstrated that the squared norm value achieved by the complex weight vector $\overline{\mathbf{w}}_c$ is equal to the square of the maximum singular value λ_{max} of the matrix $\left[\overline{\mathbf{h}}_1 \ \overline{\mathbf{h}}_2\right]$. More importantly, Figure 2 shows that almost the same norm value as λ_{max}^2 is obtained by choosing one of inner and outer weight vector based on the above selection criterion. This suggests that we may not need the complex weight vector $\overline{\mathbf{w}}_c$ to maximize the output norm for small M_t.

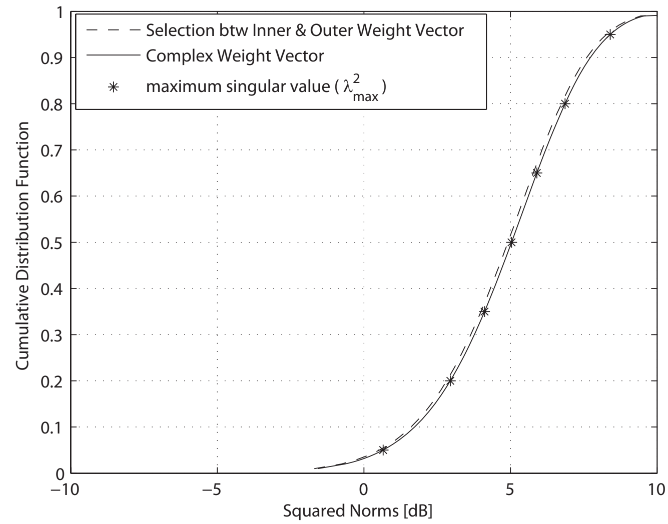

Fig. 2. Cumulative distribution function of the squared vector norm.

We consider two different types of the MNC weight vector $\overline{\mathbf{w}}_{\text{mnc}}$: TYPE 1 and TYPE 2. For TYPE 1, we employ the inner weight vector $\overline{\mathbf{w}}_{\text{mnc}} = \overline{\mathbf{w}}_I$ if $\left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_R\right| \geq \left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_I\right|$, otherwise choose the outer weight vector $\overline{\mathbf{w}}_{\text{mnc}} = \overline{\mathbf{w}}_o$. In contrast, for TYPE 2, we apply the complex weight vector. To summarize, the MNC process provides the weight vector $\overline{\mathbf{w}}_{\text{mnc}}$ as follows:

$$\overline{\mathbf{w}}_{\text{mnc}} = \begin{cases} \left.\begin{matrix}\overline{\mathbf{w}}_{\text{I}} & \text{if} \ \left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_R\right| \geq \left|\left\langle\overline{\mathbf{h}}_1, \overline{\mathbf{h}}_2\right\rangle_I\right| \\ \overline{\mathbf{w}}_{\text{o}} & \text{otherwise}\end{matrix}\right. & \text{for TYPE 1} \\ \overline{\mathbf{w}}_c & \text{for TYPE 2} \end{cases}. \tag{28}$$

In what follows, on the basis of the MNC process above, we propose a beamforming strategy on how to construct $\overline{\mathbf{t}}$. Notice that in the case of $M_t = 2$ the beamforming vector $\overline{\mathbf{t}}$ becomes the MNC weight vector $\overline{\mathbf{w}}_{\text{mnc}}$, i.e., $\overline{\mathbf{t}} = \overline{\mathbf{w}}_{\text{mnc}}$. After the beamforming vector $\overline{\mathbf{t}}$ is determined, the receive vector $\overline{\mathbf{r}}$ is simply obtained from (2). In this case, the achievable channel gain is equal to the norm of the 2×1 vector $\overline{\mathbf{h}}_1' = \left[\overline{\mathbf{h}}_1 \ \overline{\mathbf{h}}_2\right] \overline{\mathbf{w}}_{\text{mnc}}$ since the channel gain achieved by the MRC receiver is given as $|\overline{\mathbf{r}}^\dagger \, \overline{\mathbf{H}} \, \overline{\mathbf{t}}| = ||\overline{\mathbf{H}} \, \overline{\mathbf{t}}|| = ||\overline{\mathbf{h}}_1'||$ from Equation (3). It is worthy of note that this approach on the beamforming problem for $M_t = 2$ is optimal in terms of the output SNR since the achievable channel gain is equal to λ_{max} of the MIMO channel matrix when the MNC weight vector $\overline{\mathbf{w}}_{\text{mnc}}$ is TYPE 2 (See Figure 2).

