A New Beamforming Structure Based on Transmit-MRC for Closed-Loop MIMO Systems

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***Abstract*—This paper proposes an efficient beamforming scheme which attains optimality as singular value decomposition (SVD) based systems with low complexity utilizing transmit maximum-ratio combining (TMRC) techniques. The TMRC scheme is the optimum structure for single beamforming systems in terms of received signal-to-noise ratio (SNR) in multiple-input single-output (MISO) channels. In this paper, we generalize the TMRC scheme to multiple beamforming multiple-input multiple-output (MIMO) systems which support more than one data stream in coded systems. We express each beamforming vector as a linear combination of TMRC vectors whose coefficients are optimized in a successive manner. Optimization of the beamforming vector is followed by the decorrelation process. All TMRC vectors used as a basis of the remaining beamforming vectors are made orthogonal to previously computed beamforming vectors. Exploiting the concept of the gradient ascent algorithm, we propose a simple non-iterative method of computing the precoder which obtains the near optimal performance. Also we derive a closed form expression of the output SNR distribution for the proposed scheme. Simulation results demonstrate that the proposed scheme achieves the almost identical link performance as the SVD-based system for arbitrary configurations with reduced complexity.**

***Index Terms*—Closed-loop multiple-input multiple-output (MIMO) systems, transmit beamforming.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) wireless systems have widely been studied to increase the communication reliability and spectral efficiency. The capacity analysis of MIMO systems has shown significant gains over single-input single-output (SISO) systems [1][2]. The expected benefits include spatial multiplexing (SM) gain and diversity gain. The tradeoff between the SM gain and the diversity gain has been characterized in [3].

In [4] and [5], it is suggested that an additional performance gain can be extracted from multiple antennas in the presence of channel state information (CSI) at the transmitter. Most work on closed-loop MIMO systems has been carried out by performing singular value decomposition (SVD) of the channel transfer matrix. Among linear precoders, the SVD-based beamforming technique combined with a proper power allocation method is shown to be optimum in terms of capacity [6][7][8]. Besides, recent results in [9] have shown that the SVD-based transmission combined with bit interleaved coded modulation (BICM) [10] can achieve both full SM and full spatial diversity gain for any antenna configurations. However, the computational complexity of such SVD based schemes becomes problematic with the increased number of transmit and receive antennas, due to the iterative nature of the SVD computation [11][12].

The goal of this paper is to design a simple transmission scheme which achieves the advantage of the SVD-based scheme with reduced complexity. We focus on coded systems since MIMO techniques combined with powerful channel coding can provide an effective means to counteract severe impairments of fading channels. We will start with a review of transmit maximal ratio combining (TMRC) introduced in [13], [14] and [15], which is the optimal solution for single beamforming[1] multiple-input single-output (MISO) systems. Although the TMRC scheme provides quite a good link performance gain with low complexity, the number of data streams is restricted to one, since the TMRC can be applied only to systems with single receive antenna.

In this paper, we propose a new beamforming scheme which can transmit multiple data streams simultaneously in MIMO channels by generalizing the TMRC technique. We first represent beamforming vectors as a linear combination of TMRC vectors and then the coefficients of linear combination are optimized successively. Adopting the concept of the gradient ascent algorithm, the vector coefficients are optimized in a non-iterative way to achieve the near-optimal performance. Motivated by the fact that the beamforming vectors of the SVD-based system are fully orthogonal to each other, we orthogonalize the basis TMRC vectors to previously computed beamforming vectors. As we shall see later, the proposed one-shot approach provides the performance almost identical to the SVD-based beamforming scheme with reduced complexity. Also, a closed form expression of the output signal-to-noise ratio (SNR) distribution for the proposed scheme is derived. Our simulation results show that the proposed scheme achieves complexity savings of up to 90% compared to the conventional SVD scheme with a slight performance loss.

For clarity, the following notations are used for description throughout this paper. Normal letters represent scalar quanti-

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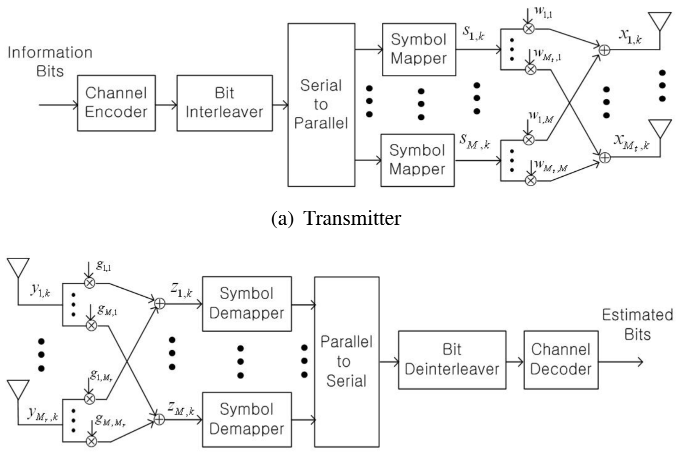
[1] A linear precoding system which transmits one data stream at a time is called single beamforming, while linear precoders which support more than one data symbols are referred to as multiple beamforming.



(a) Transmitter

(b) Receiver

Fig. 1. Schematic diagram of coded linear precoding systems with M_t transmit and M_r receive antennas.

ties, boldface letters indicate vectors and boldface uppercase letters designate matrices. The superscripts $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ represent the complex conjugate, the transpose and the Hermitian transpose, respectively.

The organization of the paper is as follows: In Section II, we present a system description and review the SVD-based beamforming system. In Section III, we introduce a new beamforming scheme established by generalizing the TMRC. Section IV presents the performance analysis of the proposed precoding scheme. In Section V, simulation results are shown comparing the proposed method with the conventional SVD scheme and other alternative schemes. Finally, the paper is terminated with conclusions in Section VI.

II. System Description

In this section, we present a general description of coded linear precoding systems combined with BICM, as shown in Figure 1. We consider a MIMO link with M_t transmit and M_r receive antennas where M independent data streams are transmitted simultaneously $(M \leq \min(M_t, M_r))$.

At the transmitter, the information bits are encoded with a code rate R_c, bit-interleaved and symbol-mapped to yield the M-dimensional symbol vector $\mathbf{s}_k = [s_{1,k}, \cdots, s_{M,k}]^T$, where the subscript k indicates the k-th time slot. The precoding matrix $\mathbf{W}$ of size M_t-by-M receives the symbol vector $\mathbf{s}_k$ and generates the linearly precoded signal vector $\mathbf{x}_k$ of length M_t as

$$\mathbf{x}_k = \mathbf{W}\mathbf{s}_k = \sum_{m=1}^{M} \mathbf{w}_m s_{m,k} \tag{1}$$

where $\mathbf{w}_m$ denotes the m-th column vector of $\mathbf{W}$. In Figure 1, $w_{i,j}$ and $g_{i,j}$ indicate the (i,j)-th element of $\mathbf{W}$ and $\mathbf{G}$, respectively. In this paper, we concentrate on column-orthonormal precoders[2] as $\mathbf{W}^\dagger\mathbf{W} = \mathbf{I}_M$ where $\mathbf{I}_d$ indicates an identity matrix of size d. The resulting precoded symbol vector $\mathbf{x}_k$ is then transmitted through M_t transmit antennas. Throughout the paper, we assume the flat fading channel model where the fading coefficients are static over a frame of transmitted symbols and independent over frames.

