Generalized Channel Inversion Methods for Multiuser MIMO Systems

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***Abstract*—Block diagonalization (BD) is a well-known precoding method in multiuser multi-input multi-output (MIMO) broadcast channels. This scheme can be considered as a extension of the zero-forcing (ZF) channel inversion to the case where each receiver is equipped with multiple antennas. One of the limitation of the BD is that the sum rate does not grow linearly with the number of users and transmit antennas at low and medium signal-to-noise ratio regime, since the complete suppression of multi-user interference is achieved at the expense of noise enhancement. Also it performs poorly under imperfect channel state information. In this paper, we propose a generalized minimum mean-squared error (MMSE) channel inversion algorithm for users with multiple antennas to overcome the drawbacks of the BD for multiuser MIMO systems. We first introduce a generalized ZF channel inversion algorithm as a new approach of the conventional BD. Applying this idea to the MMSE channel inversion for identifying orthonormal basis vectors of the precoder, and employing the MMSE criterion for finding its combining matrix, the proposed scheme increases the signal-to-interference-plus-noise ratio at each user's receiver. Simulation results confirm that the proposed scheme exhibits a linear growth of the sum rate, as opposed to the BD scheme. For block fading channels with four transmit antennas, the proposed scheme provides a 3dB gain over the conventional BD scheme at 1 % frame error rate. Also, we present a modified precoding method for systems with channel estimation errors and show that the proposed algorithm is robust to channel estimation errors.**

***Index Terms*—Multi-input multi-output (MIMO), multiuser, broadcast channels (BC), linear precoding, minimum mean-squared error (MMSE).**

I. INTRODUCTION

MULTI-INPUT multi-output (MIMO) systems have drawn a lot of attention in the past few years due to their great potential to achieve high throughput in wireless communication systems [1][2][3]. More recently, the investigation of the capacity region has been of concern in multiuser MIMO broadcast channels (BC) [4][5].

In [6] and [7], it was shown that the maximum sum rate in multiuser MIMO BC can be achieved by dirty paper coding (DPC). However, the DPC is difficult to implement in practical systems due to high computational burden of successive encodings and decodings. Even though the Tomlinson-Harashima precoding can be adopted as a suboptimal strategy of the DPC as in [8] and [9], the complexity issue still remains, since this algorithm is based on nonlinear modulo operations.

In linear processing systems where each user has multiple antennas, transmitter and receiver design methods have normally been developed in two different ways. The first approach employs an iterative method of canceling out multi-user interference (MUI), allowing multiple data subchannels per users as in classical MIMO transmission techniques. For single user MIMO channels, the optimum joint linear transmitter and receiver design was investigated in [10]. Also, authors in [11], [12] and [13] expanded the work in [10] to multiuser MIMO downlink channels by adopting a joint iterative algorithm. However, the iterative nature of these algorithms typically results in a high computational cost.

The second approach is a noniterative method. For the special case where the base station has multiple antennas but all users employ a single antenna, several practical precoding techniques have been proposed [14]. A zero-forcing channel inversion (ZF-CI) scheme [14] is one of the simplest precoding techniques for this case. However its performance is rather poor due to a transmit power boost issue. Although a minimum mean-squared error channel inversion (MMSE-CI) method [14] overcomes the drawback of the ZF-CI, this is still confined to a single receive antenna case.

For the case where the users in the network have multiple antennas, the block diagonalization (BD) is a well-known precoding algorithm [15]. As a generalization of the ZF-CI for the multiuser MIMO systems, the BD attempts to completely eliminate the MUI without any consideration on the noise. Normally, the performance of the BD is limited by the number of users and multiuser channel conditions.

In this paper, we propose a generalized MMSE-CI (GMI) algorithm which supports multiple data stream transmission to each user in multiuser MIMO BC based on a noniterative method. Unlike the conventional BD algorithm, our GMI algorithm takes the noise into account for finding each user's precoding matrix to increase the signal-to-interference-plus-noise ratio (SINR) at each user's receiver. We first introduce an expanded version of the ZF-CI [14] for the case where each user has multiple antennas as a new approach of the conventional BD scheme [15] with reduced complexity. Then, we generalize this idea to the MMSE-CI [14] so that the power boost problem can be taken care of for users with multiple antennas.

A similar idea was independently studied in [16], which

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regularizes the BD for multiuser MIMO systems. In comparison, our proposed GMI method can identify the precoding matrices with two different transmit power constraints, and we will show that one of these solutions is equivalent to the precoder in [16]. Also, compared to the singular value decomposition (SVD) based approach algorithm introduced in [16], our proposed scheme can be computed using a simple matrix inversion method, which results in low computational complexity. In addition, we propose a modified method to improve the performance in the presence of imperfect channel state information (CSI) at the transmitter.

From simulation results, we will show that the sum rate of the proposed GMI outperforms the conventional BD for all simulation scenarios. In the frame error rate (FER) comparison, the proposed GMI algorithm provides a 3dB gain over the BD scheme in block fading channels for the case of four transmit antennas and two users. We also demonstrate that the modified GMI scheme is much more robust to the channel estimate error than the BD scheme.

This paper is organized as follows: In section II, we describe a general system model for the multiuser MIMO downlink. Section III presents a summary of the BD algorithm and an alternative approach of this. Next in Section IV, we propose the GMI algorithm under perfect and imperfect CSI conditions. Section V compares the proposed scheme to prior works. In Section VI, the simulation results are presented. Finally, the paper is terminated with conclusions in Section VII.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For any general matrix $\mathbf{A}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and the conjugate transpose, respectively. $\mathrm{Tr}(\mathbf{A})$ indicates the trace and the Frobenius norm of a matrix $\mathbf{A}$ is $\|\mathbf{A}\|_F^2 = \mathrm{Tr}(\mathbf{A}\mathbf{A}^H)$. For $m \times m$ matrices $\mathbf{A}_j$, $\mathbf{A} = \mathrm{diag}\{\mathbf{A}_1, \cdots, \mathbf{A}_n\}$ denotes an $mn \times mn$ block diagonal matrix.

II. System Model

We consider multiuser MIMO downlink systems where the base station is transmitting to K independent users simultaneously and generating co-channel interference at all users as shown in Fig 1. In this system, the base station is equipped with N_t transmit antennas and user j has $n_j \geq 1$ receive antennas, referred to in the following as $\{n_1, \cdots, n_K\} \times N_t$. The total number of receive antennas at all users is defined as $N_r = \sum_{j=1}^{K} n_j$. In the discrete-time complex baseband MIMO case, the channel from the base station to the jth user is modeled by the $n_j \times N_t$ channel matrix $\mathbf{H}_j$, where the (p, q)th entry of $\mathbf{H}_j$ represents the channel gain from base antenna q to antenna p of user j. We assume that $\mathbf{H}_j$ has full row rank and the total channel matrix $\mathbf{H}_s = [\,\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \cdots \quad \mathbf{H}_K^T\,]^T$ is known perfectly at the base station.

In this paper, we focus on linear processing for multiuser MIMO systems. We first define the transmitted data symbol vector $\mathbf{s}_s$, the noise vector $\mathbf{w}_s$ and the precoding matrix $\mathbf{P}_s$

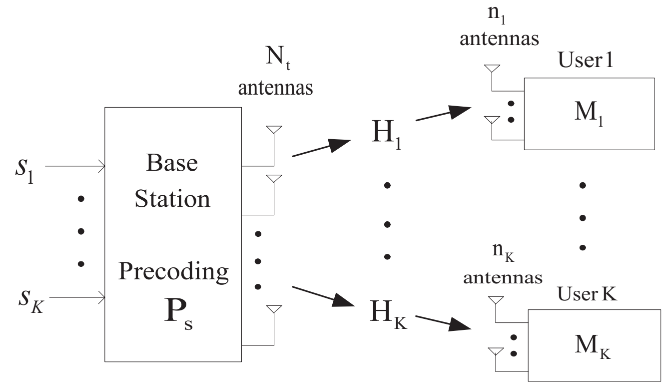

Fig. 1. Structure of a multiuser MIMO downlink system

for all users as

$$\begin{aligned}\mathbf{s}_s &= [\,\mathbf{s}_1^T \quad \mathbf{s}_2^T \quad \cdots \quad \mathbf{s}_K^T\,]^T,\\ \mathbf{w}_s &= [\,\mathbf{w}_1^T \quad \mathbf{w}_2^T \quad \cdots \quad \mathbf{w}_K^T\,]^T,\\ \mathbf{P}_s &= [\,\mathbf{P}_1 \quad \mathbf{P}_2 \quad \cdots \quad \mathbf{P}_K\,]\end{aligned}$$

where $\mathbf{s}_j$ and $\mathbf{w}_j$ are the jth user's data and noise vectors, respectively and $\mathbf{P}_j$ represents the associated precoding matrix. Here all symbols of $\mathbf{s}_j$ are assumed to be independently generated with unit variance and the components of the noise vector $\mathbf{w}_j$ have independently and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance σ_w^2 for $j = 1, \cdots, K$.