We now present a general way of constructing the transmit beamforming vector $\overline{\mathbf{t}}$ for $M_t \geq 2$. The whole algorithm for determining $\overline{\mathbf{t}}$ is carried out by applying $M_t - 1$ successive MNC processes between column vectors of the MIMO channel matrix $\overline{\mathbf{H}}$. In particular, each MNC is applied to combine the first column vector $\overline{\mathbf{h}}_1$ and the rest columns $\overline{\mathbf{h}}_k$ for $k = 2, \cdots, M_t$ so that the resulting vector norm becomes maximized, where $\overline{\mathbf{h}}_k$ denotes the k-th column of $\overline{\mathbf{H}}$.

The algorithm starts with an initialization process: $\overline{\mathbf{h}}_1^{'} = \overline{\mathbf{h}}_1$ (see the algorithm below). For the first step ($k = 2$), we perform the MNC on the pair of $\overline{\mathbf{h}}_1^{'}$ and $\overline{\mathbf{h}}_2$. The 2×1 weight vector $\overline{\mathbf{w}}_{\text{mnc}}^2$ is chosen by the MNC process as one of the three weight vectors in (26) and (27) using $\overline{\mathbf{h}}_1^{'}$ and $\overline{\mathbf{h}}_2$ according to the defined TYPE. After $\overline{\mathbf{h}}_1^{'}$ is updated as $\overline{\mathbf{h}}_1^{'} = \left[\overline{\mathbf{h}}_1^{'}\ \overline{\mathbf{h}}_2\right] \overline{\mathbf{w}}_{\text{mnc}}^2$, subsequent iterations proceed. At the following steps $k = 3, \cdots, M_t$, in order to perform the MNC on the pair of $\overline{\mathbf{h}}_1^{'}$ and $\overline{\mathbf{h}}_k$, we determine the corresponding $\overline{\mathbf{w}}_{\text{mnc}}^k$ at the same way. At each step, we update $\overline{\mathbf{h}}_1^{'}$ as $\overline{\mathbf{h}}_1^{'} \Leftarrow \left[\overline{\mathbf{h}}_1^{'}\ \overline{\mathbf{h}}_k\right] \overline{\mathbf{w}}_{\text{mnc}}^k$. This is described compactly as follows:

```
1: h'_1 = h_1
2: for  k = 2 : M_t
3:    w^k_mnc is obtained in (28) for two vectors h'_1 and h_k
4:    h'_1 ⇐ [h'_1 h_k] w^k_mnc
5: end
```

As the iteration proceeds, the norm of the $M_r \times 1$ vector $\overline{\mathbf{h}}_1^{'}$ approaches λ_{max} of $\overline{\mathbf{H}}$. The resulting norm $||\overline{\mathbf{h}}_1^{'}||$ corresponds to the effective channel gain achieved by the proposed scheme. It is easy to show that $\overline{\mathbf{h}}_1^{'}$ is related to $\overline{\mathbf{H}}$ as $\overline{\mathbf{h}}_1^{'} = \overline{\mathbf{H}}\,\overline{\mathbf{F}}_2\overline{\mathbf{F}}_3 \cdots \overline{\mathbf{F}}_{M_t}$ with

$$\overline{\mathbf{F}}_k = \left[\begin{array}{cc} \overline{\mathbf{w}}_{\text{mnc}}^k & 0_{2\times(M_t-k)} \\ 0_{(M_t-k)\times 1} & \mathbf{I}_{M_t-k} \end{array} \right], \tag{29}$$

where $0_{m\times n}$ denotes an $m \times n$ zero matrix or vector. Here the $(M_t - k + 2) \times (M_t - k + 1)$ transformation matrix $\overline{\mathbf{F}}_k$ combines the first two columns of the matrix $\overline{\mathbf{H}} \prod_{j=2}^{k-1} \overline{\mathbf{F}}_j$ by the weight vector $\overline{\mathbf{w}}_{\text{mnc}}^k$ and leaves the rest unchanged, resulting in the $M_r \times (M_t - k + 1)$ matrix $\overline{\mathbf{H}} \prod_{j=2}^{k} \overline{\mathbf{F}}_j$. As a result, the transmit beamforming vector is given as the $M_t \times 1$ vector $\overline{\mathbf{t}} = \prod_{j=2}^{M_t} \overline{\mathbf{F}}_j$.

