Degrees of Freedom of Multiple Broadcast Channels in the Presence of Inter-Cell Interference

Seok-Hwan Park, *Student Member, IEEE,* and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—In this paper, we provide lower and upper bounds for the number of degree of freedom (DOF) of B multiple-input single-output (MISO) broadcast channels (BC) where each base station (BS) equipped with M antennas supports its corresponding K single antenna users suffering from inter-cell interference. The sufficient and neccessary condition for tightness of two bounds is presented. From the derived result, it can be observed that in-cell receiver cooperation does not help in most of the cases in a multiple-input multiple-output (MIMO) interference channel (IFC) except for one special case. Even for that special case, the DOFs with and without in-cell receive cooperation approach the same value for large K. Also, in a MIMO IFC with symmetric antenna settings (i.e., $M = K$), if both transmit and receive cooperations are removed to make it a single-input single-output (SISO) IFC, we show that the DOF is not affected. In addition, the DOF is studied for two mutually interfering broadcast channels in the presence of a cognitive BS. We obtain an interesting result that disabling in-cell receive cooperation of the MIMO IFC causes no DOF loss if at least one of two transmitters is a cognitive BS.**

***Index Terms*—Interference mitigation, degrees of freedom, multi-cell.**

I. INTRODUCTION

THERE have been many research activities to study the capacity of wireless multi-user networks. While the capacity analysis has been carried out mostly for systems where the transmitter or receiver can operate in a centralized mode such as multiple access channel (MAC) and broadcast channel (BC), the capacity characterization of multi-point to multi-point communication is still an open problem [1].

As an alternative means to understand the performance limits of wireless networks, the degree of freedom (DOF) has attracted a great deal of attention from many researchers, since the sum capacity of communication networks at high signal-to-noise (SNR) regime is dominated by the DOF. The DOFs of various multi-point to multi-point channels have been analyzed in [2]–[11]. In [3], the authors derived the DOF of two-user multiple-input multiple-output (MIMO) interference channels (IFC) and it was shown in [9] that disabling receive cooperation in the two-user MIMO IFC can cause a DOF loss in the absence of symbol extension. Recently, the authors in [6] have investigated the DOF for a general B-user MIMO IFC with $B \geq 3$. As in the case of B-user single-input single-output (SISO) IFC systems in [5], interference alignment (IA) in conjunction with a zero-forcing (ZF) scheme is shown to achieve the optimal DOF of the MIMO IFC under certain conditions.

The multi-user interference channels in [5], [12] and [13] are well suited to model multi-cell downlink channels where users located in the same cell are separated in the frequency or time domain. However, as the demand for space-division multiple access (SDMA) increases which can boost the spectral efficiency, mutually interfering broadcast channels where each link has a single transmit node and multiple receive nodes become more pervasive. In [9], the DOF is studied for two mutually interfering BCs with multiple antenna transmitters and single antenna users.

In this paper, as an extension of [9], we analyze the DOF of B BCs where each base station (BS) equipped with M antennas supports its corresponding K single antenna users suffering from inter-cell interference (ICI). We refer to this channel model as the $B \times (M \times K)$ interfering broadcast channel (IFBC). The channel model of the $B \times (M \times K)$ IFBC is exactly the same as that of the MIMO IFC in [6] except that K in-cell receivers are disconnected and cannot cooperate with each other. Compared to [9], we study the DOF of the $B \times (M \times K)$ IFBC in more general view by considering the possibility of symbol extension in the frequency domain[1]. We provide lower and upper bounds on the DOF of the $B \times (M \times K)$ IFBC and show that the two bounds coincide if and only if $(M = 1)$ or $(B \geq \lfloor\beta\rfloor + 1$ and β is integer$)$ or $(B \leq \lfloor\beta\rfloor$ and $M \geq K)$ where β is defined as $\beta = \frac{\max(M,K)}{\min(M,K)}$ and $\lfloor a \rfloor$ is the largest integer not greater than a. We prove the upper bound by allowing cooperation among some nodes or increasing the number of receive antennas [3][4] and the lower bound is confirmed with the IA algorithm. From the derived results, we obtain the following observations: 1) disconnecting all receive antennas incurs a loss of the DOF for the MIMO IFC derived in [6] only for one special case. 2) Even for that case, the DOF loss becomes negligible for large K.

Furthermore, we consider two mutually interfering broadcast channels in the presence of cognitive BSs. Cognitive radio is a good candidate for compensating for a DOF loss induced by distributed signal processing in wireless networks or cellular systems [7]. By deriving the DOF of IFBC with

Paper approved by N. Jindal, the Editor for MIMO Techniques of the IEEE Communications Society. Manuscript received July 29, 2009; revised July 6, 2010 and December 5, 2010.

