Enhanced Detection with New Ordering Schemes for V-BLAST Systems

Sang-Rim Lee, Seok-Hwan Park, Sung Won Kim, and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—This letter proposes a new optimal ordering method which minimizes error propagation in the vertical Bell-lab Layered Space-Time (V-BLAST) by exploiting the whole filter output. A suboptimal ordering metric is also proposed which requires much reduced complexity compared to the optimal ordering metric. We also derive a simplified version of the suboptimal ordering metric which achieves a significant performance gain over the conventional ordering with minor additional complexity.**

***Index Terms*—Multiple-input multiple-output (MIMO) systems, vertical Bell Labs layered space-time (V-BLAST), detection ordering.**

I. INTRODUCTION

THE layered space-time architecture proposed in [1] is capable of achieving extremely high spectral efficiencies by employing multiple antennas at both transmitter and receiver sides. To realize such gains, the vertical Bell-lab Layered Space-Time (V-BLAST) structure has been proposed in [2]. A detection approach known as nulling and canceling gives a good trade-off between performance and complexity. However, its performance is largely affected by an ordering scheme which determines the order of substreams to be detected and canceled [2]. Also a performance loss due to incorrect decisions in previous substreams is inevitable. A new filter design which takes incorrect decisions into account has been studied in [3] and [4].

The detection ordering techniques introduced in [5] and [6] achieve a performance gain over the conventional mean square error (MSE) ordering schemes by exploiting the instantaneous noise. Especially, the metric of each substream of the log-likelihood ratio (LLR) based ordering scheme in [5] depends on the filter output of the corresponding substream.

Motivated by the work in [5], in this letter, we first propose an optimal ordering for the V-BLAST which minimizes the error propagation by selecting a substream corresponding to the largest reliability with an assumption of error free decisions. Unlike [5], the proposed ordering scheme provides the exact reliability of data decisions by considering correlations among noise terms at the filter output.

This optimal ordering metric may require high complexity which increases exponentially with the number of transmit antennas. To circumvent this issue, we propose a suboptimal ordering metric with reduced complexity at the expense of a little performance loss. Also, a simplified version of the suboptimal ordering metric is presented which provides a significant performance gain over the conventional scheme with slightly increased complexity. Simulation results show that the proposed ordering scheme combined with the real-valued system achieves a performance gain of up to 13 dB at a symbol error rate (SER) of 10^{-4} over conventional MSE ordering schemes.

II. SYSTEM DESCRIPTIONS

Consider V-BLAST systems with N_t transmit and N_r receive antennas. At the transmitter side, the data stream is demultiplexed into N_t parallel substreams, and each substream is transmitted via individual transmit antennas. The receiver is assumed to have the perfect knowledge of the channel state information. Throughout this letter, normal letters represent scalar quantities, boldface lowercase letters indicate vectors and boldface uppercase letters designate matrices. $E(\cdot)$ accounts for expectation. Also, $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and the conjugate transpose, respectively.

The received discrete-time complex baseband signal vector $\mathbf{r} = [r_1 \ r_2 \ \cdots \ r_{N_r}]^T$ can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the (i, j)th component of the $N_r \times N_t$ channel matrix $\mathbf{H}$, denoted by h_{ij}, represents the channel fading coefficient from the jth transmit antenna to the ith receive antenna which is modeled as an independent and identically-distributed (i.i.d.) complex Gaussian variable with zero mean and unit variance, $\mathbf{x}$ indicates the N_t-dimentional transmitted signal vector with the covariance matrix $E_s\mathbf{I}$, and $\mathbf{n}$ is assumed to be the N_r-dimensional complex noise vector whose elements are modeled as samples of independent complex Gaussian random variables with zero mean and variance σ_n^2.

The conventional minimum mean square error (MMSE) equalization matrix $\mathbf{W}$ is given as

$$\mathbf{W} = [\mathbf{w}_1^T \ \mathbf{w}_2^T \ \cdots \ \mathbf{w}_{N_t}^T]^T = \left(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}\right)^{-1}\mathbf{H}^H$$

where α denotes $\frac{\sigma_n^2}{E_s}$. Applying this equalizer matrix into Equation (1) yields the filter output

$$\mathbf{y} = \mathbf{W}\mathbf{r} = \mathbf{W}\mathbf{H}\mathbf{x} + \tilde{\mathbf{n}} \tag{2}$$

where $\tilde{\mathbf{n}} = \mathbf{W}\mathbf{n}$. Here the filter output noise $\tilde{\mathbf{n}}$ is processed by $\mathbf{W}$ and thus becomes correlated.