Then, the total received signal can be expressed as

$$\mathbf{y}_s = [\,\mathbf{y}_1^T \;\; \mathbf{y}_2^T \cdots \mathbf{y}_K^T\,]^T = \mathbf{H}_s\mathbf{P}_s\mathbf{s}_s + \mathbf{w}_s$$

where $\mathbf{y}_j$ represents the received signal vector at the jth user as

$$\mathbf{y}_j = \mathbf{H}_j\mathbf{P}_j\mathbf{s}_j + \mathbf{H}_j \sum_{k=1,\, k\neq j}^{K} \mathbf{P}_k\mathbf{s}_k + \mathbf{w}_j. \tag{1}$$

Let $\tilde{\mathbf{s}}_s = \mathbf{P}_s\mathbf{s}_s$ denote the signal vector actually transmitted at the base station, which satisfies $\mathbb{E}[\|\tilde{\mathbf{s}}_s\|^2] \leq P_{total}$ where P_{total} represents the total downlink transmitted power. Since we assume that each data symbol has unit variance, the total transmit power constraint can be expressed as $\mathrm{Tr}(\mathbf{P}_s^H\mathbf{P}_s) \leq P_{total}$.

Next, denoting the overall receive filter $\mathbf{M}_s$ as

$$\mathbf{M}_s = \mathrm{diag}\{\mathbf{M}_1, \quad \mathbf{M}_2, \quad \cdots, \quad \mathbf{M}_K\}$$

where $\mathbf{M}_j$ represents the jth user's receive filter, the receive filter output vector of the jth user $\mathbf{x}_j$ can be written as

$$\mathbf{x}_j = \mathbf{M}_j\mathbf{H}_j\mathbf{P}_j\mathbf{s}_j + \mathbf{M}_j\mathbf{H}_j \sum_{k=1,\, k\neq j}^{K} \mathbf{P}_k\mathbf{s}_k + \mathbf{M}_j\mathbf{w}_j. \tag{2}$$

III. Generalization of Zero-Forcing Channel Inversion

In this section, we represent a generalized ZF-CI algorithm for multiuser MIMO systems in two different approaches. First, we review the conventional BD algorithm presented in [15]. Then, we introduce an alternative way of representing the

BD by extending the ZF-CI method [14] for the case where each user has more than a single antenna. This representation will serve as a basis of the generalized MMSE-CI scheme which will be described in section IV.

A. Review of Block Diagonalization

The BD algorithm [15] is known as a generalization of the ZF-CI for multiuser MIMO systems. When each user has multiple antenna, the linear precoder and receive filters can be obtained through the two stage process. In the first stage, we seek a precoding matrix which suppresses the other users' interference. Applying this matrix, block channels are formulated for each user. In the second stage, each block channel is decoupled into n_j parallel subchannels in order to allow single symbol decodable receivers for each user.

The key idea of the BD algorithm is to identify the precoding matrix $\mathbf{P}_s$ which suppresses the MUI completely. To eliminate all the MUI, the following constraint is imposed.

$$\mathbf{H}_j\mathbf{P}_k = \mathbf{0} \quad \text{for all} \quad j \neq k \quad \text{and} \quad 1 \leq j, k \leq K. \tag{3}$$

In order to satisfy the zero-interference constraint (3), $\mathbf{P}_j$ should be in the null space of $\tilde{\mathbf{H}}_j$ where $\tilde{\mathbf{H}}_j$ is defined as

$$\tilde{\mathbf{H}}_j = [\mathbf{H}_1^T \quad \cdots \quad \mathbf{H}_{j-1}^T \quad \mathbf{H}_{j+1}^T \quad \cdots \quad \mathbf{H}_K^T]^T. \tag{4}$$

Denoting $\tilde{L}_j$ as $\tilde{L}_j = \text{rank}(\tilde{\mathbf{H}}_j)$, the SVD of $\tilde{\mathbf{H}}_j$ can be obtained as

$$\tilde{\mathbf{H}}_j = \tilde{\mathbf{U}}_j\tilde{\boldsymbol{\Lambda}}_j[\tilde{\mathbf{V}}_j^{(1)} \quad \tilde{\mathbf{V}}_j^{(0)}]^H$$

where the unitary matrix $\tilde{\mathbf{U}}_j$ contains left singular vectors, the matrix $\tilde{\boldsymbol{\Lambda}}_j$ consists of ordered singular values of $\tilde{\mathbf{H}}_j$, the matrix $\tilde{\mathbf{V}}_j^{(1)}$ is composed of the first $\tilde{L}_j$ right singular vectors, and the matrix $\tilde{\mathbf{V}}_j^{(0)}$ holds the last $(N_t - \tilde{L}_j)$ right singular vectors. Since $\tilde{\mathbf{V}}_j^{(0)}$ forms an orthogonal basis for the null space of $\tilde{\mathbf{H}}_j$, after applying the nullspace $\tilde{\mathbf{V}}_j^{(0)}$ for $j = 1, \cdots, K$ to the channel matrix $\mathbf{H}_s$, the jth user has a non-interfering block channel $\mathbf{H}_j\tilde{\mathbf{V}}_j^{(0)}$.

Now, in order to decouple this block channel into n_j parallel sub channels, the SVD of $\mathbf{H}_j\tilde{\mathbf{V}}_j^{(0)}$ is computed as

$$\mathbf{H}_j\tilde{\mathbf{V}}_j^{(0)} = \mathbf{U}_j^{(\mathrm{b})}\boldsymbol{\Lambda}_j^{(\mathrm{b})}\mathbf{V}_j^{(\mathrm{b})H}.$$

Then, employing $\mathbf{P}_j = \tilde{\mathbf{V}}_j^{(0)}\mathbf{V}_j^{(\mathrm{b})}$ and $\mathbf{M}_j = \mathbf{U}_j^{(\mathrm{b})H}$ in (2), the jth user's signal vector $\tilde{\mathbf{x}}_j$ becomes

$$\tilde{\mathbf{x}}_j = \boldsymbol{\Lambda}_j^{(\mathrm{b})}\mathbf{s}_j + \mathbf{U}_j^{(\mathrm{b})H}\mathbf{w}_j. \tag{5}$$

Given (5), we can compute the achievable sum rate for the BD. To this end, we maximize the sum of information rates for all users subject to the sum power constraint P_{total}. Let us denote $\boldsymbol{\Phi}_j$ as the power allocation matrix for the jth user $(j = 1, \cdots, K)$, which is a diagonal matrix of size n_j with non-negative elements. Then, the sum rate R_{BD} for the BD can be written in terms of the following maximization:

$$R_{\text{BD}} = \max_{\boldsymbol{\Phi}_j}\sum_{j=1}^{K}\log_2\det\left(\mathbf{I} + \frac{(\boldsymbol{\Lambda}_j^{(\mathrm{b})})^2\boldsymbol{\Phi}_j}{\sigma_w^2}\right)$$
$$\text{subject to}\sum_{j=1}^{K}\text{Tr}(\boldsymbol{\Phi}_j) \leq P_{total}. \tag{6}$$

The optimal power loading matrix $\boldsymbol{\Phi}_j$ can be calculated by using the water-filling (WF) method [17].

Finally, we can obtain the precoding matrix combined with the optimal power loading as

$$\mathbf{P}_s^{\text{BD}} = [\tilde{\mathbf{V}}_1^{(0)}\mathbf{V}_1^{(\mathrm{b})} \quad \tilde{\mathbf{V}}_2^{(0)}\mathbf{V}_2^{(\mathrm{b})} \quad \cdots \quad \tilde{\mathbf{V}}_K^{(0)}\mathbf{V}_K^{(\mathrm{b})}]\boldsymbol{\Phi}^{\frac{1}{2}}$$

where $\boldsymbol{\Phi} = \text{diag}\,\{\boldsymbol{\Phi}_1, \ \boldsymbol{\Phi}_2, \cdots, \boldsymbol{\Phi}_K\}$. The overall receive filter is also expressed as

$$\mathbf{M}_s^{\text{BD}} = \text{diag}\,\{\mathbf{U}_1^{(\mathrm{b})H}, \quad \mathbf{U}_2^{(\mathrm{b})H}, \quad \cdots, \quad \mathbf{U}_K^{(\mathrm{b})H}\}.$$

B. New Approach for Generalized Zero-Forcing Channel Inversion

In [15], the authors adopted the SVD operation to obtain the precoding matrix $\mathbf{P}_j$ of which columns lie in the nullspace of $\tilde{\mathbf{H}}_j$. In this subsection, we propose an alternative way of finding vectors orthonormal to all columns of $\tilde{\mathbf{H}}_j$ based on the ZF-CI method in [14] and the orthogonalization process. This will be referred to as the generalized ZF-CI (GZI).

In order to compute the nullspace of $\tilde{\mathbf{H}}_j$, we define the pseudo-inverse of the channel matrix $\mathbf{H}_s$ as

$$\hat{\mathbf{H}}_s = \mathbf{H}_s^H(\mathbf{H}_s\mathbf{H}_s^H)^{-1} = [\,\hat{\mathbf{H}}_1 \quad \hat{\mathbf{H}}_2 \quad \cdots \quad \hat{\mathbf{H}}_K\,]. \tag{7}$$

Consider the QR decomposition of the $N_t \times n_j$ matrix $\hat{\mathbf{H}}_j$ as

$$\hat{\mathbf{H}}_j = \hat{\mathbf{Q}}_j\hat{\mathbf{R}}_j \quad \text{for} \quad j = 1, \cdots, K \tag{8}$$

where $\hat{\mathbf{R}}_j$ is an $n_j \times n_j$ upper triangular matrix and $\hat{\mathbf{Q}}_j$ is an $N_t \times n_j$ matrix whose columns form an orthonormal basis for $\hat{\mathbf{H}}_j$. Recognizing in (7) that $\tilde{\mathbf{H}}_j\hat{\mathbf{H}}_j = \mathbf{0}$, we have $\tilde{\mathbf{H}}_j\hat{\mathbf{Q}}_j\hat{\mathbf{R}}_j = \mathbf{0}$ from (8). Since $\hat{\mathbf{R}}_j$ is invertible, it follows $\tilde{\mathbf{H}}_j\hat{\mathbf{Q}}_j = \mathbf{0}$. Here, we can see that the columns of $\hat{\mathbf{Q}}_j$ form an orthonormal basis for the nullspace of $\tilde{\mathbf{H}}_j$ so that the jth user's precoder of the GZI constructed by a linear combination of $\hat{\mathbf{Q}}_j$ also satisfies the zero MUI constraint in (3). If each user has a single receive antenna, this reduces to the conventional ZF-CI solution with unit norm precoding vectors.

