

# Alamouti-Codes Based Four-Antenna Transmission Schemes with Phase Feedback

Ilhwan Choi, Jong-Kyu Kim, Heunchul Lee, *Member, IEEE*, and Inkyu Lee, *Senior Member, IEEE*

**Abstract**—We present a new transmit scheme based on Alamouti codes for four transmit antenna systems with phase feedback. We propose a pre-processor for combining two Alamouti codes in terms of Frobenius norm maximization. For the proposed scheme, full diversity is achieved by the combining effect at the pre-processor. Also the phase feedback is utilized to increase an array gain. Simulation results show that the performance degradation due to the quantized feedback in the proposed scheme is small, and thus the proposed scheme exhibits a performance gain over existing closed-loop schemes with the same number of feedback bits.

**Index Terms**—Space-time block codes (STBC), phase feedback, multiple-input multiple-output (MIMO).

## I. INTRODUCTION

THE use of multiple antennas with space-time processing has been considered to increase the capacity for high data rate wireless links. In open-loop systems where channel state information (CSI) is available only at the receiver, space-time block codes (STBC) are efficient methods to achieve a transmit diversity gain for the higher link performance [1]. In contrast, closed-loop systems can utilize knowledge of the channel at the transmitter to further improve the system performance.

In open-loop systems, a rotated Quasi Orthogonal (QO)-STBC scheme [2] which requires joint two symbol maximum likelihood (ML) detection achieves full rate and full diversity by using constellation rotations for different transmitted symbols. Several closed-loop STBC schemes for four transmit antennas are introduced to achieve the full diversity and full rate with simple decoding process. A closed-loop QO-STBC scheme with 1-bit feedback has been proposed in [3]. In [4], a closed-loop Extended Orthogonal (EO)-STBC with a smaller block size of a codeword matrix is also designed, which outperforms a closed-loop QO-STBC by additional array gain. In [7], a hybrid STBC which employs the precoder based on Givens rotations to orthogonalize the channel by utilizing the phase feedback has been introduced.

In this letter, we propose a new four transmit antennas scheme based on Alamouti codes with phase feedback. We begin by designing a pre-processor which combines two

Alamouti codes embedded in the effective channel into one Alamouti code to achieve the full diversity regardless of the phase feedback. As shown in [5], since the effective channel gain of orthogonal STBC is proportional to the Frobenius norm of the effective channel matrix, the Frobenius norm is utilized as a useful performance metric for maximization of achievable signal-to-noise ratio (SNR) by the proposed scheme. In particular, in this letter we improve the overall system performance by exploiting the phase feedback to maximize the Frobenius norm of the combined Alamouti code.

We derive an expression for determining the optimum phase which maximizes the Frobenius norm. The proposed scheme requires one phase value for feedback and allows a simple maximum likelihood (ML) decoding [6] for each in-phase and quadrature component by utilizing the real-valued representation of complex orthogonal codes. Also, to further reduce the feedback overhead, a simple selection metric on quantized phase feedback is suggested. In the simulation section, we show that the performance of the 2-bit feedback case becomes identical to that of the unquantized case and the proposed scheme outperforms existing closed-loop schemes with the same number of feedback bits.

Throughout this letter, normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. With a bar accounting for complex variables, for any complex notation  $\bar{c}$ , we denote the real part of  $\bar{c}$  by  $\Re[\bar{c}]$ . The superscripts  $(\cdot)^T$  and  $(\cdot)^*$  represent the transpose and the complex conjugate operation, respectively.

## II. PROPOSED SCHEME FOR FOUR-TRANSMIT ANTENNAS

In this section, we describe the proposed scheme for multiple-input multiple-output (MIMO) systems with  $M_t = 4$  transmit and  $M_r$  receive antennas. Two Alamouti codes are embedded in the STBC system and each block encompasses two transmit antennas.