S.-H. Park was with the School of Electrical Eng., Korea University, Seoul, Korea. He is now with the Agency for Defense Development, Daejeon, Korea (e-mail: shpark@wireless.korea.ac.kr).

I. Lee is with the School of Electrical Eng., Korea University, Seoul, Korea (e-mail: inkyu@korea.ac.kr).

This research was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909). The material in this paper was presented in part at the IEEE Globecom, Honolulu, Hawaii, November 2009.

Digital Object Identifier 10.1109/TCOMM.2011.031611.090428

[1]The symbol extension in the frequency domain means a signaling over multiple frequency slots.

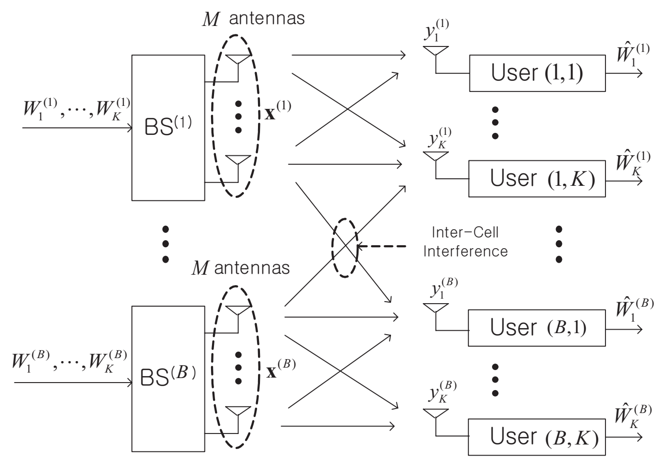

Fig. 1. The $B \times (M \times K)$ IFBC model.

cognitive message sharing [7], we observe a positive result that if at least one cognitive BS exists, the DOF of two user MIMO IFC is not degraded by disabling receive cooperation. Also, the existence of a cognitive BS allows us to achieve the optimal DOF by applying the ZF beamforming without the symbol extension.

Throughout the paper, the following notations are used for description. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The null space, transpose and conjugate transpose of a matrix or a vector are represented by $\mathcal{N}(\cdot)$, $(\cdot)^T$ and $(\cdot)^H$, respectively. A set of all complex matrices of size M-by-N is represented by $\mathbb{C}^{M\times N}$ and $1(\cdot)$ represents the indicator function. Representing x_i as the i-th element of a vector $\mathbf{x}$, we define $[\mathbf{x}]_{a:b} = [x_a\, x_{a+1}\, \cdots\, x_b]^T$.

The remainder of this paper is organized as follows: In Section II, the $B \times (M \times K)$ IFBC and two cell IFBC with cognitive BSs are introduced and the earlier works in [6] and [7] are briefly reviewed. The DOF of the $B \times (M \times K)$ IFBC will be analyzed in Section III. Also in Section IV, we derive an exact expression of the DOF for two-cell IFBC in the presence of a cognitive BS. The paper is closed with conclusions in Section V.

II. System Model

In this section, we provide a system model for multiple interfering BCs. First, the $B\times(M\times K)$ IFBC is described, and then it is followed by the exhibition of two mutually interfering BCs with cognitive BS.

A. Multiple Interfering Broadcast Channels

General multi-cell and multi-user downlink transmission can be modeled as multiple BCs interfering with each other as shown in Figure 1. There are B base stations BS$^{(1)}$, $\cdots$, BS$^{(B)}$, where BS$^{(i)}$ with M antennas supports K single antenna users ($i = 1, \cdots, B$). For simplicity, we denote the l-th user in the i-th cell by user (i, l). Then, denoting $W_l^{(i)}$ as the message intended for user (i, l), BS$^{(i)}$ has messages $W_1^{(i)}, \cdots, W_K^{(i)}$ while the information about $W_l^{(j)}$ ($j \neq i$) is not available. The trivial case of $B = 1$ which stands for a single cell environment is neglected throughout the paper.