Similar to the BD algorithm, the jth user's non-interfering block channel $\mathbf{H}_j\hat{\mathbf{Q}}_j$ can be decomposed into parallel subchannels by appling the SVD operation of $\mathbf{H}_j\hat{\mathbf{Q}}_j$ as

$$\mathbf{H}_j\hat{\mathbf{Q}}_j = \mathbf{U}_j^{(\mathrm{z})}\boldsymbol{\Lambda}_j^{(\mathrm{z})}\mathbf{V}_j^{(\mathrm{z})H}.$$

Then, the jth user's receive filter output vector of the GZI scheme $\hat{\mathbf{x}}_j$ in (2) is given as

$$\hat{\mathbf{x}}_j = \boldsymbol{\Lambda}_j^{(\mathrm{z})}\mathbf{s}_j + \mathbf{U}_j^{(\mathrm{z})H}\mathbf{w}_j. \tag{9}$$

The achievable sum rate of the GZI and the optimal power loading coefficients in $\boldsymbol{\Phi}_j$ are then found using the WF solution from (6). Since $\|\mathbf{H}_j\hat{\mathbf{Q}}_j\|_F^2 = \|\mathbf{H}_j\tilde{\mathbf{V}}_j^{(0)}\|_F^2$, the diagonal matrix $\boldsymbol{\Lambda}_j^{(\mathrm{b})}$ in (5) is the same as $\boldsymbol{\Lambda}_j^{(\mathrm{z})}$ in (9). Thus, it is obvious that the sum rate of the BD is the same as that of the GZI. Finally, the precoding matrix and the overall receive filter of the GZI are obtained by

$$\mathbf{P}_s^{\text{GZI}} = [\hat{\mathbf{Q}}_1\mathbf{V}_1^{(\mathrm{z})} \quad \hat{\mathbf{Q}}_2\mathbf{V}_2^{(\mathrm{z})} \quad \cdots \quad \hat{\mathbf{Q}}_K\mathbf{V}_K^{(\mathrm{z})}]\boldsymbol{\Phi}^{\frac{1}{2}},$$
$$\mathbf{M}_s^{\text{GZI}} = \text{diag}\,\{\mathbf{U}_1^{(\mathrm{z})H}, \quad \mathbf{U}_2^{(\mathrm{z})H}, \quad \cdots, \quad \mathbf{U}_K^{(\mathrm{z})H}\}.$$

Comparing with the BD scheme, the proposed GZI has less computational complexity, since the GZI needs the orthogonalization of the $N_t \times n_j$ matrix $\hat{\mathbf{H}}_j$ in (8), which is simpler than the SVD operation of the $(N_r - n_j) \times N_t$ matrix $\tilde{\mathbf{H}}_j$ in (4) required for the BD. A more detailed complexity analysis will be shown in Section V.

IV. Generalization of MMSE Channel Inversion

In this section, we propose a generalized MMSE-CI (GMI) algorithm. Based on the new interpretation of the GZI made in the previous section, we outline a procedure for identifying the GMI precoding matrix which could balance the MUI and the noise for each user. Note that, unlike the zero-forcing based schemes which have a dimensional constraint, i.e., $N_r \leq N_t$, the GMI can be employed when there are more receive antennas than transmit antennas, i.e., $N_r > N_t$, similar to the MMSE-CI. For notational and analytical simplicity, we assume that the number of transmit antennas is the same as the total number of receive antennas ($N_r = N_t$).

A. Generalized MMSE Channel Inversion

In the GMI scheme, the precoding matrix can be determined by applying the MMSE-CI introduced in [14]. We denote $\bar{\mathbf{H}}_s$ as

$$\bar{\mathbf{H}}_s = (\mathbf{H}_s^H \mathbf{H}_s + \alpha \mathbf{I})^{-1} \mathbf{H}_s^H = [\, \bar{\mathbf{H}}_1 \quad \bar{\mathbf{H}}_2 \quad \cdots \quad \bar{\mathbf{H}}_K \,] \tag{10}$$

where α represents the ratio of the total noise variance to the total transmit power, i.e., $\alpha = N_r \sigma_w^2 / P_{total}$ [14]. Thus, it is clear that the conventional BD solution approaches the GMI solution for high signal-to-noise ratio (SNR), which is analogous to a well-known relation of ZF and MMSE in the filter theory. For orthogonalization of $\bar{\mathbf{H}}_j$, we employ the QR decomposition as

$$\bar{\mathbf{H}}_j = \bar{\mathbf{Q}}_j \bar{\mathbf{R}}_j \quad \text{for} \quad j = 1, \cdots, K \tag{11}$$

where $\bar{\mathbf{R}}_j$ is an $n_j \times n_j$ upper triangular matrix and the matrix $\bar{\mathbf{Q}}_j$ is composed of n_j orthonormal basis vectors of $\bar{\mathbf{H}}_j$.

Similar to the GZI scheme, we can also construct the jth user's precoding matrix $\mathbf{P}_j$ of the GMI using a linear combination of columns of $\bar{\mathbf{Q}}_j$. As shown in the previous section, the GZI achieves the performance equivalent to the conventional BD. In comparison to the GZI method based on the ZF-CI, the columns of $\bar{\mathbf{Q}}_j$ are obtained from the MMSE-CI which takes the noise into account. Thus, the proposed GMI scheme is able to overcome the noise enhancement issue observed in the conventional BD, and this leads to a linear growth of the sum rate with the number of users and transmit antennas compared to the BD scheme. However, unlike the BD algorithm in (5), the jth user's precoder of the GMI constructed by a linear combination of $\bar{\mathbf{Q}}_j$ generates residual interference. Thus, a proper residual interference suppression process is needed additionally. In the following, we introduce a way of computing the transmit combining matrix with two different criteria.

First, denoting the $n_j \times n_j$ transmit combining matrix applied to $\bar{\mathbf{Q}}_j$ as $\mathbf{T}_j$, the corresponding received signal vector of the jth user can be written from (1) as

$$\bar{\mathbf{y}}_j = \mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j \mathbf{s}_j + \mathbf{H}_j \sum_{k \neq j} \bar{\mathbf{Q}}_k \mathbf{T}_k \mathbf{s}_k + \mathbf{w}_j. \tag{12}$$

Next, we define $\mathbf{U}_j^H$ and $\mathbf{V}_j$ as the matrices employed to receiver and precoder in order to decompose the block channel $\mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j$ in (12) into parallel subchannels, respectively. Then, applying these matrices to (12), the single symbol decodable signal vector of the jth user can be expressed as

$$\mathbf{x}_j = \mathbf{U}_j^H \mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j \mathbf{V}_j \mathbf{s}_j + \mathbf{U}_j^H \mathbf{H}_j \sum_{k \neq j} \bar{\mathbf{Q}}_k \mathbf{T}_k \mathbf{V}_k \mathbf{s}_k + \mathbf{U}_j^H \mathbf{w}_j. \tag{13}$$

Here, similar to the BD algorithm, $\mathbf{U}_j^H$ and $\mathbf{V}_j$ are computed by the SVD of $\mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j$. Note that, we do not know the values of the matrices' entries, since $\mathbf{T}_j$ is not determined yet. However, we can assume that $\mathbf{U}_j^H$ and $\mathbf{V}_j$ are unitary, and we use this assumption in the following derivations.

