Spatial Multiplexing Gain for Two Interfering MIMO Broadcast Channels Based on Linear Transceiver

Jaesin Kim, *Student Member, IEEE*, Seok-Hwan Park, *Student Member, IEEE*, Hakjea Sung, *Student Member, IEEE*, and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—In this letter, we provide an expression of spatial multiplexing gain (SMG) for two mutually interfering multiple-input multiple-output (MIMO) broadcast channels, referred to as MIMO-IBC, with linear transceiver. We derive the SMG with respect to user antenna distribution, and compare the systems with and without cooperation among receive antennas in each cell. Additionally, we propose a linear precoding and decoding algorithm for the MIMO-IBC which maximizes the sum rate by extending a solution for single-cell multiple-input single-output broadcast channels. Simulation results confirm the accuracy of our theoretical SMG analysis for the MIMO-IBC.**

***Index Terms*—Spatial multiplexing gain, multiple-input multiple-output (MIMO), broadcast channel (BC), transmit beamforming, minimum mean-squared error (MMSE).**

I. INTRODUCTION

WHEN evaluating multiple-input multiple-output (MIMO) multiuser systems, the number of degrees of freedom (DOF) provides the capacity scaling behavior at high signal-to-noise ratio (SNR) [1][2]. Authors in [1] focused on two-user MIMO interference channels (IC) where transmitter i with T_i antennas has a message for receiver i with R_i antennas ($i = 1, 2$), which will be referred to as (T_1, R_1, T_2, R_2) IC. By providing the precise number of the DOF for the MIMO IC, they showed that there may be a significant loss on the DOF due to distributed processing at both transmitter and receiver sides, compared to point-to-point (PTP) MIMO channels.

Since it is not trivial to analytically derive the DOFs in many IC environments, we consider a spatial multiplexing gain (SMG) measure. Here, the SMG is defined as the pre-log factor of the sum rate of the system when linear transceivers are only spatially exploited. Recently, an expression of the SMG for two mutually interfering broadcast channels (IBC) was derived as a function of arbitrary numbers of transmit antennas and users [3]. We will refer to the IBC where each transmitter equipped with T_i antennas sends messages to its corresponding R_i single-antenna receivers ($i = 1, 2$) as (T_1, R_1, T_2, R_2) IBC. The IBC differs from the MIMO IC in a sense that R_i receive antennas in cell i are disconnected and cannot cooperate with each other. Although one might think that the SMG of the MIMO IC[1] would be the same as that of the IBC, the derived result in [3] showed that disabling receive cooperation of the MIMO IC causes a SMG loss for certain antenna configurations.

In this letter, we extend the previous analysis of the two-cell IBC to the case of multiple-antenna users, which will be referred to as MIMO-IBC. Motivated by earlier works on the IC and the IBC, we derive the SMG for the MIMO-IBC based on a space-division multiplexing system with the transmit beamforming and the minimum mean-squared error (MMSE) receiver. The SMG expression of the MIMO-IBC is a general equation including the results of the IC and the IBC. Thus, we offer a quantitative analysis on how the SMG can be limited by the distribution of user antennas. Additionally, we propose a precoding and decoding method to support the derived SMG for various antenna configurations. To this end, we present an algorithm to jointly design a sum rate maximizing linear precoding and decoding filter. Simulation results show that the proposed scheme supports the derived SMG of the MIMO-IBC.

The remainder of this letter is organized as follows: In Section II, we introduce the MIMO-IBC model and the SMG of the MIMO-IBC is derived in Section III. We propose a joint transmit and receive filter design algorithm for the MIMO-IBC in Section IV, which is numerically evaluated in Section V. The letter is closed with conclusion in Section VI.

Throughout this letter, the following notations are used for description. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The transpose, Hermitian and trace of a matrix or a vector are represented by $(\cdot)^T$, $(\cdot)^H$ and $\text{tr}(\cdot)$, respectively. $\|\cdot\|^2$ and $I(\cdot)$ indicate the Euclidean 2-norm of a vector and the indicator function, respectively.

II. SYSTEM MODEL AND SPATIAL MULTIPLEXING GAIN MEASURE

In this section, we describe a system model for the MIMO-IBC illustrated in Fig. 1. Each base station (BS) equipped with M antennas supports its corresponding K_i

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J. Kim was with the School of Electrical Engineering, Korea University, Seoul 136-701, Korea. He is now with the Agency for Defense Development, Daejeon 305-152, Korea (e-mail: jaesin.kim@gmail.com).

H. Sung was with the School of Electrical Engineering, Korea University, Seoul 136-701, Korea. He is now with Samsung Electronics, Kyunggi-do 443-742, Korea (e-mail: jay.sung@samsung.com).

S.-H. Park and I. Lee are with the School of Electrical Engineering, Korea University, Seoul 136-701, Korea (e-mail: {psk1501, inkyu}@korea.ac.kr).