Then, the M_r-dimensional received signal vector can be written as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k$$

where $\mathbf{n}_k$ is the complex Gaussian noise vector with the covariance matrix $\sigma_n^2\mathbf{I}_{M_r}$. Also, the M_r-by-M_t channel matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_t} \\ h_{21} & h_{22} & \cdots & h_{2M_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_r1} & h_{M_r2} & \cdots & h_{M_rM_t} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \vdots \\ \mathbf{h}_{M_r}^T \end{bmatrix}$$

where h_{ij} represents the channel response between the j-th transmit antenna and the i-th receive antenna and $\mathbf{h}_i^T$ denotes the i-th row vector of $\mathbf{H}$. The elements of $\mathbf{H}$ are obtained from an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero-mean and unit variance and assumed to be perfectly known at the receiver. Without loss of generality, we assume that

$$||\mathbf{h}_1||^2 \geq ||\mathbf{h}_2||^2 \geq \cdots \geq ||\mathbf{h}_{M_r}||^2 \tag{2}$$

where $||\cdot||$ denotes the Euclidean norm.

At the receiver, the received signal vector $\mathbf{y}_k$ is post-filtered by the receive matrix $\mathbf{G}$ of size M-by-M_r as

$$\mathbf{z}_k = \mathbf{G}\mathbf{y}_k = \mathbf{G}\mathbf{H}\mathbf{W}\mathbf{s}_k + \tilde{\mathbf{n}}_k \tag{3}$$

where $\tilde{\mathbf{n}}_k = \mathbf{G}\mathbf{n}_k$ is the filter output noise. The output vector $\mathbf{z}_k = [z_{1,k}, \cdots, z_{M,k}]^T$ is converted to soft log-likelihood ratio values in the soft demapper to be processed in the decoder.

In what follows, we briefly review the SVD-based beamforming structure. Defining $\mathcal{U}(m,n)$ as a set of m-by-n matrices with orthonormal columns, the SVD of $\mathbf{H}$ is given by $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$ where $\mathbf{\Sigma}$ denotes an M_r-by-M_t diagonal matrix whose (i,i)-th element is the i-th largest singular value of $\mathbf{H}$ denoted by $\lambda_i(\mathbf{H})$, and $\mathbf{U}$ and $\mathbf{V}$ are represented as $\mathbf{U} \in \mathcal{U}(M_r, M_r)$ and $\mathbf{V} \in \mathcal{U}(M_t, M_t)$, respectively. Here, we denote $\mathbf{v}_i$ and $\mathbf{u}_i$ as the i-th column of $\mathbf{V}$ and $\mathbf{U}$. The optimum linear precoder in terms of capacity is given by $\mathbf{W} = \mathbf{V}_M\mathbf{\Phi}$ where $\mathbf{V}_M$ is a matrix constructed from the first M columns of $\mathbf{V}$ and $\mathbf{\Phi}$ is an M-by-M diagonal matrix with diagonal terms derived in [6]. We refer to the precoder $\mathbf{W} = \mathbf{V}_M$ as *SVD-based Beamforming* (SVD-BF). The SVD-BF technique combined with adaptive bit-loading was evaluated via throughput simulations for adaptive MIMO systems in [17] and [18]. In the following section, we present a new beamforming scheme which achieves the advantage of the SVD-BF system with reduced complexity.

III. New Beamforming Schemes

In this section, we present a new beamforming scheme, which is called generalized TMRC (G-TMRC), achieving the

[2]Note that the optimal linear precoder can be obtained by proper power allocation onto the precoder optimized within the set of column-orthonormal matrices.

performance almost identical to the SVD-BF system. We first start with the review of the TMRC scheme. For the MISO case ($M_r = 1$), the channel matrix reduces to the vector channel $\mathbf{H} = \mathbf{h}_1^T$, and thus the linear precoder can support only one data stream due to the rank-deficient MISO channel matrix. In this case, the optimal SVD-BF solution simply becomes $\mathbf{w}_1 = \rho \mathbf{h}_1^*$ where a constant ρ is chosen such that the beamforming vector $\mathbf{w}_1$ has a unit-norm. This solution is known as the TMRC technique [13][14][15]. A limitation of the TMRC is that it is only applicable to single beamforming in MISO systems ($M_r = M = 1$). To overcome this problem, we first extend this TMRC technique to systems with general configurations ($M_r \geq 2$) in this section. Then, we will further extend the proposed scheme to multiple beamforming systems ($M \geq 2$).

A. Single Beamforming (M=1)

In this section, the TMRC scheme is extended to single beamforming systems with multiple receive antennas. Denoting $\mathbf{t}_i$ as the TMRC vectors ($i = 1, \cdots, M_r$), we set the beamforming vector $\mathbf{w}_1$ in (1) as the linear combination of TMRC vectors as

$$\mathbf{w}_1 = \sum_{i=1}^{M_r} c_i \mathbf{t}_i = \mathbf{Tc}$$

where the TMRC matrix $\mathbf{T}$ and the weighting vector $\mathbf{c}$ are defined as

$$\mathbf{T} = [\mathbf{t}_1 \quad \cdots \quad \mathbf{t}_{M_r}] \quad \text{and} \quad \mathbf{c} = [c_1 \quad \cdots \quad c_{M_r}]^T.$$

In the single beamforming case, $\mathbf{t}_i$ is set to $\mathbf{h}_i^*$ which results in $\mathbf{T} = \mathbf{H}^\dagger$.

Assuming that the MRC is applied at the receiver, the output SNR of the MRC combiner becomes

$$SNR = \frac{E_s ||\mathbf{H}\mathbf{w}_1||^2}{\sigma_n^2}$$

where E_s denotes the average constellation energy. Thus, maximizing the output SNR is equivalent to the maximization of $||\mathbf{H}\mathbf{w}_1||^2 = ||\mathbf{HTc}||^2$. Note that $|\mathbf{h}_j^T \mathbf{w}_1|^2$ is maximized when $\mathbf{w}_1 = \mathbf{h}_j^* / ||\mathbf{h}_j||$. Our problem can be formulated as follows:

$$\max_{\mathbf{c}} \; F(\mathbf{c}) = ||\mathbf{HTc}||^2 \qquad (4)$$
$$\text{subject to } \; ||\mathbf{Tc}||^2 = 1.$$

This constrained optimization problem is convex. However, it is complicated to obtain the closed-form solution when $M_r \geq 3$.