We note again that the proposed MNC scheme generates optimal solutions only for $M_t = 2$. For the case of $M_t > 2$, the above procedure can be considered as the first iteration of the optimal MNC algorithm. In other words, in order to obtain the optimum beamforming vector, we may further repeat the above algorithm with increased computational complexity [16][18]. However, we do not consider subsequent iterations in this paper since simulation results show that the performance improvement is marginal in spite of the increase in complexity.

We will briefly address the computational complexity issue by evaluating the number of floating point multiplications required for the proposed method based on the MNC. The overall computational cost in the proposed scheme can be assessed as follows:

1) Determining the weight vector $\overline{\mathbf{w}}_{\text{mnc}}^k$ in line 3 of the above algorithm requires $(8M_r+3)+(6M_r+3)(M_t-2)$ and $(8M_r + 10) + (6M_r + 10)(M_t - 2)$ real multiplications for TYPE 1 and TYPE 2, respectively. Note that from Equations (14), (19), and (22), the weight vector $\overline{\mathbf{w}}_{\text{mnc}}^k \in \{\overline{\mathbf{w}}_c, \overline{\mathbf{w}}_I, \overline{\mathbf{w}}_o\}$ can be generated through simple computations without any intermediate calculation of the optimal angles θ_I, θ_o and θ_c as follows: $\cos\theta_I = \frac{1}{\sqrt{t_I^2+1}}$ and $\sin\theta_I = t_I \cos\theta_I$ with $t_I = \frac{A}{2B} + \text{sign}(B)\sqrt{(\frac{A}{2B})^2 + 1}$, $\cos\theta_o = \frac{1}{\sqrt{t_o^2+1}}$ and $\sin\theta_o = t_o \cos\theta_o$ with $t_o = \frac{A}{2C} + \text{sign}(C)\sqrt{(\frac{A}{2C})^2 + 1}$, $\cos\theta_c = \frac{1}{\sqrt{t_c^2+1}}$ and $\sin\theta_c = t_c \cos\theta_c$ with $t_c = \frac{A'}{2B} + \text{sign}(B)\sqrt{(\frac{A'}{2B})^2 + 1}$, where $A = ||\overline{\mathbf{h}}_1^{'}||^2 - ||\overline{\mathbf{h}}_k||^2$, $B = \left\langle \overline{\mathbf{h}}_1^{'}, \overline{\mathbf{h}}_k \right\rangle_R$, $C = \left\langle \overline{\mathbf{h}}_1^{'}, \overline{\mathbf{h}}_k \right\rangle_I$, $A' = \sqrt{A^2 + 4B^2}$. Here, sign$(\cdot)$ denotes the signum function. Therefore, the main computational cost is in the evaluation of $||\overline{\mathbf{h}}_k||$ and $\left\langle \overline{\mathbf{h}}_1^{'}, \overline{\mathbf{h}}_k \right\rangle$ since the norm values of $||\overline{\mathbf{h}}_1^{'}||$ for $k \geq 3$ is easily obtained from Equations (15), (20) and (24) during the preceding step.
2) The update $\overline{\mathbf{h}}_1^{'} \Leftarrow \left[\overline{\mathbf{h}}_1^{'}\ \overline{\mathbf{h}}_k\right] \overline{\mathbf{w}}_{\text{mnc}}^k$ in line 4 requires $4M_r(M_t - 1)$ and $8M_r(M_t - 1)$ real multiplications, respectively, for TYPE 1 and TYPE 2.
3) When determining the beamforming vector $\overline{\mathbf{t}} = \prod_{j=2}^{M_t} \overline{\mathbf{F}}_j$, we need to perform $2(M_t-2)$ and $8(M_t-2)$ real multiplications, respectively, for TYPE 1 and TYPE 2. Note that each transformation $\overline{\mathbf{F}}_k$ involves only two non-zero elements considered in the matrix product $\prod_{j=2}^{M_t} \overline{\mathbf{F}}_j$, as can be seen in Equation (29).

We assume that for both the proposed scheme and the iterative power method, the square root operation is performed by a lookup table. In the simulation section, we will compare the computational complexity with the beamforming scheme based on the power method and show that our proposed method achieves a significant reduction in the complexity to find the transmit weight vector.