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[1] Since the DOF of the two-user MIMO IC allows only integer values with respect to the number of antennas [4], we can readily show that the SMG of the IC is the same as the DOF presented in [1].

$$\mathbf{y}_l^{(i)} = \mathbf{H}_l^{(i)} \sum_{k=1}^{N} \mathbf{w}_{l,k}^{(i)} s_{l,k}^{(i)} + \mathbf{H}_l^{(i)} \sum_{\substack{m=1 \\ m\neq l}}^{K_i} \sum_{k=1}^{N} \mathbf{w}_{m,k}^{(i)} s_{m,k}^{(i)} + \mathbf{Z}_l^{(i)} \sum_{m=1}^{K_{\bar{i}}} \sum_{k=1}^{N} \mathbf{w}_{m,k}^{(\bar{i})} s_{m,k}^{(\bar{i})} + \mathbf{n}_l^{(i)} \tag{2}$$

$$\mathbf{g}_{l,k}^{(i)} = \mathbf{w}_{l,k}^{(i)H} \mathbf{H}_l^{(i)H} \left[\mathbf{H}_l^{(i)} \left(\sum_{m=1}^{K_i} \sum_{n=1}^{N} \mathbf{w}_{m,n}^{(i)} \mathbf{w}_{m,n}^{(i)H} \right) \mathbf{H}_l^{(i)H} + \mathbf{Z}_l^{(i)} \left(\sum_{m=1}^{K_{\bar{i}}} \sum_{n=1}^{N} \mathbf{w}_{m,n}^{(\bar{i})} \mathbf{w}_{m,n}^{(\bar{i})H} \right) \mathbf{Z}_l^{(i)H} + \mathbf{I} \right]^{-1} \tag{3}$$

$$R_l^{(i)} = \sum_{k=1}^{N} \log \frac{\gamma + \sum_{m=1}^{K_i} \sum_{n=1}^{N} |\mathbf{h}_{l,k}^{(i)} \tilde{\mathbf{w}}_{m,n}^{(i)}|^2 + \sum_{m=1}^{K_{\bar{i}}} \sum_{n=1}^{N} |\mathbf{z}_{l,k}^{(i)} \tilde{\mathbf{w}}_{m,n}^{(\bar{i})}|^2}{\gamma + \sum_{\substack{m=1 \\ (m,n)\neq(l,k)}}^{K_i} \sum_{n=1}^{N} |\mathbf{h}_{l,k}^{(i)} \tilde{\mathbf{w}}_{m,n}^{(i)}|^2 + \sum_{m=1}^{K_{\bar{i}}} \sum_{n=1}^{N} |\mathbf{z}_{l,k}^{(i)} \tilde{\mathbf{w}}_{m,n}^{(\bar{i})}|^2} \tag{4}$$

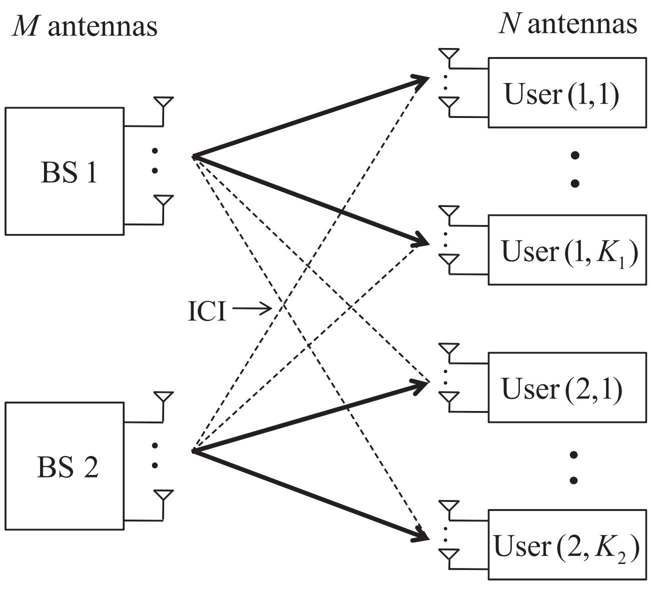

Fig. 1. The (M, K_1N, M, K_2N) MIMO-IBC model.