One of the simplest suboptimum solution is to choose $\mathbf{w}_1$ as

$$\mathbf{w}_1 = \frac{\mathbf{t}_1}{||\mathbf{t}_1||} = \frac{\mathbf{h}_1^*}{||\mathbf{h}_1||} \qquad (5)$$

which is equivalent to $\mathbf{c} = \mathbf{c}_{(0)} \triangleq \frac{\mathbf{1}_1}{||\mathbf{t}_1||}$. Here, $\mathbf{1}_i$ denotes an M_r-by-1 unit vector of 0's with one at the i-th position. This choice of (5) is based on the observation made in [19], where it is shown that employing the TMRC vector matched to only one row vector $\mathbf{h}_{\hat{m}}^T$ (out of $\mathbf{H}$) provides the near-optimal performance of the SVD-BF by properly selecting $\hat{m} \in \{1, \cdots, M_r\}$. In [19], $\hat{m}$ is selected according to the following criterion:

$$\hat{m} = \arg \max_{i \in \{1, \cdots, M_r\}} \frac{||\mathbf{H}\mathbf{h}_i^*||^2}{||\mathbf{h}_i||^2}. \qquad (6)$$

The selection metric in (6) is the MRC output SNR with the beamforming vector matched to the i-th row of $\mathbf{H}$. This choice of $\hat{m}$ based on the MRC output SNR is optimum in terms of the link performance. However, because of the assumption in (2), the choice of $\hat{m} = 1$ in (5) means that the prefixed beamforming vector $\mathbf{w}_1 = \frac{\mathbf{h}_1^*}{||\mathbf{h}_1||}$ is matched to the channel row vector with the largest energy. Note that this becomes the optimum selection of (6) only when $M_r = 2$. Nevertheless, in the case of $M_r \geq 3$, the performance loss of this energy-based selection of $\hat{m}$ is small as will be shown later. We call the proposed scheme with this simple prefixed coefficient as "G-TMRC Type-I". The analysis of this algorithm will be presented in Section IV.

The performance of the proposed G-TMRC Type-I is not optimum due to its simple choice of $\mathbf{c}$. We can further improve the proposed scheme by incorporating the idea of the gradient ascent algorithm [20]. The original gradient ascent method is an iterative process. However, we propose a method which does not require iterations. The gradient ascent algorithm is based on the observation that the objective function $F(\mathbf{c})$ in (4) increases fastest if one goes from $\mathbf{c}_{(0)}$ in the direction of $\nabla F(\mathbf{c}_{(0)})$, where $\nabla F(\mathbf{c}_{(0)})$ represents a gradient of $F(\mathbf{c})$ at $\mathbf{c} = \mathbf{c}_{(0)}$ as

$$\nabla F(\mathbf{c}_{(0)}) = \left[\nabla ||\mathbf{HTc}||^2\right]_{\mathbf{c}=\mathbf{c}_{(0)}} = 2\mathbf{T}^\dagger \mathbf{H}^\dagger \mathbf{HT}\mathbf{c}_{(0)}.$$

Then, it follows that $F(\mathbf{c}_{(1)}) \geq F(\mathbf{c}_{(0)})$ with

$$\mathbf{c}_{(1)} \triangleq \mathbf{c}_{(0)} + \gamma \nabla F(\mathbf{c}_{(0)})$$

for a small positive real number γ. In general, to achieve the near-optimal performance with $\mathbf{w}_1 = \mathbf{T}\mathbf{c}_{(1)}$, the optimal γ should be chosen judiciously [20][21]. In order to extract the advantage of the normalized least-mean-square (NLMS) algorithm [22], we model γ as

$$\gamma = \frac{\delta}{||\nabla F(\mathbf{c}_{(0)})||^2}$$

where δ is a positive real number. The proposed near-optimal method with the new coefficient $\mathbf{c} = \mathbf{c}_{(1)}$ is referred to as "G-TMRC Type-II". The whole algorithm of the proposed single beamforming system is summarized as follows:

Initialization:

$\mathbf{T} = \mathbf{H}^\dagger$

Main Body:

$$\mathbf{c} = \begin{cases} \frac{\mathbf{1}_1}{||\mathbf{t}_1||} & \text{(Type-I)} \\ \frac{\mathbf{1}_1}{||\mathbf{t}_1||} + \delta \frac{\mathbf{T}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{t}_1}{||\mathbf{T}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{t}_1||} & \text{(Type-II)} \end{cases}$$

$$\mathbf{w}_1 = \frac{\mathbf{Tc}}{||\mathbf{Tc}||}$$

Figure 2 compares the cumulative distribution functions (CDF) of the MRC output SNR for the G-TMRC and the SVD-BF for 2-by-2, 4-by-4 and 6-by-6 systems with $M = 1$, where the SNR of the G-TMRC Type-II is evaluated with $\delta = 150$. Although the continuous change of δ may provide

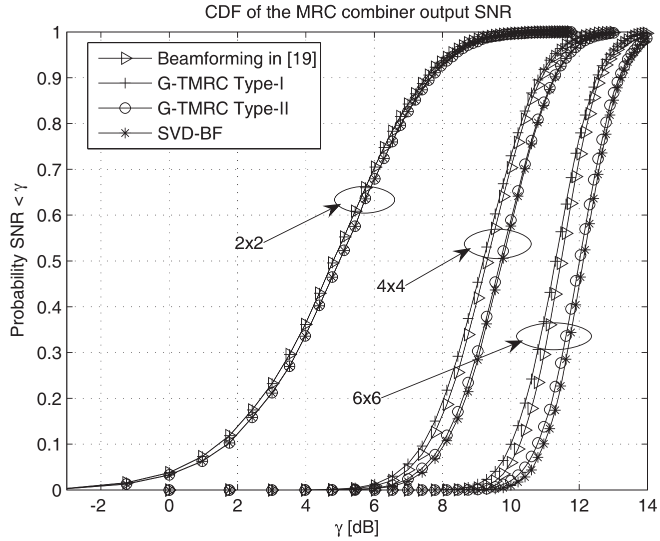


Fig. 2. CDF of SNR at the output of the MRC combiner.

an enhanced performance gain, we simply choose δ as a fixed value to minimize the processing complexity. It is clear from the plot that the performance of the G-TMRC Type-I is less than 1 dB away from the optimum SVD scheme with much reduced complexity. Also, the proposed G-TMRC Type-II provides the performance very close to the SVD-BF for all antenna configurations. A similar pattern can be obtained in the frame error rate (FER) analysis in Section V. Note that the proposed schemes are much simpler than the SVD-BF for these configurations. Also, we observe that the proposed G-TMRC Type-I shows a slight performance loss compared to the optimal selection criterion of (6).

B. Multiple Beamforming ($M > 1$)

From now on, we generalize the proposed G-TMRC schemes to the multiple beamforming system where more than one data stream is supported to increase the throughput. We now revisit the SVD-BF system. As mentioned earlier, the precoding matrix of the SVD-BF is set to $\mathbf{W} = \mathbf{V}_M = [\mathbf{v}_1 \quad \cdots \quad \mathbf{v}_M]$. Here, the right singular vectors $\mathbf{v}_1, \cdots, \mathbf{v}_M$ have the following property [23], which is a key basis for establishing our proposed G-TMRC schemes:

$$\mathbf{v}_n = \arg\max_{\mathbf{v} \in S_n} ||\mathbf{H}\mathbf{v}||^2 \quad (n = 1, \cdots, M) \tag{7}$$

where S_1 is a set of all unit vectors of length M_t and S_n ($n = 2, \cdots, M$) indicates the set of all unit vectors orthogonal to $\mathbf{v}_1, \cdots, \mathbf{v}_{n-1}$. This means that $\mathbf{v} = \mathbf{v}_1$ maximizes $||\mathbf{H}\mathbf{v}||^2$ among all M_t-dimensional unit vectors, while $\mathbf{v} = \mathbf{v}_n$ ($n = 2, \cdots, M$) achieves the maximum $||\mathbf{H}\mathbf{v}||^2$ within the vector space orthogonal to $\mathbf{v}_1, \cdots, \mathbf{v}_{n-1}$.