1) Method 1: Minimum total MSE criterion under total transmit power constraint

Let us define the jth user's combining matrix $\mathbf{T}_j$ as $\beta \bar{\mathbf{T}}_j$ where the scaling parameter β is a positive real number. Then, the mean-squared error (MSE) of the jth user is represented as [9][13]

$$\mathbb{E}\Big[\| \mathbf{U}_j^H \mathbf{G}_j \mathbf{V}_j \mathbf{s}_j - \frac{1}{\beta} \mathbf{x}_j \|^2 \Big] \tag{14}$$

where $\mathbf{G}_j$ is the $n_j \times n_j$ target channel matrix based on the MMSE criterion. From (13) and (14), the problem of minimizing the total MSE under the total transmit power constraint can be written as

$$\begin{aligned} \min_{\mathbf{T}_j} \sum_{j=1}^{K} \mathbb{E}\Big[\| \mathbf{U}_j^H \mathbf{G}_j \mathbf{V}_j \mathbf{s}_j & \\ - \frac{1}{\beta} (\mathbf{U}_j^H \mathbf{H}_j \sum_{k=1}^{K} \bar{\mathbf{Q}}_k \mathbf{T}_k \mathbf{V}_k \mathbf{s}_k + \mathbf{U}_j^H \mathbf{w}_j) \|^2 \Big] & \\ \text{subject to} \sum_{j=1}^{K} \mathrm{Tr}(\mathbf{V}_j^H \mathbf{T}_j^H \bar{\mathbf{Q}}_j^H \bar{\mathbf{Q}}_j \mathbf{T}_j \mathbf{V}_j) = P_{total}. & \end{aligned} \tag{15}$$

In order to solve the above equation, we first convert this problem into an unconstrained minimization problem [18]. In the constraint function of the above equation, we substitute $\beta \bar{\mathbf{T}}_j$ for $\mathbf{T}_j$. Then, since $\mathbf{V}_j$ is unitary and $\bar{\mathbf{Q}}_j^H \bar{\mathbf{Q}}_j = \mathbf{I}_{nj}$, the scaling parameter β is given by

$$\beta = \sqrt{P_{total}} \Bigg[\sum_{j=1}^{K} \mathrm{Tr}(\bar{\mathbf{T}}_j^H \bar{\mathbf{T}}_j) \Bigg]^{-\frac{1}{2}}. \tag{16}$$

Applying (16) to the cost function in (15), the unconstrained total MSE minimization problem on $\bar{\mathbf{T}}_j$ is written as

$$\begin{aligned} \min_{\bar{\mathbf{T}}_j} \sum_{j=1}^{K} \mathbb{E}\Big[\| \mathbf{U}_j^H \mathbf{G}_j \mathbf{V}_j \mathbf{s}_j - \mathbf{U}_j^H \mathbf{H}_j \sum_{k=1}^{K} \bar{\mathbf{Q}}_k \bar{\mathbf{T}}_k \mathbf{V}_k \mathbf{s}_k & \\ - \frac{1}{\sqrt{P_{total}}} \sqrt{\sum_{k=1}^{K} \mathrm{Tr}(\bar{\mathbf{T}}_k^H \bar{\mathbf{T}}_k)} \;\; \mathbf{U}_j^H \mathbf{w}_j \|^2 \Big]. & \end{aligned}$$

Also, this problem can be formulated as [12]

$$\min_{\bar{\mathbf{T}}_j} \sum_{j=1}^{K} \mathrm{Tr}\Big(\mathbf{U}_j^H \mathbf{H}_j \sum_{k=1}^{K} \bar{\mathbf{Q}}_k \bar{\mathbf{T}}_k \bar{\mathbf{T}}_k^H \bar{\mathbf{Q}}_k^H \mathbf{H}_j^H \mathbf{U}_j + \frac{\sigma_w^2}{P_{total}} \sum_{k=1}^{K} \mathrm{Tr}(\bar{\mathbf{T}}_k^H \bar{\mathbf{T}}_k)\, \mathbf{I}_{nj} - \mathbf{U}_j^H \mathbf{G}_j \bar{\mathbf{T}}_j^H \bar{\mathbf{Q}}_j^H \mathbf{H}_j^H \mathbf{U}_j - \mathbf{U}_j^H \mathbf{H}_j \bar{\mathbf{Q}}_j \bar{\mathbf{T}}_j \mathbf{G}_j^H \mathbf{U}_j + \mathbf{U}_j^H \mathbf{G}_j \mathbf{G}_j^H \mathbf{U}_j\Big).$$

Now, we take a derivative of the above equation with respect to $\bar{\mathbf{T}}_j^H$ and set it to zero. Then, this results in

$$\bar{\mathbf{T}}_j = \Big(\bar{\mathbf{Q}}_j^H \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \bar{\mathbf{Q}}_j + \alpha \mathbf{I}_{n_j}\Big)^{-1} \bar{\mathbf{Q}}_j^H \mathbf{H}_j^H \mathbf{H}_j \bar{\mathbf{Q}}_j. \quad (17)$$

Here, we employ $\mathbf{H}_j \bar{\mathbf{Q}}_j \bar{\mathbf{T}}_{high,j}$ as the target channel matrix of the jth user $\mathbf{G}_j$ where $\bar{\mathbf{T}}_{high,j}$ is denoted as the jth user's combining matrix at high SNR, and we set $\bar{\mathbf{T}}_{high,j}$ to $\mathbf{I}_{n_j}$. Note that, from (10), (11) and (17), it is obvious that $\bar{\mathbf{T}}_j$ converges to an identity matrix as SNR increases. Finally, the jth user's transmit combining matrix $\mathbf{T}_j$ which minimizes the total MSE is given by $\mathbf{T}_j = \beta \bar{\mathbf{T}}_j$ where β is determined by the equation (16).

2) Method 2: Minimum interference-plus-noise power criterion under per-user power constraint

In Method 1, the assigned transmit power to the jth user, $\mathrm{Tr}(\mathbf{T}_j^H \mathbf{T}_j)$, is determined from (16) and (17) by itself. However, in some scenarios, it may be necessary to control the power allocation to each user for various purposes. For instance, we can allocate more power to a weaker user to improve the overall FER performance in coded systems. Also, for a cross-layer system design, a different user power allocation should be needed in order to support different Quality of Service demands of each user. For these cases, it is better to compute the transmit combining matrix with the per-user power constraint.

First, we define the jth user's transmit combining matrix $\mathbf{T}_j$ as $p_j \bar{\mathbf{T}}_j$ where $\mathrm{Tr}(\bar{\mathbf{T}}_j^H \bar{\mathbf{T}}_j) = n_j$ for $j = 1, \cdots, K$ and p_j is a positive real number with constraint $\sum_{j=1}^{K} n_j p_j^2 \leq P_{total}$. Then, the allocated power to the jth user is given by $\mathrm{Tr}(\mathbf{T}_j^H \mathbf{T}_j) = n_j p_j^2$.

When the individual power constraint is imposed to each user's precoding matrix, the transmit combining matrix of each user can be found separately. From (12), it is clear that the transmit combining matrix affects other users' interference. Thus, we define the power of other users' interference induced by the jth user's precoder plus the total noise power of its receiver as

$$\mathbb{E}\left[\sum_{k\neq j} \|\mathbf{U}_k^H \mathbf{H}_k \bar{\mathbf{Q}}_j \mathbf{T}_j \mathbf{V}_j \mathbf{s}_j\|^2\right] + \mathbb{E}\Big[\|\mathbf{U}_j^H \mathbf{w}_j\|^2\Big]. \quad (18)$$

Substituting $p_j \bar{\mathbf{T}}_j$ for $\mathbf{T}_j$ in (18), the minimization problem of (18) on $\bar{\mathbf{T}}_j$ can be expressed by

$$\min_{\bar{\mathbf{T}}_j} p_j^2 \left[\mathrm{Tr}\,(\bar{\mathbf{T}}_j^H (\bar{\mathbf{Q}}_j^H \sum_{k\neq j} \mathbf{H}_k^H \mathbf{H}_k \bar{\mathbf{Q}}_j + \frac{\sigma_w^2}{p_j^2} \mathbf{I}_{n_j}) \bar{\mathbf{T}}_j)\right]. \quad (19)$$

Since the matrix $\bar{\mathbf{Q}}_j^H \sum_{k\neq j} \mathbf{H}_k^H \mathbf{H}_k \bar{\mathbf{Q}}_j + (\sigma_w^2/p_j^2) \mathbf{I}_{n_j}$ in (19) is Hermitian and positive definite, we can decompose this matrix using Cholesky factorization as

$$\bar{\mathbf{Q}}_j^H \sum_{k\neq j} \mathbf{H}_k^H \mathbf{H}_k \bar{\mathbf{Q}}_j + \frac{\sigma_w^2}{p_j^2} \mathbf{I}_{n_j} = \bar{\mathbf{L}}_j^H \bar{\mathbf{L}}_j. \quad (20)$$

Then, (19) can be given by

$$\min_{\bar{\mathbf{T}}_j} p_j^2 \Big[\mathrm{Tr}\,(\bar{\mathbf{T}}_j^H \bar{\mathbf{L}}_j^H \bar{\mathbf{L}}_j \bar{\mathbf{T}}_j)\Big]. \quad (21)$$

Now, we can obtain the jth user's transmit combining matrix which minimizes (18) as $\mathbf{T}_j = p_j \bar{\mathbf{T}}_j$ where $\bar{\mathbf{T}}_j = \gamma \bar{\mathbf{L}}_j^{-1}$ and γ is determined by $\mathrm{Tr}(\bar{\mathbf{T}}_j^H \bar{\mathbf{T}}_j) = n_j$ [16]. Similar to Method 1, when SNR increases, $\bar{\mathbf{T}}_j$ converges to $\mathbf{I}_{n_j}$.

Note that, in this method, we have computed $\mathbf{T}_j$ with the assumption that p_j^2 is given. For simplicity, we consider an equal power allocated case, i.e., $p_j^2 = P_{total}/N_r$ for $j = 1, \cdots, K$. In this case, the term σ_w^2/p_j^2 in (19) and (20) becomes equal to α.