users with N antennas ($i = 1, 2$), which will be referred to as (M, K_1N, M, K_2N) MIMO-IBC. Also, we represent the l-th user in the i-th cell as user (i, l). Denoting $\mathbf{y}_l^{(i)}$ as the signal vector received by user (i, l), the MIMO-IBC is mathematically described as

$$\mathbf{y}_l^{(i)} = \mathbf{H}_l^{(i)} \mathbf{x}^{(i)} + \mathbf{Z}_l^{(i)} \mathbf{x}^{(\bar{i})} + \mathbf{n}_l^{(i)} \tag{1}$$

where $\mathbf{x}^{(i)}$ stands for the signal vector of length M transmitted from BS i, $\mathbf{n}_l^{(i)}$ is the additive Gaussian noise for user (i, l) with unit variance, $\mathbf{H}_l^{(i)}$ denotes the $N \times M$ channel matrix from BS i to user (i, l) and $\mathbf{Z}_l^{(i)}$ indicates the channel matrix representing the interference from BS $\bar{i}$ to user (i, l). Here we define $\bar{1} = 2$ and $\bar{2} = 1$. It is assumed that the channel elements are sampled from independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance so that the probability of the channel being rank-deficient converges to zero. Also, all channel realizations are assumed to be perfectly known at all nodes. Although the elements in $\mathbf{H}_l^{(i)}$ are generally distributed with power larger than those in $\mathbf{Z}_l^{(i)}$ due to a path loss, we consider the most challenging case where all of them have unit power. This situation arises for users located in cell boundaries.

For realistic implementations, we consider a space-division multiplexing system with linear transceiver in the two-cell environment. Thus, the transmitted signal vector $\mathbf{x}^{(i)}$ in (1) is precoded as $\mathbf{x}^{(i)} = \sum_{l=1}^{K_i} \mathbf{x}_l^{(i)}$. Here the transmitted signal vector $\mathbf{x}_l^{(i)}$ intended for user (i, l) is expressed as $\mathbf{x}_l^{(i)} = \sum_{k=1}^{N} \mathbf{w}_{l,k}^{(i)} s_{l,k}^{(i)}$ where the k-th data symbol $s_{l,k}^{(i)}$ for user (i, l) is multiplied by the beamforming vector $\mathbf{w}_{l,k}^{(i)}$. Assuming $E|s_{l,k}^{(i)}|^2 = 1$ for all i, l and k, the total power constraint P should be satisfied such that $\sum_{i=1}^{2} E\|\mathbf{x}^{(i)}\|^2 = \sum_{(i,l)\in\Phi} \sum_{k=1}^{N} \|\mathbf{w}_{l,k}^{(i)}\|^2 \leq P$ where $\Phi \triangleq \{(1, 1), \cdots, (1, K_1), (2, 1), \cdots, (2, K_2)\}$.

With the linear precoding in $\mathbf{x}_l^{(i)}$, the received signal of user (i, l) in (1) can be written as (2) at the bottom of the page. Notice that the third term in (2) indicates inter-cell interference (ICI). At the l-th user in BS i, the MMSE receive combining vector $\mathbf{g}_{l,k}^{(i)}$ for decoding $s_{l,k}^{(i)}$ is performed as $\hat{s}_{l,k}^{(i)} = \mathbf{g}_{l,k}^{(i)T} \mathbf{y}_l^{(i)}$ for $k = 1, 2, \cdots, N$, which is defined as (3) at the bottom of the page. We denote the effective channels after filtering as $\mathbf{g}_{l,k}^{(i)T} \mathbf{H}_l^{(i)} \triangleq \mathbf{h}_{l,k}^{(i)}$ and $\mathbf{g}_{l,k}^{(i)T} \mathbf{Z}_l^{(i)} \triangleq \mathbf{z}_{l,k}^{(i)}$ where $\mathbf{h}_{l,k}^{(i)}$ and $\mathbf{z}_{l,k}^{(i)}$ are row vectors of length M. Expressing each beamforming vector as $\mathbf{w}_{l,k}^{(i)} = \frac{1}{\sqrt{\gamma}} \tilde{\mathbf{w}}_{l,k}^{(i)}$ where $\gamma = \left(\sum_{(i,l)\in\Phi} \sum_{k=1}^{N} \|\tilde{\mathbf{w}}_{l,k}^{(i)}\|^2 \right) / P$ is introduced to satisfy the total power constraint, the achievable individual rate of user (i, l) is given as (4) at the bottom of the page.

Then, the maximum sum rate for given power constraint P, denoted by $R_\Sigma^{\max}(P)$, is given as

$$R_\Sigma^{\max}(P) = \max_{\left\{R_1^{(1)}, \cdots, R_{K_1}^{(1)}, R_1^{(2)}, \cdots, R_{K_2}^{(2)}\right\} \in \mathcal{R}(P)} \sum_{(i,l)\in\Phi} R_l^{(i)} \tag{5}$$

where the achievable rate region $\mathcal{R}(P)$ is defined as

$$\mathcal{R}(P) = \bigcup_{\sum_{(i,l)\in\Phi} \sum_{k=1}^{N} \|\mathbf{w}_{l,k}^{(i)}\|^2 \leq P} \left\{ R_1^{(1)}, \cdots, R_{K_1}^{(1)}, R_1^{(2)}, \cdots, R_{K_2}^{(2)} \right\}.$$

Finally, different from the definition of the original DOF with respect to the sum capacity, the SMG of the MIMO-IBC is defined as

$$\eta_{\text{MIMO-IBC}} \triangleq \lim_{P\to\infty} \frac{R_\Sigma^{\max}(P)}{\log(P)},$$

which provides the pre-log factor of the MIMO-IBC with respect to the maximum sum rate of (5). Unlike our assumptions of the system model, the transceiver structures optimized under certain constraints might produce different SMG results.