Observing the relations (7), we try to compute the beamforming vector $\mathbf{w}_m$ ($m = 1, \cdots, M$) for the m-th data stream without relying on the SVD operation. We express all beamforming vectors in (1) as a linear combination of the TMRC vectors, that is,

$$\mathbf{w}_m = \sum_{i=1}^{M_r} c_i^m \mathbf{t}_i^m = \mathbf{T}^m \mathbf{c}^m, \qquad m = 1, \cdots, M$$

where the superscript m is introduced to indicate multiple beamforming vectors. We start with the initialization $\mathbf{T}^1 = \mathbf{H}^\dagger$. The first beamforming vector $\mathbf{w}_1$ can be written as $\mathbf{w}_1 = \mathbf{T}^1\mathbf{c}^1$ where $\mathbf{c}^1$ is optimized using the single beamforming solution, that is,

$$\mathbf{c}^1 = \begin{cases} \frac{\mathbf{1}_1}{||\mathbf{t}_1^1||} & \text{(G-TMRC Type-I)} \\ \rho_1 \left(\frac{\mathbf{1}_1}{||\mathbf{t}_1^1||} + \delta \frac{\mathbf{T}^{1\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_1^1}{||\mathbf{T}^{1\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_1^1||} \right) & \text{(G-TMRC Type-II)} \end{cases}$$

where ρ_1 is chosen to satisfy $||\mathbf{w}_1||^2 = ||\mathbf{T}^1\mathbf{c}^1||^2 = 1$.

Recognizing the relation (7), we want to obtain $\mathbf{w}_2$ which is orthogonal to $\mathbf{w}_1$, and this can be achieved by applying orthogonal projection. To this end, we compute $\mathbf{T}^2 = \mathbf{T}^1 - \mathbf{w}_1\mathbf{w}_1^\dagger\mathbf{T}^1$, which is equivalent to

$$\mathbf{t}_i^2 = \mathbf{t}_i^1 - \mathbf{w}_1\mathbf{w}_1^\dagger\mathbf{t}_i^1, \quad i = 1, \cdots, M_r \tag{8}$$

where $\mathbf{t}_i^m$ denotes the i-th column of $\mathbf{T}^m$. As the second term in the right-hand side of (8) represents the orthogonal projection of $\mathbf{t}_i^1$ onto $\mathbf{w}_1$, it is obvious that $\mathbf{w}_1 \perp \mathbf{t}_i^2$ for $i = 1, \cdots, M_r$. This means that the second beamforming vector $\mathbf{w}_2$ is optimized in the subspace orthogonal to $\mathbf{w}_1$ since $\mathbf{w}_2$ is a linear combination of $\mathbf{t}_i^2$.

Now, for computing $\mathbf{w}_2 = \mathbf{T}^2\mathbf{c}^2$, we have to optimize the weighting vector $\mathbf{c}^2$. For the case of the G-TMRC Type-I, $\mathbf{w}_2$ is set to

$$\mathbf{w}_2 = \frac{\mathbf{t}_2^2}{||\mathbf{t}_2^2||}$$

which is equivalent to $\mathbf{c}^2 = \frac{\mathbf{1}_2}{||\mathbf{t}_2^2||}$. Note that we choose $\mathbf{w}_2$ matched to the second largest row vector $\mathbf{h}_2^T$ rather than the largest one $\mathbf{h}_1^T$, because a large fraction of a gain from $\mathbf{h}_1^T$ has already been delivered to the first stream. As in the case of the single beamforming system, we can further improve the performance by utilizing the idea of the gradient ascent algorithm. Thus, the G-TMRC Type-II chooses $\mathbf{c}^2$ as

$$\mathbf{c}^2 = \rho_2 \left(\frac{\mathbf{1}_2}{||\mathbf{t}_2^2||} + \delta \frac{\mathbf{T}^{2\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_2^2}{||\mathbf{T}^{2\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_2^2||} \right)$$

where ρ_2 is determined such that $||\mathbf{w}_2||^2 = ||\mathbf{T}^2\mathbf{c}^2||^2 = 1$. At the subsequent steps, we determine $\mathbf{w}_m$ ($m = 3, \cdots, M$) using the same way. The proposed G-TMRC scheme with M greater than one is described as follows:

Initialization:
$\mathbf{T}^1 = \mathbf{H}^\dagger$
Main Body:
for $m = 1 : M$

$$\mathbf{c}^m = \begin{cases} \frac{\mathbf{1}_m}{||\mathbf{t}_m^m||} & \text{(G-TMRC Type-I)} \\ \frac{\mathbf{1}_m}{||\mathbf{t}_m^m||} + \delta \frac{\mathbf{T}^{m\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_m^m}{||\mathbf{T}^{m\dagger}\mathbf{H}^\dagger\mathbf{H}\mathbf{t}_m^m||} & \text{(G-TMRC Type-II)} \end{cases}$$

$$\mathbf{w}_m = \frac{\mathbf{T}^m\mathbf{c}^m}{||\mathbf{T}^m\mathbf{c}^m||}$$

$$\mathbf{T}^{m+1} = \mathbf{T}^m - \mathbf{w}_m\mathbf{w}_m^\dagger\mathbf{T}^m$$

end

We briefly summarize the required amount of CSI at the transmitter (CSIT) for the SVD-BF and the proposed G-TMRC schemes. In frequency division duplexing systems, the G-TMRC Type-I scheme can reduce the feedback overhead

TABLE I
THE NUMBER OF FLOATING POINT MULTIPLICATIONS

	Multiplications
Power Method	$(8M_rM_t+4M_r+4M_t)MN_{itr}+4M_rM_t(M-1)$
Jacobi SVD	$(24M_r+4)(MM_t-\frac{M^2+M}{2})N_{sw}$
G-TMRC Type-I	$(8M-6)M_tM_r+M_t(4M-2)$
G-TMRC Type-II	$M_tM_r(24M-6)+(4M_t+4M_r+2)M-2M_t$

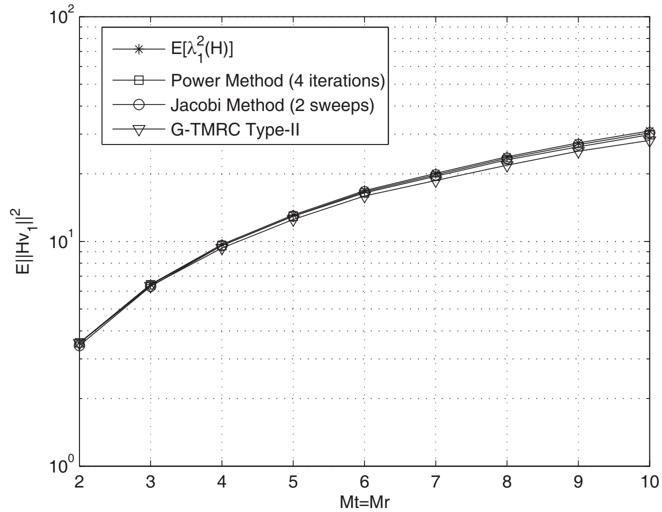


Fig. 3. Expected value of $||\mathbf{H}\hat{\mathbf{v}}_1||^2$ as a function of $M_t = M_r$.

