Distributed Beamforming Techniques for Weighted Sum-Rate Maximization in MISO Interference Channels

Seok-Hwan Park, *Student Member, IEEE,* Haewook Park, *Student Member, IEEE,* and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—This letter proposes a beamforming technique for weighted sum-rate (WSR) maximization in multiple-input single-output (MISO) interference channels. In order to solve the WSR maximization problem in a distributed manner, we obtain a decoupled problem by applying high signal-to-interference-plus-noise-ratio (SINR) approximation. When there are more than two users, further approximation is employed to fully decouple the problem. Then, the decoupled problems are solved by using a zero-gradient (ZG) based algorithm which converges to a local optimal point with only a few iterations. Unlike the conventional distributed schemes where additional information should be exchanged at each iteration, each transmitter of the proposed scheme utilizes only local channel information to compute its beamformer.**

***Index Terms*—Distributed beamforming, weighted sum-rate maximization, interference channel.**

I. Introduction

It is important to study interference channels (IC) since well designed cellular systems are interference-limited. In this letter, we focus on a beamforming design in multiple-input single-output (MISO) IC under the assumption of single-user detection (SUD)[1]. In [1], the authors provided interesting parameterization of Pareto-optimal beamformers using linear combination of maximal-ratio transmission (MRT) and zero-forcing (ZF) beamforming vectors. In [2] and [5], distributed approaches for sum-rate maximization were proposed. However, these algorithms are not fully distributed in a sense that the receivers should report their interference power which depends on other users' beamformers at each iteration. In [3], a virtual signal-to-interference-plus-noise ratio (VSINR) framework which guarantees a certain point in Pareto boundary[2] was proposed. However, it guarantees the Pareto-optimal point only for the two-user case and the weighted sum-rate performance with various weights is significantly degraded compared to the exhaustive search.

In this letter, we propose a fully distributed beamforming approach which efficiently computes near-optimal solutions for the WSR maximization problem. The beamforming vectors of the proposed scheme can be computed using only local channel state information (CSI) without a need for any additional information exchange. We first decouple the problem by applying high signal-to-interference-plus-noise ratio (SINR) approximation as in [3] and [6]. Since it is difficult to obtain a closed-form solution for the decoupled problems, we employ a zero-gradient (ZG) algorithm which converges with only a few iterations. To the best of our knowledge, the proposed scheme is the first algorithm which requires only local CSIs to generate the near optimal weighted sum-rate performance.

II. System Model

We consider the K-user MISO IC where transmitter i uses M transmit antennas to support receiver i ($i = 1, \cdots, K$). Especially, we focus on the case of $M \geq K$ for simplicity as in [1]. At transmitter k, the symbol $s_k \sim \mathcal{CN}(0, 1)$ intended for receiver k is multiplied by the beamforming vector $\mathbf{v}_k$ where $\mathbf{v}_k$ is subject to $||\mathbf{v}_k||^2 \leq 1$ to satisfy the per-transmitter power constraint. Then, the received signal at receiver k can be written as

$$y_k = \mathbf{h}_{k,k}^H \mathbf{v}_k s_k + \sum_{j \neq k} \mathbf{h}_{k,j}^H \mathbf{v}_j s_j + n_k,$$

where $\mathbf{h}_{k,j} \in \mathbb{C}^M$ denotes the channel vector from transmitter j to receiver k and $n_k \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise at receiver k. We will refer to the pair of transmitter k and receiver k as user k.

The individual rate of user k under the assumption of the SUD is given as $R_k(\mathbf{v}_1, \cdots, \mathbf{v}_K) = \log(1 + \text{SINR}_k(\mathbf{v}_1, \cdots, \mathbf{v}_K))$ where $\text{SINR}_k(\mathbf{v}_1, \cdots, \mathbf{v}_K)$ represents the individual SINR defined as

$$\text{SINR}_k(\mathbf{v}_1, \cdots, \mathbf{v}_K) = \frac{|\mathbf{h}_{k,k}^H \mathbf{v}_k|^2}{N_0 + \sum_{j \neq k} |\mathbf{h}_{k,j}^H \mathbf{v}_j|^2}.$$

In this letter, our goal is to find the beamforming vectors which maximize the WSR defined as $R_{\sum}(\mathbf{v}_1, \cdots, \mathbf{v}_K) = \sum_{k=1}^{K} w_k R_k(\mathbf{v}_1, \cdots, \mathbf{v}_K)$ where the weight term w_k is determined depending on the required quality of service for applications. Then, the problem can mathematically be written as

$$\max_{\mathbf{v}_1, \cdots, \mathbf{v}_K} R_{\sum}(\mathbf{v}_1, \cdots, \mathbf{v}_K) \text{ s.t. } ||\mathbf{v}_i||^2 \leq 1 \ \forall i. \quad (1)$$