III. Spatial Multiplexing Gain of Two Interfering MIMO Broadcast Channels

In order to highlight the work which does not considered in [3], we start with establishing the relation of the

SMGs among the IC, the IBC and the MIMO-IBC with (M_1, K_1N, M_2, K_2N) configuration. First, it is important to notice that the SMGs of the IC and the IBC with (T_1, R_1, T_2, R_2) configuration are given as

$$\eta_{\text{IC}} = \min\left\{ \begin{array}{c} T_1+T_2, R_1+R_2, \\ \max(T_1,R_2), \max(T_2,R_1) \end{array} \right\} \quad (6)$$

$$\eta_{\text{IBC}} = \min\left\{ \begin{array}{c} \max(T_1,T_2), R_1+R_2, \\ \max(T_1,R_2), \max(T_2,R_1) \end{array} \right\} \quad (7)$$

respectively [1][3]. Setting $T_i = M$ and $R_i = K_iN$ $(i = 1, 2)$ in (6) and (7), we observe that the IBC can achieve the SMG of IC as long as $M \geq \min(K_1N, K_2N)$ despite the lack of receive cooperations. Here, as the MIMO-IBC can be considered as a system with partial cooperation among receive antennas compared to the IBC, it follows $\eta_{\text{IBC}} = \eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}$ through the Sandwich principle. In contrast, for $M < \min(K_1N, K_2N)$, we conclude that $\eta_{\text{IBC}} \leq \eta_{\text{MIMO-IBC}} < \eta_{\text{IC}}$ or $\eta_{\text{IBC}} < \eta_{\text{MIMO-IBC}} \leq \eta_{\text{IC}}$. Therefore, we focus primarily on the SMG analysis for the case where the number of total user antennas in each cell is greater than that of the BS antennas, i.e., $M < \min(K_1N, K_2N)$.

In this section, we will show that the SMG in the (M, K_1N, M, K_2N) MIMO-IBC is exactly given as

$$\eta_{\text{MIMO-IBC}} = \min\Big\{2M, (K_1+K_2)N, \max(M,N)\Big\}. \quad (8)$$

To prove the above result, we first derive upper and lower bounds, and show that they coincide with each other. Note that when $N = 1$ in (8), it becomes the SMG equation of the (M, K_1, M, K_2) IBC, whereas the expression with $K_1 = K_2 = 1$ in (8) reduces to the SMG equation for the (M, N, M, N) IC.

A. Upper Bound

In order to prove the upper bound, we establish the following proposition.

Proposition 1: The SMG of the (M, K_1N, M, K_2N) MIMO-IBC is upper-bounded as $\eta_{\text{MIMO-IBC}} \leq \max(M, N)$.

Proof: Since full cooperations among nodes does not decrease the SMG, we allow receive cooperation among all users in the MIMO-IBC except only one user, e.g., user $(1, 1)$, and refer to the connected users as a *supernode*. Then, we can construct a channel with two BSs and two receivers, user $(1, 1)$ and supernode, where BS 1 has messages for both receivers and BS 2 has a message for a supernode only. Here, user $(1, 1)$ and supernode have N and $(K_1 + K_2 - 1)N$ antennas, respectively. We also suppose that a supernode knows a message transmitted from BS 1. This assumption does not make the rate region smaller. If $M \leq N$, the sum rate of the preceding configuration is upper-bounded by a multiple access channel (MAC) with N receive antennas [1]. Therefore, we have $\eta_{\text{MIMO-IBC}} \leq N$ for $M \leq N$. Now, for $N < M$, let us add more antennas to each user so that there are total M receive antennas. As additional receive antennas cannot hurt, the upper bound argument is not violated. With M receive antennas at each user, once again the above MAC bound can be applied to the new MIMO-IBC. Therefore, the SMG is upper-bounded as $\eta_{\text{MIMO-IBC}} \leq M$ when $N < M$. Combining the two cases, we obtain the upper-bound as $\max(M, N)$. ■

As mentioned before, the trivial upper bound of the MIMO-IBC can be obtained from the SMG of the IC as (6). Putting these bounds together, the upper bound of the SMG in the (M, K_1N, M, K_2N) MIMO-IBC is given as

$$\begin{aligned}\eta_{\text{MIMO-IBC}} &\leq \min\left\{ \begin{array}{c} 2M, (K_1+K_2)N, \max(M,K_1N), \\ \max(M,K_2N), \max(M,N) \end{array} \right\} \\ &\overset{(a)}{=} \min\Big\{2M, (K_1+K_2)N, \max(M,N)\Big\} \quad (9)\end{aligned}$$

where equality (a) holds since $\min\{\max(M, K_1N), \max(M, K_2N), \max(M, N)\} = \max(M, N)$ for all possible cases.

