Transceiver Design Based on Blockwise Uniform Channel Decomposition for Coded MIMO Systems

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***Abstract*—In this paper, we investigate the transceiver design for coded multiple-input multiple-output (MIMO) systems assuming channel knowledge at both transmitter and receiver. First we derive an expression of the diversity order of singular value decomposition (SVD) based systems with arbitrary channel coding configurations. Motivated by this analysis, we propose a blockwise design based on uniform channel decomposition (UCD) which utilizes a successive interference cancellation (SIC) receiver. To eliminate error propagation inherent in the SIC structure, the proposed scheme applies the UCD precoder for a pair of subchannels to achieve single-symbol decodable maximum likelihood detection (MLD) instead of the SIC receiver. From the analysis, we demonstrate that the proposed scheme has an enhanced diversity order compared to the SVD scheme by exploiting the feature of the UCD. Also, in the presence of imperfect channel knowledge at the transmitter, we describe an appropriate receive filter design for the proposed scheme. The simulation results show that the proposed transceiver technique outperforms both the SVD scheme and the conventional UCD by about 6dB in 4 by 4 MIMO systems at the spectral efficiency of 12bps/Hz.**

***Index Terms*—Multiple-input multiple-output (MIMO), joint transceiver design, uniform channel decomposition (UCD), geometric mean decomposition (GMD), maximum likelihood detection (MLD), channel coded system.**

I. Introduction

IT is well known that multiple-input multiple-output (MIMO) systems are capable of improving the system performance as the number of antennas grows [1] [2]. The expected benefits include higher system capacity and improved quality of service (QoS) as a result of spatial multiplexing and diversity gain. In order to fully exploit both potentials of MIMO channels, the channel state information at the transmitter (CSIT) should be employed to optimize the transmission scheme according to instant channel conditions as well as the channel state information at the receiver (CSIR). The information theoretic analysis suggests that an additional performance gain can be extracted in the presence of the CSIT [3]. Based on the knowledge of the full CSIT, several precoding methods have been proposed [4].

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Most work on linear precoding and decoding for such closed-loop MIMO systems has been carried out based on singular value decomposition (SVD) of the channel transfer matrix. The SVD converts the MIMO channel into parallel subchannels on which multiple streams are transmitted. Although the SVD technique with water-filling is optimal from an information theoretic point of view [4], this requires complex bit allocation schemes because of vastly different signal-to-noise ratios (SNRs) of the subchannels.

Linear precoding and decoding schemes based on the SVD have been optimized in [5], [6] and [7] to enhance the error probability. The error performance of uncoded SVD systems was analyzed in terms of the diversity gain in [8] and [9], which showed that the subchannel with the smallest SNR dominates the performance. In most communication systems, channel coding is applied to enhance transmission reliability. It is shown in [10] that the SVD scheme combined with an optimized interleaver and a proper channel coding can provide a full diversity gain by compensating for the effect of the worst subchannel.

Recently, uniform channel decomposition (UCD) [11] has been proposed, which transforms the effective channel into an upper triangular matrix with equal gain diagonal elements. By employing a successive interference cancellation (SIC) receiver, the UCD scheme attains full diversity and thus exhibits better performance than the SVD scheme in uncoded systems. Also, it was shown that the UCD is the optimum design with the SIC receiver [12], while the SVD is optimal for linear receivers [7]. However, in practical wireless links, the conventional UCD scheme may not work well when channel coding is applied, since the decoder performance substantially degrades due to error propagation inherent in the SIC receiver. Various methods have been proposed to deal with the error propagation in the SIC receiver in open-loop systems [13] [14] [15], and the methods may be directly applied to the UCD schemes (e.g., [16]). However, those approaches are mostly based on iterative processing between detection and decoding, which is subject to increased complexity and latency issues.

In this paper, based on the results in [17] and [18], we propose a design framework for closed-loop MIMO systems. First, we analyze the maximum achievable diversity order of coded SVD systems with arbitrary system configurations. Our analysis determines a tradeoff between diversity gain and code rate in the coded SVD scheme. It is shown that the error performance of the SVD is limited by low diversity gains when high rate channel codes are employed. Motivated by this observation, we propose a transceiver design which applies the UCD precoder to a pair of subchannels of the effective channel matrix to enhance the performance. We demonstrate that a simple maximum-likelihood detection (MLD) is possible by utilizing the block diagonal property of the real-valued equivalent channel matrix in the proposed scheme. As a result, the proposed scheme attains single-symbol decodability, and is able to exploit a salient feature of the UCD without the error propagation problem.

In the proposed blockwise scheme, we combine two subchannels of a large singular value and a small singular value. Then parallel subchannels are generated which have smaller deviations in channel gains compared to the SVD scheme. Thus an improved gain of the worst subchannel results in the enhanced performance for the proposed scheme. A subspace beamforming technique adopting a similar idea was presented in [19]. From the analytical investigation, we also derive the diversity order of our blockwise technique combined with channel coding. The analysis shows that the proposed scheme achieves full diversity for a larger range of code rates compared to the SVD scheme. Finally, in the presence of channel estimation error at the transmitter, an appropriate receive filter design is investigated to maintain the block diagonality of the effective channel in the proposed technique. These analysis result and the receiver design presented in Sections V and VI have not been addressed in [17] and [18]. Simulation results demonstrate that the proposed scheme exhibits a performance advantage over both the SVD and the conventional UCD in terms of frame error rate (FER) and that the proposed scheme is robust to channel estimation errors.

The paper is organized as follows: Section II describes the system model for coded spatial multiplexing schemes in closed-loop MIMO systems. We analyze the maximum achievable diversity order of coded SVD systems in Section III. In Section IV, the proposed blockwise scheme with a single-symbol decodable MLD is presented, and the achievable diversity order of our scheme is analyzed in Section V. In Section VI, a receive filter for the blockwise scheme is proposed assuming imperfect CSIT. Section VII provides simulation results comparing the proposed method with the conventional schemes. Finally, the paper is terminated with conclusions in Section VIII.

II. System Model

In this section, we present a general description of coded precoding systems equipped with N_t transmit and N_r receive antennas as shown in Fig. 1, where it is assumed that N independent data streams are transmitted simultaneously ($N \leq \min\{N_t, N_r\}$)[1], and both transmitter and receiver have perfect CSI. Channel encoding is employed by means of bit-interleaved coded modulation (BICM) [20] [21], where a single channel encoder supports all transmit antennas through an interleaver as shown in Fig. 1 (a). At the transmitter, the information bits are encoded with a code rate R_c, bit-wise interleaved and modulated to produce N dimensional symbol vector $\mathbf{s}_k = [s_{k,1} \ \ s_{k,2} \cdots s_{k,N}]^T$, where the subscript k represents the kth time slot. We define $s_{k,i} \triangleq s^I_{k,i} + j s^Q_{k,i}$ where the superscripts I and Q denote the inphase and quadrature

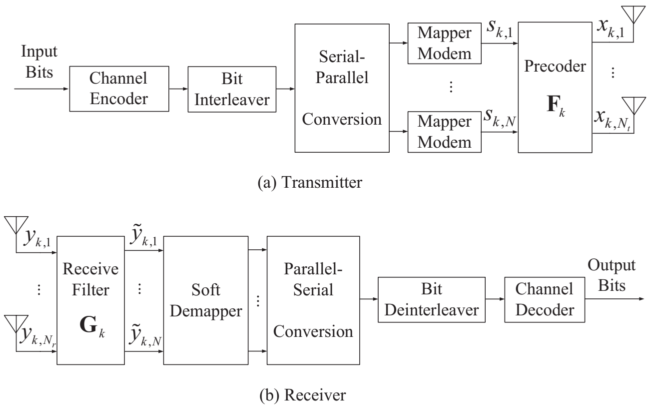

Fig. 1. Schematic diagram of a closed-loop spatial multiplexing scheme for N_t transmit and N_r receive antennas

component, respectively, and j indicates $\sqrt{-1}$. We use an M-QAM Gray mapping constellation $\mathcal{X}$ with $s^I_{k,i}$ and $s^Q_{k,i}$ chosen from a $\sqrt{M}$-PAM constellation. The spectral efficiency is then given by $R_c \cdot N \cdot \log_2 M$ bps/Hz.