TABLE I
The Lower Bound and Upper Bound on the DOF of the $B \times (M \times K)$ IFC

$M \geq K$		$M < K$	
$B \leq \lfloor\beta\rfloor$	$B \geq \lfloor\beta\rfloor + 1$	$B \leq \lfloor\beta\rfloor$	$B \geq \lfloor\beta\rfloor + 1$
BK	UB: $\frac{\beta}{\lfloor\beta\rfloor+1}BK$ LB: $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BK$	BM	UB: $\frac{\beta}{\lfloor\beta\rfloor+1}BM$ LB: $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM$

At the specific frequency slot f, the received signal of user (i, l) denoted by $y_l^{(i)}(f)$ is given as

$$y_l^{(i)}(f) = \mathbf{h}_l^{(i,i)}(f)\mathbf{x}^{(i)}(f) + \sum_{j\neq i}\mathbf{h}_l^{(i,j)}(f)\mathbf{x}^{(j)}(f) + n_l^{(i)}(f) \quad (1)$$

where $\mathbf{x}^{(i)}(f) \in \mathbb{C}^{M\times 1}$ stands for the signal vector transmitted from BS$^{(i)}$, $n_l^{(i)}(f)$ is the additive Gaussian noise with unit variance for user (i, l) and $\mathbf{h}_l^{(i,j)}(f) \in \mathbb{C}^{1\times M}$ denotes the channel response vector between BS$^{(j)}$ and user (i, l). It is assumed that the channel elements are sampled from independent identically distributed (i.i.d.) complex Gaussian random varianbles with zero mean and unit variance and vary at every channel use f. Also, all channel realizations are assumed to be perfectly known at all nodes. We notice that the channel use index f can equivalently be used to indicate time slots or a time-frequency tuple if coding is performed in both time and frequency [5]. It is the frequency-selective nature of channel coefficients that is the most important assumption for deriving the lower bound in Section III.

We define the spatial DOF as

$$\eta \triangleq \lim_{\rho\to\infty}\frac{C_\Sigma(\rho)}{\log\rho}$$

where $C_\Sigma(\rho)$ is the sum rate capacity at SNR ρ. Note that the DOF equals the multiplexing gain for a given system configuration. The DOF measure can be used for evaluating the approximate sum capacity as the sum capacity at high SNR regime can be expressed as $C_\Sigma(\rho) = \eta\log(\rho) + o(\log(\rho))$ where $o(\log(\rho))$ is the approximation error with $\lim_{\rho\to\infty}\frac{o(\log(\rho))}{\log(\rho)} = 0$.

If we allow in-cell receive cooperation among users $(i, 1), \cdots (i, K)$ for each $i \in \{1, \cdots, B\}$, the IFBC becomes the MIMO IFC investigated in [6]. Throughout the paper, the MIMO IFC with this configuration is referred to as $B \times (M \times K)$ IFC. The authors in [6] analyzed the DOF of the $B \times (M \times K)$ IFC denoted by $\eta_{\text{IFC}}(B \times (M \times K))$ as summraized in Table I. Interestingly, if $B \leq \lfloor\beta\rfloor$, we can achieve the interference-free DOF of $\eta_{\text{PTP}}(BM, BK) = B \cdot \min(M, K)$ where $\eta_{\text{PTP}}(N_T, N_R)$ indicates the DOF of the $N_T \times N_R$ point-to-point (PTP) MIMO channels. However, if B is greater than $\lfloor\beta\rfloor$, we cannot achieve the DOF more than $\frac{\beta}{\lfloor\beta\rfloor+1}\eta_{\text{PTP}}(BM, BK)$ which is strictly lower than the interference-free DOF. In Section III, we will study the DOF of the $B \times (M \times K)$ IFBC where all receive antennas are distributed and show that a DOF loss occurs in comparison to the $B \times (M \times K)$ IFC only when $M < K$ and $B \leq \lfloor\beta\rfloor$. Even for that case, the DOFs of IFC and IFBC converge to the same value as K increases.

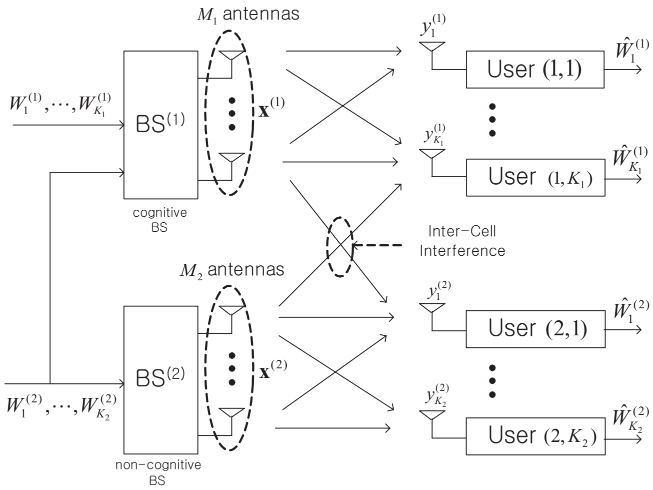

Fig. 2. (M_1, K_1, M_2, K_2) IFBC with $[1, 0]$.