B. Lower Bound

For the (M, K_1N, M, K_2N) MIMO-IBC, we prove the lower bound with respect to the available SMG by using the results of the IC and the IBC. To this end, all possible cases of M, K_1, K_2, N are classified into the following two cases: $M \geq \min(K_1N, K_2N)$ and $M < \min(K_1N, K_2N)$. First, in the case of $M \geq \min(K_1N, K_2N)$, by observing that disabling receive cooperation cannot increase the SMG, we can obtain the following lower bound.

$$\eta_{\text{MIMO-IBC}} \geq \eta_{\text{IBC}} \overset{(a)}{=} \begin{cases} \min\Big(M, (K_1+K_2)N\Big) \\ \qquad \text{for } \max(K_1N, K_2N) \leq M \\ M \\ \qquad \text{for } K_iN \leq M \leq K_{\bar{i}}N \ (i=1,2) \end{cases}$$

where equality (a) is computed from (7). Note that the above lower bound coincides with the upper bound in (9).

The remaining case of $M < \min(K_1N, K_2N)$ can be again partitioned into two cases. First, for $N \leq M < \min(K_1N, K_2N)$, by disabling receive cooperation, we have the following lower bound as

$$\eta_{\text{MIMO-IBC}} \geq \eta_{\text{IBC}} = M \quad \text{for} \quad N \leq M < \min(K_1N, K_2N),$$

which matches with (9). Also, if $M \leq N$, equation (9) results in $\min(2M, N)$. Then, the MIMO-IBC can achieve the SMG by supporting only one user per cell, e.g. $K_1 = K_2 = 1$. Thus, we have

$$\eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}(M, N, M, N) = \min(2M, N) \quad \text{for} \quad M \leq N.$$

This result implies that adding users beyond one user in each cell does not increase the total SMG, which will be verified in the simulation section. Here, the $M = N$ case can belong to either $N \leq M < \min(K_1N, K_2N)$ or $M \leq N$. Therefore, by using the achievability results of [1] and [3] proven earlier through the zero-forcing scheme, the proof for the achievability of (9) is now completed for all system configurations.

C. Comparison of the SMGs among IC, IBC and MIMO-IBC

So far we have established upper and lower bounds with respect to the SMG of the MIMO-IBC, and have shown that two bounds coincide with each other. Thus, we have the exact SMG for all possible M, N, K_1, K_2 as shown in (8). Based on (6), (7) and (8), a couple of observations can be made about the SMG, which are summarized in Table I. In a two-cell environment, a main result of [3] is that disabling the receive

TABLE I
COMPARISON OF THE SMGS FOR IC, IBC AND MIMO-IBC WITH (M, K_1N, M, K_2N)

No.	Condition			SMG
1	$M \geq \min(K_1N, K_2N)$			$\eta_{\text{IBC}} = \eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}$
2	$M < \min(K_1N, K_2N)$	$N \leq M < \min(K_1N, K_2N)$		$\eta_{\text{IBC}} = \eta_{\text{MIMO-IBC}} < \eta_{\text{IC}}$
3		$M < N$	$M < N < 2M$	$\eta_{\text{IBC}} < \eta_{\text{MIMO-IBC}} < \eta_{\text{IC}}$
4			$2M \leq N$	$\eta_{\text{IBC}} < \eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}$

cooperation incurs a SMG loss for the case of full cooperations as long as $M < \min(K_1N, K_2N)$. This fact is equivalent to the SMG result when $N = 1$ in condition 2 of Table I. Also, it shows that there is nothing to be gained by multiple antenna users with respect to the SMG as far as the number of user antennas is not greater than that of BS antennas. For example, consider the $(3, 4, 3, 4)$ IBC with the SMG of 3. If full cooperation is possible between two users with a single antenna, the resulting MIMO-IBC has $K_1 = K_2 = N = 2$. However, the SMG is still 3. That is, there is no benefit with respect to the SMG even when $N \leq M$. On the other hand, there is a SMG gain compared to the IBC if $N > M$ as shown in condition 3 and 4. Specifically, if $N \geq 2M$, the effect of partially disabling the receive cooperation can be offset by having enough spatial dimensions at user nodes. Thus, the SMG of the MIMO-IBC for this case can be equal to that of the IC.

IV. ZERO-GRADIENT BASED SUM RATE MAXIMIZATION

In this section, we propose a new linear precoding and decoding scheme for the MIMO-IBC. The proposed scheme attempts to identify the transmit beamforming and receive combining vectors iteratively such that the sum rate is maximized. To this end, we employ the zero-gradient condition based linear precoding and decoding schemes in the MIMO-IBC environments. One part of this scheme is motivated by iterative precoding techniques in [5] and [6], both of which do not consider the ICI and multiple antenna users.