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S.-R. Lee, S.-H. Park, and I. Lee are with the School of Electrical Eng., Korea University, Seoul, Korea (e-mail: {srlee, shpark}@wireless.korea.ac.kr; inkyu@korea.ac.kr).

S. W. Kim is with the Dept. of Information and Communication Eng., Yeungnam University, Gyeongsangbuk-do, 712-749, Korea (e-mail: swon@yu.ac.kr).

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The covariance matrix $\mathbf{R_e}$ of the estimation error $\mathbf{e} = \mathbf{x} - \mathbf{W}\mathbf{y}$ is given as

$$\mathbf{R_e} = E\left(\mathbf{e}\mathbf{e}^H\right) = \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}\right)^{-1}.$$

Denoting σ_j^2 as the MSE of the jth symbol estimate, σ_j^2 is equal to the jth diagonal element of $\mathbf{R_e}$. The conventional MSE ordering selects a substream with the minimum σ_j^2.

III. Optimal Detection Ordering

In this section, we propose a new optimal detection ordering scheme. Let $x_{(1)}$ and $\hat{x}_{(1)}$ be the transmitted symbol and the estimated symbol, respectively, for the first detected layer. Given the received signal vector $\mathbf{y}$, the symbol error probability of the first detected layer, denoted by $P(x_{(1)} \neq \hat{x}_{(1)}|\mathbf{y})$, can be minimized by selecting a substream with the maximum $P(x_i = \hat{x}_i|\mathbf{y})$ and this results in

$$P(x_{(1)} \neq \hat{x}_{(1)}|\mathbf{y}) = 1 - \max_i P(x_i = \hat{x}_i|\mathbf{y}). \tag{3}$$

Now we derive a metric to determine (3). Considering an M-ary constellation $\mathcal{M} = \{s_1, s_2, \cdots, s_M\}$, we define the pairwise LLR as

$$\zeta_{i,m} = \ln\frac{P(x_i = \hat{x}_i|\mathbf{y})}{P(x_i = s_m|\mathbf{y})}. \tag{4}$$

Assuming that each symbol is transmitted with an equal probability, (4) can be expressed by

$$\zeta_{i,m} = \ln\frac{p(\mathbf{y}|x_i = \hat{x}_i)}{p(\mathbf{y}|x_i = s_m)} = \ln\frac{\sum_{\mathbf{x}\in S_i^{\hat{x}_i}} p(\mathbf{y}|\mathbf{x})}{\sum_{\mathbf{x}\in S_i^{s_m}} p(\mathbf{y}|\mathbf{x})} \tag{5}$$

where S_i^d represents the transmitted signal vector set whose symbol at the ith substream is $d \in \mathcal{M}$. Since $\tilde{\mathbf{n}}$ is a Gaussian random vector, the conditional pdf $p(\mathbf{y}|\mathbf{x})$ in (5) can be computed as

$$p(\mathbf{y}|\mathbf{x}) = p_{\tilde{\mathbf{n}}}(\mathbf{y} - \mathbf{WHx}) = \frac{1}{\pi^n(\det\mathbf{R}_{\tilde{\mathbf{n}}})} \times \exp\left(-(\mathbf{y} - \mathbf{WHx})^H\mathbf{R}_{\tilde{\mathbf{n}}}^{-1}(\mathbf{y} - \mathbf{WHx})\right) \tag{6}$$

where the covariance matrix of $\tilde{\mathbf{n}}$, defined by $\mathbf{R}_{\tilde{\mathbf{n}}}$, is given as

$$\mathbf{R}_{\tilde{\mathbf{n}}} = E\left(\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right) = \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}\right)^{-1}.$$