B. Two Interfering Broadcast Channels with Cognitive Message Sharing

A DOF loss caused by distributed processing at both the transmitter and the receiver may be compensated by user cooperation via noisy link or cognitive message sharing [2][7]. Especially, we focus on the cognitive message sharing among transmit nodes. The cognitive message sharing means that some messages are made available to non-intended nodes non-causally[2] [7]. As shown in Figure 2, we consider the (M_1, K_1, M_2, K_2) IFBC with $[1_{T_1}, 1_{T_2}]$ where the BS$^{(i)}$ serves K_i single antenna receivers using M_i transmit antennas ($i = 1, 2$). The indicator variable 1_{T_i} takes 1 if the BS$^{(i)}$ is a cognitive BS and 0 otherwise. Then, the dimensions of $\mathbf{x}^{(i)}(f)$ and $\mathbf{h}_l^{(i,j)}(f)$ are given as $\mathbf{x}^{(i)}(f) \in \mathbb{C}^{M_i \times 1}$ and $\mathbf{h}_l^{(i,j)}(f) \in \mathbb{C}^{1 \times M_j}$ for $i, j \in \{1, 2\}$. Figure 2 illustrates the case of $1_{T_1} = 1$ and $1_{T_2} = 0$ where the BS$^{(1)}$ knows $W_l^{(1)}$ ($l = 1, \cdots, K_1$) as well as $W_l^{(2)}$ ($l = 1, \cdots, K_2$). This scenario accounts for the case where all receive antennas of the two-user MIMO IFC in [7] are distributed with the cognitive message sharing between two transmitters, which will be referred to as (M_1, K_1, M_2, K_2) IFC with $[1_{T_1}, 1_{T_2}]$.

The DOF for this IFC model denoted by $\eta_{\text{IFC}}^{[1_{T_1}, 1_{T_2}]}(M_1, K_1, M_2, K_2)$ is derived as [7]

$$\eta_{\text{IFC}}^{[1_{T_1}, 1_{T_2}]}(M_1, K_1, M_2, K_2) = \min \begin{bmatrix} M_1 + M_2, \\ K_1 + K_2, \\ (1 - 1_{T_2}) \max(M_1, K_2) + 1_{T_2}(M_1 + M_2), \\ (1 - 1_{T_1}) \max(M_2, K_1) + 1_{T_1}(M_1 + M_2) \end{bmatrix}.$$

In this paper, we focus on the partial cognitive case of $[1_{T_1}, 1_{T_2}] = [1, 0]$ since the scenarios with $1_{T_1} = 1_{T_2} = 0$ and $1_{T_1} = 1_{T_2} = 1$ correspond to the non-cognitive setting and single-cell BC systems, respectively. Substituting $[1_{T_1}, 1_{T_2}] = [1, 0]$ into the above equation, $\eta_{\text{IFC}}^{[1,0]}$ is computed as

$$\eta_{\text{IFC}}^{[1,0]}(M_1, K_1, M_2, K_2) = \min\{M_1 + M_2, K_1 + K_2, \max(M_1, K_2)\}. \tag{2}$$

In Section IV, we will derive the exact DOF of the (M_1, K_1, M_2, K_2) IFBC with $[1_{T_1}, 1_{T_2}] = [1, 0]$, denoted by $\eta_{\text{IFBC}}^{[1,0]}(M_1, K_1, M_2, K_2)$ to show that there is no loss in the DOF regardless of the receive cooperation, i.e., $\eta_{\text{IFBC}}^{[1,0]} = \eta_{\text{IFC}}^{[1,0]}$, if at least one transmitter is a cognitive BS.

III. DOF of Multiple Interfering Broadcast Channels

In this section, we derive the lower and upper bounds on the DOF of the $B \times (M \times K)$ IFBC denoted by $\eta_{\text{IFBC}}(B \times (M \times K))$. First, we show that it is possible to achieve the DOF of $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1} B \cdot \min(M, K)$ if $B \geq \lfloor\beta\rfloor + 1$ and $1(M \geq K)BK + 1(M < K)\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM$ if $B \leq \lfloor\beta\rfloor$. Also, we will provide the upper bound coinciding with the lower bound if and only if $(M = 1)$ or $(B \geq \lfloor\beta\rfloor + 1$ and β is integer) or $(B \leq \lfloor\beta\rfloor$ and $M \geq K)$. The result is summarized in Table II.

A. Lower Bound

In this subsection, we show that one can achieve the lower bound on the DOF summarized in Table II in the $B \times (M \times K)$ IFBC with the ZF beamforming or the IA scheme discussed in [4], [6] and [8].