For all i, l, and k, we have two variables $\mathbf{w}_{l,k}^{(i)}$ and $\mathbf{g}_{l,k}^{(i)}$ to be optimized in terms of maximizing the sum rate. The optimal $\mathbf{g}_{l,k}^{(i)}$ depends on the whole transmit beamforming vectors. On the other hand, the optimal $\mathbf{w}_{l,k}^{(i)}$ depends on the whole receive combining vectors and other beamforming vectors. Hence, we build an iterative optimization method in what follows. First, consider the fixed beamforming vectors. Then, for decoding the k-th data symbol at user (i, l), the MMSE receive vector is applied as (3). In addition, we normalize these combining vectors as $\mathbf{g}_{l,k}^{(i)} = \mathbf{g}_{l,k}^{(i)}/\|\mathbf{g}_{l,k}^{(i)}\|$ in order to make the noise variance unchanged after filtering. To prevent a numerical problem of computing $\mathbf{g}_{l,k}^{(i)}$ in (3) when $\mathbf{w}_{l,k}^{(i)}$ converges to a zero vector, we do not utilize streams corresponding to a zero vector $\mathbf{w}_{l,k}^{(i)}$ by nulling $\mathbf{g}_{l,k}^{(i)}$ as $\mathbf{g}_{l,k}^{(i)} = \mathbf{0}$.

Next, for the fixed receive combining vectors, we identify the optimal transmit beamforming vectors under the total power constraint P. To solve the sum rate maximization problem in an unconstrained manner, the resulting problem can be formulated as

$$\max_{\tilde{\mathbf{w}}_{l,k}^{(i)}} R_\Sigma \tag{10}$$

where $R_\Sigma \triangleq \sum_{(i,l)\in\Phi} R_l^{(i)}$ and $R_l^{(i)}$ is defined as (4).

To solve this problem, we apply a zero-gradient condition. If $\tilde{\mathbf{w}}_{l,k}^{(i)}$'s are solutions of (10), they should satisfy the zero-gradient condition as $\frac{\partial R_\Sigma}{\partial \tilde{\mathbf{w}}_{p,q}^{(j)}} = \mathbf{0}$ for all j, p, and q. First, we compute the gradient $\frac{\partial R_\Sigma}{\partial \tilde{\mathbf{w}}_{p,q}^{(j)}}$. Defining $d_{l,k}^{(i)}$ as the denominator of the $\log$ term in (4), the gradient can be computed as (11) at the top of the next page where $\mathbf{m}_{l,k}^{ij}$ is defined as $\mathbf{m}_{l,k}^{ij} \triangleq \mathbf{h}_{l,k}^{(i)} I(i = j) + \mathbf{z}_{l,k}^{(i)} I(i \neq j)$, $\tilde{d}_{l,k}^{(i)} = \frac{|\mathbf{h}_{l,k}^{(i)}\tilde{\mathbf{w}}_{l,k}^{(i)}|^2}{d_{l,k}^{(i)}(d_{l,k}^{(i)}+|\mathbf{h}_{l,k}^{(i)}\tilde{\mathbf{w}}_{l,k}^{(i)}|^2)}$ and $\tilde{\mathbf{D}} = \text{diag}(\tilde{d}_{l,k}^{(i)})$. Substituting (11) into the preceding zero-gradient condition, we obtain (12) at the top of the next page. Since it is complicated to obtain the closed-form solution for (12) over all j, p, and q, we propose an iterative algorithm as shown in Algorithm 1.

Algorithm 1 Iteration for optimizing the Tx and Rx vectors

Initialization:
1) Initialize $\tilde{\mathbf{w}}_{l,k}^{(i)}$ as arbitrary Tx vectors of length M.
2) $R_\Sigma^{\max} \leftarrow 0$.

Outer Iteration:
3) For all i, l and k, update $\mathbf{g}_{l,k}^{(i)}$ by using (3).
4) Normalize $\mathbf{g}_{l,k}^{(i)} \leftarrow \mathbf{g}_{l,k}^{(i)}/\|\mathbf{g}_{l,k}^{(i)}\|$ $\left(\mathbf{g}_{l,k}^{(i)} \leftarrow \mathbf{0} \text{ if } \|\mathbf{w}_{l,k}^{(i)}\| < \epsilon\right)$.
5) Make the effective channels with the updated Rx vectors as $\mathbf{h}_{l,k}^{(i)} \leftarrow \mathbf{g}_{l,k}^{(i)T}\mathbf{H}_l^{(i)}$ and $\mathbf{z}_{l,k}^{(i)} \leftarrow \mathbf{g}_{l,k}^{(i)T}\mathbf{Z}_l^{(i)}$.
6) $R_{\Sigma,0}^{\text{local}} \leftarrow 0$ and set $s = 1$.