Using (4) and the equality $\sum_{m=1}^{M} P(x_i = s_m|\mathbf{y}) = 1$, the symbol error probability of the ith symbol $P(x_i \neq \hat{x}_i|\mathbf{y})$ is given by

$$P(x_i \neq \hat{x}_i|\mathbf{y}) = 1 - \frac{1}{\sum_{m=1}^{M}\exp\left(-\zeta_{i,m}\right)}.$$

Then it follows that the optimal ordering is equivalent to detecting a substream with

$$\min\sum_{m=1}^{M}\exp\left(-\zeta_{i,m}\right).$$

Thus, by exploiting the whole received vector in (4), the optimal ordering can enhance the performance over the conventional MSE ordering. Moreover, as the proposed ordering extracts the exact reliability information of the decision on x_i based on the whole relevant information $\mathbf{y}$, the optimal ordering outperforms the LLR-based ordering in [5] as will be verified in the simulation section. For high modulation level, the optimal ordering may require high computational complexity, as the search candidate number of the elements in $S_i^{s_m}$ in (5) grows exponentially with the number of transmit antennas.

IV. Suboptimal Detection Ordering

In this section, we derive suboptimal ordering schemes with reduced complexity compared with the optimal ordering presented in Section III. First, we consider a suboptimal ordering scheme by taking y_i only instead of $\mathbf{y}$. Then, we further introduce a simplified version of the suboptimal scheme without any performance loss.

Denoting $\mathbf{h}_i$ as the i-th column of $\mathbf{H}$, from Equation (2), the ith filter output y_i is expressed as

$$y_i = \mathbf{w}_i\mathbf{r} = \mathbf{w}_i\mathbf{h}_i x_i + \sum_{j=1,j\neq i}^{N_t}\mathbf{w}_i\mathbf{h}_j x_j + \mathbf{w}_i\mathbf{n} = \beta_i x_i + w_i. \tag{7}$$

where $\beta_i = \mathbf{w}_i\mathbf{h}_i$ is a bias introduced by the MMSE equalizer and $w_i = \sum_{j=1,j\neq i}^{N_t}\mathbf{w}_i\mathbf{h}_j x_j + \mathbf{w}_i\mathbf{n}$ denotes the residual interference plus noise. From Equation (7), the variance $\sigma_{i,w}^2$ of w_i is given by

$$\sigma_{i,w}^2 = \sum_{j=1,j\neq i}^{N_t}|\mathbf{w}_i\mathbf{h}_j|^2 E_s + ||\mathbf{w}_i||^2\sigma_n^2.$$

A. Neighbor's Likelihood Sum Ordering

It is straightforward to show that

$$\arg\max_i P(x_i = \hat{x}_i|\mathbf{y}) = \arg\max_i p(\mathbf{y}|x_i = \hat{x}_i) = \arg\min_i \sum_{s_m\neq\hat{x}_i} p(\mathbf{y}|x_i = s_m).$$

Then, the above expression can be approximated by

$$\arg\min_i \sum_{s_m\neq\hat{x}_i} p(y_i|x_i = s_m). \tag{8}$$

We assume that w_i in (7) has a complex Gaussian distribution with zero mean and variance $\sigma_{i,w}^2$. Thus, the conditional pdf $p(y_i|x_i = s_m)$ in (8) can be computed as

$$p(y_i|x_i = s_m) = \frac{1}{\pi\sigma_{i,w}^2}\exp\left(-\frac{|y_i - \beta_i s_m|^2}{\sigma_{i,w}^2}\right).$$

To simplify the computation, the term $\frac{1}{\pi\sigma_{i,w}^2}$ in the above pdf can be neglected. Then, (8) becomes

$$\arg\min_i \sum_{s_m\neq\hat{x}_i}\exp\left(-\frac{|y_i - \beta_i s_m|^2}{\sigma_{i,w}^2}\right). \tag{9}$$

This suboptimal ordering thus selects a substream with the minimum sum of likelihood[11] for candidate symbols except the detected symbol $\hat{x}_i$. We refer to this ordering as the

[11] Likelihood is the hypothetical probability that an event which has already occurred would yield a specific outcome.