1) $\underline{M \geq K}$

All possible cases can be classified into the following 3 cases.

1-1) $\underline{B \leq \lfloor\beta\rfloor}$

There are BK users in total. Since $B \leq \lfloor\frac{M}{K}\rfloor \leq \frac{M}{K}$, we obtain $M \geq BK$, which means that each BS has transmit antennas more than total users. Thus, with the ZF beamforming, each BS can support its K users while nulling the interference leakaging to other cells.

Conclusively, we have the following lower bound as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \geq BK.$$

1-2) $\underline{B = \lfloor\beta\rfloor + 1}$

If one cell is restricted to be turned off, we obtain the inequality of

$$\eta_{\text{IFBC}}\left((\lfloor\beta\rfloor + 1) \times (M \times K)\right) \geq \eta_{\text{IFBC}}\left(\lfloor\beta\rfloor \times (M \times K)\right),$$

where the lower bound on $\eta_{\text{IFBC}}\left(\lfloor\beta\rfloor \times (M \times K)\right)$ has been derived in the previous subsection as

$$\eta_{\text{IFBC}}\left(\lfloor\beta\rfloor \times (M \times K)\right) \geq \lfloor\beta\rfloor K.$$

Thus, we arrive at the lower bound of

$$\eta_{\text{IFBC}}\left((\lfloor\beta\rfloor + 1) \times (M \times K)\right) \geq \lfloor\beta\rfloor K$$

which matches with Table II since $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BK = \lfloor\beta\rfloor K$ if $B = \lfloor\beta\rfloor + 1$.

1-3) $\underline{B > \lfloor\beta\rfloor + 1}$

From $\lfloor\beta\rfloor = \lfloor\frac{M}{K}\rfloor \leq \frac{M}{K}$, we have $M \geq K\lfloor\beta\rfloor$. Thus, it follows

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \geq \eta_{\text{IFBC}}\left(B \times (K\lfloor\beta\rfloor \times K)\right). \tag{3}$$

[2]This kind of system model with data-coordinated transmitters is well suited for the network MIMO systems also termed as coordinated multi-point transmission (CoMP) or collaborative MIMO.

TABLE II
THE LOWER BOUND AND UPPER BOUND ON THE DOF OF THE $B \times (M \times K)$ IFBC

$M \geq K$		$M < K$	
$B \leq \lfloor\beta\rfloor$	$B \geq \lfloor\beta\rfloor + 1$	$B \leq \lfloor\beta\rfloor$	$B \geq \lfloor\beta\rfloor + 1$
BK	UB: $\frac{\beta}{\lfloor\beta\rfloor+1}BK$ LB: $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BK$	UB: $\frac{\beta}{\beta+\frac{1}{M}}BM$ LB: $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM$	UB: $\frac{\beta}{\lfloor\beta\rfloor+1}BM$ LB: $\frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM$

By separating each BS with $K\lfloor\beta\rfloor$ antennas into K nodes each with $\lfloor\beta\rfloor$ antennas, we obtain

$$\begin{aligned}\eta_{\text{IFBC}}\left(B \times (K\lfloor\beta\rfloor \times K)\right) &\geq \eta_{\text{IFC}}\left(BK \times (\lfloor\beta\rfloor \times 1)\right)\\ &\geq \frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BK \end{aligned} \tag{4}$$

where the last step follows from [6]. To achieve this lower bound, we can apply the IA scheme with the symbol extension in the frequency domain. From (3) and (4), we get the following lower bound as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \geq \frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BK.$$

2) $\underline{M < K}$
Since $\lfloor\beta\rfloor = \lfloor\frac{K}{M}\rfloor \leq \frac{K}{M}$ (i.e., $K \geq \lfloor\beta\rfloor M$), we get

$$\begin{aligned}\eta_{\text{IFBC}}\left(B \times (M \times K)\right) &\geq \eta_{\text{IFBC}}\left(B \times (M \times \lfloor\beta\rfloor M)\right)\\ &\geq \eta_{\text{IFBC}}\left(BM \times (1 \times \lfloor\beta\rfloor)\right) \end{aligned} \tag{5}$$

where the last inequality is obtained by disallowing transmit cooperation. The right-hand side (RHS) of (5) was derived in [4] and [8] as

$$\eta_{\text{IFBC}}\left(BM \times (1 \times \lfloor\beta\rfloor)\right) = \frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM \tag{6}$$

which can be achieved using the IA scheme with the symbol extension.

Combining (5) and (6) results in the following lower bound as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \geq \frac{\lfloor\beta\rfloor}{\lfloor\beta\rfloor+1}BM.$$

B. Upper Bound

We derive the upper bound on the DOF for the $B\times(M\times K)$ IFBC by partitioning all possible cases of M, K and B into the following two cases.

