Modulo Loss Reduction for Vector Perturbation Systems

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***Abstract*—In this letter, we present an improved precoding technique which reduces a modulo loss in vector perturbation with low complexity. Instead of searching perturbation vectors in the infinite lattice, the proposed scheme restricts the search range by utilizing the distribution of the perturbation vector depending on transmitted data. As a result, we can achieve significant complexity savings at the transmitter while providing better performance compared to the original vector perturbation.**

***Index Terms*—Multiple antennas, precoding, vector perturbation.**

I. Introduction

In high data rate wireless systems, multiple-input multiple-output (MIMO) techniques have received considerable attention. The use of multiple antennas at both transmitter and receiver in single-user MIMO channels has been shown to improve the system throughput and capacity [1][2]. More recently, MIMO broadcast channels [3][4][5] have been studied as an important research topic. In the MIMO broadcast channel, a base station employs multiple antennas to communicate with several co-channel users in the same frequency and time slots. Therefore, it is necessary to eliminate co-channel interference without receiver cooperation. A simple channel inversion technique was proposed to eliminate the co-channel interference and allow independent signals to be directed to various users [6]. However, the channel inversion cannot approach the capacity due to noise enhancement.

To enhance the performance, the vector perturbation was introduced in [7], which adopts a modulo operator at both transmitter and receiver. With the modulo operator, the original constellation can be represented as the multiples of constellation in the infinite lattice. Then any point in the infinite lattice can be recovered in the original constellation at the receiver. Utilizing this property of the modulo operator, the transmitter gains a degree of freedom to choose an element in the multiple of constellation to be transmitted as a desired value for the modulo operator input. In the vector perturbation technique, the data is perturbed such that the transmit power is minimized. Finding a perturbation vector is a lattice closest-point problem that can be solved via sphere encoder [8][9].

Paper approved by D. J. Love, the Editor for MIMO and Adaptive Techniques of the IEEE Communications Society. Manuscript received September 21, 2009; revised April 1, 2010.

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909). The material in this paper was presented in part at the IEEE International Conference on Communications (ICC) in June 2009.

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Digital Object Identifier 10.1109/TCOMM.2010.092810.090187

It is well known that the modulo operator causes a modulo loss. For example, Tomlinson-Harashima precoding (THP) [10][11] has been shown to suffer from a 4-5 dB modulo loss at low signal-to-noise-ratio (SNR) [12], and a similar loss is observed in the vector perturbation. To eliminate such a modulo loss, we characterize the effect of the modulo loss which results from the increased number of nearest neighbors. Using the anti-symmetry property [13] of the perturbed data, we restrict the range of perturbation vectors in the transmitter. Such a restriction results in a reduction in both complexity of finding perturbation vectors and the modulo loss. Thus the proposed scheme accomplishes unusual features that better performance is obtained with reduced complexity compared to conventional schemes. For the coded system with four transmit antennas and four users, the proposed scheme achieves a 0.2 dB gain with 4QAM over the original vector perturbation. Moreover, the proposed scheme provides a complexity reduction of 38% compared to the original vector perturbation in terms of the average number of search candidates.

This letter is organized as follows: In Section II, we present the system model of MIMO broadcast channels and review the conventional vector perturbation techniques. In section III, we propose a low complexity vector perturbation scheme with a reduced modulo loss based on the anti-symmetry property. Next, Sections IV demonstrates the simulation results. Finally, the conclusions are presented in Section V.

Throughout this letter, $(.)^T$ and $(.)^H$ denote transpose and complex conjugate transpose, respectively. The subscript $(\cdot)_k$ and $||.||^2$ indicate the kth element in vectors and the Frobenius norm, respectively.

II. Conventional Vector Perturbation Systems

We consider a multiuser downlink system where a base station with M transmit antennas transmits independent data streams to K users with a single antenna. Let us define the M dimensional complex transmitted signal vector $\overline{\mathbf{x}}$, and the K dimensional complex received signal vector $\overline{\mathbf{y}}$. Then, the corresponding complex vector equation can be written as

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\overline{\mathbf{x}} + \overline{\mathbf{n}} \quad (1)$$

where $\overline{\mathbf{n}} \in \mathbb{C}^{K\times 1}$ is the white Gaussian noise vector at the users with zero mean and the covariance matrix $\sigma_n^2\mathbf{I}_K$, and the channel matrix $\overline{\mathbf{H}}$ consists of $K \times M$ independent and identically distributed (i.i.d.) complex Gaussian coefficients with zero mean and unit variance.