Defining $\mathbf{F}_k$ as the $N_t \times N$ precoder matrix, the data symbol vector $\mathbf{s}_k$ is precoded by $\mathbf{F}_k$ to form the N_t dimensional complex transmit signal vector $\mathbf{x}_k = \mathbf{F}_k\mathbf{s}_k = [x_{k,1} \ x_{k,2} \ \cdots \ x_{k,N_t}]^T$. Denoting L as the size of the frame, the received signal vector $\mathbf{y}_k = [y_{k,1} \ y_{k,2} \ \cdots \ y_{k,N_r}]^T$ for $k = 1, 2, \cdots, L$ is written by

$$\begin{aligned} \mathbf{y}_k &= \mathbf{H}_k\mathbf{x}_k + \mathbf{w}_k \\ &= \mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{w}_k \end{aligned}$$

where $\mathbf{H}_k$ is the $N_r \times N_t$ channel matrix whose elements have independent and identically distributed (i.i.d.) complex Gaussian distribution with unit variance and $\mathbf{w}_k$ denotes the additive white Gaussian noise vector with zero mean and the covariance matrix $\sigma^2_w \mathbf{I}_{N_r}$. Here $\mathbf{I}_d$ indicates an identity matrix of size d. The autocorrelation matrix of $\mathbf{s}_k$ is assumed to be $E[\mathbf{s}_k\mathbf{s}_k^\dagger] = \sigma^2_s \mathbf{I}_N$ where $E[\cdot]$ accounts for expectation and $(\cdot)^\dagger$ denotes the complex conjugate transpose of a vector or matrix. We also assume $\text{Tr}(\mathbf{F}_k^\dagger\mathbf{F}_k) = N$, where $\text{Tr}(\cdot)$ indicates the trace of a matrix. The SNR ρ is defined as

$$\rho = \frac{E[\mathbf{s}_k^\dagger\mathbf{F}_k^\dagger\mathbf{F}_k\mathbf{s}_k]}{\sigma^2_w} = \frac{\sigma^2_s}{\sigma^2_w}\text{Tr}(\mathbf{F}_k^\dagger\mathbf{F}_k) = \frac{1}{\alpha}N$$

where $\alpha \triangleq \sigma^2_w/\sigma^2_s$.

At the receiver, the $N \times N_r$ receive filter $\mathbf{G}_k$ is applied into the received signal $\mathbf{y}_k$ and then its output $\tilde{\mathbf{y}}_k = \mathbf{G}_k\mathbf{y}_k = [\tilde{y}_{k,1} \ \tilde{y}_{k,2} \ \cdots \ \tilde{y}_{k,N}]^T$ is converted to soft values in the soft demapper as shown in Fig. 1 (b). The receive filter output signal vector $\tilde{\mathbf{y}}_k$ is expressed as

$$\tilde{\mathbf{y}}_k = \mathbf{G}_k\mathbf{y}_k = \mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{G}_k\mathbf{w}_k. \tag{1}$$

In this article, we will focus on flat fading channel models where the fading coefficients are static over a frame of transmitted symbols and independent over frames. Thus, the time index k is omitted for the rest of this paper for simplicity.

[1]The number of data streams N is determined by fulfilling both the data rate and the QoS required in a system.

III. Diversity Analysis of Coded SVD Schemes

In this section, we will provide an analysis on the diversity order of coded SVD schemes for arbitrary system configurations through the pairwise error probability (PEP) derivation. The SVD technique generates independent parallel substreams with gains equal to singular values. The SVD of $\mathbf{H}$ is then given by

$$\mathbf{H} = \underline{\mathbf{U}}\,\underline{\mathbf{\Lambda}}\,\underline{\mathbf{V}}^\dagger \triangleq [\mathbf{u}_1\ \mathbf{u}_2\ \cdots\ \mathbf{u}_{N_r}]\ \underline{\mathbf{\Lambda}}\ [\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_{N_t}]^\dagger \quad (2)$$

where $\underline{\mathbf{U}} \in \mathbb{C}^{N_r \times N_r}$ and $\underline{\mathbf{V}} \in \mathbb{C}^{N_t \times N_t}$ are unitary matrices, and $\underline{\mathbf{\Lambda}} \in \mathbb{R}^{N_r \times N_t}$ denotes a nonnegative matrix with the ith diagonal element equal to the ith largest singular value λ_i of $\mathbf{H}$ at entry (i,i).

Utilizing N subchannels for transmission, we define $\mathbf{U} \in \mathbb{C}^{N_r \times N}$ and $\mathbf{V} \in \mathbb{C}^{N_t \times N}$ as $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \cdots\ \mathbf{u}_N]$ and $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_N]$, respectively. Since power allocation at the transmitter cannot improve diversity order as shown in [8], the precoder $\mathbf{F}_{\text{SVD}} = \mathbf{V}$ and the receive filter $\mathbf{G}_{\text{SVD}} = \mathbf{U}^\dagger$ are used. The filter output signal $\tilde{\mathbf{y}}_{\text{SVD}} \triangleq \mathbf{U}^\dagger \mathbf{y}$ is then written as

$$\tilde{\mathbf{y}}_{\text{SVD}} = \mathbf{U}^\dagger \mathbf{H} \mathbf{V} \mathbf{s} + \mathbf{U}^\dagger \mathbf{w} = \mathbf{\Lambda}\,\mathbf{s} + \tilde{\mathbf{w}}$$

where $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_N\} \in \mathbb{R}^{N \times N}$ is a nonnegative diagonal matrix ($\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$) and the filtered noise $\tilde{\mathbf{w}}$ is denoted as $\tilde{\mathbf{w}} \triangleq \mathbf{U}^\dagger \mathbf{w} = [\tilde{w}_1\ \tilde{w}_2\ \cdots\ \tilde{w}_N]^T$. It is well known that the error performance of the SVD scheme is mainly limited by the subchannel with the smallest singular value λ_N [5] [8] [9]. In practice, the SVD schemes utilizing multiple streams should be combined with channel coding to guarantee the system performance. It was shown in [10] that the SVD scheme combined with the BICM structure can achieve full diversity under a specific assumption on the employed channel coding.

Now, we analyze the maximum diversity order for the coded SVD scheme with any coding configurations. We consider the average pairwise error probability (PEP) that ML decoder chooses the erroneous coded bit sequence $\hat{\underline{\mathbf{c}}}$ over the transmitted correct coded bit sequence $\underline{\mathbf{c}}$. If $\hat{\underline{\mathbf{s}}}$ and $\underline{\mathbf{s}}$ are the symbol sequences associated with the coded bit sequences $\hat{\underline{\mathbf{c}}}$ and $\underline{\mathbf{c}}$, respectively, then the PEP given $\mathbf{H}$ is expressed by

$$\begin{aligned} &P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}} \mid \mathbf{H}) \\ &= P\left(\sum_{k=1}^{L}\sum_{i=1}^{N}\left(|\tilde{y}_{k,i} - \lambda_i s_{k,i}|^2 - |\tilde{y}_{k,i} - \lambda_i \hat{s}_{k,i}|^2\right) > 0\right). \end{aligned}$$

Using a Chernoff bound after some mathematical manipulations, it follows

$$P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}} \mid \mathbf{H}) \leq \exp\left(-\frac{\sigma_s^2}{4\sigma_w^2}\sum_{i=1}^{N}\lambda_i^2 d_{E,i}\right) \quad (3)$$

where $d_{E,i} \triangleq \sum_{k=1}^{L} |s_{k,i} - \hat{s}_{k,i}|^2 / \sigma_s^2$ is defined as the sum of the normalized squared Euclidean distances computed on all the symbols transmitted over the ith spatial subchannel. Here, we are primarily interested in the maximum achievable diversity order and the system parameters which affect it.

By noting that the maximal achievable diversity order depends on the worst case of all possible combinations of $\hat{\underline{\mathbf{c}}}$ and $\underline{\mathbf{c}}$, we will show that the diversity order is determined only by the largest singular value with nonzero Euclidean distance sum $d_{E,i}$. Let us define p as the maximum number of the spatial subchannels with zero $d_{E,i}$. In other words, in the worst case p subchannels result in $d_{E,i} = 0$. In an uncoded system, since the minimum Hamming distance between $\hat{\underline{\mathbf{c}}}$ and $\underline{\mathbf{c}}$ is one, there exists only one nonzero $d_{E,i}$ in the worst case. Thus, p in uncoded systems always equals $N-1$. However, if channel coding is employed, p can be smaller than or equal to $N-1$.

If the singular values are placed in a decreasing order, the worst case occurs in the instance where $d_{E,i}$ becomes zero for $1 \leq i \leq p$ and nonzero for $p+1 \leq i \leq N$. In this case, we get

$$\sum_{i=1}^{N}\lambda_i^2 d_{E,i} \ = \sum_{i=p+1}^{N}\lambda_i^2 d_{E,i} \ \geq \ d_{E,\min}\sum_{i=p+1}^{N}\lambda_i^2 \quad (4)$$

where $d_{E,\min}$ denotes the minimum of all nonzero $d_{E,i}$ values. Substituting (4) into the bound (3), the PEP given $\mathbf{H}$ is upper-bounded by

$$P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}|\mathbf{H}) \leq P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}|\mathbf{H}) \triangleq \exp\left(-\frac{\rho d_{E,\min}}{4N}\sum_{i=p+1}^{N}\lambda_i^2\right).$$

It is clear that $P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}})$ determines the diversity order of the average error probability since $P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}})$ represents the worst case of all possible combinations of $\hat{\underline{\mathbf{c}}}$ and $\underline{\mathbf{c}}$.