Solving (1) is quite complicated due to non-convexity. However, since it is obvious that the optimal solution lies on the Pareto boundary of the achievable rate region, we can find the optimal point by exhaustive search using the parameterizations in [1] and [3]. Here, the rate region is defined as the set of all possible rate-tuples $(R_1(\mathbf{v}_1, \cdots, \mathbf{v}_K), \cdots, R_K(\mathbf{v}_1, \cdots, \mathbf{v}_K))$ which can be achieved simultaneously while satisfying $||\mathbf{v}_i||^2 \leq 1$ for all i.

Manuscript received September 4, 2010. The associate editor coordinating the review of this letter and approving it for publication was Y. Chen.

The authors are with the School of Electrical Eng., Korea University, Seoul, Korea (e-mail: shpark@wireless.korea.ac.kr, {jetaime01, inkyu}@korea.ac.kr).

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909).

Digital Object Identifier 10.1109/LCOMM.2010.12.101635

[1]Receivers in SUD treat the interference signals as noise [1]–[4].

[2]The Pareto boundary consists of rate-tuples where one can improve a particular user's rate only by simultaneously decreasing the rate of at least one of the other users and its detailed definition is provided in [1].

III. Proposed Distributed Beamforming

In this section, we propose a near optimal beamforming scheme which can operate in a fully distributed manner. First of all, we notice that there is no need to apply power control as long as $M \geq K$ [1]. This means that we can achieve all Pareto-optimal points by considering only the beamforming vectors corresponding to full power transmission. Then, the WSR maximization problem can be reformulated as

$$\max_{\mathbf{v}_1,\cdots,\mathbf{v}_K} R_{\sum}(\mathbf{v}_1,\cdots,\mathbf{v}_K) \text{ s.t. } ||\mathbf{v}_i||^2 = 1\ \forall i \tag{2}$$

where the feasible set involves only the beamforming vectors with unit power. For illustrative purposes, we begin by presenting the proposed scheme for the 2-user case and it will be extended to general $K \geq 3$ cases later.

A. Two-User Case ($K = 2$)

For the two-user case, the WSR is given as

$$R_{\sum}(\mathbf{v}_1,\mathbf{v}_2) = \sum_{l=1}^{2} w_l \log\left(1 + \frac{|\mathbf{h}_{l,l}^H \mathbf{v}_l|^2}{N_0 + |\mathbf{h}_{l,\bar{l}}^H \mathbf{v}_{\bar{l}}|^2}\right)$$

where $\bar{1} = 2$ and $\bar{2} = 1$. Computing the beamformers maximizing the above metric is complicated since each individual rate contains both $\mathbf{v}_1$ and $\mathbf{v}_2$. To obtain the decoupled problems, we apply a high-SINR approximation as in [3] and [6]

$$\begin{aligned} R_{\sum}(\mathbf{v}_1,\mathbf{v}_2) &\approx w_1 \log\frac{|\mathbf{h}_{1,1}^H\mathbf{v}_1|^2}{N_0 + |\mathbf{h}_{1,2}^H\mathbf{v}_2|^2} + w_2 \log\frac{|\mathbf{h}_{2,2}^H\mathbf{v}_2|^2}{N_0 + |\mathbf{h}_{2,1}^H\mathbf{v}_1|^2} \\ &= \log\frac{\left(|\mathbf{h}_{1,1}^H\mathbf{v}_1|^2\right)^{w_1}}{\left(N_0 + |\mathbf{h}_{2,1}^H\mathbf{v}_1|^2\right)^{w_2}} + \log\frac{\left(|\mathbf{h}_{2,2}^H\mathbf{v}_2|^2\right)^{w_2}}{\left(N_0 + |\mathbf{h}_{1,2}^H\mathbf{v}_2|^2\right)^{w_1}}. \end{aligned} \tag{3}$$

From (3), we now have two decoupled problems as

$$\max_{||\mathbf{v}_l||^2=1} \log\frac{\left(|\mathbf{h}_{l,l}^H\mathbf{v}_l|^2\right)^{w_l}}{\left(N_0 + |\mathbf{h}_{\bar{l},l}^H\mathbf{v}_l|^2\right)^{w_{\bar{l}}}} \quad \text{for } l = 1, 2. \tag{4}$$