After finding $\bar{\mathbf{Q}}_j$ and $\mathbf{T}_j$, we now make each user's received signal vector single symbol decodable. The term $\mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j$ in (12) represents the interference suppressed block channel of the jth user. In order to decompose this channel into parallel subchannels, we apply the SVD of $\mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j$ as

$$\mathbf{H}_j \bar{\mathbf{Q}}_j \mathbf{T}_j = \mathbf{U}_j^{(\mathrm{m})} \mathbf{\Lambda}_j^{(\mathrm{m})} \mathbf{V}_j^{(\mathrm{m})H}.$$

Then, the precoding matrix and the overall receive filter of the GMI scheme are obtained as

$$\begin{aligned} \mathbf{P}_s^{\mathrm{GMI}} &= [\bar{\mathbf{Q}}_1 \mathbf{T}_1 \mathbf{V}_1^{(\mathrm{m})} \quad \bar{\mathbf{Q}}_2 \mathbf{T}_2 \mathbf{V}_2^{(\mathrm{m})} \quad \cdots \quad \bar{\mathbf{Q}}_K \mathbf{T}_K \mathbf{V}_K^{(\mathrm{m})}], \\ \mathbf{M}_s^{\mathrm{GMI}} &= \mathrm{diag}\{\mathbf{U}_1^{(\mathrm{m})H}, \quad \mathbf{U}_2^{(\mathrm{m})H}, \quad \cdots, \quad \mathbf{U}_K^{(\mathrm{m})H}\}. \end{aligned}$$

Finally, after applying the above solutions to (2), the receive filter output signal vector at the jth user can be written as

$$\mathbf{x}_j = \mathbf{\Lambda}_j^{(\mathrm{m})} \mathbf{s}_j + \mathbf{M}_j^{\mathrm{GMI}} \mathbf{H}_j \sum_{k\neq j} \mathbf{P}_k^{\mathrm{GMI}} \mathbf{s}_k + \mathbf{M}_j^{\mathrm{GMI}} \mathbf{w}_j. \quad (22)$$

We now briefly address the sum rate of the GMI. For the BD scheme, the WF solution in (6) is utilized to maximize the sum rate, since the precoding matrix with the WF method still satisfies the zero-interference constraint (3). In contrast, for the GMI scheme, it is not needed to allocate different power to each stream, since each combining matrix has already been computed to meet its corresponding criterion. For instance, in Method 1, if we apply an additional power loading matrix to the precoding matrix, the minimum total MSE condition is no longer satisfied. Also, in the MMSE based scheme, it is not easy to identify the optimal WF solution, since the residual interference varies according to the power allocation. Thus, we do not consider the WF algorithm when we compute the sum rate of the GMI.

Denoting the ith diagonal element of $\mathbf{\Lambda}_j^{(\mathrm{m})}$ as $\lambda_{j,i}^{(\mathrm{m})}$, each received signal in (22) contains in part the signal of interest with the channel gain $(\lambda_{j,i}^{(\mathrm{m})})^2$ and in part the interference from the other users plus Gaussian noise. Since the matrix $\mathbf{M}_j^{\mathrm{GMI}}$ is unitary, the SINR of each stream can be expressed as [5]

$$\mathrm{SINR}_{j,i} = \frac{\left(\lambda_{j,i}^{(\mathrm{m})}\right)^2}{\sigma_w^2 + \sum_{k\neq j} \|\mathbf{m}_{j,i} \mathbf{H}_j \mathbf{P}_k^{\mathrm{GMI}}\|^2} \quad (23)$$

where $\mathbf{m}_{j,i}$ is the ith row vector of $\mathbf{M}_j^{\text{GMI}}$. Then, the sum rate of the proposed GMI scheme is given by

$$R_{\text{GMI}} = \sum_{j=1}^{K}\sum_{i=1}^{n_j} \log_2\left(1 + \text{SINR}_{j,i}\right). \tag{24}$$

B. Design with imperfect channel information

So far, we have assumed that the base station has knowledge of full CSI. In a system with time-division duplexing, the CSI can be obtained by exploiting uplink-downlink reciprocity, while in a frequency-division duplexing, the quantized channel information can be fed back to the transmitter. In any case, the mismatch between the actual CSI and the transmitter's estimate of the CSI is inevitable, and this can result in serious performance degradation. Here, we illustrate how the GMI algorithm can be modified to mitigate such losses when the CSI at the base station is inaccurate.

We consider the channel estimation error model introduced in [19][20]

$$\mathbf{H}_s = \mathbf{H}_{est} + \mathbf{H}_{err} \tag{25}$$

where $\mathbf{H}_s$, $\mathbf{H}_{est}$ and $\mathbf{H}_{err}$ represent the true channel matrix, the estimated channel matrix and the estimation error matrix, respectively. We assume that $\mathbf{H}_{err}$ is uncorrelated with $\mathbf{H}_{est}$ and $\mathbf{s}_s$, and that $\mathbf{H}_{err}$ in (25) has i.i.d. elements with zero mean and the estimation error variance $\sigma_{e,h}^2$. The entries of $\mathbf{H}_s$ are also i.i.d. with zero mean and unit variance.

In this system model, the received signal vector is given by

$$\mathbf{y}_s = \mathbf{H}_{est}\mathbf{P}_s\mathbf{s}_s + \mathbf{H}_{err}\mathbf{P}_s\mathbf{s}_s + \mathbf{w}_s$$

where $\mathbf{H}_{err}\mathbf{P}_s\mathbf{s}_s$ results from the estimation error. Defining the error term as $\mathbf{e} = \mathbf{H}_{err}\mathbf{P}_s\mathbf{s}_s + \mathbf{w}_s$, the total error variance σ_e^2 can be computed as

$$\sigma_e^2 = \mathbb{E}[\|\mathbf{e}\|^2] = N_t\,\sigma_{e,h}^2\,\text{Tr}\left(\mathbf{P}_s^H\mathbf{P}_s\right) + N_r\,\sigma_w^2. \tag{26}$$

From (10) and (26), the precoding matrix can be obtained by applying the MMSE-CI of the estimated channel matrix with the above total error variance. We denote $\bar{\mathbf{H}}_{est}$ as

$$\begin{aligned}\bar{\mathbf{H}}_{est} &= (\mathbf{H}_{est}^H\mathbf{H}_{est} + \alpha_e\mathbf{I})^{-1}\mathbf{H}_{est}^H \\ &= [\,\bar{\mathbf{H}}_{est,1} \quad \bar{\mathbf{H}}_{est,2} \quad \cdots \quad \bar{\mathbf{H}}_{est,K}\,]\end{aligned}$$

where α_e is given by $\alpha_e = \sigma_e^2/P_{total} = N_t\sigma_{e,h}^2 + \alpha$. After employing the QR-decomposition to $\bar{\mathbf{H}}_{est,j}$ as $\bar{\mathbf{H}}_{est,j} = \bar{\mathbf{Q}}_{est,j}\bar{\mathbf{R}}_{est,j}$, the jth user's power scaled transmit combining matrix $\bar{\mathbf{T}}_{est,j}$ for Method 1 in (17) is given as

$$\begin{aligned}\bar{\mathbf{T}}_{est,j} = \Big(\bar{\mathbf{Q}}_{est,j}^H \sum_{k=1}^{K} \mathbf{H}_{est,k}^H\,\mathbf{H}_{est,k}\,\bar{\mathbf{Q}}_{est,j} + \alpha_e\mathbf{I}_{n_j}\Big)^{-1} \\ \times\,\bar{\mathbf{Q}}_{est,j}^H\,\mathbf{H}_{est,j}^H\,\mathbf{H}_{est,j}\,\bar{\mathbf{Q}}_{est,j}.\end{aligned}$$

Also, in Method 2, the Cholesky factorization in (20) can be expressed by

$$\bar{\mathbf{Q}}_{est,j}^H \sum_{k\neq j} \mathbf{H}_{est,k}^H\,\mathbf{H}_{est,k}\bar{\mathbf{Q}}_{est,j} + \alpha_e\mathbf{I}_{n_j} = \bar{\mathbf{L}}_{est,j}^H\,\bar{\mathbf{L}}_{est,j}$$

and $\bar{\mathbf{T}}_{est,j}$ is computed as $\bar{\mathbf{T}}_{est,j} = \gamma\bar{\mathbf{L}}_{est,j}^{-1}$ where $\gamma^2 = n_j/\text{Tr}(\mathbf{L}_{est,j}^{-H}\mathbf{L}_{est,j}^{-1})$. A performance gain over the conventional BD in the presence of channel estimation errors will be verified in the following simulation section.

Note that, for the above derivations, we have assumed that the value of $\sigma_{e,h}^2$ is perfectly known to the base station. In practical systems, the channel error variance can be estimated through the stochastic process or approximated according to the channel quantization rate [21][22]. The study of an estimation method of the channel error variance is outside the scope of this paper and remains as future work.

V. Comparison with Existing Schemes

An idea similar to the proposed GMI scheme was independently studied in [16], which regularizes the BD scheme in multiuser MIMO systems named as regularized block diagonalization (RBD). In this section, we compare the proposed GMI scheme and the RBD scheme. Especially, we prove that Method 2 in the proposed GMI algorithm is equivalent to the RBD scheme and demonstrate computational complexity benefits of the proposed GMI scheme compared to the prior works.