Using the bounds $(N-p)\lambda_{p+1}^2 \geq \sum_{i=p+1}^{N}\lambda_i^2 \geq \lambda_{p+1}^2$, we now have

$$\begin{aligned} &\exp\left(-\frac{\rho d_{E,\min}}{4N}(N-p)\lambda_{p+1}^2\right) \\ &\qquad \leq P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}|\mathbf{H}) \leq \exp\left(-\frac{\rho d_{E,\min}}{4N}\lambda_{p+1}^2\right). \end{aligned} \quad (5)$$

Note the ith squared singular value λ_i^2 of $\mathbf{H}$ equals the ith largest eigenvalue μ_i of $\mathbf{H}^\dagger\mathbf{H}$ or $\mathbf{H}\mathbf{H}^\dagger$. Thus, taking an expectation in (5) with respect to $\mathbf{H}$ results in

$$\begin{aligned} &\int_0^\infty \exp\left(-\frac{\rho d_{E,\min}}{4N}(N-p)\mu_{p+1}\right) f_{\underline{\mu}_{p+1}}(\mu_{p+1})\,d\mu_{p+1} \\ &\leq P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}) \leq \int_0^\infty \exp\left(-\frac{\rho d_{E,\min}}{4N}\mu_{p+1}\right) f_{\underline{\mu}_{p+1}}(\mu_{p+1})\,d\mu_{p+1} \end{aligned} \quad (6)$$

where $f_{\underline{\mu}_i}(\mu_i)$ denotes the marginal probability density function (pdf) of μ_i.

The first-order expansion of the marginal pdf $f_{\underline{\mu}_i}(\mu_i)$ can be approximated for small value by [8] [9] [10]

$$f_{\underline{\mu}_i}(\mu_i) \approx \kappa \mu_i^{(N_t-i+1)(N_r-i+1)-1} \quad \text{for } \mu_i \to 0 \quad (7)$$

where κ is a constant. As shown in [8] and [22], the error probability at high SNR depends only on the first-order expansion of the marginal pdf of μ_{p+1} since as $\mu_{p+1} \to \infty$, the indefinite integral in (6) approaches zero due to an exponential factor with a negative exponent.

Hence, by substituting (7) to (6), the upper bound of the PEP at high SNR is expressed as

$$\begin{aligned} &\gamma_1 \cdot \left(\frac{(N-p)d_{E,\min}}{4N}\rho\right)^{-(N_t-p)(N_r-p)} \\ &\qquad \leq P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}) \leq \gamma_2 \cdot \left(\frac{d_{E,\min}}{4N}\rho\right)^{-(N_t-p)(N_r-p)} \end{aligned} \quad (8)$$

where γ_1 and γ_2 are constants. We now see that the diversity orders in both bounds are equal to $(N_t - p)(N_r - p)$. Consequently, the maximum achievable diversity order is given as $(N_t - p)(N_r - p)$. [2] In the following theorem, by determining p for various code rates R_c, we derive a general expression on the diversity order for coded SVD systems.

Theorem 1: For coded SVD schemes with code rate R_c and N subchannels in N_t by N_r MIMO systems, the maximum achievable diversity order is given by

$$D_{\text{SVD}} = (N_t - \lceil R_c \cdot N \rceil + 1)(N_r - \lceil R_c \cdot N \rceil + 1) \qquad (9)$$

where $\lceil x \rceil$ equals the smallest integer value not less than x.

Proof: Consider a convolutional code as its equivalent block code by terminating codeword sequences. Then, neglecting the number of the terminating bits, a rate R_c convolutional code with input word length m can be viewed as a $(m/R_c, m)$ block code. This block code consists of 2^m codewords with length m/R_c. Without loss of generality, consider a linear systematic block code where a codeword is divided into the information part and the parity part. If we divide a codeword of length m/R_c into N subcodewords of length $m_s = m/(R_c \cdot N)$, then the number of subcodewords to cover the information part is $\lceil m/m_s \rceil = \lceil R_c \cdot N \rceil$. In the worst case, the information part has only one non-zero weight subcodeword, and there are $\lceil R_c \cdot N \rceil - 1$ subcodewords with zero weight in the information part. From the definition of $d_{E,i}$, the number of the subcodewords with zero Hamming distance is equal to the number of the subchannels with zero Euclidean distance $d_{E,i}$. Thus, we obtain the maximum number of the subchannels with zero $d_{E,i}$ as

$$p = \lceil R_c \cdot N \rceil - 1. \qquad (10)$$

Then, from the result in (8), we get $D_{\text{SVD}} = (N_t - \lceil R_c \cdot N \rceil + 1)(N_r - \lceil R_c \cdot N \rceil + 1)$. ■

Note that the above result matches with the Singleton bound [24] [25]. To illustrate this result, we plot the maximum achievable diversity order for various code rates for different antenna configurations in Fig. 2. In order to achieve the full diversity order of $N_t \cdot N_r$ for SVD systems, the condition $R_c \cdot N \leq 1$ should be satisfied in (9), i.e., we should choose the code rate R_c less than or equal to $1/N$. According to this theorem, we can see that the SVD schemes with $N_t = N_r = N$ fail to achieve any diversity gain when codes with R_c higher than $(N-1)/N$ is employed. For example, the SVD schemes with $N_t = N_r = N = 2$ have a diversity order of 1 when $R_c > 1/2$. In the case of $R_c = 1$, Theorem 1 indicates $D_{\text{SVD}} = (N_t - N + 1)(N_r - N + 1)$, which agrees with the diversity analysis for uncoded SVD systems in [8] and [9].

Consequently, Theorem 1 provides an insight on the relation between code construction and the diversity order. Channel codes with high rate are normally employed to support high data rates when high spectral efficiencies are desirable. It is clear from the plot that the performance of the coded SVD scheme may suffer from the reduced diversity order when channel codes with high code rates are applied. This analysis will be confirmed through simulations in the simulation section. In the following section, we introduce a transceiver scheme which can compensate for the diversity loss in coded MIMO systems.

[2]This result is more general than the analysis in [10] which presents only the case achieving full diversity of $N_t \cdot N_r$. In [10], the code polynomial and the interleaver are assumed to be chosen such that each subchannel created by SVD is utilized at least once within distinct bits between all $\underline{c}$ and $\hat{\underline{c}}$, and thus p becomes zero. However, the assumption of [10] cannot be satisfied for all error patterns if the code rate is higher than $1/N$ as shown in [23].

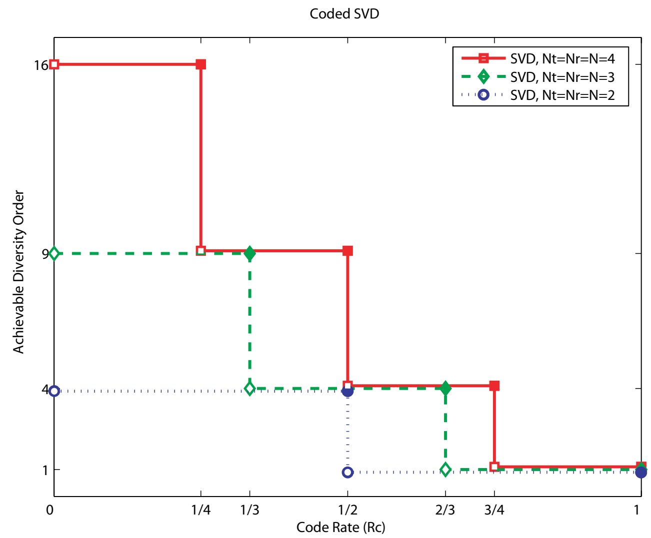

Fig. 2. Maximum achievable diversity order of the SVD scheme with various code rate R_c

IV. Blockwise Uniform Channel Decomposition

In this section, we briefly review the conventional UCD scheme introduced in [11] which is the optimal transceiver structure with the SIC receiver and achieves the full diversity gain. Then, we present a blockwise UCD (BL-UCD) technique which eliminates the error propagation issue in the conventional UCD scheme.

A. Review of UCD Schemes

In [11], the UCD scheme is combined with a SIC receiver based on the minimum mean-squared-error (MMSE) criterion. First consider the optimal precoder from an information theoretic point of view designed by $\mathbf{V\Phi}$, where $\mathbf{\Phi} \in \mathbb{R}^{N\times N}$ denotes a power allocation matrix as $\mathbf{\Phi} \triangleq \text{diag}\{\phi_1, \phi_2, \cdots, \phi_N\}$. Here ϕ_i is found via the water filling process as [2]

$$\phi_i = \left(\mu - \frac{\alpha}{\lambda_i^2}\right)_+^{\frac{1}{2}}$$

where $(a)_+ \triangleq \max\{0, a\}$ and μ is chosen such that $\sum_{i=1}^N \phi_i^2 = N$. This precoder yields the effective channel as $\mathbf{HV\Phi} = \mathbf{U\Sigma}$, where $\mathbf{\Sigma} \triangleq \mathbf{\Lambda\Phi}$ denotes an $N \times N$ nonnegative diagonal matrix as $\mathbf{\Sigma} = \text{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_N\}$ and $\sigma_i = \lambda_i\phi_i$.