For the unweighted sum-rate case ($w_l = w_{\bar{l}}$), the above problem (4) has a closed-form solution as shown in [3] and [6]. However, since the problem is typically non-convex as long as $w_l \neq w_{\bar{l}}$, it is difficult to obtain the closed-form solution. Thus, we propose to solve (4) using the ZG algorithm introduced in [7] which utilizes the fact that the beamformers at a stationary point always satisfy the ZG conditions. First, by expressing $\mathbf{v}_l = \frac{\tilde{\mathbf{v}}_l}{||\tilde{\mathbf{v}}_l||}$ in (4), we get an unconstrained problem with respect to $\tilde{\mathbf{v}}_l$ as

$$\max_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right)$$

where $f_l\left(\tilde{\mathbf{v}}_l\right)$ is defined as

$$f_l\left(\tilde{\mathbf{v}}_l\right) = \log\frac{\left(\frac{|\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l|^2}{||\tilde{\mathbf{v}}_l||^2}\right)^{w_l}}{\left(N_0 + \frac{|\mathbf{h}_{\bar{l},l}^H\tilde{\mathbf{v}}_l|^2}{||\tilde{\mathbf{v}}_l||^2}\right)^{w_{\bar{l}}}}.$$

Then, the local optimal point occurs at $\nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right) = \mathbf{0}$ where $\nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right)$ is the gradient of $f_l\left(\tilde{\mathbf{v}}_l\right)$ with respect to $\tilde{\mathbf{v}}_l$ computed as

$$\begin{aligned} \nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right) &= \frac{w_l}{|\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l|^2}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l - \frac{w_l}{||\tilde{\mathbf{v}}_l||^2}\tilde{\mathbf{v}}_l + \frac{w_{\bar{l}}}{||\tilde{\mathbf{v}}_l||^2}\tilde{\mathbf{v}}_l \\ &- \frac{w_{\bar{l}}}{N_0||\tilde{\mathbf{v}}_l||^2 + |\mathbf{h}_{\bar{l},l}^H\tilde{\mathbf{v}}_l|^2}\left(N_0\mathbf{I} + \mathbf{h}_{\bar{l},l}\mathbf{h}_{\bar{l},l}^H\right)\tilde{\mathbf{v}}_l. \end{aligned}$$

After some manipulations, we see that $\nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right) = \mathbf{0}$ is satisfied only when

$$\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l \propto \left(\alpha_l\mathbf{I} + \mathbf{h}_{\bar{l},l}\mathbf{h}_{\bar{l},l}^H\right)\tilde{\mathbf{v}}_l \tag{5}$$

where α_l is defined as $\alpha_l = N_0 + \left(N_0 + \frac{|\mathbf{h}_{\bar{l},l}^H\tilde{\mathbf{v}}_l|^2}{||\tilde{\mathbf{v}}_l||^2}\right)\left(\frac{w_l}{w_{\bar{l}}} - 1\right)$.

Thus, in our proposed scheme, each transmitter l computes its beamformer $\mathbf{v}_l$ according to the following iterative algorithm.

Initialize $\tilde{\mathbf{v}}_l$.

for $n = 1 : N_{\text{itr}}$,

Compute $\alpha_l = N_0 + \left(N_0 + \frac{|\mathbf{h}_{\bar{l},l}^H\tilde{\mathbf{v}}_l|^2}{||\tilde{\mathbf{v}}_l||^2}\right)\left(\frac{w_l}{w_{\bar{l}}} - 1\right)$.

Update $\tilde{\mathbf{v}}_l = \left(\alpha_l\mathbf{I} + \mathbf{h}_{\bar{l},l}\mathbf{h}_{\bar{l},l}^H\right)^{-1}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l$.

end

Normalize $\mathbf{v}_l = \frac{\tilde{\mathbf{v}}_l}{||\tilde{\mathbf{v}}_l||}$.

Here, N_{itr} indicates the predetermined number of iterations.