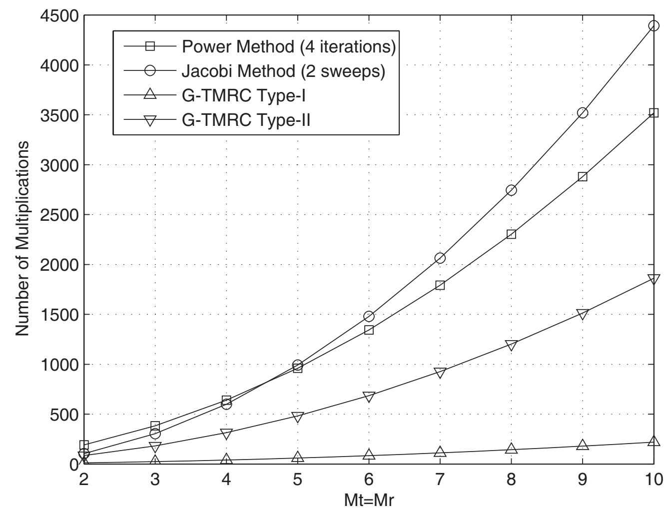


Fig. 4. The number of floating-point multiplications as a function of $M_t = M_r$.

when the number of data streams for transmission is less than M_r ($M < M_r$), since the transmitter needs only M row vectors for calculating the precoder. In contrast, when the G-TMRC Type-II algorithm is utilized, the full CSI should be reported to the transmitter as in the SVD-BF. Nonetheless, the complexity associated with the precoding matrix computation of the proposed schemes is significantly small compared to the SVD operation. For systems with time division duplexing (TDD), the receiver of G-TMRC Type-I needs only M antennas out of M_r receive antennas for uplink channel sounding which leads to further savings of the system cost.

C. Complexity Comparison

In this section, we address the complexity issue in detail. Table I shows the number of floating-point multiplications required for the proposed G-TMRC schemes and two SVD computation algorithms (i.e. *power method* [24] and *Jacobi SVD* [25]). The power method is known to be particularly suited for computing the SVD where only the dominant singular components are of interest [26]. In contrast, the Jacobi-based SVD algorithm is an alternative way which exhibits a higher degree of potential parallelism [27]. We consider the one-sided Jacobi SVD computation with cyclic-by-row ordering, since only the right singular vectors are necessary. In Table I, N_{itr} indicates the number of iterations in the power method and N_{sw} denotes the number of sweeps in the Jacobi method.

Before examining the complexity savings of the proposed schemes in the case of $M = 1$, we first confirm that $N_{itr} = 4$ iterations and $N_{sw} = 2$ sweeps are enough for the power method and the Jacobi-SVD, respectively, to achieve the value of $||\mathbf{H}\hat{\mathbf{v}}_1||^2$ close to $\lambda_1^2(\mathbf{H})$ as shown in Figure 3. Here, $\hat{\mathbf{v}}_1$ is an estimate of $\mathbf{v}_1$ obtained from the SVD methods or the proposed G-TMRC algorithms. For the one-sided Jacobi SVD, we have used the complex rotation introduced in [28].

Since the power method with $N_{itr} = 4$ and the Jacobi SVD with $N_{sw} = 2$ exhibit the performance comparable to the proposed schemes as checked in Figure 3, we compare the number of multiplications of those schemes as a function of the number of transmit/receive antennas in Figure 4. Here, we consider the case of $M_t = M_r$. We can see that, compared to the power method, the proposed G-TMRC Type-I and Type-II show a complexity savings of about 90% and 50%, respectively.

IV. Performance Analysis

In this section, we investigate our choice of $\mathbf{w}_1$ in (5) by analyzing the SNR distribution of the proposed G-TMRC Type-I in the case of $M = 1$. Then, we compare the result with the optimal SVD-BF derived in [29] and [30]. We do not make the assumption of (2) in this section for the sake of clarity of the analysis. We focus on the case of $M_r = 2$ for simplicity.

Consider the G-TMRC Type-I which prefixes $\mathbf{c}_{(0)}$ as $\frac{\mathbf{1}_{\hat{m}}}{||\mathbf{t}_{\hat{m}}||}$ where $\hat{m} \in \{1, \cdots, M_r\}$ is selected based on the energy of the channel row vectors. We will show that this energy-based criterion is optimum when one of $M_r = 2$ receive antennas is selected for $\hat{m}$. The received signal vector for $M_r = 2$ can be written as

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \end{bmatrix} \mathbf{w}_1 s_{1,k} + \mathbf{n}_k = \begin{bmatrix} \mathbf{h}_1^T\mathbf{w}_1 \\ \mathbf{h}_2^T\mathbf{w}_1 \end{bmatrix} s_{1,k} + \mathbf{n}_k.$$

Note that $|\mathbf{h}_i^T\mathbf{w}_1|^2$ ($i = 1, 2$) is maximized for $\mathbf{c} = \mathbf{1}_i/||\mathbf{t}_i||$ (i.e. $\mathbf{w}_1 = \mathbf{t}_i/||\mathbf{t}_i||$).

Denoting α_i as the normalized SNR obtained by $\mathbf{c} = \mathbf{1}_i/||\mathbf{t}_i||$, we have

$$\begin{aligned} \alpha_1 &= \left|\left| \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \end{bmatrix} \frac{\mathbf{h}_1^*}{||\mathbf{h}_1||} \right|\right|^2 = ||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\beta \\ &= \beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) + (1-\beta)\,||\mathbf{h}_1||^2 \end{aligned} \quad (9)$$

where β indicates the normalized inner product of $\mathbf{h}_1$ and $\mathbf{h}_2$ as

$$\beta = \frac{|\mathbf{h}_1^\dagger \mathbf{h}_2|^2}{||\mathbf{h}_1||^2||\mathbf{h}_2||^2} \in [0,1].$$

Similarly, α_2 is given by

$$\alpha_2 = \beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) + (1-\beta)\,||\mathbf{h}_2||^2. \quad (10)$$

We briefly explain the meaning of β. For the effective channel gain α_1 with the beamforming vector matched to $\mathbf{h}_1$, β can be considered as the weighting factor of $\mathbf{h}_2$. When $\mathbf{h}_1$ and $\mathbf{h}_2$ are strongly correlated, β is close to 1 and a large fraction of $||\mathbf{h}_2||^2$ is converted to the effective channel gain. In contrast, if the correlation between $\mathbf{h}_1$ and $\mathbf{h}_2$ is weak, β becomes small and the gain from $\mathbf{h}_2$ almost disappears.