Then, we define the $(N_r + N) \times N$ matrix $\mathbf{J}$ as

$$\mathbf{J} \triangleq \begin{bmatrix} \mathbf{U\Sigma} \\ \sqrt{\alpha}\mathbf{I}_N \end{bmatrix}.$$

This can be decomposed by geometric mean decomposition (GMD) [26] as

$$\mathbf{J} = \mathbf{Q}\mathbf{R}\mathbf{P}^\dagger \triangleq \begin{bmatrix} \mathbf{Q}_\mathrm{U} \\ \mathbf{Q}_\mathrm{L} \end{bmatrix} \mathbf{R}\mathbf{P}^\dagger \tag{11}$$

where $\mathbf{Q} \in \mathbb{C}^{(N_r+N)\times N}$ is a semi-unitary matrix (i.e., $\mathbf{Q}^\dagger\mathbf{Q} = \mathbf{I}_N$) which consists of an $N_r \times N$ submatrix $\mathbf{Q}_\mathrm{U}$ and an $N \times N$ submatrix $\mathbf{Q}_\mathrm{L}$, $\mathbf{P} \in \mathbb{C}^{N\times N}$ indicates a unitary matrix, and $\mathbf{R} \in \mathbb{R}^{N\times N}$ denotes an upper triangular matrix whose all diagonal elements equal the geometric mean value of the singular values of the matrix $\mathbf{J}$ (i.e., $[\mathbf{R}]_{ii} = \left(\prod_{n=1}^{N}\sqrt{\sigma_n^2+\alpha}\right)^{1/N}$ for $i = 1, 2, \cdots, N$). Here $[\mathbf{A}]_{ii}$ denotes the (i, i) element of a matrix $\mathbf{A}$.

Applying the precoder $\mathbf{F}_\mathrm{UCD} = \mathbf{V}\mathbf{\Phi}\mathbf{P}$ and the receive filter $\mathbf{G}_\mathrm{UCD} = \mathbf{Q}_\mathrm{U}^\dagger$ in (1) yields

$$\begin{aligned} \tilde{\mathbf{y}}_\mathrm{UCD} &= \mathbf{Q}_\mathrm{U}^\dagger\mathbf{H}\mathbf{V}\mathbf{\Phi}\mathbf{P}\mathbf{s} + \mathbf{Q}_\mathrm{U}^\dagger\mathbf{w} \\ &= \mathbf{R}\mathbf{s} - \sqrt{\alpha}\mathbf{Q}_\mathrm{L}^\dagger\mathbf{P}\mathbf{s} + \mathbf{Q}_\mathrm{U}^\dagger\mathbf{w}. \end{aligned}$$

Note that the residual interference-plus-noise vector $-\sqrt{\alpha}\mathbf{Q}_\mathrm{L}^\dagger\mathbf{P}\mathbf{s} + \mathbf{Q}_\mathrm{U}^\dagger\mathbf{w}$ can be assumed to have a Gaussian distribution with zero mean and covariance matrix $\sigma_w^2\mathbf{I}_N$ because $\mathbf{Q}_\mathrm{U}^\dagger\mathbf{Q}_\mathrm{U} + \mathbf{Q}_\mathrm{L}^\dagger\mathbf{Q}_\mathrm{L} = \mathbf{I}_N$. Since the effective channel $\mathbf{R}$ is an upper triangular matrix, the transmitted symbols are detected by the SIC operation. Then, the UCD operation results in equal subchannel gains for all diagonal elements of $\mathbf{R}$.

Consequently, it is shown in [11] that the UCD can obtain full diversity gain as $D_\mathrm{UCD} = N_t \cdot N_r$. However, the conventional UCD method suffers from a performance loss due to the error propagation inherent in the SIC process. Furthermore, in the coded systems this loss becomes more pronounced [14] [15]. In the following, we propose a blockwise structure based on the UCD precoder which does not need the SIC receiver.

B. Blockwise UCD Scheme

In this subsection, we propose the BL-UCD scheme to avoid the error propagation issue. Subblocks in our scheme are formed by pairing two subchannels and then are precoded by the UCD method in order to increase the minimum value of the subchannel gains. The real-valued representation of the effective channel allows a simple MLD which achieves single-symbol decodability.

For simplicity of notation, we assume that the number of transmitted streams N is even. By rearranging the order of the first N singular values in (2), we define the reordered $\bar{\mathbf{U}}$, $\bar{\mathbf{V}}$ and $\bar{\mathbf{\Lambda}}$ as $\bar{\mathbf{U}} \triangleq [\mathbf{u}_1\, \mathbf{u}_N\, \mathbf{u}_2\, \mathbf{u}_{N-1} \cdots \mathbf{u}_{N/2}\, \mathbf{u}_{N/2+1}] \in \mathbb{C}^{N_r\times N}$, $\bar{\mathbf{V}} \triangleq [\mathbf{v}_1\ \mathbf{v}_N\ \mathbf{v}_2\ \mathbf{v}_{N-1} \cdots \mathbf{v}_{N/2}\ \mathbf{v}_{N/2+1}] \in \mathbb{C}^{N_t\times N}$ and $\bar{\mathbf{\Lambda}} \triangleq \mathrm{diag}\{\lambda_1, \lambda_N, \lambda_2, \lambda_{N-1}, \cdots, \lambda_{N/2}, \lambda_{N/2+1}\} \in \mathbb{R}^{N\times N}$, respectively.

The proposed BL-UCD scheme employs the precoder $\mathbf{F}_\mathrm{BL} \triangleq \bar{\mathbf{V}}\bar{\mathbf{\Phi}}\mathbf{P}_\mathrm{BL}$ where $\bar{\mathbf{\Phi}}$ indicates the $N \times N$ diagonal matrix with power loading parameters as $\bar{\mathbf{\Phi}} \triangleq \mathrm{diag}\{\phi_1, \phi_N, \phi_2, \phi_{N-1}, \cdots, \phi_{N/2}, \phi_{N/2+1}\}$. Here our goal is to find a unitary matrix $\mathbf{P}_\mathrm{BL}$ which improves the performance of the UCD with a single-symbol decodable MLD. By applying the precoder $\mathbf{F}_\mathrm{BL}$, the received signal $\mathbf{y}$ is given as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{F}_\mathrm{BL}\mathbf{s} + \mathbf{w} \\ &= \bar{\mathbf{U}}\bar{\mathbf{\Sigma}}\,\mathbf{P}_\mathrm{BL}\,\mathbf{s} + \mathbf{w} \end{aligned} \tag{12}$$

where $\bar{\mathbf{\Sigma}}$ is defined as $\bar{\mathbf{\Sigma}} \triangleq \bar{\mathbf{\Lambda}}\bar{\mathbf{\Phi}} = \mathrm{diag}\{\sigma_1, \sigma_N, \sigma_2, \sigma_{N-1}, \cdots, \sigma_{N/2}, \sigma_{N/2+1}\}$.

Denoting the ith 2×2 effective channel submatrix $\bar{\mathbf{\Sigma}}_i$ as $\bar{\mathbf{\Sigma}}_i \triangleq \mathrm{diag}\{\sigma_i, \sigma_{N-i+1}\}$ for $i = 1, 2, \cdots, N/2$, $\bar{\mathbf{\Sigma}}$ can be expressed as $\bar{\mathbf{\Sigma}} = \mathrm{diag}\{\bar{\mathbf{\Sigma}}_1, \bar{\mathbf{\Sigma}}_2, \cdots, \bar{\mathbf{\Sigma}}_{N/2}\}$. Note that the ith subblock $\bar{\mathbf{\Sigma}}_i$ pairs two subchannels with the ith largest and the ith smallest subchannel gain. This strategy will be shown to be effective in improving the diversity gain in Section V. For the ith subblock, we define the 4×2 matrix $\mathbf{J}_i$ as

$$\mathbf{J}_i \triangleq \begin{bmatrix} \bar{\mathbf{\Sigma}}_i \\ \sqrt{\alpha}\mathbf{I}_2 \end{bmatrix}.$$

Then the GMD operation [26] is applied to obtain

$$\mathbf{J}_i = \mathbf{Q}_i\mathbf{R}_i\mathbf{P}_i^\dagger$$

where $\mathbf{Q}_i \in \mathbb{R}^{4\times 2}$ is a semi-unitary matrix, $\mathbf{P}_i \in \mathbb{R}^{2\times 2}$ represents a unitary matrix and $\mathbf{R}_i \in \mathbb{R}^{2\times 2}$ denotes an upper triangular matrix with diagonal elements equal to the geometric mean value of the singular values of the matrix $\mathbf{J}_i$ (i.e., $[\mathbf{R}_i]_{jj} = \sqrt[4]{(\sigma_i^2+\alpha)(\sigma_{N-i+1}^2+\alpha)}$ for $j = 1, 2$). It should be noted that the computation of $\mathbf{Q}_i$ is not necessary since we will employ a simple MLD instead of the SIC receiver which requires $\mathbf{Q}_i$.