1) $\underline{M \geq K}$
We provide the proof of an upper bound for the case of $B \leq \lfloor\beta\rfloor$, $B = \lfloor\beta\rfloor + 1$ and $B > \lfloor\beta\rfloor + 1$ separately.

1-1) $\underline{B \leq \lfloor\beta\rfloor}$
Since enabling transmit and receive cooperation cannot decrease the DOF, it is obvious that we have

$$\begin{aligned}\eta_{\text{IFBC}}\left(B \times (M \times K)\right) &\leq \eta_{\text{PTP}}\left(BM, BK\right)\\ &= \min\{BM, BK\} = BK.\end{aligned}$$

1-2) $\underline{B = \lfloor\beta\rfloor + 1}$
By making $\lfloor\beta\rfloor$ BSs co-located, the following inequality holds.

$$\begin{aligned}\eta_{\text{IFBC}}\left((\lfloor\beta\rfloor+1) \times (M \times K)\right) &\leq \eta_{\text{IFBC}}^{[0,0]}(\lfloor\beta\rfloor M, \lfloor\beta\rfloor K, M, K)\\ &\leq \eta_{\text{IFC}}^{[0,0]}(\lfloor\beta\rfloor M, \lfloor\beta\rfloor K, M, K)\end{aligned} \tag{7}$$

where the last inequality comes from enabling receive cooperation. In [3], the RHS of (7) is derived as

$$\begin{aligned}&\eta_{\text{IFC}}^{[0,0]}(\lfloor\beta\rfloor M, \lfloor\beta\rfloor K, M, K) =\\ &\min\{(\lfloor\beta\rfloor+1)M, (\lfloor\beta\rfloor+1)K, \max(\lfloor\beta\rfloor M, K), \max(M, \lfloor\beta\rfloor K)\}.\end{aligned} \tag{8}$$

As $\frac{M}{\lfloor\beta\rfloor K} = \frac{\beta}{\lfloor\beta\rfloor} \geq 1$, we obtain $\max(M, \lfloor\beta\rfloor K) = M$. Since both $(\lfloor\beta\rfloor+1)M$ and $\max(\lfloor\beta\rfloor M, K)$ cannot be less than M, the RHS of (8) is given as

$$\begin{aligned}&\min\{\lfloor\beta\rfloor M, (\lfloor\beta\rfloor+1)K, \max(\lfloor\beta\rfloor M, K), \max(M, \lfloor\beta\rfloor K)\}\\ &\qquad = \min\{(\lfloor\beta\rfloor+1)K, M\} = M\end{aligned} \tag{9}$$

where the last step follows from $\frac{M}{(\lfloor\beta\rfloor+1)K} = \frac{\beta}{\lfloor\beta\rfloor+1} < 1$.

Combining (7), (8) and (9) results in

$$\eta_{\text{IFBC}}\left((\lfloor\beta\rfloor+1) \times (M \times K)\right) \leq M$$

which coincides with Table II since $\frac{\beta}{\lfloor\beta\rfloor+1}BK = M$ for $B = \lfloor\beta\rfloor + 1$.

1-3) $\underline{B > \lfloor\beta\rfloor + 1}$
Picking any distinct $\lfloor\beta\rfloor + 1$ variables $\eta_{i_1}, \cdots, \eta_{i_{\lfloor\beta\rfloor+1}} \in \{\eta_1, \cdots, \eta_B\}$ with $i_1 < i_2 < \cdots < i_{\lfloor\beta\rfloor+1}$, we get the inequality of

$$\eta_{i_1} + \cdots + \eta_{i_{\lfloor\beta\rfloor+1}} \leq \eta_{\text{IFBC}}\left((\lfloor\beta\rfloor+1) \times (M \times K)\right). \tag{10}$$

If we sum all the above inequalities after substituting $\eta_{\text{IFBC}}\left((\lfloor\beta\rfloor+1) \times (M \times K)\right) \leq M$ which is proven in the previous subsection, we arrive at the inequality of

$$\binom{B-1}{\lfloor\beta\rfloor}\sum_{i=1}^{B}\eta_i \leq \binom{B}{\lfloor\beta\rfloor+1}M.$$

Then, dividing both sides by $\binom{B-1}{\lfloor\beta\rfloor}$ leads to the upper bound as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \leq \frac{1}{\lfloor\beta\rfloor+1}BM = \frac{\beta}{\lfloor\beta\rfloor+1}BK.$$

2) $\underline{M < K}$
We partition all possible cases as follows.