Equivalently, the real-valued representation of system (1) is given as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

where $\mathbf{y} = [\Re[\overline{\mathbf{y}}]^T \ \Im[\overline{\mathbf{y}}]^T]^T$, $\mathbf{x} = [\Re[\overline{\mathbf{x}}]^T \ \Im[\overline{\mathbf{x}}]^T]^T$, $\mathbf{n} = [\Re[\overline{\mathbf{n}}]^T \ \Im[\overline{\mathbf{n}}]^T]^T$, and

$$\mathbf{H} = \begin{bmatrix} \Re[\overline{\mathbf{H}}] & -\Im[\overline{\mathbf{H}}] \\ \Im[\overline{\mathbf{H}}] & \Re[\overline{\mathbf{H}}] \end{bmatrix}.$$

The desired signal vectors for K users are denoted as $\overline{\mathbf{u}} = [\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_K]^T$ which is chosen from an M-ary quadrature amplitude modulation (QAM). In the channel inversion, the transmit signal vector is selected as

$$\overline{\mathbf{x}} = \frac{1}{\sqrt{\gamma}} \overline{\mathbf{P}}\overline{\mathbf{u}} \tag{3}$$

where $\overline{\mathbf{P}}$ is the right pseudoinverse of $\overline{\mathbf{H}}$, i.e., $\overline{\mathbf{P}} = \overline{\mathbf{H}}^H(\overline{\mathbf{H}}\,\overline{\mathbf{H}}^H)^{-1}$ and $\gamma = ||\overline{\mathbf{P}}\overline{\mathbf{u}}||^2$ denotes the normalization factor to satisfy the transmit power constraint. Here, $1/\sqrt{\gamma}$ can be considered as a channel gain.

After passing through the channel, the kth user's received signal can be written as

$$\overline{y}_k = \frac{1}{\sqrt{\gamma}} \overline{u}_k + \overline{n}_k$$

where $\overline{y}_k$ is the kth user's received signal, $\overline{u}_k$ denotes the kth user's desired signal, and $\overline{n}_k$ represents the kth user's additive noise. We assume that each user knows γ [7]. Here, γ can be very large on ill-conditioned channels, and large γ degrades the performance.

To reduce γ, the vector perturbation technique is proposed in [7] to adopt

$$\overline{\mathbf{x}} = \frac{1}{\sqrt{\gamma}} \overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}}) \tag{4}$$

where $\overline{\mathbf{l}}$ is a K dimensional complex vector with real and imaginary integer parts, a positive real number τ denotes a design parameter that may be chosen to provide a symmetric decoding region around every signal constellation point, and $\gamma = ||\overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}})||^2$ represents the normalization factor. The integer vector $\overline{\mathbf{l}}$ which minimizes γ can be found as

$$\overline{\mathbf{l}} = \arg \min_{\overline{\mathbf{l}}' \subset \mathbb{Z}^K + \mathrm{j}\mathbb{Z}^K} ||\overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}}')||^2. \tag{5}$$

To solve (5), the sphere encoder was introduced in [8], which requires exponential complexity in the number of users on average [14].

At the receiver, the received signal for the kth user becomes

$$\overline{y}_k = \frac{1}{\sqrt{\gamma}}(\overline{u}_k + \tau\overline{l}_k) + \overline{n}_k. \tag{6}$$

To remove the effect of the integer multiple of τ, the modulo operation is employed as

$$f_\tau(y) = y - \left\lfloor \frac{y + \tau/2}{\tau} \right\rfloor \tau$$

where the function $\lfloor x \rfloor$ is the largest integer less than or equal to x.