Inner Iteration:
7) Compute $d_{l,k}^{(i)}$, $\tilde{d}_{l,k}^{(i)}$ and $\tilde{\mathbf{D}}$ for all i, l and k.
8) For all j, p and q, update $\tilde{\mathbf{w}}_{l,k}^{(i)}$ by using (12).
9) Compute $R_{\Sigma,s}^{\text{local}}$ with the updated Tx vectors.
10) If $|R_{\Sigma,s}^{\text{local}} - R_{\Sigma,s-1}^{\text{local}}| < \delta$, stop the inner iteration. Otherwise $s \leftarrow s + 1$ and go back to step 7).
11) If $|R_{\Sigma,s}^{\text{local}} - R_\Sigma^{\max}| < \delta$, stop the outer iteration. Otherwise $R_\Sigma^{\max} \leftarrow R_{\Sigma,s}^{\text{local}}$ and go back to step 3).

In this algorithm, ϵ denotes an arbitrary small positive value and δ is the tolerance factor for terminating the iteration. With Algorithm 1, we have a non-decreasing $R_\Sigma^{\max}$ with respect to the number of iterations [6]. However, the convergence to a global optimal point cannot be guaranteed due to non-convexity of the optimization problem in (10). Therefore, by repeating Algorithm 1 with multiple randomly chosen initial beamforming vectors and selecting the one leading to

$$\frac{\partial R_\Sigma}{\partial \tilde{\mathbf{w}}_{p,q}^{(j)}} = \frac{1}{d_{p,q}^{(j)}} \mathbf{h}_{p,q}^{(j)H} \mathbf{h}_{p,q}^{(j)} \tilde{\mathbf{w}}_{p,q}^{(j)} - \frac{\sigma_n^2 \mathrm{tr}(\tilde{\mathbf{D}})}{P} \tilde{\mathbf{w}}_{p,q}^{(j)} - \sum_{(i,l)\in\Phi} \sum_{k=1}^{N} \tilde{d}_{l,k}^{(i)} \mathbf{m}_{l,k}^{ijH} \mathbf{m}_{l,k}^{ij} \tilde{\mathbf{w}}_{p,q}^{(j)} \tag{11}$$

$$\tilde{\mathbf{w}}_{p,q}^{(j)} = \frac{1}{d_{p,q}^{(j)}} \left(\frac{\sigma_n^2 \mathrm{tr}(\tilde{\mathbf{D}})}{P} \mathbf{I} + \sum_{(i,l)\in\Phi} \sum_{k=1}^{N} \tilde{d}_{l,k}^{(i)} \mathbf{m}_{l,k}^{ijH} \mathbf{m}_{l,k}^{ij} \right)^{-1} \mathbf{h}_{p,q}^{(j)H} \mathbf{h}_{p,q}^{(j)} \tilde{\mathbf{w}}_{p,q}^{(j)} \tag{12}$$

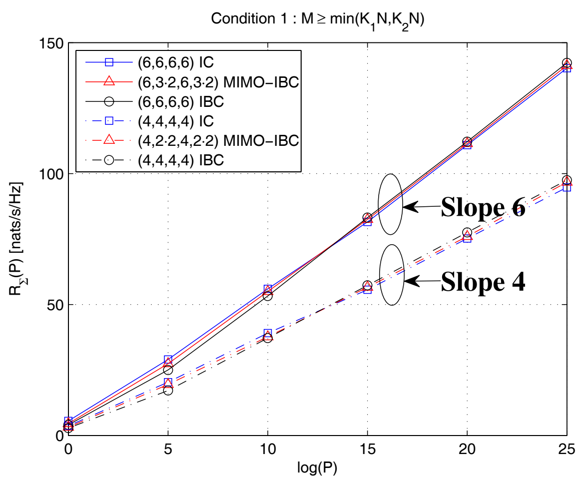

Fig. 2. Average sum rate performance with $\eta_{\text{IBC}} = \eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}$.

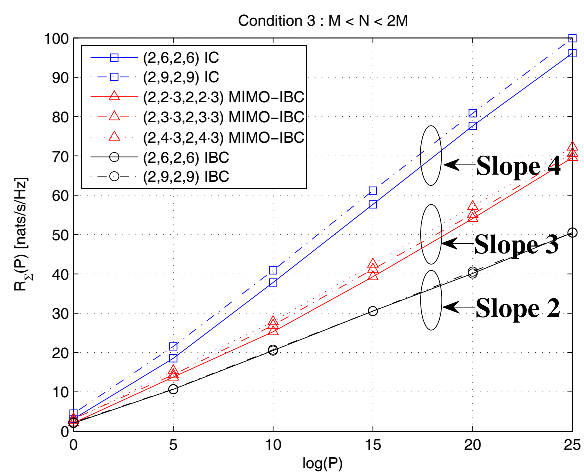

Fig. 4. Average sum rate performance with $\eta_{\text{IBC}} < \eta_{\text{MIMO-IBC}} < \eta_{\text{IC}}$.