B. Extension to General $K \geq 3$

In this subsection, we extend our proposed scheme to the case of general $K \geq 3$. After applying the high-SINR approximation to the WSR expression for general K, all terms dependent on $\tilde{\mathbf{v}}_l$ are given as

$$f(\tilde{\mathbf{v}}_l) = w_l \log\left(|\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l|^2\right) - w_l \log\left(||\tilde{\mathbf{v}}_l||^2\right) - \sum_{k\neq l} w_k \log\left(N_0 + I_k\right)$$

where $I_k = \sum_{j\neq k}\frac{|\mathbf{h}_{k,j}^H\tilde{\mathbf{v}}_j|^2}{||\tilde{\mathbf{v}}_j||^2}$ represents the interference power at receiver k. This is not a decoupled problem because I_k is a function of all beamforming vectors. In order to fully decouple the problem, we additionally apply a high signal-to-noise-ratio (SNR) approximation. Notice that at high SNR, since the ZF beamforming achieves the near-optimal WSR performance, the interference power caused by other beamformers $\tilde{\mathbf{v}}_j$ ($j \neq k, l$) expressed as $\sum_{j\neq k,l}\frac{|\mathbf{h}_{k,j}^H\tilde{\mathbf{v}}_j|^2}{||\tilde{\mathbf{v}}_j||^2}$ can be approximated to 0 with a little performance loss. Then, the last term in $f(\tilde{\mathbf{v}}_l)$ can be replaced with $\sum_{k\neq l} w_k \log\left(N_0 + \frac{|\mathbf{h}_{k,l}^H\tilde{\mathbf{v}}_l|^2}{||\tilde{\mathbf{v}}_l||^2}\right)$.

Now, the gradient expression for the fully decoupled problem is computed as

$$\nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right) = \frac{w_l}{|\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l|^2}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l - \left(\alpha_l\mathbf{I} + \sum_{k\neq l}\frac{w_k}{D_k}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\right)\tilde{\mathbf{v}}_l$$

where the coefficients D_k and α_l are defined as $D_k = N_0||\tilde{\mathbf{v}}_l||^2 + |\mathbf{h}_{k,l}^H\tilde{\mathbf{v}}_l|^2$ and $\alpha_l = \frac{w_l}{||\tilde{\mathbf{v}}_l||^2} +$

$\sum_{k\neq l}\left(\frac{N_0}{D_k}-\frac{1}{||\tilde{\mathbf{v}}_l||^2}\right)w_k$, respectively. The ZG condition $\nabla_{\tilde{\mathbf{v}}_l} f_l\left(\tilde{\mathbf{v}}_l\right)=\mathbf{0}$ occurs only when

$$\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l \propto \left(\alpha_l\mathbf{I}+\sum_{k\neq l}\frac{w_k}{D_k}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\right)\tilde{\mathbf{v}}_l. \tag{6}$$

Thus, at each iteration, the newly updated beamformer for transmitter l becomes

$$\left(\alpha_l\mathbf{I}+\sum_{k\neq l}\frac{w_k}{D_k}\mathbf{h}_{k,l}\mathbf{h}_{k,l}^H\right)^{-1}\mathbf{h}_{l,l}\mathbf{h}_{l,l}^H\tilde{\mathbf{v}}_l. \tag{7}$$

C. Comparison to Conventional Distributed Algorithms

Now, we compare the proposed beamforming scheme with the conventional distributed algorithms in [2] and [5]. In these schemes, transmitter l computes its beamformer as a function of $\{\mathbf{h}_{k,l}, I_k, \forall k\}$ where $\mathbf{h}_{k,l}$'s denote the local channel state information (CSI) and $I_k=\sum_{k\neq l}|\mathbf{h}_{k,l}^H\mathbf{v}_l|^2$ is the interference power at receiver k. Since I_k is a function of all the beamformers, the receivers should report their interference power to the transmitters until convergence. In contrast, in our proposed scheme, there is no need for information exchange during iterations since each transmitter l can compute its beamformer using its own local CSIs $\{\mathbf{h}_{k,l}, \forall k\}$. Thus, the overhead for information exchange is lower in our scheme compared to the conventional schemes in [2] and [5].

IV. Simulation Results

In this section, we confirm the effectiveness of the proposed scheme by observing the average weighted sum-rate performance. In all simulations, spatially uncorrelated Rayleigh fading channels with unit variance are assumed. For a fair comparison, we consider only the beamforming schemes based on the local CSI except for exhaustive search algorithms [1] [8]. The performance of [2] and [5] is excluded since they require additional information exchange. For the two-user case, the exhaustive search using parameterization in [1] is simulated to check the upper bound on the performance of the beamforming schemes. For $K\geq 3$, the simulation time of the exhaustive search [1] becomes prohibitive since it requires a search over K complex parameters. Instead, we present the performance of the gradient-descent (GD) algorithm which guarantees a local optimal solution [8]. In order to increase the probability of finding the global optimal solution, two initial points (MRT and ZF) are applied to the GD. For the proposed scheme, we have chosen the ZF beamforming as an initial point.