First, we briefly describe the RBD algorithm [16] in the followings. Let us define the jth user's RBD precoding matrix $\mathbf{P}_j^{\text{RBD}}$ as

$$\mathbf{P}_j^{\text{RBD}} = \tilde{\beta}\tilde{\mathbf{P}}_j^{\text{RBD}}\bar{\mathbf{P}}_j^{\text{RBD}}$$

where $\tilde{\mathbf{P}}_j^{\text{RBD}}$ is an $N_t \times N_t$ matrix, $\bar{\mathbf{P}}_j^{\text{RBD}}$ is an $N_t \times n_j$ matrix and the scaling factor $\tilde{\beta}$ represents a real positive number. Then, $\tilde{\mathbf{P}}_j^{\text{RBD}}$ can be determined by the following optimization

$$\min_{\tilde{\mathbf{P}}_j^{\text{RBD}}} \mathbb{E}\left[\sum_{j=1}^{K}\|\tilde{\mathbf{H}}_j\tilde{\mathbf{P}}_j^{\text{RBD}}\|^2 + \frac{\|\mathbf{w}_s\|^2}{\tilde{\beta}^2}\right]. \tag{27}$$

Defining the SVD of $\tilde{\mathbf{H}}_j$ as $\tilde{\mathbf{H}}_j = \tilde{\mathbf{U}}_j\tilde{\mathbf{\Lambda}}_j\tilde{\mathbf{V}}_j^H$, the solution of the minimization (27) results in

$$\tilde{\mathbf{P}}_j^{\text{RBD}} = \tilde{\mathbf{V}}_j(\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1/2}. \tag{28}$$

Also, from the SVD of the jth user's equivalent channel

$$\mathbf{H}_j\tilde{\mathbf{P}}_j^{\text{RBD}} = \bar{\mathbf{U}}_j[\bar{\mathbf{\Lambda}}_j \;\; \mathbf{O}][\bar{\mathbf{V}}_j^{(1)} \;\; \bar{\mathbf{V}}_j^{(0)}]^H \tag{29}$$

where $\mathbf{O}$ is a zero matrix, the matrix $\bar{\mathbf{P}}_j^{\text{RBD}}$ is obtained as $\bar{\mathbf{P}}_j^{\text{RBD}} = \bar{\mathbf{V}}_j^{(1)}$ where $\bar{\mathbf{V}}_j^{(1)}$ denotes the set of right singular vectors corresponding to non-zero singular values. Finally, the jth user's precoding matrix of the RBD can be written as

$$\mathbf{P}_j^{\text{RBD}} = \tilde{\beta}\tilde{\mathbf{V}}_j(\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1/2}\bar{\mathbf{V}}_j^{(1)}. \tag{30}$$

Now, we prove that the jth user's precoding matrices of the RBD and the GMI with Method 2 in Section IV yield the identical performance. First, let us denote $\mathbf{A}$ as an arbitrary $m \times n$ matrix where $m \geq n$. Then, when the matrix $\mathbf{A}$ is used as a precoding matrix, we can define that the precoding matrices $\mathbf{A}$ and $\mathbf{AB}$ are *equivalent* if $\mathbf{B}$ is an $n \times n$ unitary matrix, since $\mathbf{B}$ has no effect on the performance of precoding methods.

Next, we define $\mathbb{C}(\mathbf{A})$ as an $m \times n$ matrix which consists of the orthonormal basis vectors that span the column space of

$\mathbf{A}$. For example, denoting the QR-decomposition of $\mathbf{A}$ as $\mathbf{A} = \mathbf{Q}_a\mathbf{R}_a$ and the SVD of $\mathbf{A}$ as $\mathbf{A} = [\mathbf{U}_a^{(1)}\ \mathbf{U}_a^{(0)}]\mathbf{\Lambda}_a\mathbf{V}_a^H$, both $\mathbf{Q}_a$ and $\mathbf{U}_a^{(1)}$ can be $\mathbb{C}(\mathbf{A})$, and also $\mathbf{Q}_a$ and $\mathbf{U}_a^{(1)}$ are equivalent. Using these definitions, we describe some theorems in the following.

Lemma 1: $m \times n$ matrices $\mathbb{C}(\mathbf{AB})$, $\mathbb{C}(\mathbf{AC})$ and $\mathbb{C}(\mathbf{A})$ are equivalent with each other when $\mathbf{B}$ and $\mathbf{C}$ are $n \times n$ square matrices.

Proof: Since $\mathbf{AB}$ and $\mathbf{AC}$ can be constructed by a linear combination of the columns of $\mathbb{C}(\mathbf{A})$, the matrices $\mathbf{AB}$, $\mathbf{AC}$ and $\mathbf{A}$ have the same column space. ■

Theorem 1: Let us set $\tilde{\beta}$ in (30) to 1 for notational simplicity. Then, the jth user's precoding matrix of the RBD in (30) can be rewritten as

$$\mathbf{P}_j^{\mathrm{RBD}} = \mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)\mathbf{F} \tag{31}$$

where $\mathbf{F}$ is an $n_j \times n_j$ square matrix which satisfies

$$\mathbf{F}^H\,\mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)^H\left(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I}\right) \times \mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)\mathbf{F} = \mathbf{I}_{n_j}. \tag{32}$$

Proof: From (28) and (29), since $\bar{\mathbf{U}}_j$ and $\bar{\mathbf{\Lambda}}_j$ are invertible, the $N_t \times n_j$ matrix $\bar{\mathbf{V}}_j^{(1)}$ can be expressed as

$$\bar{\mathbf{V}}_j^{(1)} = (\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1/2}\tilde{\mathbf{V}}_j^H\mathbf{H}_j^H\bar{\mathbf{U}}_j\bar{\mathbf{\Lambda}}_j^{-1}.$$

Substituting this to (30), since $(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1} = \tilde{\mathbf{V}}_j(\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1}\tilde{\mathbf{V}}_j^H$, $\mathbf{P}_j^{\mathrm{RBD}}$ is given by

$$\mathbf{P}_j^{\mathrm{RBD}} = (\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\bar{\mathbf{U}}_j\bar{\mathbf{\Lambda}}_j^{-1}.$$

Here $(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H$ is an $N_t \times n_j$ matrix and $\mathbf{U}_j\bar{\mathbf{\Lambda}}_j^{-1}$ is an $n_j \times n_j$ matrix. Hence, from lemma 1, the jth user's precoding matrix of RBD can be given by (31). Also, applying $\mathbf{P}_j^{\mathrm{RBD}}$ in (30) to $(\mathbf{P}_j^{\mathrm{RBD}})^H(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})\,\mathbf{P}_j^{\mathrm{RBD}}$, it can be shown that

$$\mathbf{V}_j^{(1)H}(\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1/2}\tilde{\mathbf{V}}_j^H(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I}) \times \tilde{\mathbf{V}}_j(\tilde{\mathbf{\Lambda}}_j^H\tilde{\mathbf{\Lambda}}_j + \alpha\mathbf{I})^{-1/2}\bar{\mathbf{V}}_j^{(1)} = \mathbf{V}_j^{(1)H}\mathbf{V}_j^{(1)} = \mathbf{I}_{n_j}.$$

Thus, $\mathbf{F}$ in (31) satisfies the equation (32). ■

Theorem 2: Let us set p_j in Method 2 of Section IV to 1 for notational simplicity. The jth user's precoding matrix of the GMI in Method 2 can be rewritten as

$$\mathbf{P}_j^{\mathrm{GMI}} = \mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)\mathbf{E} \tag{33}$$

where $\mathbf{E}$ is an $n_j \times n_j$ square matrix which satisfies

$$\mathbf{E}^H\,\mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)^H\left(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I}\right) \times \mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)\mathbf{E} = \mathbf{I}_{n_j}. \tag{34}$$

Proof: From (10) and (11), the $N_t \times n_j$ matrix $\bar{\mathbf{Q}}_j$ is computed by the orthogonalization of the $N_t \times n_j$ matrix $(\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H$. Thus, $\bar{\mathbf{Q}}_j$ can be expressed as $\mathbb{C}((\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H)$. Using the matrix inversion lemma [23], $(\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H$ can be rewritten as[1]

$$\begin{aligned}(\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H &= (\mathbf{H}_j^H\mathbf{H}_j + \tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H \\ &= (\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H(\mathbf{I}_{n_j} + \mathbf{H}_j(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H)\end{aligned} \tag{35}$$

where $(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H$ is an $N_t \times n_j$ matrix and $(\mathbf{I}_{n_j} + \mathbf{H}_j(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H)$ is an $n_j \times n_j$ matrix. Thus, from lemma 1, the $N_t \times n_j$ matrices $\mathbb{C}((\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H)$ and $\mathbb{C}((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H)$ are equivalent, and the jth user's precoding matrix of the GMI in Method 2, $\bar{\mathbf{Q}}_j\mathbf{T}_j$, can be expressed as (33). Also, it is shown from (20) and (21) that

$$\begin{aligned}&(\mathbf{P}_j^{\mathrm{GMI}})^H(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})\,\mathbf{P}_j^{\mathrm{GMI}} \\ &= \mathbf{T}_j^H(\bar{\mathbf{Q}}_j^H\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j\bar{\mathbf{Q}}_j + \alpha\mathbf{I}_{n_j})\mathbf{T}_j = \mathbf{I}_{n_j}.\end{aligned}$$

Thus, $\mathbf{E}$ in (33) satisfies the equation (34). ■

Theorem 3: The $n_j \times n_j$ matrices $\mathbf{F}$ in (32) and $\mathbf{E}$ in (34) are equivalent.