Comparing α_1 and α_2, we need to choose $\hat{m}$ which maximizes the channel gain as

$$\hat{m} = \arg\max_{i\in\{1,2\}} \left\{\beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) + (1-\beta)\,||\mathbf{h}_i||^2\right\}. \quad (11)$$

Since $\beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right)$ is a common term in α_1 and α_2 and $(1-\beta)$ is a nonnegative real number, the selection algorithm of (11) reduces to the following simple energy-based selection:

$$\hat{m} = \arg\max_{i\in\{1,2\}} ||\mathbf{h}_i||^2.$$

Therefore, the energy-based selection of the row vector matched by the beamforming vector is equivalent to the optimal selection criterion derived in [19] when $M_r = 2$.

Now, we analyze the CDF of the normalized SNR in the proposed G-TMRC Type-I where the beamforming vector is matched to the row vector of the largest energy. Since the effective channel gain with $\mathbf{w}_1 = \mathbf{t}_i/||\mathbf{t}_i||$ is given by $\alpha_i = \beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) + (1-\beta)\,||\mathbf{h}_i||^2$ from (9) and (10), the normalized SNR with the energy-based selection algorithm, denoted by α_{sel}, is represented by

$$\begin{aligned}\alpha_{sel} &= \max(\alpha_1,\alpha_2)\\ &= \beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) + (1-\beta)\max\left(||\mathbf{h}_1||^2, ||\mathbf{h}_2||^2\right).\end{aligned}$$

Here, $||\mathbf{h}_1||^2$ and $||\mathbf{h}_2||^2$ are i.i.d. Chi-square random variables with $2M_t$ degrees of freedom and β is beta-distributed with parameters 1 and $M_t - 1$ [31]. Thus, the marginal probability density functions (PDF) are given by

$$f_{||\mathbf{h}_i||^2}(x) = \frac{1}{(M_t-1)!}\, x^{M_t-1}e^{-x}, \quad x>0, \quad i=1,2 \quad (12)$$

and

$$f_\beta(z) = (M_t-1)\,(1-z)^{M_t-2}, \quad 0<z<1. \quad (13)$$

Since $\mathbf{h}_1$ and $\mathbf{h}_2$ are independent isotropic random vectors, the normalized inner product β which depends only on the direction of $\mathbf{h}_1$ and $\mathbf{h}_2$ is independent of the magnitudes of $\mathbf{h}_1$ and $\mathbf{h}_2$ [31]. Hence, the joint PDF of $||\mathbf{h}_1||^2$, $||\mathbf{h}_2||^2$ and β is equal to the product of three marginal PDFs. Then, we can obtain the CDF of α_{sel} as

$$\begin{aligned}F_{\alpha_{sel}}(\gamma) = Pr\Big[&\beta\left(||\mathbf{h}_1||^2 + ||\mathbf{h}_2||^2\right) \quad (14)\\ &+ (1-\beta)\max\left(||\mathbf{h}_1||^2, ||\mathbf{h}_2||^2\right) \le \gamma\Big].\end{aligned}$$

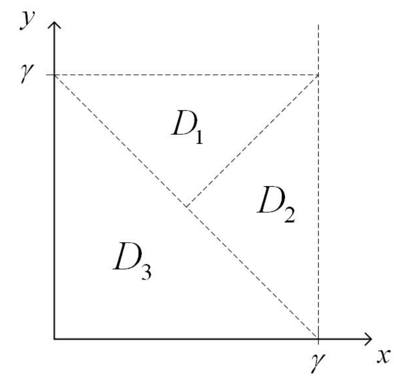


Fig. 5. Integration Region for the CDF of α_{sel}.

Applying the total probability theorem onto (14) yields

$$\begin{aligned}F_{\alpha_{sel}}(\gamma) &= \iint_{D_1\cup D_2\cup D_3} Pr\left[\beta(x+y) + (1-\beta)\max(x,y) \le \gamma\right]\\ &\quad\times f_{||\mathbf{h}_2||^2}(y) f_{||\mathbf{h}_1||^2}(x)\,\mathrm{d}y\,\mathrm{d}x\\ &= \iint_{D_1\cup D_2\cup D_3} Pr\left[\beta\left(x+y-\max(x,y)\right) \le \gamma - \max(x,y)\right]\\ &\quad\times f_{||\mathbf{h}_2||^2}(y) f_{||\mathbf{h}_1||^2}(x)\,\mathrm{d}y\,\mathrm{d}x\\ &= \iint_{D_1\cup D_2\cup D_3} Pr\left[\beta \le \frac{\gamma - \max(x,y)}{\min(x,y)}\right]\\ &\quad\times f_{||\mathbf{h}_2||^2}(y) f_{||\mathbf{h}_1||^2}(x)\,\mathrm{d}y\,\mathrm{d}x \quad (15)\end{aligned}$$

where D_1, D_2 and D_3 are depicted in Figure 5. From (13), we know that

$$Pr\left[\beta \le \frac{\gamma - \max(x,y)}{\min(x,y)}\right] = \begin{cases} 1-\left(1-\frac{\gamma-y}{x}\right)^{M_t-1}, \\ \quad (\text{if } (x,y)\in D_1) \\ 1-\left(1-\frac{\gamma-x}{y}\right)^{M_t-1}, \\ \quad (\text{if } (x,y)\in D_2) \\ 1, \\ \quad (\text{if } (x,y)\in D_3) \end{cases}. \quad (16)$$

Substituting (12) and (16) into (15) results in

$$\begin{aligned}F_{\alpha_{sel}}(\gamma) = \frac{1}{((M_t-1)!)^2}\Bigg[&\iint_{D_1\cup D_2\cup D_3} (xy)^{M_t-1}e^{-(x+y)}\,\mathrm{d}y\,\mathrm{d}x\\ &-\iint_{D_1}\left(1-\frac{\gamma-y}{x}\right)^{M_t-1}(xy)^{M_t-1}e^{-(x+y)}\,\mathrm{d}y\,\mathrm{d}x\\ &-\iint_{D_2}\left(1-\frac{\gamma-x}{y}\right)^{M_t-1}(xy)^{M_t-1}e^{-(x+y)}\,\mathrm{d}y\,\mathrm{d}x\Bigg]. \quad (17)\end{aligned}$$

By applying multinomial series and solving the integral in

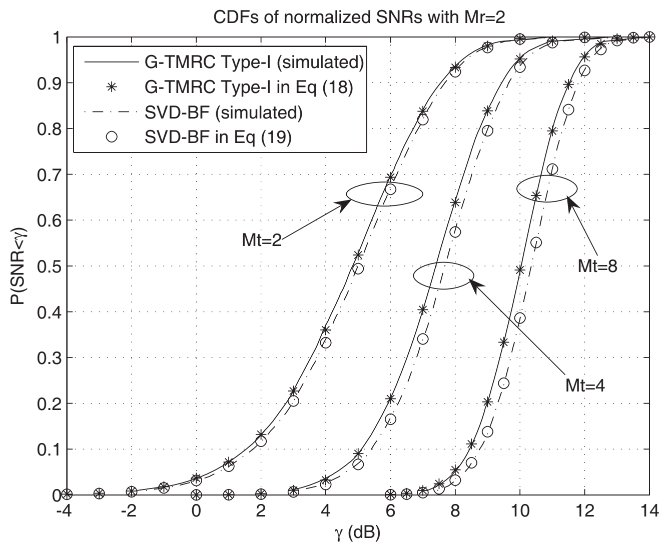


Fig. 6. CDF of SNRs in the SVD-BF and the proposed G-TMRC Type-I with $M = 1$ and $M_r = 2$.