TABLE I
PROBABILITY OF PERTURBATION VECTORS ($M = K = 4$, 4QAM)

	Original vector perturbation	
	$p(l_k \mid u_k = -1)$	$p(l_k \mid u_k = 1)$
$l_k = -1$	0.0200	0.1859
$l_k = 0$	0.7941	0.7941
$l_k = 1$	0.1859	0.0200
	Lattice limit	
	$p(l_k \mid u_k = -1)$	$p(l_k \mid u_k = 1)$
$l_k = -1$	0	0.1926
$l_k = 0$	0.8074	0.8074
$l_k = 1$	0.1926	0

TABLE II
PROBABILITY OF DIFFERENT WEIGHTS ($M = K = 4$, 4QAM)

Weight of Candidates	L	$p(\mathbf{l} \in$ Candidate Set$)$
$w = 0$	$\sum_{i=0}^{0} {}_8\mathrm{C}_i = 1$	0.1806
$w \leq 1$	$\sum_{i=0}^{1} {}_8\mathrm{C}_i = 9$	0.5252
$w \leq 2$	$\sum_{i=0}^{2} {}_8\mathrm{C}_i = 37$	0.8129
$w \leq 3$	$\sum_{i=0}^{3} {}_8\mathrm{C}_i = 93$	0.9502
$w \leq 4$	$\sum_{i=0}^{4} {}_8\mathrm{C}_i = 163$	0.9911
$w \leq 5$	$\sum_{i=0}^{5} {}_8\mathrm{C}_i = 219$	0.9990
$w \leq 6$	$\sum_{i=0}^{6} {}_8\mathrm{C}_i = 247$	0.9999
$w \leq 7$	$\sum_{i=0}^{7} {}_8\mathrm{C}_i = 255$	1.000
$w \leq 8$	$\sum_{i=0}^{8} {}_8\mathrm{C}_i = 256$	1.000

III. Proposed Lattice Limit Scheme

In this section, we present a new search scheme, which is called the lattice limit algorithm, achieving lower complexity and better performance compared to the original vector perturbation. First, we introduce the lattice limit using the anti-symmetry property [13]. Next, we characterize the modulo loss of the vector perturbation which degrades the performance. Finally, the complexity comparison for the original vector perturbation and the lattice limit is presented.

A. Proposed method

In [13], it was shown by simulation that the conditional probability of perturbation vectors and data vectors has an anti-symmetry property. As we observe in Table I, elements of the perturbation vector mainly consist of -1, 0, and 1 for the $M = K = 4$ system with 4QAM. Furthermore, when the element of transmit data has a positive sign, elements of the perturbation vector are mostly selected from -1 and 0. On the contrary, when the element of the transmit data is negative, elements of the perturbation vector are mostly chosen as $+1$ and 0. Utilizing this anti-symmetry property, our proposed scheme limits the search range of the perturbation vector based on the sign of the transmit data. In other words, we restrict the search range to $l_k \in \{0, -\mathrm{sgn}(u_k)\}$ for the proposed lattice limit scheme.

Let us denote L as the candidate size. By limiting the search size in the proposed scheme, the infinite search set of the original vector perturbation can be reduced to $L = 2^{2K}$ candidates for the system with K users, since $2K$ elements in real-valued vectors have two choices. Thus, perturbation vectors can be obtained within a limited search size for the proposed lattice limit scheme, whereas the original vector perturbation needs the infinite size.

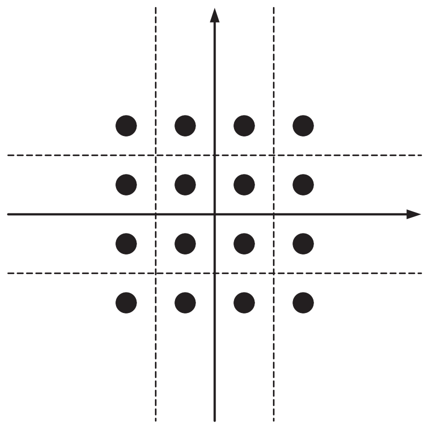

Fig. 1. Effective constellations for proposed scheme with 4QAM.

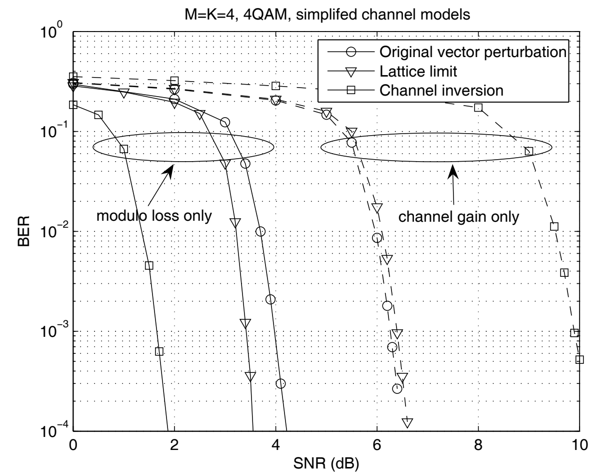

Fig. 2. Comparison in simplified channel models with a rate 1/2 turbo code.