Then, the precoding matrix for the ith subblock $\mathbf{P}_i$ is given by a rotation matrix as [26]

$$\mathbf{P}_i = \begin{bmatrix} \mathcal{C}_i & \mathcal{S}_i \\ -\mathcal{S}_i & \mathcal{C}_i \end{bmatrix} \quad \text{for } i = 1, 2, \cdots, \frac{N}{2} \tag{13}$$

where

$$\mathcal{C}_i = \sqrt{\frac{\sqrt{(\sigma_i^2+\alpha)(\sigma_{N-i+1}^2+\alpha)} - (\sigma_{N-i+1}^2+\alpha)}{\sigma_i^2 - \sigma_{N-i+1}^2}}$$

and

$$\mathcal{S}_i = \sqrt{1-\mathcal{C}_i^2}.$$

Denoting the $N \times N$ matrix $\mathbf{P}_\mathrm{BL}$ as

$$\mathbf{P}_\mathrm{BL} \triangleq \mathrm{diag}\left\{\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_{\frac{N}{2}}\right\}, \tag{14}$$

the precoder and the receive filter of the proposed BL-UCD are designed as $\mathbf{F}_\mathrm{BL} = \bar{\mathbf{V}}\bar{\mathbf{\Phi}}\mathbf{P}_\mathrm{BL}$ and $\mathbf{G}_\mathrm{BL} = \bar{\mathbf{U}}^\dagger$, respectively. Applying the orthogonal matrix $\mathbf{P}_\mathrm{BL}$ in (14) and the receive filter $\bar{\mathbf{U}}^\dagger$ to (12) yields

$$\tilde{\mathbf{y}} = \bar{\mathbf{U}}^\dagger\mathbf{y} = \bar{\mathbf{\Sigma}}\,\mathbf{P}_\mathrm{BL}\,\mathbf{s} + \tilde{\mathbf{w}},$$

$$\begin{bmatrix} \tilde{\mathbf{y}}_1 \\ \tilde{\mathbf{y}}_2 \\ \vdots \\ \tilde{\mathbf{y}}_{\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_{\frac{N}{2}} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{w}}_1 \\ \tilde{\mathbf{w}}_2 \\ \vdots \\ \tilde{\mathbf{w}}_{\frac{N}{2}} \end{bmatrix} \tag{15}$$

where we have $\tilde{\mathbf{y}}_i \triangleq [\tilde{y}_{2i-1}\ \tilde{y}_{2i}]^T$, $\mathbf{s}_i \triangleq [s_{2i-1}\ s_{2i}]^T$ and $\tilde{\mathbf{w}}_i \triangleq [\tilde{w}_{2i-1}\ \tilde{w}_{2i}]^T$, and the 2×2 subblock matrix $\mathbf{B}_i$ is defined as $\mathbf{B}_i \triangleq \bar{\mathbf{\Sigma}}_i\mathbf{P}_i$. Thus the effective channel of the proposed scheme is easily separated into $N/2$ subblock channels as

$$\tilde{\mathbf{y}}_i = \mathbf{B}_i\mathbf{s}_i + \tilde{\mathbf{w}}_i \quad \text{for } i = 1, 2, \cdots, \frac{N}{2}.$$

It is important to note that since $\mathbf{B}_i$ is a 2×2 real matrix, the real-valued representation for the above ith subblock channel matrix can be equivalently written as [27] [28]

$$\begin{bmatrix} \tilde{\mathbf{y}}_i^I \\ \tilde{\mathbf{y}}_i^Q \end{bmatrix} = \begin{bmatrix} \mathbf{B}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_i \end{bmatrix} \begin{bmatrix} \mathbf{s}_i^I \\ \mathbf{s}_i^Q \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{w}}_i^I \\ \tilde{\mathbf{w}}_i^Q \end{bmatrix} \tag{16}$$

where the superscripts I and Q for a complex vector or matrix indicate the inphase and the quadrature parts of the vector or matrix, respectively (i.e., $\mathbf{b}^I \triangleq \Re\{\mathbf{b}\}$ and $\mathbf{b}^Q \triangleq \Im\{\mathbf{b}\}$).

Utilizing the block diagonality, the ML solution $\hat{\mathbf{s}}_i = \hat{\mathbf{s}}_i^I + j\hat{\mathbf{s}}_i^Q$ for $i = 1, 2, \cdots, N/2$ in equation (15) can be individually found as

$$\hat{\mathbf{s}}_i^I = \arg \min_{\mathbf{s}_i^I \in \mathcal{X}} \left|\left| \, \tilde{\mathbf{y}}_i^I - \mathbf{B}_i \, \mathbf{s}_i^I \right|\right|^2 \tag{17}$$

and

$$\hat{\mathbf{s}}_i^Q = \arg \min_{\mathbf{s}_i^Q \in \mathcal{X}} \left|\left| \, \tilde{\mathbf{y}}_i^Q - \mathbf{B}_i \, \mathbf{s}_i^Q \right|\right|^2 \tag{18}$$

where $\hat{\mathbf{s}}_i^I$ and $\hat{\mathbf{s}}_i^Q$ indicate $\hat{\mathbf{s}}_i^I = [\hat{s}_{2i-1}^I \ \hat{s}_{2i}^I]^T$ and $\hat{\mathbf{s}}_i^Q = [\hat{s}_{2i-1}^Q \ \hat{s}_{2i}^Q]^T$, and $\|\cdot\|$ denotes the Euclidean norm.

The proposed BL-UCD generates N independent parallel streams for the pair of two inphase or quadrature components, which are separated from two complex symbols. If N is odd, the last symbol s_N does not have any pair and has the channel gain for the middle singular value $\lambda_{(N+1)/2}$. In this case, the symbol s_N can be independently detected. The MLD equations in the real-valued representation (17) and (18) show that with the proposed scheme, the MLD can be done by searching for a single symbol (called *single-symbol decodable*), while the traditional MLD requires searching all transmitted symbols. Hence, the proposed BL-UCD method accomplishes both single-symbol decodability and the optimum ML performance for the effective channel at the receiver. Note again that the conventional UCD with channel coding needs more complex joint process between detection and decoding to reach the optimum performance, since the suboptimal SIC receiver is employed. In the next section, we investigate the error performance of the BL-UCD scheme by deriving the diversity order.

V. Diversity Analysis of Coded BL-UCD

In this section, the proposed BL-UCD scheme with channel coding will be analyzed in terms of diversity and be compared with conventional schemes. By applying a Chernoff bound for the PEP given $\mathbf{H}$ derived from the ML equations (17) and (18) similar to Section III, we have

$$P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}} \mid \mathbf{H}) \le \exp\left(-\frac{1}{4\sigma_w^2} \sum_{k=1}^{L} \sum_{i=1}^{N/2} \sum_{l \in \{I,Q\}} \left\| \mathbf{B}_i \, \mathbf{e}_{k,i}^l \right\|^2 \right) \tag{19}$$

where $\mathbf{e}_{k,i}^l$ is defined as $\mathbf{e}_{k,i}^l \triangleq \hat{\mathbf{s}}_{k,i}^l - \mathbf{s}_{k,i}^l = [e_{k,2i-1}^l \ e_{k,2i}^l]^T$ with $e_{k,j}^l \triangleq \hat{s}_{k,j}^l - s_{k,j}^l$.

To simplify the analysis, power loading is not considered ($\bar{\mathbf{\Phi}} = \mathbf{I}_N$). By using the equality $\|\mathbf{B}_i \, \mathbf{e}_{k,i}^l\|^2 = \lambda_i^2 \, (\mathcal{C}_i e_{k,2i-1}^l + \mathcal{S}_i e_{k,2i}^l)^2 + \lambda_{N-i+1}^2 \, (\mathcal{C}_i e_{k,2i}^l - \mathcal{S}_i e_{k,2i-1}^l)^2$, the above bound (19) is written as

$$P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}} \mid \mathbf{H}) \le \exp\left(-\frac{\sigma_s^2}{4\sigma_w^2} \sum_{i=1}^{N/2} \left(\lambda_i^2 \bar{d}_{E,i} + \lambda_{N-i+1}^2 \tilde{d}_{E,i} \right) \right) \tag{20}$$

where $\bar{d}_{E,i}$ and $\tilde{d}_{E,i}$ for $i = 1, 2, \cdots, N/2$ are denoted as

$$\bar{d}_{E,i} \triangleq \frac{1}{\sigma_s^2} \sum_{k=1}^{L} \sum_{l \in \{I,Q\}} \left(\mathcal{C}_i \, e_{k,2i-1}^l + \mathcal{S}_i \, e_{k,2i}^l \right)^2 \tag{21}$$

and

$$\tilde{d}_{E,i} \triangleq \frac{1}{\sigma_s^2} \sum_{k=1}^{L} \sum_{l \in \{I,Q\}} \left(\mathcal{C}_i \, e_{k,2i}^l - \mathcal{S}_i \, e_{k,2i-1}^l \right)^2 .$$