TABLE I
EQUIVALENT EXPRESSIONS OF $|y_i - \beta_i \bar{s}_m|^2$ FOR 4PAM

$\Re[y_i]$	$\lvert y_i - \beta_i \bar{s}_m\rvert^2$
$0 \leq \lvert\Re[y_i]\rvert < \sqrt{\frac{E_s}{5}}$	$\lvert y_i\rvert^2 + \beta_i^2 \frac{E_s}{5} + 2\beta_i \mathrm{sgn}\left(\Re[y_i]\right)\sqrt{\frac{E_s}{5}}\Re[y_i]$
$\sqrt{\frac{E_s}{5}} \leq \lvert\Re[y_i]\rvert < 2\sqrt{\frac{E_s}{5}}$	$\lvert y_i\rvert^2 + 9\beta_i^2 \frac{E_s}{5} - 6\beta_i \mathrm{sgn}\left(\Re[y_i]\right)\sqrt{\frac{E_s}{5}}\Re[y_i]$
$\lvert\Re[y_i]\rvert \geq 2\sqrt{\frac{E_s}{5}}$	$\lvert y_i\rvert^2 + \beta_i^2 \frac{E_s}{5} - 2\beta_i \mathrm{sgn}\left(\Re[y_i]\right)\sqrt{\frac{E_s}{5}}\Re[y_i]$

TABLE II
THE NUMBER OF FLOATING POINT MULTIPLICATIONS FOR THE PROPOSED ORDERING SCHEMES

	NLS	Simplified NLS
Compute $\mathbf{W}, \beta_i, \sigma_{i,w}^2$	$5N^4 + 8N^3 + 3N^2$	$5N^4 + 8N^3 + 3N^2$
Compute y_i	$2N^3 + 2N^2$	$2N^3 + 2N^2$
Compute the metrics	$(3.5M - 0.5)\,N^2$	$(0.5M + 3.5)\,N^2$
Cancelling	$4N^2$	$4N^2$
Total	$5N^4 + 10N^3 + (3.5M + 8.5)N^2$	$5N^4 + 10N^3 + (0.5M + 12.5)N^2$

neighbor's likelihood sum (NLS) ordering. The number of candidate search in (9) equals $(M-1) \cdot N_t$, while that for the optimal ordering in (5) is M^{N_t}. Thus the complexity is significantly reduced at the expense of a small performance loss.

B. Simplified NLS Ordering

We may further simplify (9) by considering only the dominant term. Let $\bar{s}_m$ be the second nearest symbol from y_i. Then Equation (9) in the NLS ordering can be approximated as

$$\arg\min_i \sum_{s_m \neq \hat{x}_i} \exp\left(-\frac{|y_i - \beta_i s_m|^2}{\sigma_{i,w}^2}\right) \approx \arg\max_i \left(\frac{|y_i - \beta_i \bar{s}_m|^2}{\sigma_{i,w}^2}\right). \quad (10)$$

Note that the computation of (10) still requires searching $\bar{s}_m$ with the second nearest distance from y_i.

In what follows, we introduce equivalent expressions for the computation of Equation (10) for BPSK and 4QAM which do not require searching $\bar{s}_m$. The numerator term $|y_i - \beta_i \bar{s}_m|^2$ in Equation (10) can be expanded as

$$|y_i|^2 + \beta_i^2 |\bar{s}_m|^2 - 2\beta_i |y_i||\bar{s}_m| \cos\left(\theta_{y_i} - \theta_{\bar{s}_m}\right)$$

where θ_x denotes the phase of x. Here, it can be shown that $\cos\left(\theta_{y_i} - \theta_{\bar{s}_m}\right)$ is simplified to $-|\cos\left(\theta_{y_i}\right)|$ and $|\cos\left(\left(\theta_{y_i} \bmod \frac{\pi}{2}\right) + \frac{\pi}{4}\right)|$ for BPSK and 4QAM, respectively. Therefore, we can compute (10) for BPSK as

$$\arg\max_i \left(\frac{|y_i|^2 + \beta_i^2 E_s + 2\beta_i \sqrt{E_s}|y_i||\cos\theta_{y_i}|}{\sigma_{i,w}^2}\right). \quad (11)$$

Similarly, the ordering metric (10) for 4QAM can be expressed as

$$\arg\max_i \left(\frac{|y_i|^2 + \beta_i^2 E_s - 2\beta_i \sqrt{E_s}|y_i||\cos\left(\left(\theta_{y_i} \bmod \frac{\pi}{2}\right) + \frac{\pi}{4}\right)|}{\sigma_{i,w}^2}\right) \quad (12)$$

where 'mod' denotes the modulo operation.