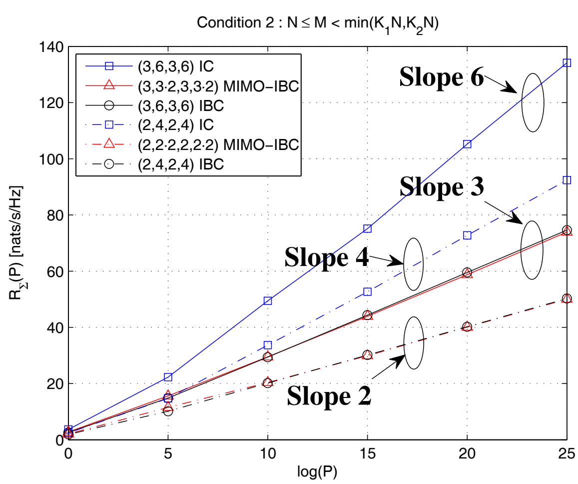

Fig. 3. Average sum rate performance with $\eta_{\text{IBC}} = \eta_{\text{MIMO-IBC}} < \eta_{\text{IC}}$.

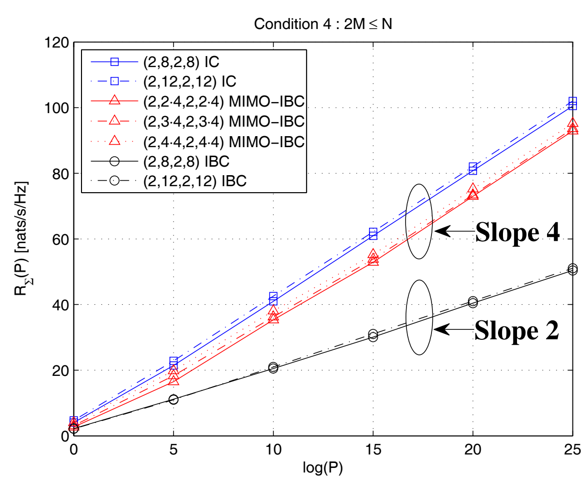

Fig. 5. Average sum rate performance with $\eta_{\text{IBC}} < \eta_{\text{MIMO-IBC}} = \eta_{\text{IC}}$.

the highest sum rate value, we can increase the chance of achieving the global optimal point. In the following simulation results, we verify that the SMG of the proposed scheme is exactly the same as (8) for various system configurations.

V. Simulation Results

In this section, we confirm the accuracy of our theoretical SMG analysis for the MIMO-IBC through simulations. To this end, we provide numerical results evaluating the average sum rate performance of the precoding and decoding scheme proposed in the previous section. Then, we demonstrate that the proposed scheme attains the SMG derived in Section III for all conditions in Table I. First of all, it is observed that adopting initial points more than 10, no performance gain is achieved. Thus, throughout the simulations, we have evaluated the proposed scheme with 10 random initial transmit beamforming vectors. Also the threshold values for convergence are set to $\epsilon = \delta = 10^{-5}$.

Figure 2 shows the average sum rate performance as a function of $\log(P)$. According to the condition 1 in Table I, the curves correspond to the antenna configurations with the same SMG regardless of receive cooperations. As expected, the solid lines with $M = 6$, $N = 2$ and $K_1 = K_2 = 3$ show the slope of 6. Similarly, the dashed lines with $M = 4$ and

$N = K_1 = K_2 = 2$ also increase linearly with the slope of 4. Figures 3, 4, and 5 plot the average sum rate curves for the condition 2, 3 and 4 of Table I, respectively. As the number of user antennas increases, we can observe that the slope of the MIMO-IBC curve approaches that of the IC as shown in the figures. In Fig. 3, the solid lines show that if the number of user antennas is smaller than that of BS antennas, the sum rate slope of the MIMO-IBC is equal to that of the IBC with the corresponding total user antennas. Also, the dashed lines indicate the case where the number of user antennas is the same as that of BS antennas. In particular, in the MIMO-IBC sum rate curves of Figs. 4 and 5, as the number of users grows, we can observe that the SMG does not change when the number of user antennas is greater than that of BS antennas.

VI. Conclusion

In this letter, a two-cell multiuser MIMO downlink system has been modeled as two mutually interfering MIMO broadcast channels. Here, the SMG expression of the MIMO-IBC has been analytically derived. Moreover, we have verified the derived results through the numerical simulation. Based on this, we have characterized the effect of the distributed nature of user antennas with respect to the SMG by comparing the MIMO-IBC with the IC and the IBC.

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