Note that the distribution of the perturbation vectors changes in the proposed scheme. Table I shows the new distribution for the lattice limit obtained from simulations. For example, if we transmit $u_k = 1$, then $l_k = 1$ will not be selected in our proposed lattice limit.

Let w denote the weight of a vector, which is the number of nonzero elements in perturbation vectors. We can further reduce the candidate size using the fact that the perturbation vector with large weights are seldom selected. The probability that the perturbation vector has the weight $w = i$ becomes

$$p(w = i) = {}_{2K}\mathrm{C}_i \cdot p(l_k \neq 0)^i \cdot p(l_k = 0)^{2K-i}. \tag{7}$$

Thus, the probability of $w \leq j$ can be computed as the sum of (7)

$$p(w \leq j) = \sum_{i=0}^{j} p(w = i).$$

Table II shows the sum of the probability of different weights for the case of four transmit antennas and four users with 4QAM. Considering that $p(w \leq 2)$ is 0.8129, we may choose the perturbation vector on the candidates which have a weight w less than or equal to two with a little loss on the performance. The search size of the candidates for this case is only 37, so we can search the perturbation vectors with reduced candidate size. The case of $w \leq 8$ achieves the best performance for the proposed scheme. Also, simulation results indicate that $w \leq 2$ shows a good tradeoff between performance and complexity. Thus, we will consider two cases of $w \leq 2$ and $w \leq 8$ in the following.

B. Modulo Loss for Vector Perturbation

Now, we will explain a loss of the vector perturbation caused by the modulo operation and address that the proposed scheme can compensate for this loss. When compared with the performance of the standard pulse-amplitude modulation (PAM) on an additive white gaussian noise channel, the THP incurs some non-negligible performance losses. In [15], the performance losses of the THP are categorized into three classes: a shaping loss, a modulo loss and a power loss. The modulo loss is caused by the modulo operation at the receiver. Due to noise corruption, received symbols at the boundary of a constellation may be mistaken for symbols at the opposite boundary of the constellation, which may result in the modulo loss.

The vector perturbation experiences similar modulo losses, since the receiver adopts the same modulo operation and the received symbol is viewed as multiple representations of the same constellation as in the THP. For example, at the channel inversion, the symbols of the 4QAM have two nearest neighbors, while each boundary symbol at the vector perturbation is surrounded by four nearest neighbors. In contrast, for the proposed scheme in Figure 1, the boundary symbols have two or three nearest neighbors. This change of the number of nearest neighbors becomes a dominant factor especially for low SNR regime. Thus for coded systems where the operating SNR range is usually low, systems with small nearest neighbor exhibit better performance, and this will be confirmed through simulation in the following.

Now, we will evaluate the effect of the modulo loss and the channel gain $1/\sqrt{\gamma}$ for the original vector perturbation, the proposed lattice limit scheme, and the channel inversion. To observe how much modulo loss the vector perturbation experiences, we employ two simplified channel models. First, we consider an artificial system which includes the modulo loss only without the channel gain. As can be seen in the coded BER results for 4QAM in Figure 2 with a rate 1/2 turbo code, the lattice limit reduces the modulo loss by 0.6 dB at a BER of 10^{-3} compared to the original vector perturbation. As mentioned earlier, this is due to fact that the lattice limit has the smaller number of nearest neighbors than the original vector perturbation. Clearly, the simulation shows that the lattice limit exhibits a smaller modulo loss than the original vector perturbation.

Next, we consider a system with the channel gain $1/\sqrt{\gamma}$ by omitting the modulo operation at both the transmitter and receiver. The mean values of the channel gain $1/\sqrt{\gamma}$ are computed as 0.429, 0.425, and 0.313 for the original vector perturbation, the proposed lattice limit scheme, and the channel inversion, respectively. Figure 2 shows that the proposed lattice limit scheme exhibits little loss associated

TABLE III
COMPLEXITY COMPARISON WITH 10000 CHANNEL REALIZATIONS ($M = K = 4$, 4QAM)

	Average Number of Candidate	Maximum Number of Candidate
Original vector perturbation	64	5553
Lattice limit ($L = 256$)	40	256
Lattice limit ($L = 37$)	20	37

with the channel gain compared to the original vector perturbation. Moreover, the original vector perturbation and the lattice limit outperform the channel inversion for the channel with the channel gain only. Therefore, we can expect that the lattice limit achieves the better performance than the original vector perturbation for normal channels which contain both the channel gain and the modulo operation.