We assume that the elements of the channel matrix are obtained from an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance. Let  $\bar{C}$  denote the codeword matrix of a  $T$ -by- $M_t$  code design where the  $(i, j)$ th element is the signal transmitted from antenna  $j$  at time  $i$ , and  $T$  represents the block size of the STBC. We also assume that the feedback period equals the block size  $T$ . The codeword matrix  $\bar{C}$  is given as

$$\bar{C} = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 & \bar{x}_3 & \bar{x}_4 \\ -\bar{x}_2^* & \bar{x}_1^* & -\bar{x}_4^* & \bar{x}_3^* \end{bmatrix}. \quad (1)$$

By properly applying the conjugate operation of the received signal [1], the  $2M_r$ -by- $M_t$  effective channel matrix  $\bar{H}$

Manuscript received February 5, 2009. The associate editor coordinating the review of this letter and approving it for publication was G. Mazzini.

I. Choi and I. Lee are with the School of Electrical Eng., Korea University, Seoul, Korea (e-mail: {ariyan, inkyu}@korea.ac.kr).

J.-K. Kim is with the Agency for Defense Development, Daejeon, Korea (e-mail: jongkyu@add.re.kr).

H. Lee is with the Information Systems Lab., Stanford University, CA, USA (e-mail: heunchul@stanford.edu).

This research was supported by the MKE(Ministry of Knowledge Economy), Korea, under the ITRC(Information Technology Research Center) support program supervised by the IITA(Institute for Information Technology Advancement) (IITA-2009-C1090-0902-0013).

Digital Object Identifier 10.1109/LCOMM.2009.090229

can be written as

$$\bar{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{h}}_1 & \bar{\mathbf{h}}_2 & \bar{\mathbf{h}}_3 & \bar{\mathbf{h}}_4 \\ \bar{\mathbf{h}}_2^* & -\bar{\mathbf{h}}_1^* & \bar{\mathbf{h}}_4^* & -\bar{\mathbf{h}}_3^* \end{bmatrix} = [\bar{\mathbf{H}}_1 \ \bar{\mathbf{H}}_2]$$

where  $\bar{\mathbf{h}}_i$  is defined as  $\bar{\mathbf{h}}_i = [\bar{h}_{1i} \cdots \bar{h}_{M_r i}]^T$ ,  $\bar{h}_{ij}$  denotes the path gain from the  $j$ th transmit antenna to the  $i$ th receive antenna. Here,  $\bar{\mathbf{H}}_1$  and  $\bar{\mathbf{H}}_2$  indicate the matrices which consist of the first two and the last two column vectors of  $\bar{\mathbf{H}}$ , respectively.

The system model based on the code (1) can be obtained by

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}} \quad (2)$$

where  $\bar{\mathbf{y}}^T = [\bar{y}_1 \ \bar{y}_2^*]$ ,  $\bar{\mathbf{n}}^T = [\bar{n}_1 \ \bar{n}_2^*]$ , and the complex transmitted signal vector  $\bar{\mathbf{x}}$  denotes  $\bar{\mathbf{x}} = [\bar{x}_1 \ \bar{x}_2 \ \bar{x}_3 \ \bar{x}_4]^T$ . Here,  $\bar{y}_i$  and  $\bar{n}_i$  denote the received signal and noise complex column vector of length  $M_r$  at time  $i$ , respectively.

At the transmitter, two data symbols  $\bar{\mathbf{s}} = [\bar{s}_1 \ \bar{s}_2]^T$  are pre-processed by  $\mathbf{P}$  to form the four dimensional complex transmitted signal vector  $\bar{\mathbf{x}}$  as

$$\bar{\mathbf{x}} = \mathbf{P}\bar{\mathbf{s}} = \begin{bmatrix} \cos \theta \mathbf{I}_2 \\ \sin \theta \mathbf{I}_2 \end{bmatrix} \bar{\mathbf{s}} \quad (3)$$

where  $\mathbf{I}_n$  indicates an identity matrix of size  $n$ .