2-1) $\underline{B \leq \lfloor\beta\rfloor}$
Let $\eta_{\text{IFBC}}\left(B \times \{M \times (K, N)\}\right)$ denote the DOF of B interfering BCs where each BS equipped with M antennas supports its corresponding K users each with N antennas. By increasing the number of receive antennas, we obtain the upper bound on $\eta_{\text{IFBC}}\left(B \times (M \times K)\right)$ as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \leq \eta_{\text{IFBC}}\left(B \times \{M \times (K, M)\}\right). \tag{11}$$

From the results of [4] and [8], the RHS of (11) is upper bounded as

$$\eta_{\text{IFBC}}\left(B \times \{M \times (K, M)\}\right) \leq \frac{K}{K+1}BM = \frac{\beta}{\beta+\frac{1}{M}}BM. \tag{12}$$

TABLE III
DOF COMPARISON WITH INTEGER β

	$M \geq K$		$M < K$	
	$B \leq \beta$	$B \geq \beta+1$	$B \leq \beta$	$B \geq \beta+1$
η_{IFC}	BK	$\frac{\beta}{\beta+1}BK$	BM	$\frac{\beta}{\beta+1}BM$
η_{IFBC}	BK	$\frac{\beta}{\beta+1}BK$	UB: $\frac{\beta}{\beta+\frac{1}{M}}BM$ LB: $\frac{\beta}{\beta+1}BM$	$\frac{\beta}{\beta+1}BM$

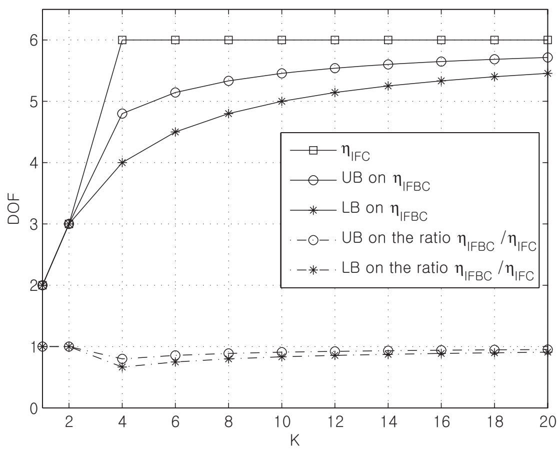

Fig. 3. DOF comparison for $B = 3$ and $M = 2$.

By combining (11) and (12), we obtain the upper bound as

$$\eta_{\text{IFBC}}\left(B \times (M \times K)\right) \leq \frac{\beta}{\beta + \frac{1}{M}} BM.$$

2-2) $\underline{B \geq \lfloor \beta \rfloor + 1}$

Since enabling receive cooperation cannot decrease the DOF, we obtain

$$\eta_{\text{IFBC}}(B \times (M \times K)) \leq \eta_{\text{IFC}}(B \times (M \times K)) \leq \frac{\beta}{\lfloor \beta \rfloor + 1} BM$$

where the last inequality comes from [6].

C. Comparison with MIMO Interference Channels

In this subsection, we compare the derived result with the DOF of the $B \times (M \times K)$ IFC studied in [6]. For the sake of clarity in comparison, we focus on the case of an integer β. Then, the upper bound and lower bound on $\eta_{\text{IFC}}(B \times (M \times K))$ are tight which are summarized in Table III. First of all, we can observe an interesting result that in-cell receiver cooperation does not help in increasing the DOF in most of the cases except for one special case of $M < K$ and $B \leq \beta$. In other words, we have $\eta_{\text{IFBC}} = \eta_{\text{IFC}}$ as long as $M \geq K$ or $B > \beta$. Also, it should be emphasized that even for the case of $\eta_{\text{IFBC}} < \eta_{\text{IFC}}$, the loss of η_{IFBC} compared to η_{IFC} becomes negligible as K increases.

In Figure 3, $\eta_{\text{IFC}}\left(B \times (M \times K)\right)$ and the lower and upper bounds on $\eta_{\text{IFBC}}\left(B \times (M \times K)\right)$ are plotted as a function of K for fixed $B = 3$ and $M = 2$. We display the DOF only for $K = 1$ or even K since β is assumed to be an integer number. For $K \leq M$, the lower and upper bounds on $\eta_{\text{IFBC}}\left(B \times (M \times K)\right)$ not only coincide with each other but also match with the DOF of the MIMO IFC, i.e., $\eta_{\text{IFBC}}\left(B \times (M \times K)\right) = \eta_{\text{IFC}}\left(B \times (M \times K)\right)$. Also, for the case of $K \geq 4$ where the $B \times (M \times K)$ IFC achieves the interference-free DOF of $\eta_{\text{PTP}}(BM, BK) = \min(BM, BK) = \min(6, 3K) = 6$, two bounds on $\eta_{\text{IFBC}}\left(B \times (M \times K)\right)$ do not coincide, but the gap between two bounds becomes smaller as K increases. This is because the gap can be computed as