C. Complexity Comparison

In this subsection, we analyze the complexity of the original vector perturbation and the proposed scheme by counting the mean and maximum number of candidates. For fair comparison of the complexity, the proposed lattice limit scheme also employ the sphere encoder within limited search size. Table III shows the complexity comparison of the original vector perturbation and the lattice limit over 10000 channel realizations.

As can be seen in Table III, the lattice limit with $L = 37$ and 256 achieve a reduction of 69% and 38% in the average number of candidates compared to the original vector perturbation, respectively. Also, the lattice limit with $L = 37$ and 256 achieve a significant reduction of 99% and 95% in the maximum number of candidates compared to the original vector perturbation, respectively. It is important to note that in practical implementation, the maximum number of candidates determines the overall complexity. Thus, our proposed scheme achieves a substantial complexity savings in terms of the maximum number of candidates.

IV. SIMULATION RESULTS

In this section, we present the coded BER performance results of the proposed schemes, the original vector perturbation and the channel inversion. We assume independent fading channels throughout simulations. In all simulations, a rate $1/2$ turbo code with polynomial (15,13) in octal is employed. The number of decoding iterations is set to 6. One frame is assumed to consist of 1000 and 2000 information bits for 4QAM and 16QAM, respectively. The channel input after passing through the precoder is normalized to 1. Thus, we define SNR as $1/\sigma_n^2$. Also we employ $M = 4$ transmit antennas and $K = 4$ users.

Figure 3 illustrates the results for coded systems with 4QAM and 16QAM. In the case of 4QAM, as expected in the analysis made above, the lattice limit with $L = 256$ performs 0.2 dB better than the original vector perturbation at a BER of 10^{-3}. Moreover, the lattice limit with $L = 37$ outperforms the original vector perturbation. This is due to a fact that the lattice limit has a smaller modulo loss than the original vector perturbation. In the case of 16QAM, we can see that

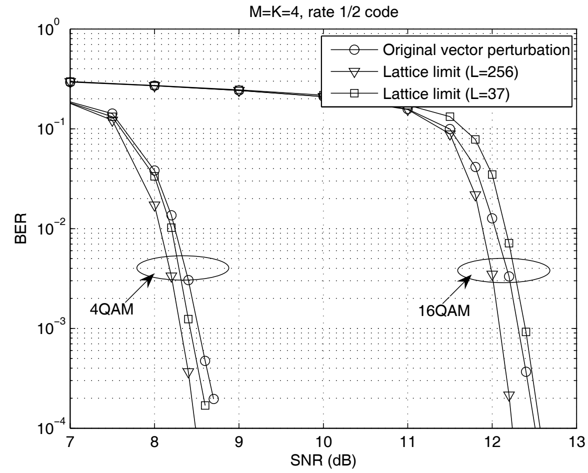

Fig. 3. Performance comparison with $M = K = 4$.

the performance of the lattice limit with $L = 256$ is better than the original vector perturbation. Also, lattice limit with $L = 37$ achieves almost same the performance as the original vector perturbation.

V. CONCLUSION

In this letter, we have proposed the lattice limit algorithm for a multiuser MIMO downlink using the vector perturbation technique. Instead of finding the perturbation vector in the infinite lattice, in the proposed scheme, we have restricted a search range according to the anti-symmetry property. As we limit the search range, we can identify the integer vector with much reduced candidate size. Using the fact that the perturbation vectors with large weights are seldom selected, we have further reduced the candidate size. By adjusting the weight, we have offered the tradeoff with performance and complexity. Moreover, we have shown that, since the modulo loss becomes smaller compared to the original vector perturbation, the proposed scheme achieves the better performance than the the original vector perturbation with reduced complexity. Also, we have investigated the complexity comparison by counting the average and maximum number of the candidates. We conclude that our proposed scheme exhibits unusual characteristics that a better performance gain is obtained with reduced complexity.

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