From the equations (2) and (3), the received signal  $\bar{\mathbf{y}}$  is written as  $\bar{\mathbf{y}} = \bar{\mathbf{H}}_e \bar{\mathbf{s}} + \bar{\mathbf{n}}$  where

$$\begin{aligned} \bar{\mathbf{H}}_e &= \bar{\mathbf{H}}\mathbf{P} \\ &= [\bar{\mathbf{H}}_1 \ \bar{\mathbf{H}}_2] \begin{bmatrix} \cos \theta \mathbf{I}_2 \\ \sin \theta \mathbf{I}_2 \end{bmatrix} \\ &= \cos \theta \bar{\mathbf{H}}_1 + \sin \theta \bar{\mathbf{H}}_2 \\ &= \begin{bmatrix} \bar{\mathbf{h}}_1 \cos \theta + \bar{\mathbf{h}}_3 \sin \theta & \bar{\mathbf{h}}_2 \cos \theta + \bar{\mathbf{h}}_4 \sin \theta \\ \bar{\mathbf{h}}_2^* \cos \theta + \bar{\mathbf{h}}_4^* \sin \theta & -\bar{\mathbf{h}}_1^* \cos \theta - \bar{\mathbf{h}}_3^* \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \bar{\mathbf{h}}_{e1} & \bar{\mathbf{h}}_{e2} \\ \bar{\mathbf{h}}_{e2}^* & -\bar{\mathbf{h}}_{e1}^* \end{bmatrix}. \end{aligned} \quad (4)$$

Note that  $\bar{\mathbf{H}}_e$  results in a new Alamouti code regardless of the phase  $\theta$ . In other words, a simple ML decoding can be employed regardless of the phase  $\theta$  due to Alamouti structures.

As shown in [5], for the purpose of improving the system performance, we can utilize the phase  $\theta$  to maximize the Frobenius norm of the effective channel  $\bar{\mathbf{H}}_e$ . Thus, we need to maximize the Frobenius norm of  $\bar{\mathbf{H}}_e$ , defined as

$$\begin{aligned} f(\theta) &= \|\bar{\mathbf{H}}_e\|_F^2 \\ &= \|\cos \theta \bar{\mathbf{H}}_1 + \sin \theta \bar{\mathbf{H}}_2\|_F^2 \\ &= \cos^2 \theta \|\bar{\mathbf{H}}_1\|_F^2 + \sin^2 \theta \|\bar{\mathbf{H}}_2\|_F^2 \\ &\quad + 2 \cos \theta \sin \theta \text{Tr}\{\Re[\bar{\mathbf{H}}_1 \bar{\mathbf{H}}_2^H]\} \end{aligned} \quad (5)$$

where  $\text{Tr}\{\cdot\}$  indicates the trace of a matrix and  $(\cdot)^H$  represents the complex conjugate transpose of a matrix.

Now we derive an expression for determining the phase  $\theta$  which maximizes the Frobenius norm of  $\bar{\mathbf{H}}_e$  by differentiation. After differentiating the equation (5) with respect to  $\theta$ , we have the first and second derivatives of  $f(\theta)$  as

$$\dot{f}(\theta) = 2\alpha \cos \theta \sin \theta + 2\beta(\cos^2 \theta - \sin^2 \theta) \quad (6)$$

$$\ddot{f}(\theta) = 2\alpha(\cos^2 \theta - \sin^2 \theta) - 8\beta \cos \theta \sin \theta \quad (7)$$

where  $\alpha = \|\bar{\mathbf{H}}_2\|_F^2 - \|\bar{\mathbf{H}}_1\|_F^2$  and  $\beta = \text{Tr}\{\Re[\bar{\mathbf{H}}_1 \bar{\mathbf{H}}_2^H]\}$ .