We define q as the maximum number of the paired subchannels with zero $\bar{d}_{E,i}$ related with λ_i^2 in (20) ($0 \le q < N/2$). Similar to Section III, we consider an upper bound of the PEP $P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}})$ with the worst case of all combinations of $\underline{\mathbf{c}}$ and $\hat{\underline{\mathbf{c}}}$. In this case, we get $\bar{d}_{E,i} = 0$ for $1 \le i \le q$ and $\bar{d}_{E,i} > 0$ for $q+1 \le i \le N/2$ in (20) due to the order of the singular values $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_N$. Then, we obtain the bounds as

$$\bar{d}_{E,q+1}\lambda_{q+1}^2 \le \sum_{i=1}^{N/2} \left(\lambda_i^2 \, \bar{d}_{E,i} + \lambda_{N-i+1}^2 \, \tilde{d}_{E,i} \right) \le (N-q)\bar{d}_{E,\max}\lambda_{q+1}^2 \tag{22}$$

where $\bar{d}_{E,\max}$ denotes the maximum of all $\bar{d}_{E,i}$ and $\tilde{d}_{E,i}$ values. By substituting the bounds (22) into (20) and taking the expectation with respect to $\mathbf{H}$, the PEP bounds at high SNR are given by

$$\bar{\gamma}_1 \cdot \left(\frac{\bar{d}_{E,\max}}{4N} (N-q)\rho \right)^{-(N_t-q)(N_r-q)} \le P_u(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}) \le \bar{\gamma}_2 \cdot \left(\frac{\bar{d}_{E,q+1}}{4N} \rho \right)^{-(N_t-q)(N_r-q)} \tag{23}$$

where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are constants. It is clear from this result that the diversity order of the coded BL-UCD is $(N_t - q)(N_r - q)$. Now, deriving a general expression of q, the diversity order of the BL-UCD is determined in the following theorem.

Theorem 2: For coded BL-UCD schemes with code rate R_c and N subchannels in N_t by N_r MIMO systems, the maximum achievable diversity order is given by

$$D_{\text{BL-UCD}} = \left(N_t - \left\lceil R_c \cdot \frac{N}{2} \right\rceil + 1 \right) \left(N_r - \left\lceil R_c \cdot \frac{N}{2} \right\rceil + 1 \right).$$

Proof: In the BL-UCD scheme, each coded frame is separated into $N/2$ subcodewords, where the ith subcodeword which affects the value of $\bar{d}_{E,i}$ is transmitted through the ith effective channel $\mathbf{B}_i$. If proper $\mathcal{C}_i$ and $\mathcal{S}_i$ in (21) are determined by the UCD operation in (13), $\bar{d}_{E,i}$ becomes zero if and only if the Hamming distance of the ith subcodeword equals zero. Hence, by considering that there are $N/2$ subcodewords in (10), q is equal to $\lceil R_c \cdot N/2 \rceil - 1$. From (23), we have $D_{\text{BL-UCD}} = (N_t - \lceil R_c \cdot N/2 \rceil + 1)(N_r - \lceil R_c \cdot N/2 \rceil + 1)$. ■

In Fig. 3, we plot the diversity order $D_{\text{BL-UCD}}$ with respect to code rates. Comparing Fig. 2 and Fig. 3, we can see that the proposed BL-UCD has a diversity advantage over the SVD scheme at code rates higher than $1/N$. The higher the code rate is, the more gain the BL-UCD achieves. It is also interesting to note from Fig. 3 that for code rates of $R_c > (N-1)/N$, the BL-UCD is able to attain diversity gains as opposed to the SVD scheme. For example, for the BL-UCD scheme with

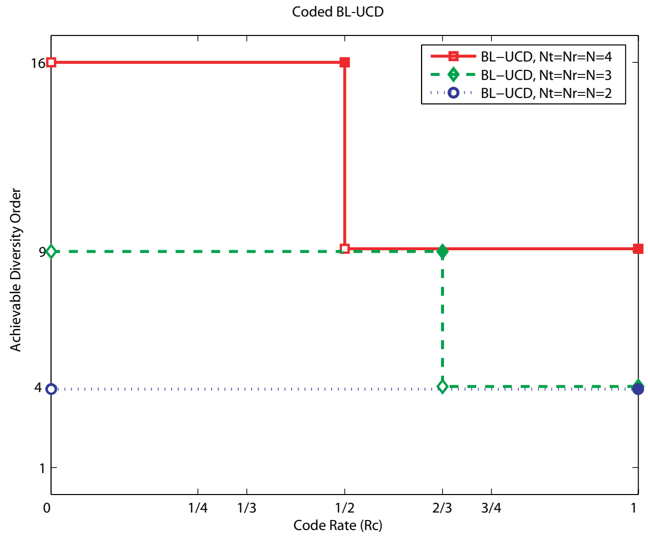

Fig. 3. Maximum achievable diversity order of the BL-UCD scheme with various code rate R_c

$N_t = N_r = N = 2$, a full diversity order of 4 is obtained regardless of code rates. This is a natural result, since in the $N = 2$ case the BL-UCD is exactly the same as the conventional UCD with a joint ML receiver.

Since the ith largest and the ith smallest singular value are combined by the UCD precoder in the ith subblock in (15), the worst effective SNR of the paired subchannels in the proposed BL-UCD is greater than that of the subchannels obtained from the SVD. From the analysis, we can see that two paired subchannels in each subblock have the diversity order corresponding to a larger singular value as a result of the UCD process. Consequently, our strategy which combines the largest and the smallest subchannel gains is effective in terms of the achievable diversity order. Since the lowest diversity order of subchannels increases, we expect that the proposed scheme provides the improved error probability, and this will be confirmed in the simulation section.

The proposed technique and the conventional schemes are summarized in Table I for the case of $N_t = N_r = N$. Recall that the conventional UCD with channel coding can achieve the full diversity N^2 when the joint MLD is employed instead of the SIC due to an error propagation problem. The proposed BL-UCD needs to compute the 2×2 matrices $\mathbf{P}_1$, $\mathbf{P}_2$, $\cdots$, $\mathbf{P}_{N/2}$ from (13) in order to obtain the precoding matrix $\mathbf{P}_{\mathrm{BL}}$, while the conventional UCD should calculate the $N \times N$ matrix $\mathbf{P}$ and the $N_r \times N$ matrix $\mathbf{Q}_{\mathrm{U}}$ using the GMD operation in (11). Hence, as listed in Table I, the computational complexity of the BL-UCD related to the precoder and the receive filter is smaller than that of the conventional UCD.

VI. Receive Filter Design of BL-UCD with Imperfect CSIT

So far, the CSI has been assumed to be perfect at both the transmitter and the receiver. However, in practical systems, the CSIT may not be accurate due to an inherent delay between the channel estimation and the transmission of data. Also, incorrect CSIT is caused by either a limited feedback in frequency division multiplex (FDD) systems or imperfect reciprocity in time division multiplex (TDD) systems. For the SVD systems with the imperfect CSIT, it is shown in [29] that the MMSE linear filter which can suppress the non-diagonal terms in the effective channel matrix should be employed at the receiver instead of the left singular matrix $\mathbf{U}^\dagger$ to mitigate the performance degradation. In this section, we illustrate an appropriate receive filter design for the BL-UCD scheme in the presence of imperfect CSIT.

We consider the channel estimation error model in [30] as $\mathbf{H} = \mathbf{H}_{\mathrm{est}} + \mathbf{E}$ where $\mathbf{H}$, $\mathbf{H}_{\mathrm{est}}$ and $\mathbf{E}$ represent the true channel matrix, the estimated channel matrix and the estimation error matrix, respectively. We assume that the entries of $\mathbf{E}$ are independent of $\mathbf{H}$, and have independent complex Gaussian distributions with mean zero and the estimation error variance σ_e^2. In this channel model, the received signal vector of the BL-UCD scheme (12) is modified as

$$\mathbf{y} = \mathbf{H}\mathbf{F}_{\mathrm{BL}}^{\mathrm{est}}\mathbf{s} + \mathbf{w} \triangleq \mathbf{H}_{\mathrm{eff}}\mathbf{s} + \mathbf{w} \tag{24}$$

where $\mathbf{F}_{\mathrm{BL}}^{\mathrm{est}}$ denotes the BL-UCD precoder calculated from the estimated channel $\mathbf{H}_{\mathrm{est}}$. Now, at the receiver, the left singular matrix $\bar{\mathbf{U}}^\dagger$ cannot decompose the effective channel into a real-valued block diagonal matrix such as in (15) due to the mismatch in $\mathbf{F}_{\mathrm{BL}}^{\mathrm{est}}$.