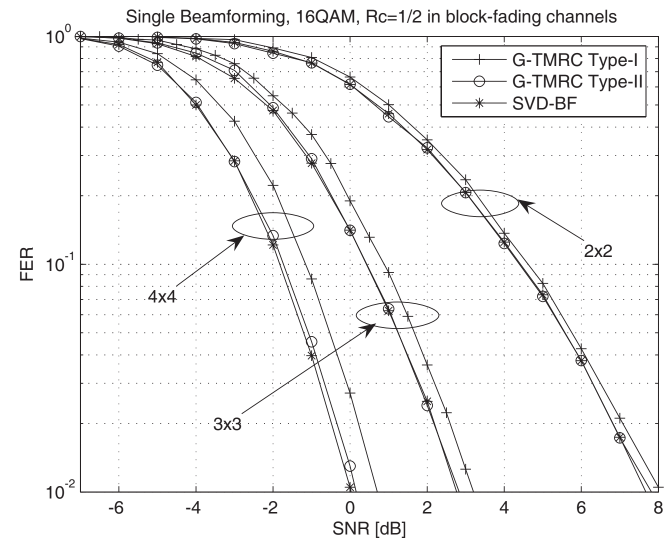


Fig. 7. Coded FER comparison for single beamforming with 16QAM and Rc=1/2 over block-fading channels.

(17), we obtain

$$\begin{aligned} F_{\alpha_{sel}}(\gamma) = & \left[1 - \sum_{l=0}^{M_t-1} \frac{\gamma^l e^{-\gamma}}{l!}\right]^2 \\ & -\frac{2}{(M_t-1)!} \sum_{k_1=0}^{M_t-1} \sum_{k_2=0}^{M_t-1-k_1} \sum_{l=0}^{2M_t-2-k_1-k_2} \frac{(-\gamma)^{k_2}}{k_1! k_2!} \times \\ & \frac{(2M_t-2-k_1-k_2)! e^{-\gamma}}{(M_t-1-k_1-k_2)! l!} \left(\sum_{m=0}^{k_1} \frac{\gamma^{l+m} k_1! e^{-\gamma}}{m!} \right. \\ & + \sum_{m=0}^{l} \frac{l!(-1)^{l-m} \gamma^{k_1+l+1}}{m!(l-m)!(k_1+l-m+1) 2^{k_1+l-m+1}} - k_1! \gamma^l \\ & \left. + \sum_{m=0}^{k_1+l} \frac{(k_1+l)!}{m! 2^{k_1+l-m+1}} \left[\left(\frac{\gamma}{2}\right)^m - \gamma^m e^{-\gamma} \right] \right). \end{aligned} \tag{18}$$

Next, for comparison purposes, we present the CDF of SNR in the SVD-BF system. The normalized SNR of the SVD-BF, denoted by α_{SVD}, is given as $\alpha_{SVD} = \lambda_1^2(\mathbf{H})$. Defining $r = \min\{M_t, M_r\}$ and $q = \max\{M_t, M_r\}$, we can obtain the CDF of α_{SVD} as [29][30]

$$F_{\alpha_{SVD}}(\gamma) = K_{r,q} \det\{\mathbf{G}(\gamma)\} \tag{19}$$

where $K_{r,q}$ is given by

$$K_{r,q} = \left(\prod_{i=1}^{r} (r-i)! \prod_{i=1}^{r} (q-i)! \right)^{-1}$$

and $\mathbf{G}(\gamma)$ is the $r \times r$ Hankel matrix whose (i,j)th element is written by

$$\{\mathbf{G}(\gamma)\}_{i,j} = (q-r+i+j-2)! - \Gamma(q-r+i+j-1, \gamma).$$

Here, $\Gamma(l,\mu)$ represents the complementary incomplete Gamma function defined as

$$\Gamma(l,\mu) = \int_{\mu}^{\infty} e^{-t} t^{l-1} \mathrm{d}t.$$

Figure 6 compares the CDFs of the SNRs for the SVD-BF and the proposed G-TMRC Type-I with $M_t = 2, 4,$ and 8 and $M_r = 2$. We can see that the curves of the G-TMRC Type-I are only a few tenth of a dB away from that of the SVD-BF. Also, the plots indicate that our analysis derived above for CDFs of α_{sel} is accurate compared to the simulated results. From this figure, it can be expected that our proposed scheme is capable of achieving a good performance gain with reduced complexity in comparison to the optimal SVD-BF, and this will be verified in the following section.

V. Simulation Results

In this section, we present simulation results to demonstrate the efficiency of the proposed scheme in flat fading channels. In all simulations, a rate 1/2 binary convolutional code with polynomial $(133, 171)$ in octal notation is applied. We use the interleaver optimized for the SVD-BF suggested in [9] for a fair comparison and the frame length is set to 64. At the receiver, a minimum mean square error (MMSE) equalizer is used except for the single beamforming case where the MRC combining is applied. For the G-TMRC Type-II scheme, we use $\delta = 150$.

Figure 7 shows the coded FER comparison for single beamforming systems ($M = 1$) with 16QAM constellation. We can see that the G-TMRC Type-II achieves almost the same performance as the SVD-BF system for all antenna configurations, as expected in Figure 2. In Figure 8, the coded FER performance for multiple beamforming systems with $M = 4$ and 4QAM modulation is compared for the SVD-BF and the proposed G-TMRC scheme. As in the case of the single beamforming, the G-TMRC Type-I shows a performance loss of less than 1 dB, which almost disappears with the G-TMRC Type-II for all configurations. It should be emphasized that the complexity of the proposed schemes is substantially smaller than that of the SVD-BF.

Until now, we have investigated the performance comparison for systems where the number of receive antennas

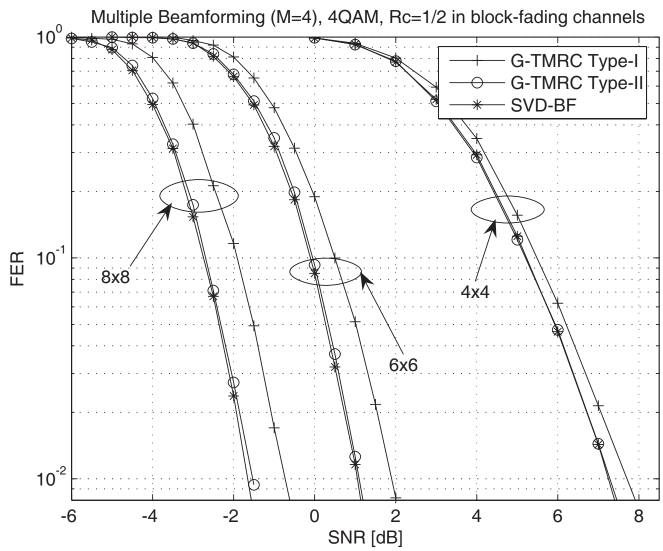


Fig. 8. Coded FER comparison for multiple beamforming ($M = 4$) with 4QAM and Rc=1/2 over block-fading channels.