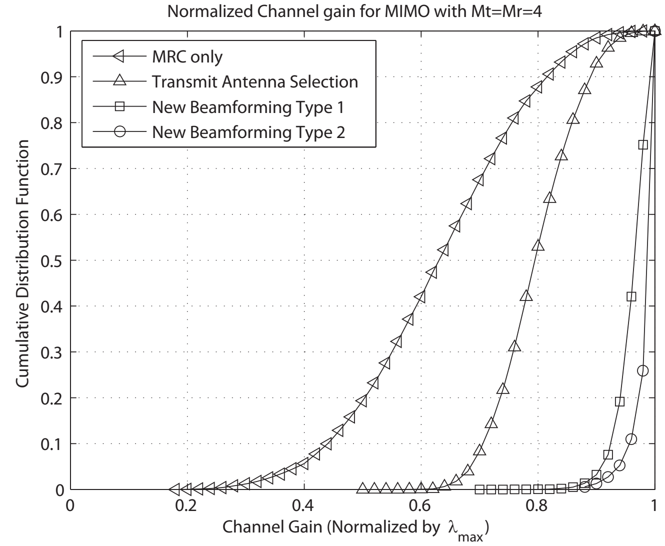

Fig. 3. Cumulative distribution function of the normalized channel gain for different beamforming schemes.

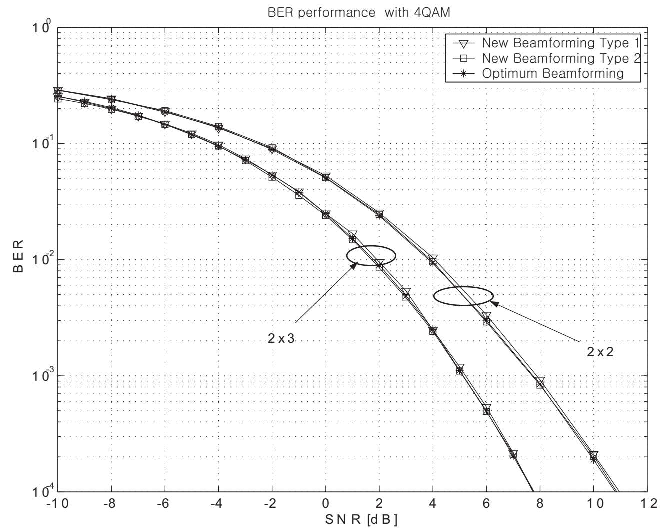

Fig. 4. Bit-error-rate performance comparison with two transmit and two/three receive antennas.

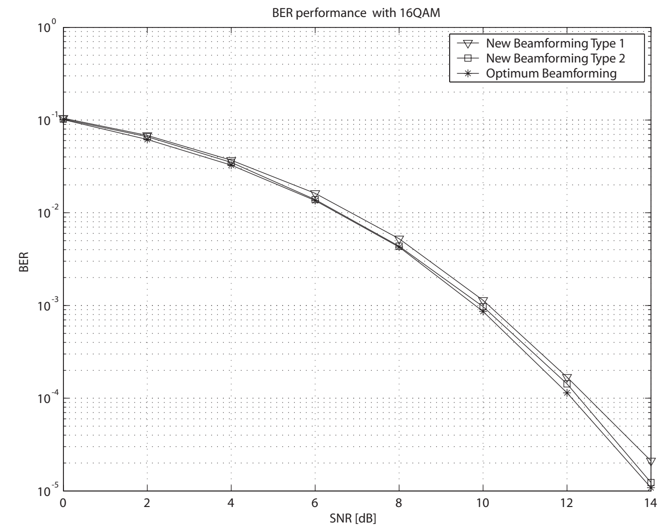

Fig. 5. Bit-error-rate performance comparison with three transmit and three receive antennas.

Figure 3 depicts the CDF of the relative channel gain of the proposed beamforming methods normalized by the maximum channel gain λ_{max} for 4-by-4 MIMO channels. For comparison purposes, we consider the performance of the 1-by-4 SIMO system with an MRC receiver. Also a simple transmit antenna selection scheme is compared where the antenna is selected to provide the highest effective channel gain [21]. In Figure 3, we can see that the proposed scheme is able to achieve more than 90% of the theoretical maximum channel gain. It can also be noticed that a significant gain is obtained compared with the MRC only and the simple transmit antenna selection schemes.

V. Simulation Results

In this section, we present the performance of the proposed beamforming method, the beamforming scheme based on the iterative power method, and the optimal beamforming scheme in flat fading channels. We assume that the transmitter has perfect knowledge of the MIMO channel. Note that the optimal beamforming vector is obtained from the dominant right singular vector of $\overline{\mathbf{H}}$. In this case, the effective channel gain is equal to the largest singular value λ_{max} of $\overline{\mathbf{H}}$, as shown in Section II.