TABLE I  
SIMPLE SELECTION METRIC ON QUANTIZED FEEDBACK  $\theta_q$

	$\beta > 0$		$\beta < 0$	
	$\alpha > 0$	$\alpha < 0$	$\alpha < 0$	$\alpha > 0$
1 bit	$\pi/4$		$-\pi/4$	
2 bits	$3\pi/8$	$\pi/8$	$-\pi/8$	$-3\pi/8$

Solving (6) and (7) with the conditions of  $\dot{f}(\theta_{max}) = 0$  and  $\ddot{f}(\theta_{max}) < 0$  yields the following optimum phase

$$\theta_{max} = \tan^{-1} \left( \frac{\alpha + \sqrt{\alpha^2 + 4\beta^2}}{2\beta} \right). \quad (8)$$

Thus,  $\bar{\mathbf{H}}_e$  has its maximum Frobenius norm when  $\mathbf{P}$  is applied with the phase  $\theta_{max}$  in (8). Note that with  $\theta_{min} = \tan^{-1}(\frac{\alpha - \sqrt{\alpha^2 + 4\beta^2}}{2\beta})$ ,  $\|\bar{\mathbf{H}}_e\|_F^2$  is minimized. From the result of (8), it is possible to combine two Alamouti codes to one resulting Alamouti code with the maximized Frobenius norm by  $\mathbf{P}$  with only one phase value  $\theta$ .

It was shown in [6] that a simple ML decoding can be applied for each in-phase and quadrature component in complex orthogonal codes by utilizing real-valued representation. By using the real-valued representation method in [6], the system model (2) can be reorganized as

$$\mathbf{y} = \begin{bmatrix} y_{1,I} \\ y_{2,I} \\ y_{1,Q} \\ y_{2,Q} \end{bmatrix} = \mathbf{H}_e \begin{bmatrix} s_{1,I} \\ s_{2,I} \\ s_{1,Q} \\ s_{2,Q} \end{bmatrix} + \begin{bmatrix} n_{1,I} \\ n_{2,I} \\ n_{1,Q} \\ n_{2,Q} \end{bmatrix}$$

where  $(\cdot)_I$  and  $(\cdot)_Q$  represent the in-phase and the quadrature component of the complex variable, respectively, and  $\mathbf{H}_e$  denotes a real representation of  $\bar{\mathbf{H}}_e$  in (4) as

$$\mathbf{H}_e = \begin{bmatrix} \mathbf{h}_{e1,I} & \mathbf{h}_{e2,I} & -\mathbf{h}_{e1,Q} & -\mathbf{h}_{e2,Q} \\ \mathbf{h}_{e2,I} & -\mathbf{h}_{e1,I} & \mathbf{h}_{e2,Q} & -\mathbf{h}_{e1,Q} \\ \mathbf{h}_{e1,Q} & \mathbf{h}_{e2,Q} & \mathbf{h}_{e1,I} & \mathbf{h}_{e2,I} \\ \mathbf{h}_{e2,Q} & -\mathbf{h}_{e1,Q} & -\mathbf{h}_{e2,I} & \mathbf{h}_{e1,I} \end{bmatrix}.$$

Since all columns in  $\mathbf{H}_e$  are orthogonal to each other, a simple ML estimate for  $\hat{\mathbf{s}} = [\hat{s}_{1,I} \ \hat{s}_{2,I} \ \hat{s}_{1,Q} \ \hat{s}_{2,Q}]^T$  can be individually given by

$$\hat{s}_{1,I} = \arg \min_{s_{1,I} \in \mathbf{u}} \|\tilde{\mathbf{h}}_1^T \mathbf{y} - \gamma s_{1,I}\|^2 \quad (9)$$

$$\hat{s}_{2,I} = \arg \min_{s_{2,I} \in \mathbf{u}} \|\tilde{\mathbf{h}}_2^T \mathbf{y} - \gamma s_{2,I}\|^2 \quad (10)$$

$$\hat{s}_{1,Q} = \arg \min_{s_{1,Q} \in \mathbf{u}} \|\tilde{\mathbf{h}}_3^T \mathbf{y} - \gamma s_{1,Q}\|^2 \quad (11)$$