Proof: In (32) and (34), we apply Cholesky factorization as

$$\mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right)^H\left(\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I}\right) \times \mathbb{C}\left((\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H\right) = \tilde{\mathbf{L}}^H\tilde{\mathbf{L}}.$$

Then, $\mathbf{F}$ and $\mathbf{E}$ should be $\tilde{\mathbf{L}}^{-1}\tilde{\mathbf{F}}$ and $\tilde{\mathbf{L}}^{-1}\tilde{\mathbf{E}}$, respectively, where $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{E}}$ are unitary matrices. ■

From theorems 1, 2 and 3, we have shown that the jth user's precoding matrices $\mathbf{P}_j^{\mathrm{RBD}}$ and $\mathbf{P}_j^{\mathrm{GMI}}$ have the same column space, and the combining matrices $\mathbf{F}$ and $\mathbf{E}$ are equivalent. Thus, without loss of generality, it is clear that $\mathbf{P}_j^{\mathrm{RBD}}$ and $\mathbf{P}_j^{\mathrm{GMI}}$ in Method 2 are equivalent.

As shown in the above derivations, we can generalize the MMSE-CI scheme with a different approach to the RBD algorithm. The RBD scheme first computes the matrix $\tilde{\mathbf{P}}_j^{\mathrm{RBD}}$ in (28) that whitens the other user's interference plus the noise, and then calculates a set of the row basis vectors of $\mathbf{H}_j\tilde{\mathbf{P}}_j^{\mathrm{RBD}}$ in (29). In contrast, the proposed GMI scheme first identifies the orthonormal basis vectors $\bar{\mathbf{Q}}_j$ which span the vector space constructed by projecting the jth user's channel matrix $\mathbf{H}_j$ onto the inverse space of the other user's effective channel matrix while taking the noise into account. Based on this, we can obtain the transmit combining matrices with different power constraints. Moreover, the proposed GMI computes a set of orthonormal basis vectors of the precoding matrix with only one inverse operation in (10) and a few orthogonalization operations, and thus the complexity becomes lower compared to the RBD approach based on the SVD operations.

Next, we will compare the computational complexity of the proposed schemes and the prior works. As it is difficult to calculate the exact number of operations for various schemes, we compute the complexity in terms of the required floating point operations (flops). According to [23], the required flops of each matrix operation are described as follows:

[1] Denoting an $m \times m$ invertible matrix as $\mathbf{A}$ and an $n \times m$ matrix as $\mathbf{B}$ where $n \leq m$, the $m \times n$ matrix $(\mathbf{A} + \mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H$ can be expressed as $(\mathbf{A} + \mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H = (\mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}^H(\mathbf{I}_n + \mathbf{B}\mathbf{A}^{-1}\mathbf{B}^H)^{-1}\mathbf{B}\mathbf{A}^{-1})\mathbf{B}^H = \mathbf{A}^{-1}\mathbf{B}^H(\mathbf{I}_n + \mathbf{B}\mathbf{A}^{-1}\mathbf{B}^H)^{-1}$. Substituting $\tilde{\mathbf{H}}_j^H\tilde{\mathbf{H}}_j + \alpha\mathbf{I}$ for $\mathbf{A}$ and $\mathbf{H}_j$ for $\mathbf{B}$ in the above equation, the $N_t \times n_j$ matrix $(\mathbf{H}_s^H\mathbf{H}_s + \alpha\mathbf{I})^{-1}\mathbf{H}_j^H$ can be given by (35).

TABLE I
THE NUMBER OF FLOATING POINT OPERATIONS OF EXISTING SCHEMES

	BD [15]	RBD [16]
SVD of (4)	$K \cdot (4N_t^2\tilde{N}_r + 13\tilde{N}_r^3)$	$K \cdot (4N_t^2\tilde{N}_r + 13\tilde{N}_r^3)$
Calculating $\bar{\mathbf{P}}_j^{\text{RBD}}$ in (29)	-	$K \cdot ((2n+1)N_t^2 + (2n^2+2)N_t + 2\tilde{N}_r - \frac{2}{3}n^3)$

TABLE II
THE NUMBER OF FLOATING POINT OPERATIONS OF PROPOSED SCHEMES

	GZI	GMI (Method 1)	GMI (Method 2)
Channel Inversion	$\frac{11}{3}N_t^3 + \frac{5}{3}N_t^2$	$\frac{11}{3}N_t^3 + \frac{5}{3}N_t^2 + N_t$	$\frac{11}{3}N_t^3 + \frac{5}{3}N_t^2 + N_t$
Orthogonalization	$K \cdot 2n^2(N_t - \frac{1}{3}n)$	$K \cdot 2n^2(N_t - \frac{1}{3}n)$	$K \cdot 2n^2(N_t - \frac{1}{3}n)$
Calculating $\mathbf{T}_j$	-	$K \cdot (2nN_t^2 + (3n^2+n)N_t + \frac{11}{3}n^3 + \frac{5}{3}n^2 + n)$	$K \cdot (2nN_t\tilde{N}_r + (n^2+n)\tilde{N}_r + n^3 + \frac{2}{3}n^2 + n)$

- Multiplication of an $m \times n$ matrix and an $n \times p$ matrix: $2mnp$.
- SVD of an $m \times n$ matrix $(m \leq n)$ where only $\mathbf{\Lambda}$ and $\mathbf{V}$ are obtained: $4n^2m + 13m^3$.
- Orthogonalization of an $m \times n$ matrix $(m \geq n)$: $2n^2(m - n/3)$.
- Cholesky factorization of an $m \times m$ matrix: $m^3/3$.
- Inversion of an $m \times m$ matrix using Gauss-Jordan elimination: $4m^3/3$.

Note that, when the results of multiplication and inversion are $m \times m$ Hermitian matrices, it is possible to reduce the complexity proportional to $(m+1)/2m$. Here, the required flops are considered in the case of a real matrix and approximated by omitting the low order terms. Also, each function, such as addition, multiplication, division and square root, is counted as one flop.

We compare the required flops of each precoding algorithm in Tables I and II where we assume $n_j = n$ for all j, and $\tilde{N}_r = N_r - n$. For instance, in case of $\{2, 2, 2, \} \times 6$, the required flops of the BD, the RBD, the GZI, the GMI (Method 1) and the GMI (Method 2) are counted as 4224, 4952, 980, 1784 and 1384, respectively. As shown in this result, performing the SVD operation needs high computational complexity. Thus, the proposed algorithms exhibit lower complexity than the conventional BD and the RBD approaches, and the complexity advantage grows as N_t and K increase.

VI. NUMERICAL RESULTS

In this section, we compare the performance of the proposed GMI scheme with the BD scheme in [15] through Monte carlo simulations. For all simulations, spatially uncorrelated MIMO channels with its elements generated by i.i.d. complex Gaussian random variables with zero-mean and unit-variance are used. In the plots, the GMI-1 and the GMI-2 indicate systems with the transmit combining matrix obtained using Method 1 and Method 2, respectively.

First, in Fig. 2, we present the comparison of the cumulative distribution functions (CDFs) of the average received SINR for the proposed schemes and prior works for the $\{2, 2\} \times 4$ case where the average received SINR is defined as $\frac{1}{N_r}\sum_{j=1}^{K}\sum_{i=1}^{n_j}\text{SINR}_{j,i}$ from (23). In this plot, we confirm that the GMI-2 and the RBD are equivalent, and the average received SINR of the GMI-1 is greater than that of other precoding algorithms.

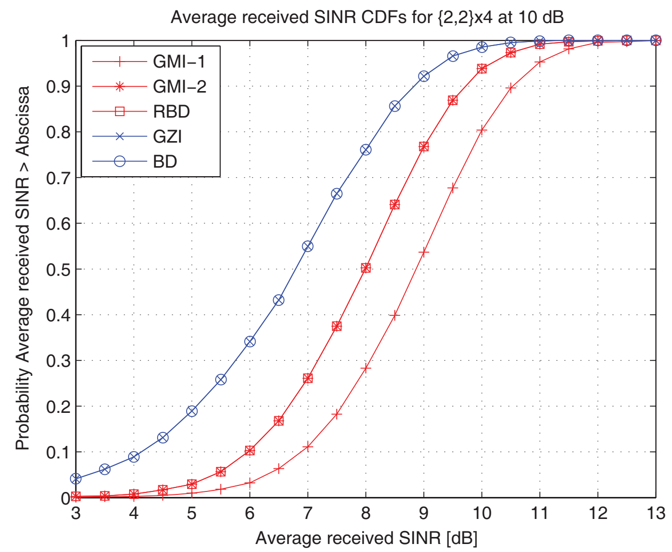

Fig. 2. CDF of the average received SINR for $\{2, 2\} \times 4$ multiuser MIMO systems

In Figures 3, 4 and 5, we compare the sum rates of various multiuser MIMO schemes under the perfect link adaptation and full CSI assumptions. The sum capacity is obtained by calculating the sum power iterative water-filling (SP-IWF) algorithm in [24], and the sum rate of the GMI scheme is computed using (23) and (24). Figures 3 and 4 illustrate the sum rates as a function of SNR for the $\{2, 2\} \times 4$ and $\{2, 2, 2\} \times 6$ cases, respectively. These figures clearly show a sum rate gain of the proposed GMI schemes over the BD algorithm, and this gain decreases for high SNR as expected. Also, as shown in Fig. 2, we observe that the GMI-1 slightly outperforms the GMI-2. Fig. 5 compares the sum capacity and sum rates for the proposed GMI-1 and conventional BD schemes in terms of the number of users K with $n_j = 2$ for all j and $N_t = 2K$. Unlike the BD scheme, the sum rate slope of the GMI-1 is much steeper than the BD scheme and exhibits a linear growth with K. It is clear from the plot that the capacity gain of the GMI-1 over the BD grows as the number of users increases.