In the first simulation, we verify that the proposed beamforming is optimal in the case of two transmit antennas regardless of M_r in terms of bit-error rate (BER) performance. In Figure 4, we present the BER performance with respect to SNR in dB with M_t = 2 and 4QAM. This figure shows that the proposed TYPE 2 beamforming provides the same performance as the optimal beamforming. More importantly, with much simpler TYPE 1 beamforming, the new beamforming provides the performance almost identical to the optimum beamforming.

Next, Figures 5 and 6 display the BER performance for MIMO systems with $M_t = 3$ and $M_t = 4$, respectively, using 16QAM. Comparing with Figure 4, Figures 5 and 6 show that there exists a small performance difference between TYPE 2 and the optimum case for $M_t > 2$. Nevertheless, for all combinations of transmit and receive antennas, the proposed TYPE 1 and TYPE 2 beamforming methods still perform within 0.3 dB of the optimum beamforming at a BER of 10^{-4} with reduced complexity.

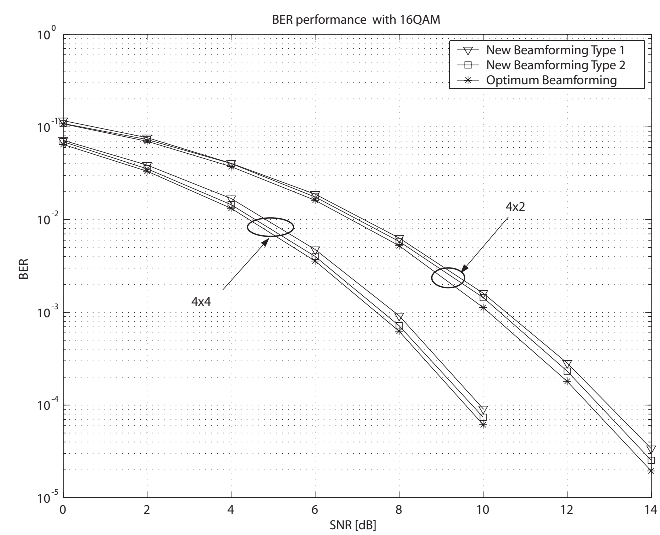

Fig. 6. Bit-error-rate performance comparison with four transmit and two/four receive antennas.

Finally, we make a complexity comparison between the proposed method and the iterative power method. We assume that for both the proposed scheme and the iterative power method, the square root operation is performed by a lookup table. For a fair comparison, we first confirm that the proposed beamforming methods with TYPE 1 and TYPE 2 outperform the beamforming scheme based on the power method with 2 and 3 iterations, respectively, as shown in Figure 7, where the BER curves versus the number of transmit and receive antennas (M_t = M_r) are shown for 16QAM at a SNR of 10dB. To verify the complexity saving of the proposed scheme, Figure 8 presents the number of real multiplications

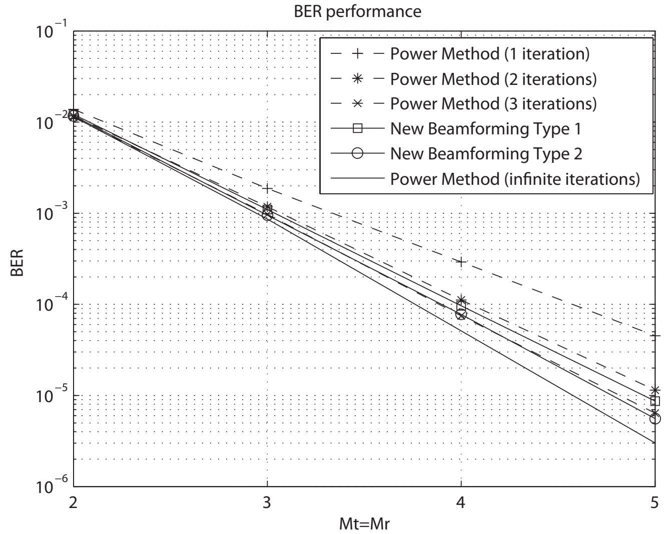

Fig. 7. BER performance of the proposed method and the iterative power method.