$$\hat{s}_{2,Q} = \arg \min_{s_{2,Q} \in \mathbf{u}} \|\tilde{\mathbf{h}}_4^T \mathbf{y} - \gamma s_{2,Q}\|^2 \quad (12)$$

where  $\mathbf{u} = \{\pm 1, \pm 3, \dots, \pm 2^{\eta/2-1}\}$ ,  $\eta$  denotes the number of bits per symbol, and  $\gamma$  denotes  $\gamma = \|\tilde{\mathbf{h}}_1\|^2 = \|\tilde{\mathbf{h}}_2\|^2 = \|\tilde{\mathbf{h}}_3\|^2 = \|\tilde{\mathbf{h}}_4\|^2$ . Here,  $\tilde{\mathbf{h}}_i$  denotes the  $i$ th column vector of  $\mathbf{H}_e$ . It is important to note that the simple ML detection in (9)-(12) can be carried out regardless of the value of  $\theta$ .

We now consider the quantization effect on the phase feedback in the limited feedback systems. From simulations, we learn that the phase  $\theta$  has a uniform distribution over a

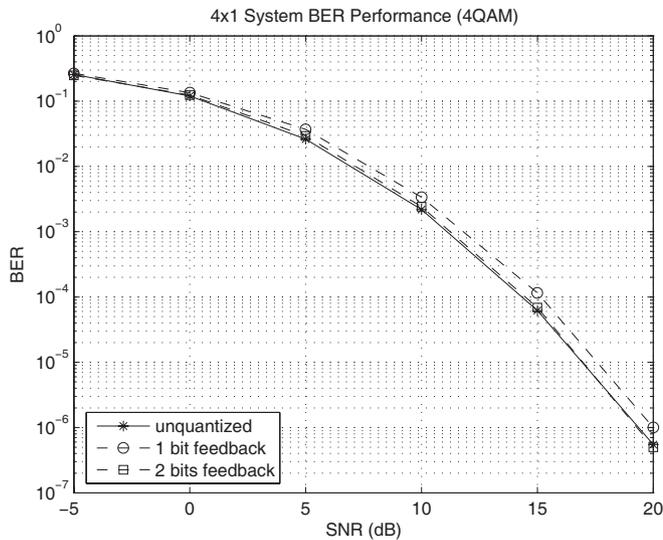


Fig. 1. The effect of quantization in the proposed scheme.

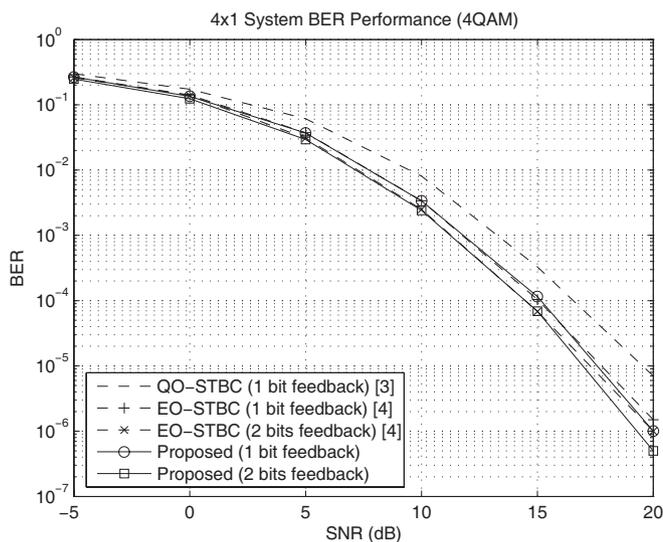


Fig. 2. Bit error rate performance of 4x1 systems with quantized feedback.