To make the effective channel block diagonal, we apply the MMSE block diagonal (MMSE-BD) filter which was proposed in [31] to suppress interferences in multi-user MIMO downlink channels. The real-valued equivalent representation of (24) is given as [27] [28]

$$\begin{aligned}\begin{bmatrix}\mathbf{y}^I \\ \mathbf{y}^Q\end{bmatrix} &= \begin{bmatrix}\mathbf{H}_{\mathrm{eff}}^I & -\mathbf{H}_{\mathrm{eff}}^Q \\ \mathbf{H}_{\mathrm{eff}}^Q & \mathbf{H}_{\mathrm{eff}}^I\end{bmatrix}\begin{bmatrix}\mathbf{s}^I \\ \mathbf{s}^Q\end{bmatrix} + \begin{bmatrix}\mathbf{w}^I \\ \mathbf{w}^Q\end{bmatrix} \\ &\triangleq \widetilde{\mathbf{H}}_{\mathrm{eff}}\begin{bmatrix}\mathbf{s}^I \\ \mathbf{s}^Q\end{bmatrix} + \begin{bmatrix}\mathbf{w}^I \\ \mathbf{w}^Q\end{bmatrix}.\end{aligned} \tag{25}$$

Now, the MMSE linear filter for the real-valued effective channel $\widetilde{\mathbf{H}}_{\mathrm{eff}}$ is calculated by

$$\widetilde{\mathbf{W}} = \left[\widetilde{\mathbf{W}}_1^T \ \widetilde{\mathbf{W}}_2^T \cdots \widetilde{\mathbf{W}}_N^T\right]^T = \widetilde{\mathbf{H}}_{\mathrm{eff}}^T\left(\widetilde{\mathbf{H}}_{\mathrm{eff}}\widetilde{\mathbf{H}}_{\mathrm{eff}}^T + \alpha\mathbf{I}_{2N}\right)^{-1}$$

where $\widetilde{\mathbf{W}}$ is a $2N \times 2N_r$ real matrix and $\widetilde{\mathbf{W}}_i$ is the ith $2 \times 2N_r$ submatrix of $\widetilde{\mathbf{W}}$. Then, the real-valued MMSE-BD filter for the BL-UCD $\widetilde{\mathbf{G}}_{\mathrm{BL}}$ is given as [31]

$$\widetilde{\mathbf{G}}_{\mathrm{BL}} = \left[\widetilde{\mathbf{Q}}_1 \ \widetilde{\mathbf{Q}}_2 \cdots \widetilde{\mathbf{Q}}_N\right]^T \tag{26}$$

where $\widetilde{\mathbf{G}}_{\mathrm{BL}}$ is a $2N \times 2N_r$ real matrix and $\widetilde{\mathbf{Q}}_i$ denotes the $2N_r \times 2$ orthonormal matrix spanning the column space of $\widetilde{\mathbf{W}}_i^T$. Thus, the orthonormal matrix $\widetilde{\mathbf{Q}}_i$ for $i = 1, 2, \cdots, N$ can be obtained by QR decomposition $\widetilde{\mathbf{W}}_i^T = \widetilde{\mathbf{Q}}_i\widetilde{\mathbf{R}}_i$, where $\widetilde{\mathbf{R}}_i$ is a 2×2 upper triangular matrix.

By multiplying this receive filter $\widetilde{\mathbf{G}}_{\mathrm{BL}}$ to the real-valued received signal in (25), the effective channel is decomposed into block-diagonal matrices in the real-valued representation such as (16) while residual interferences remain due to the MMSE process. Consequently, we can accomplish the single-symbol decodable MLD such as in (17) and (18) by separating the effective channel into subblock channels, although the soft

TABLE I
COMPARISON OF CLOSED-LOOP MIMO SYSTEMS WITH $N_t = N_r = N$

	SVD	BL-UCD	Conventional UCD
Precoder $\mathbf{F}$	$\mathbf{V}$	$\bar{\mathbf{V}}\bar{\mathbf{\Phi}}\mathbf{P}_{\text{BL}}$	$\mathbf{V\Phi P}$
Receive Filter $\mathbf{G}$	$\mathbf{U}^\dagger$	$\bar{\mathbf{U}}^\dagger$	$\mathbf{Q}_{\text{U}}^\dagger$
Computational Complexity	$O\left(N^3\right)$	$O\left(N^3+2N^2+6N\right)$	$O\left(N^3+7N^2+2N\right)$
Effective Channel	diagonal	block diagonal (real-valued)	upper triangular
Detection Method	linear receiver	single-symbol decodable MLD	SIC
Diversity Order	$(N-\lceil R_c \cdot N\rceil+1)^2$	$\left(N-\left\lceil R_c \cdot \frac{N}{2}\right\rceil+1\right)^2$	N^2 (w/ joint MLD)

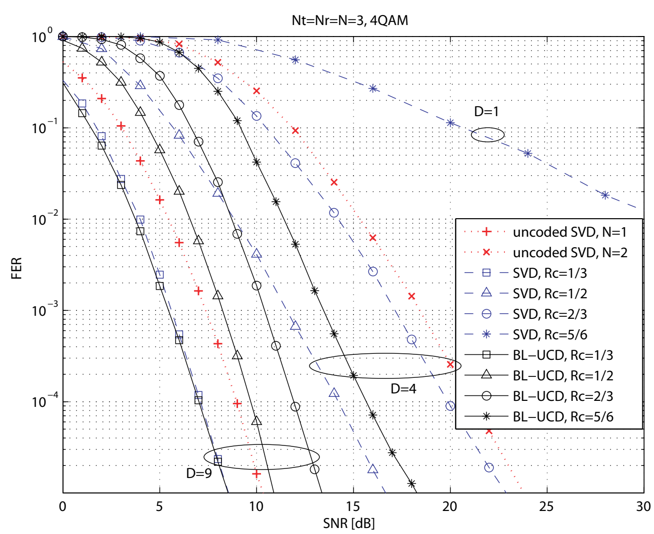

Fig. 4. Performance comparison for various code rates in 3×3 MIMO systems over flat fading channels

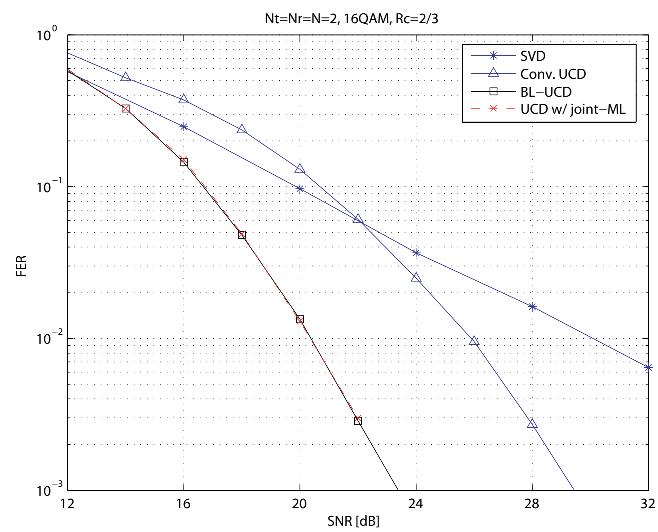

Fig. 5. Performance comparison in 2×2 MIMO systems over flat fading channels

demapper should handle the non-white residual interference-plus-noise term. The performance of the BL-UCD combined with this MMSE-BD filter in the presence of channel estimation error will be evaluated through simulations in the next section.

VII. Simulation Results

In this section, we present simulation results to support the analysis derived in the previous section and compare the proposed blockwise scheme with conventional systems in flat fading channels. Throughout the paper, we employ a rate $1/3$ convolutional encoder with polynomials (133,171,145) in octal notation as a mother code where high rate codes are obtained by the puncturing patterns in [32].[3] To make a fair comparison, we use the interleaver optimized for the SVD scheme in [10] which transmits consecutive coded bits over different subchannels and randomly interleaves the bits in each subchannel. The size of the interleaver is determined by $L \cdot N \cdot \log_2 M$ and the frame length L is set to 64 in all simulations.

[3]The channel codes employed in this paper have the minimum Hamming distances $d_{\text{free}} = 14$, 10, 6, 5 and 4 for $R_c = 1/3$, $1/2$, $2/3$, $3/4$ and $5/6$, respectively.

Fig. 4 shows the performance of the proposed BL-UCD and the SVD schemes with various code rates in the case of $N_t = N_r = N = 3$ to demonstrate the validity of our diversity analysis. Also, the curves of uncoded SVD systems with one and two subchannels (dotted lines) are provided as references for the diversity order 9 and 4, respectively, which was proved in [8] and [9]. The dashed lines for the coded SVD scheme exhibit distinct performance slopes for each code rate, where the curves with code rates of $1/3$, $1/2$, $2/3$, and $5/6$ correspond to the diversity orders of 9, 4, 4 and 1, respectively. This matches well with the diversity analysis derived in Section III. Also, the BL-UCD schemes show the FER slopes of 9, 9, 9 and 4 at high SNR for the same code rates, as predicted in Section V. Consequently, this plot confirms the accuracy of our analysis.