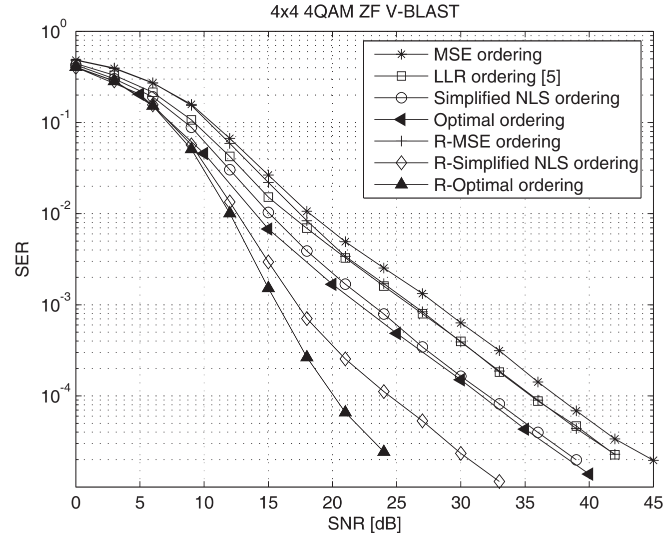

Fig. 1. Symbol error probability for 4QAM with ZF filters.

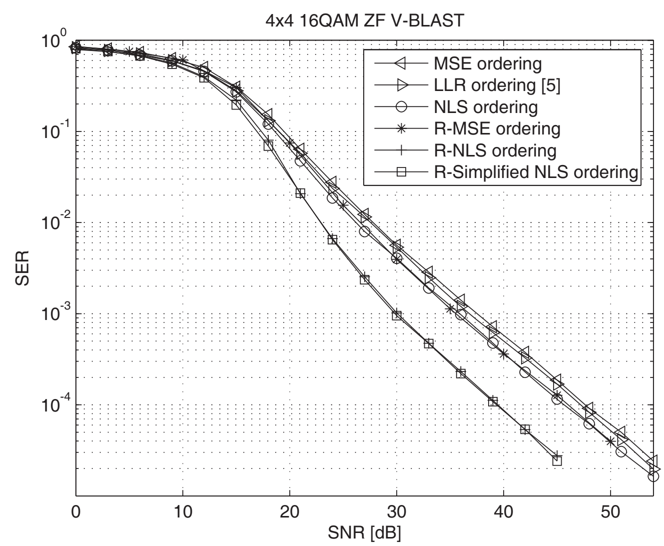

Fig. 2. Symbol error probability for 16QAM with ZF filters.

As a result, these simplified ordering (11) and (12) do not need to perform any candidate search for $\bar{s}_m$. Also, the complexity associated with this ordering is even lower than the LLR-based ordering. This ordering will be referred to as the simplified NLS ordering. The computation $|y_i - \beta_i \bar{s}_m|^2$ for 4PAM listed in Table I can be applied to the 16QAM case. It is straightforward to extend this method to other constellations.

In Table II, we present the number of floating point multiplications required in the proposed NLS ordering schemes for the case of $N_t = N_r = N$. We have assumed that the inversion of an L-by-L matrix takes $\mathcal{O}\left(L^3\right) \approx L^3$ multiplications [7]. Also, the operations of order smaller than N^2 are neglected for simplicity. We can see that the simplified NLS ordering shows the complexity savings of order N^2 over the NLS ordering.