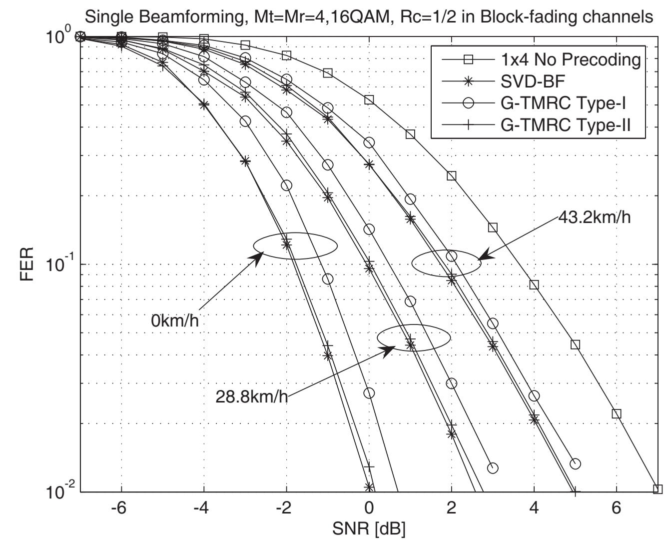


Fig. 10. Coded FER comparison for single beamforming with 16QAM and Rc=1/2 over 4-by-4 time-varying channels.

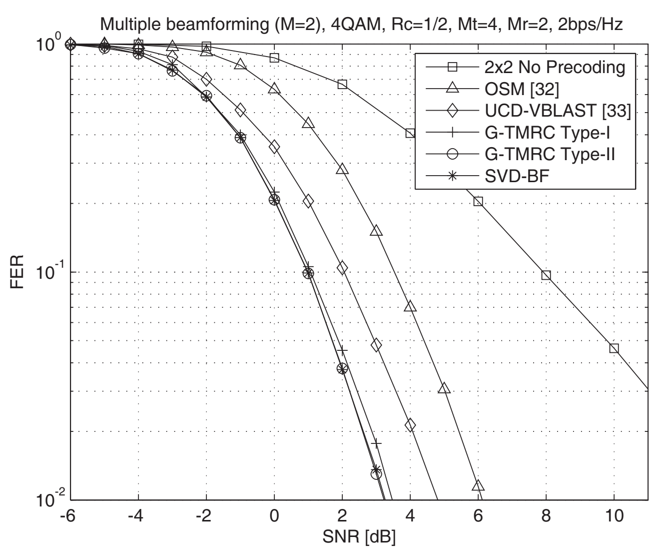


Fig. 9. Coded FER comparison for multiple beamforming ($M = 2$) with 4QAM and Rc=1/2 in 2bps/Hz over 4-by-2 block-fading channels.

is the same as that of transmit antennas. One might think that the performance loss of the G-TMRC compared to the SVD-BF would increase when there is less receive antennas than transmit antennas, since all possible linear combinations of TMRC vectors span only the M_r dimensional subspace. However, we will show that this is not the case. As shown in Figure 9, the performance loss of the G-TMRC Type-I compared to the SVD-BF is still less than 1 dB in 4-by-2 systems. Again, the G-TMRC Type-II shows the FER performance almost identical to the SVD-BF. For comparison, we plot the performance of other closed-loop systems such as orthogonalized spatial multiplexing (OSM) [32] and uniform channel decomposition combined with VBLAST (UCD-VBLAST) [33] schemes. The G-TMRC Type-II exhibits a performance gain of about 2.8 dB over the OSM. This is due to the fact that the OSM is designed for uncoded systems which maximize the minimum Euclidean distance of the received signals. Moreover, the UCD-VBLAST requires high system cost since it adopts the SVD computation of the channel matrix and the additional precoding process. To observe the precoding gain of the G-TMRC schemes, the FER curve of no precoding system with a linear MMSE receiver is also plotted.

In Figure 10, we examine the performance degradation of the proposed single beamforming schemes due to imperfect CSIT. We consider TDD systems over time-varying channels where the CSI at the transmitter is in error due to a time delay τ between estimation of the channel and transmission of the data. We adopt the first-order Markov process [34] where the outdated channel matrix $\hat{\mathbf{H}}$, based on which the transmitter performs beamforming, is related to the actual channel matrix $\mathbf{H}$ as

$$\hat{\mathbf{H}} = \rho\mathbf{H} + \sqrt{1-\rho^2}\mathbf{\Xi}$$

where the error matrix $\mathbf{\Xi}$ has i.i.d. complex Gaussian entries with zero mean and unit variance. Here, the correlation coefficient ρ can be computed from the Jakes' model [35] as $\rho = J_0(2\pi f_D \tau)$ where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind and f_D denotes the Doppler frequency shift. In simulations, we use a carrier frequency of 5.725 GHz and a typical delay of 1 ms as expected in the Hiperlan 2 standard [36]. It is observed that the performance loss of the proposed G-TMRC Type-I compared to the SVD-BF reduces to a few tenth of a dB as the mobility of the terminal increases. This simulation shows that the proposed G-TMRC Type-I is more robust to the time-varying channels compared to the SVD-BF. This tells us that the SVD-BF is sensitive to imperfect CSIT caused by time-varying channels or channel estimation errors. We notice that a small variation in the channel matrix results in major shifts of the right singular vectors because the SVD is a nonlinear function with nonunique outputs [37].

From these simulation results, we observe that the proposed G-TMRC Type-II can achieve almost the same FER performance as the SVD-BF in all simulation environments with reduced complexity and outperforms other closed-loop MIMO techniques. Also, the G-TMRC Type-I achieves a significant reduction of 90% of the computational complexity compared to the SVD-BF at the expense of a less than 1 dB performance loss.

VI. CONCLUSION

In this paper, we have presented a new beamforming scheme for MIMO systems which approaches the link performance of the SVD-BF with reduced complexity. Conventional TMRC techniques are computationally efficient but restricted to single beamforming systems, since it can be applied only to MISO channels. To optimize the beamforming vectors with low complexity, we have generalized conventional TMRC schemes to multiple beamforming systems in MIMO channels. In the optimization process, we have incorporated the idea of the gradient ascent algorithm. In order to obtain the near-optimal performance while avoiding iterative property of the gradient ascent, an initialization beamforming vector has been proposed based on the observations made in [19]. Also, we have performed the statistical analysis of the output SNR for the proposed scheme. Simulation results demonstrate that the proposed G-TMRC schemes yield the performance almost identical to that of the SVD-BF regardless of system configurations with reduced complexity. It is expected that the proposed precoding method can achieve almost the same system throughput as the SVD-BF when combined with adaptive bit loading and power allocation.

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