In Fig. 1, the WSR performance is plotted as a function of the SNR $\frac{1}{N_0}$ for the 2-user MISO IC with $M=2$. The proposed scheme shows a near-optimal performance with only 2 iterations regardless of weight terms. As the ratio between w_1 and w_2 increases, the performance gain over the VSINR maximization scheme grows. Figure 2 presents the performance for the 3-user IC with $M=3$. It is observed that the proposed scheme exhibits a very little performance loss compared to the exhaustive GD method for all simulated configurations with small number of iterations.

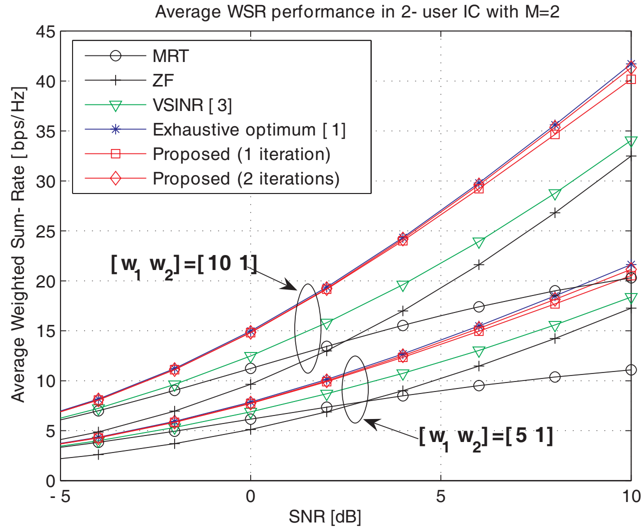

Fig. 1. Average WSR performance in 2-user MISO IC with $M=2$.

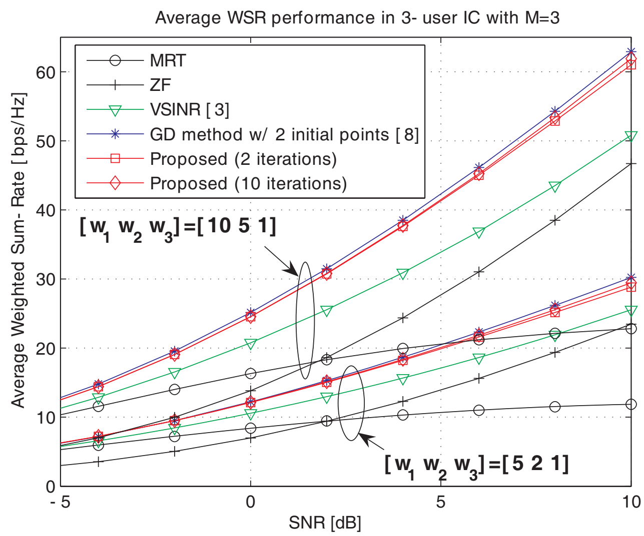

Fig. 2. Average WSR performance in 3-user MISO IC with $M=3$.

References

[1] E. A. Jorswieck and E. G. Larsson, "Complete characterization of Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, vol. 56, pp. 5292–5296, Oct. 2008.
[2] Z. K. M. Ho and D. Gesbert, "Spectrum sharing in multiple-antenna channels: a distributed cooperative game theoretic approach," in *Proc. IEEE PIMRC '08*, Sep. 2008.
[3] R. Zakhour and D. Gesbert, "Coordination on the MISO interference channel using the virtual SINR framework," in *Proc. ITG/IEEE Workshop on Smart Antennas '09*, Feb. 2009.
[4] C. Shi, R. A. Berry, and M. L. Honig, "Distributed interference pricing with MISO channels," in *Proc. IEEE Allerton Conference '08*, Sep. 2008.
[5] J. Lindblom and E. Karipidis, "Cooperative beamforming for the MISO interference channel," in *Proc. IEEE European Wireless Conference '10*, pp. 631–638, Apr. 2010.
[6] B. O. Lee, H. W. Je, O.-S. Shin, and K. B. Lee, "A novel uplink MIMO transmission scheme in a multicell environment," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 4981–4987, Oct. 2009.
[7] M. Kobayashi and G. Caire, "Joint beamforming and scheduling for a multi-antenna downlink with imperfect transmitter channel knowledge," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 1468–1477, Sep. 2007.
[8] H. Sung, S.-H. Park, K.-J. Lee, and I. Lee, "Linear precoder designs for K-user interference channels," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 291–301, Jan. 2010.