range  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ <sup>1</sup>. Thus, for the 1-bit feedback, we can simply select  $\theta_q = \frac{\pi}{4}$  or  $-\frac{\pi}{4}$  depending on the sign of  $\theta$ . When two feedback bits are available, the nearest  $\theta_q$  corresponding to  $\theta$  can be chosen from the set of  $\{-\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}\}$ . Since  $\theta_q$  can be selected based on the sign and the magnitude of  $\theta$ , we suggest a simple selection metric to determine the quantized feedback as follows. The sign of  $\theta$  is equal to that of the argument  $(\alpha + \sqrt{\alpha^2 + 4\beta^2})/2\beta$  in (8). Since the numerator  $(\alpha + \sqrt{\alpha^2 + 4\beta^2})$  of the argument is always positive, the magnitude and the sign of  $\theta$  are determined by the sign of  $\alpha$  and  $\beta$ , respectively. Thus, for quantized feedback,  $\theta_q$  can simply be chosen as in Table I. In this case, since  $\theta_q$  is selected not by the exact value but by the sign of  $\alpha$  and  $\beta$ , the proposed quantized feedback scheme can be robust to the feedback error due to imperfect estimation or time variation of the channel.

The proposed scheme has another advantage on quantization issues. Since the phase feedback  $\theta$  needs to be quantized, the

phase feedback which has a broader dynamic range results in a larger quantization error. We notice that the optimal phase  $\theta_{max}$  of the proposed scheme is chosen in the range of  $[-\pi/2, \pi/2]$ , while that of the EO-STBC scheme [4] is given in the region of  $[-\pi, \pi]$ . This means that the proposed scheme is superior to the EO-STBC scheme in terms of the feedback amount. In addition, the proposed scheme can be easily extended to more than four transmit antennas.

### III. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the efficiency of the proposed scheme in quasi-static flat fading channels. We evaluate the bit error rate (BER) performance of the proposed scheme in 4x1 systems. In Figures 1 and 2, we present the BER performance with respect to SNR in dB with 4QAM. Figure 1 illustrates that a performance loss due to the quantization on the phase feedback in the proposed scheme is small. With 1-bit feedback, it performs within 0.5dB of the unquantized feedback case, while the 2-bit feedback yields the curve identical to that of the unquantized feedback case.

In Figure 2, for the same number of feedback bits, the proposed scheme shows a performance gain of 0.6dB over the EO-STBC scheme [4] at a BER of  $10^{-6}$ . We conclude from these figures that the proposed scheme outperforms the conventional schemes for quantized feedback cases.

### REFERENCES

- [1] S. M. Alamouti, "A simple transmit diversity scheme for wireless communications," *IEEE Journal on Selected Areas in Communications*, pp. 1451–1458, October 1998.
- [2] H. Jafarkhani, *Space-Time Coding: Theory and Practice*. Cambridge University Press, 2005.
- [3] J. Kim and S. L. Ariyavisitakul, "Optimum 4-Transmit-Antenna STBC/SFBC with Angle Feedback and a Near-Optimum 1-Bit Feedback Scheme," *IEEE Communications Letters*, vol. 11, pp. 868–870, November 2007.
- [4] N. M. Eltayeb, S. Lambotharan, and J. A. Chambers, "A phase feedback based extended space-time block code for enhancement of diversity," in *Proc. IEEE VTC '07*, pp. 2296 – 2299, April 2007.
- [5] C.-Y. Chen, A. Sezgin, J. M. Cioffi, and A. Paulraj, "A Low-Complexity Algorithm for Antenna Selection in Space-Time Block Coded Systems," in *Proc. IEEE GLOBECOM '07*, pp. 4221–4225, November 2007.
- [6] H. Lee, J. Cho, J.-K. Kim, and I. Lee, "Real-Domain Decoder for Full-Rate Full-Diversity STBC with Multidimensional Constellations," *IEEE Transactions on Communications*, vol. 57, pp. 17–21, January 2009.
- [7] J.-K. Kim, H. Lee, and I. Lee, "A new hybrid space-time block codes and spatial multiplexing scheme with precoding," in *Proc. ICC 07*, pp. 5276–5281, June 2007.

<sup>1</sup>Due to the page limitation, we cannot include the details in the letter.