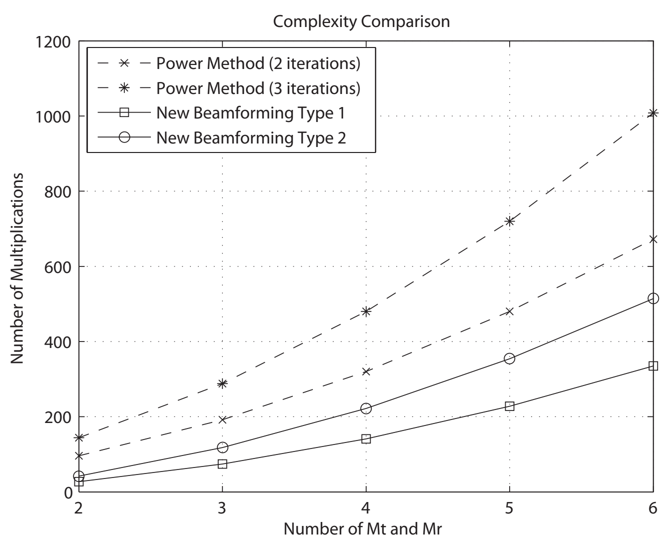

Fig. 8. Complexity comparison between the proposed method and the iterative power method.

required for the beamforming schemes. We can see that the proposed method shows a complexity saving of up to 70% for the same performance in comparison to the power method.

VI. Conclusion

In this paper, we propose a new beamforming method based on maximum-norm combining of columns for MIMO wireless channels. A new form of complex matrices has been derived as a general expression of rotation transformations for the complex vector orthogonalization. As a result, three different weight vectors have been proposed for maximum-norm combining between two complex vectors. We have shown that the channel gain comparable to the maximum singular value of the channel matrix can be achieved by employing successive maximum-norm combining to the MIMO channel matrix. The simulation results confirm that the proposed beamforming method is quite effective in approaching the performance of the optimum beamforming with much reduced complexity.

The proposed method can be applied to spatial multiplexing MIMO systems based on a minimum Euclidean distance criterion [22].

References

[1] G. J. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311–335, Mar. 1998.
[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.
[3] D. Gesbert and *et al.*, "From theory to practice: an overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 281–302, Apr. 2003.
[4] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antennas channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1073–1096, May 2003.
[5] S. M. Alamouti, "A simple transmit diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, pp. 1451–1458, Oct. 1998.
[6] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. IT-44, pp. 744–765, Mar. 1998.
[7] H. Lee, J. Cho, J.-K. Kim, and I. Lee, "An efficient decoding algorithm for STBC with multidimensional rotated constellations," in *Proc. ICC '06*, June 2006.
[8] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.
[9] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inform. Theory*, vol. 43, pp. 1986–1992, Nov. 1997.
[10] A. J. Goldsmith, S. A. Jafar, N. J. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 684–702, June 2003.
[11] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, pp. 1458–1461, Oct. 1999.
[12] X. Feng and C. Leung, "A new optimal transmit and receive diversity scheme," in *Proc. IEEE PACRIM '01*, vol. 2, pp. 538–541, Aug. 2001.
[13] P. A. Dighe, R. K. Malik, and S. S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," *IEEE Trans. Commun.*, vol. 51, pp. 694–703, Apr. 2003.
[14] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. The Johns Hopkins University Press, 1996.
[15] M. R. Hestenes, "Inversion of matrices by biorthogonalization and related results," *J. Society for Industrial and Applied Mathematics*, vol. 6, no. 1, pp. 51–90, Mar. 1958.
[16] J. C. Nash, "A one-sided transformation method for the singular value decomposition and algebraic eigenproblem," *The Computer J.*, vol. 18, no. 1, pp. 74–76, Feb. 1975.
[17] S. Shlien, "A method for computing the partial singular value decomposition," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 4, pp. 671–676, Nov. 1982.
[18] H. Lee, S.-H. Park, and I. Lee, "A new MIMO beamforming technique based on rotation transformation," in *Proc. IEEE ICC '07*, June 2007.
[19] Z. Drmac, V. Hari, and I. Slapnicar, "Advances in Jacobi methods," in *Proc. Third Conference on Applied Mathematics and Scientific Computing*, pp. 63–90, June 2001.
[20] A. M. Chan and I. Lee, "A new reduced-complexity sphere decoder for multiple antenna systems," in *Proc. IEEE ICC '02*, vol. 1, pp. 460–464, Apr. 2002.
[21] S. Sanayei and A. Nosratinia, "Antenna selection in MIMO systems," *IEEE Commun. Mag.*, pp. 68–73, Oct. 2004.
[22] H. Lee, S. Park, and I. Lee, "A new two-step precoding strategy for closed-loop MIMO systems," *IEEE Trans. Commun.*, to be published.

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