In Fig. 5, the BL-UCD is compared with the conventional schemes for 2×2 MIMO systems with 16-QAM and $R_c = 2/3$. Note that the proposed schemes with $N = 2$ use the same precoder as the UCD and employ a single-symbol decodable MLD over the real-valued blockwise channels at the receiver. This plot shows that the proposed MLD provides the identical performance as the UCD combined with the joint MLD while the receiver complexity is reduced. The conventional UCD scheme exhibits a performance loss of about 5.5dB at 1% FER

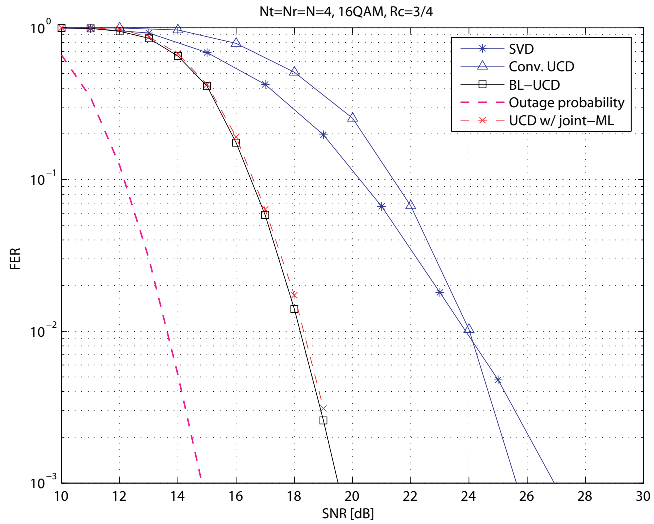

Fig. 6. Performance comparison in 4×4 MIMO systems over flat fading channels

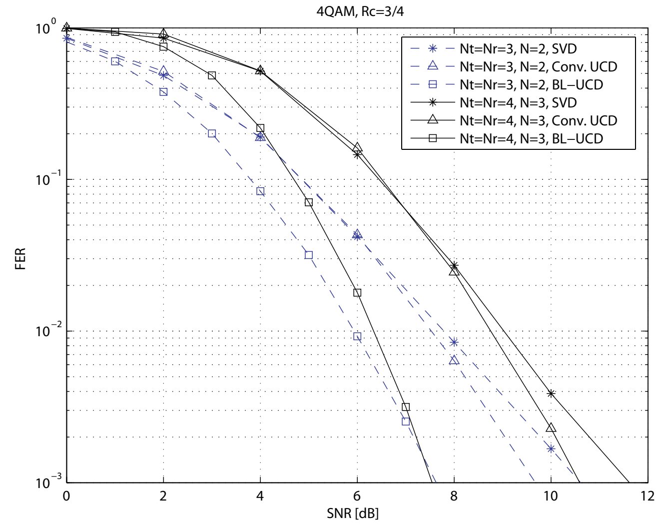

Fig. 8. Performance comparison with partial transmission over flat fading channels

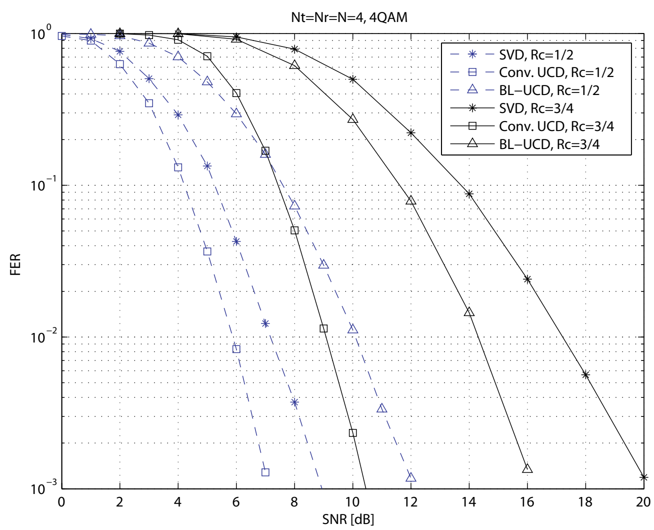

Fig. 7. Performance comparison for various code rates in 4×4 MIMO systems over flat fading channels

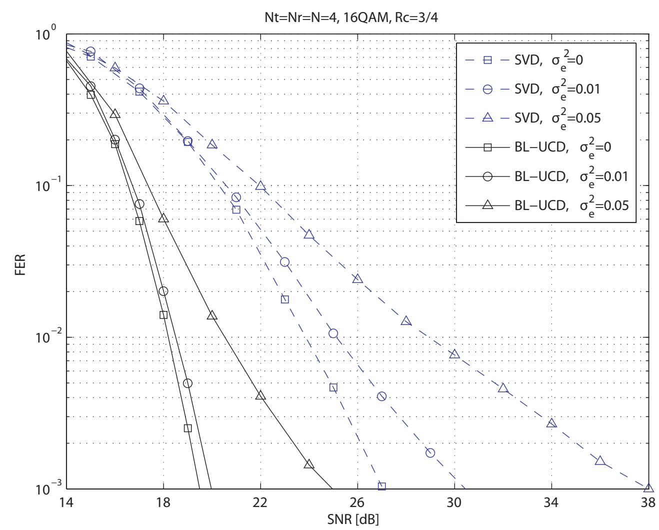

Fig. 9. Effect of imperfect CSIT over flat fading channels

due to error propagation in the SIC. Also, the BL-UCD scheme outperforms the SVD scheme which shows no diversity gain by about 10dB at 1% FER.

Fig. 6 depicts the performance comparison in 4×4 MIMO systems with 16-QAM and $R_c = 3/4$ which provide the spectral efficiency of 12bps/Hz. The BL-UCD scheme outperforms both the SVD scheme and the conventional UCD scheme by about 6dB at 1% FER, and still shows the similar FER to the UCD with the joint MLD. It is expected that at high SNR region the BL-UCD will become inferior because of the reduced diversity order. We also plot the outage probability of the closed-loop capacity for comparison. The proposed scheme performs within 4.5dB away from the outage probability. If more powerful channel codes like Turbo code or low density parity check (LDPC) codes are employed, the gap should be reduced.

Fig. 7 compares the FER performance for 4×4 systems with 4-QAM and code rates of $1/2$ and $3/4$. The dashed lines represents the cases with $R_c = 1/2$, and the SVD has better performance than the conventional UCD since the conventional UCD still suffers from error propagation. In this system configuration, the BL-UCD scheme exhibits a higher diversity gain than the SVD as predicted in the analysis, and the proposed BL-UCD scheme outperforms the SVD scheme and the conventional UCD technique by 1.5dB and 4dB at 1% FER, respectively. In the case of $R_c = 3/4$, the BL-UCD scheme shows increased performance gains of 8dB and 5dB over the SVD and the conventional UCD, respectively.

In Fig. 8, we compare the FER performance for the case where only a subset of subchannels are transmitted ($N < \min\{N_t, N_r\}$). Although the error probability of the SVD scheme is improved by discarding the worst subchannel, the proposed BL-UCD scheme has a better diversity order than the SVD scheme. Still the BL-UCD scheme outperforms the SVD scheme by 2dB and 2.5dB for 3×3 and 4×4 systems, respectively.

Finally, Fig. 9 presents the effect of the channel estimation error at the transmitter, where the BL-UCD scheme employs the MMSE-BD receive filter (26) while the SVD scheme is combined with the MMSE linear receiver. From Fig. 6, we can see that with the perfect CSIT ($\sigma_e^2 = 0$), the MMSE-BD filter obtains the same FER performance as the left singular matrix $\bar{\mathbf{U}}^{\dagger}$. As the channel error variance σ_e^2 increases, the SNR gain of the proposed BL-UCD over the SVD scheme increases. This result demonstrates the efficacy of the BL-UCD scheme in practical environments with imperfect CSIT.

VIII. Conclusion

In this paper, we have proposed a transceiver design based on the UCD operation in coded MIMO systems. We have first derived the diversity order of the coded SVD scheme by carrying out the PEP analysis. Then, the BL-UCD has been proposed which achieves a higher diversity order than the SVD. The blockwise structure of the proposed scheme increases the minimum gain of subchannels which dominates the error performance. Furthermore, we show that single-symbol decodable MLD is possible by utilizing the real-valued representation of the effective channel matrix. Therefore, the proposed blockwise scheme attains the optimal ML performance for the UCD precoder without the error propagation problem inherent in the SIC receiver. In addition, we observe from the analysis that the proposed BL-UCD achieves full diversity for a larger range of code rates compared to the SVD scheme. Finally, for systems with imperfect CSIT, the modified receive filter of the BL-UCD scheme has been presented. The simulation results confirm our analysis, and show that the proposed blockwise schemes substantially improve the FER performance of the conventional closed-loop schemes in practical coded systems.

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