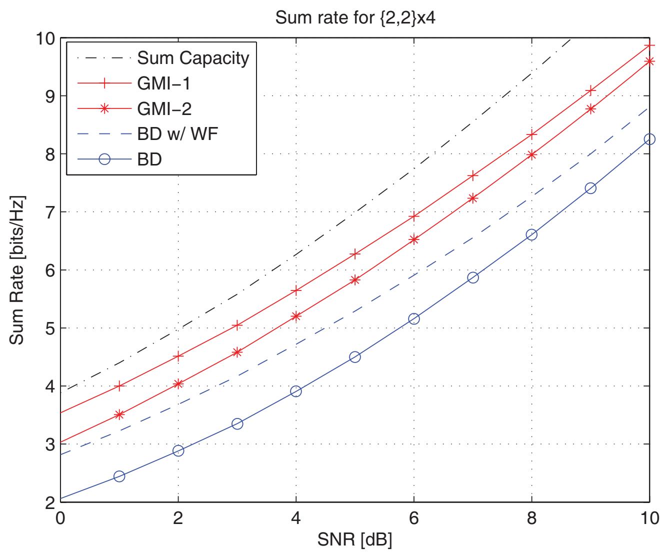

Fig. 3. Comparison of the sum rate as a function of SNR for $\{2,2\} \times 4$ multiuser MIMO downlink

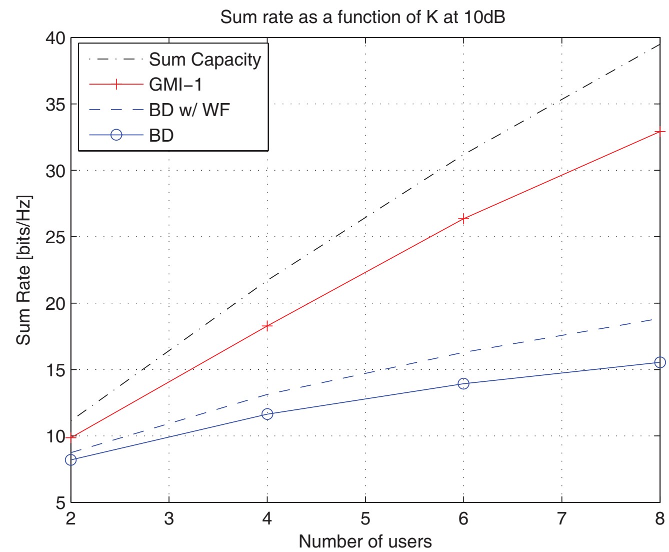

Fig. 5. Comparison of the sum rate as a function of K for SNR = 10dB

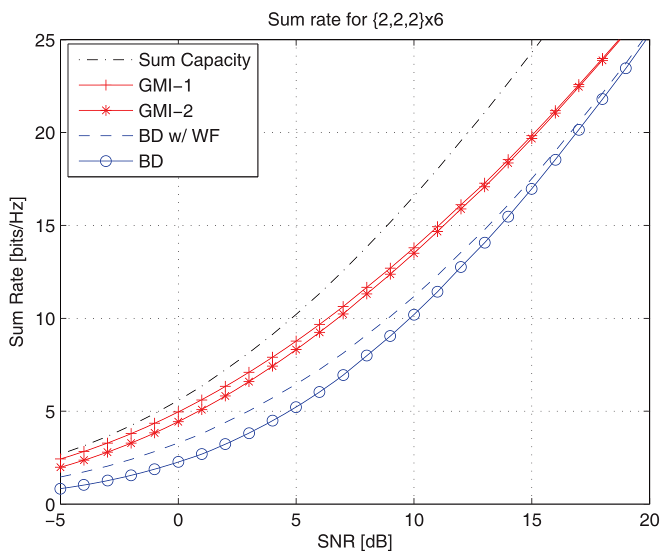

Fig. 4. Comparison of the sum rate as a function of SNR for $\{2,2,2\} \times 6$ multiuser MIMO downlink

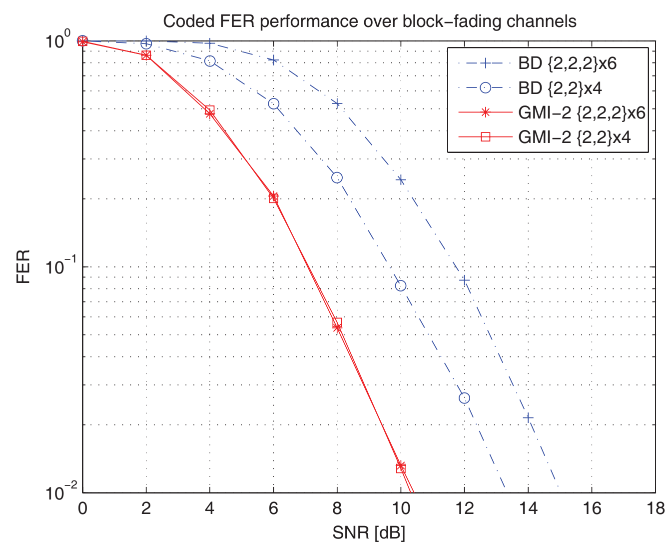

Fig. 6. FER with turbo codes for $\{2,2\}\times 4$ and $\{2,2,2\}\times 6$ in block-fading MIMO channels

In Figures 6 and 7, we show the simulation results of coded systems for various channel models in terms of FER with respect to SNR in dB for the $\{2,2\} \times 4$ and $\{2,2,2\} \times 6$ cases. For all FER simulations, a rate-1/2 turbo code based on parallel concatenated code with polynomial (15,13) in octal notation is employed with 6 iterations. We use the optimized interleaver to maximize the spatial diversity gain in MIMO systems [25][26]. We adopt 4-QAM to achieve the spectral efficiency of 4 bps/Hz and 6 bps/Hz for $\{2,2\} \times 4$ and $\{2,2,2\} \times 6$, respectively.

In Fig. 6, the spatially uncorrelated flat fading network channel $\mathbf{H}_s$ is generated by an ergodic random process at each frame and is fixed during the transmission of the frame. A 3dB SNR gain at 1% FER for the $\{2,2\}\times 4$ case is observed in Fig. 6. Also, the figure shows that the proposed GMI algorithm outperforms the BD by more than 5dB at 1% FER for the $\{2,2,2\} \times 6$ case. Note that, as expected from the sum rate analysis, the FER performance gap increases as the number of users grows.

Finally, in Fig. 7, we evaluate the performance in flat fading channels with various channel estimation error variance values $\sigma^2_{e,h}$. We plot the bit error rate (BER) performance of the proposed GMI-2 and the BD schemes. In the presence of the channel estimation error, we can see from (26) that the performance becomes limited by the error variance as SNR increases. Hence, as shown in the figure, both schemes exhibit error floors with $\sigma^2_{e,h} = 0.05$. Nevertheless, it is clear that the proposed GMI scheme is much more robust to the estimation errors compared to the conventional BD scheme.

VII. Conclusion

In this paper, we have generalized the channel inversion methods for multiuser MIMO downlink systems where each

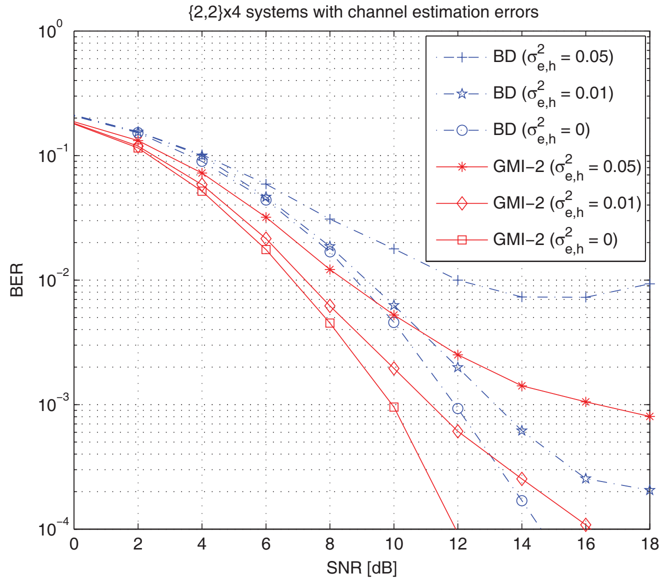

Fig. 7. BER with turbo codes for $\{2, 2\} \times 4$ in block-fading MIMO channels with various $\sigma^2_{e,h}$

user has more than one antenna. An alternative approach has been introduced by using the ZF channel inversion and the orthogonalization process for establishing the block diagonal channel matrix. The proposed GMI precoders are obtained by employing the MMSE channel inversion and combining methods, and as a result, the SINR is increased at each user's receiver. We have shown that one of the GMI schemes is equivalent to the RBD and demonstrated that the proposed algorithms have lower complexity compared to the conventional BD and RBD methods. Through Monte Carlo simulations, we have exhibited that the proposed GMI outperforms the conventional BD in terms of sum rate and FER. Also, we have proposed a modified method to improve the performance in the presence of the channel estimation error and illustrated that the proposed scheme is robust to the estimation error.

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