V. SIMULATION RESULTS

In this section, we present the simulation results for various ordering schemes. Fig. 1 shows the SER for 4QAM with

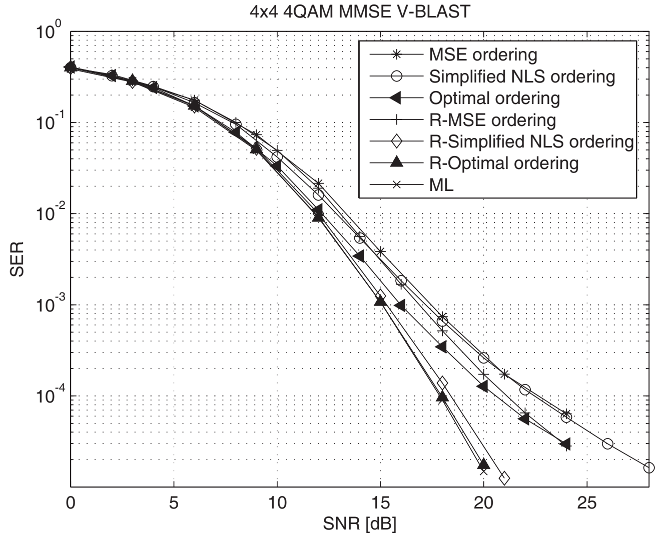

Fig. 3. Symbol error probability for 4QAM with MMSE filters.

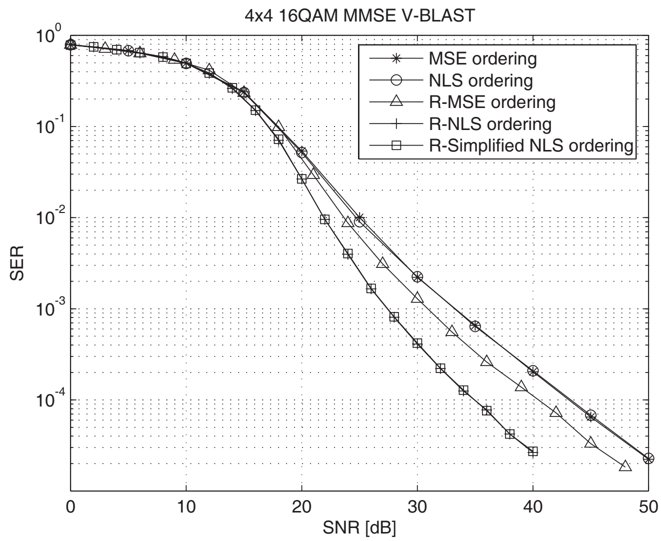

Fig. 4. Symbol error probability for 16QAM with MMSE filters.

$N_t = N_r = 4$ with zero forcing (ZF) filters. In the 4QAM case, the curve of the NLS ordering is not depicted since it shows almost the same performance as the simplified NLS ordering. In the legend, the prefix "R-" indicates the ordering scheme combined with the real-valued system. The application of the equivalent real-valued model to V-BLAST detection was introduced in [8]. The proposed NLS ordering scheme combined with the real-valued model outperforms the conventional MSE ordering by 13 dB at a SER of 10^{-4}. The gain of the proposed ordering scheme over the conventional LLR method increases from 4.5 dB to 11 dB at a SER of 10^{-4} by adopting the real-valued model instead of the complex-valued model. Similarly, Fig. 2 provides simulation results for the case of 16QAM. It is clear from the plot that the real-valued system provides a 6.5 dB gain when combined with the proposed ordering schemes.

In Figures 3 and 4, we consider the V-BLAST with an MMSE filter in the 4×4 system for 4QAM and 16QAM. When combined with the real-valued system, the proposed optimal ordering scheme achieves the optimal maximum likelihood (ML) performance. Also the performance of the proposed simplified NLS ordering scheme with much reduced complexity is just less than 1 dB away from the ML performance in Fig. 3. The simplified NLS ordering results for the 16QAM in Fig. 4 are obtained using Table I. Compared to the conventional MSE ordering, the proposed ordering achieves a 5 dB gain with only minor additional complexity.

VI. Conclusions

In this letter, we have proposed an enhanced ordering scheme for V-BLAST systems which minimizes the overall SER performance. By utilizing the whole filter output, we have derived the optimal ordering which extracts the exact reliability information of decisions. Also, the NLS ordering is proposed which requires much reduced complexity than the optimal ordering. Further, we have presented a simplified NLS ordering which can obtain a significant performance gain over the LLR-based ordering with reduced complexity. The simulation results confirm that the proposed ordering schemes are quite effective in reducing the effect of error propagation.

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