$$BM\left(\frac{\beta}{\beta + \frac{1}{M}} - \frac{\beta}{\beta+1}\right) = \frac{BM(M-1)}{M+1+K+\frac{M}{K}}$$

which is monotonically decreasing with respect to K if $K > \sqrt{M}$. In addition, we notice that for $K \geq 4$ where the IFBC shows a DOF loss compared to the IFC, the ratio of η_{IFBC} to η_{IFC} is bounded as

$$\frac{\beta}{\beta + \frac{1}{M}} \leq \frac{\eta_{\text{IFBC}}}{\eta_{\text{IFC}}} \leq \frac{\beta}{\beta+1}.$$

As K goes to infinity, both two bounds approach unity, which means that the DOF loss of the $B \times (M \times K)$ IFBC compared to the $B \times (M \times K)$ IFC becomes negligible with large K.

Lastly, we consider the symmetric antenna setting, i.e., $M = K$, where both η_{IFBC} and η_{IFC} are derived as $\frac{1}{2}BM$ for all B and $M = K$. This result indicates that in a MIMO IFC which has the same number of transmit and receive antennas in total, if the receive cooperation is removed, the DOF is not degraded. This coincides with the result in [5] which showed that if both the transmitters and receivers in the L-user IFC with every nodes equipped with N antennas do not cooperate, the channel still has the DOF of $\frac{LN}{2}$. The interference networks with the maximum achievable DOF of $\frac{BM}{2}$ including the $B \times (M \times M)$ IFC, $B \times (M \times M)$ IFBC and (BM)-user SISO IFC are illustrated in Figure 4.

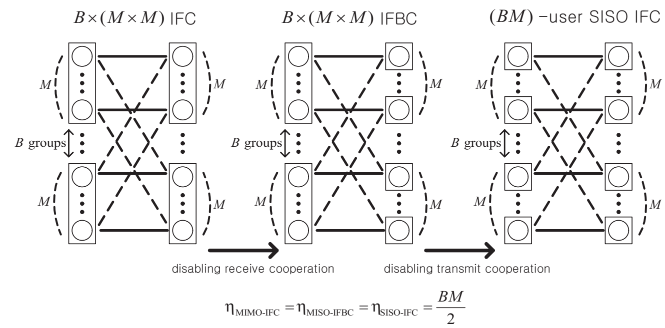

Fig. 4. The interference networks with symmetric antenna settings.

IV. DOF FOR TWO MUTUALLY INTERFERING BROADCAST CHANNELS WITH COGNITIVE BASE STATION

In this section, we study the effect of cognitive BS on the DOF performance of the IFBC.

A. DOF with Cognitive Base Station

In this subsection, we show that a precise expression of the DOF for the (M_1, K_1, M_2, K_2) IFBC with $[1_{T_1}, 1_{T_2}] = [1, 0]$ is given as

$$\eta_{\text{IFBC}}^{[1,0]}(M_1, K_1, M_2, K_2) = \min\{M_1 + M_2, K_1 + K_2, \max(M_1, K_2)\}, \quad (13)$$

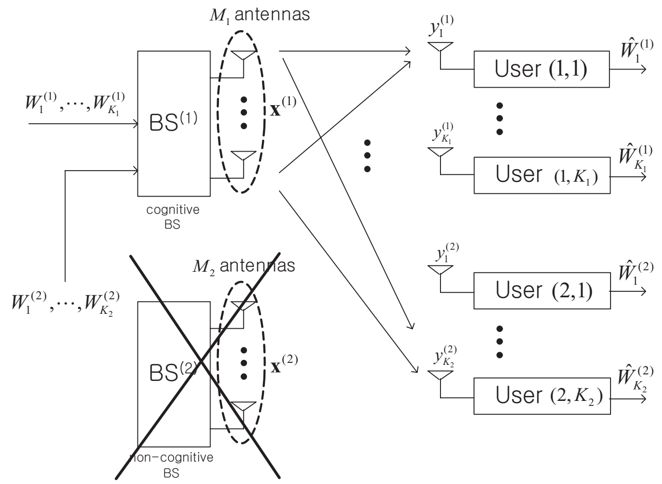

Fig. 5. Establishing a single-cell $M_1 \times (K_1 + K_2)$ BC for the case of $M_1 \geq K_2$.

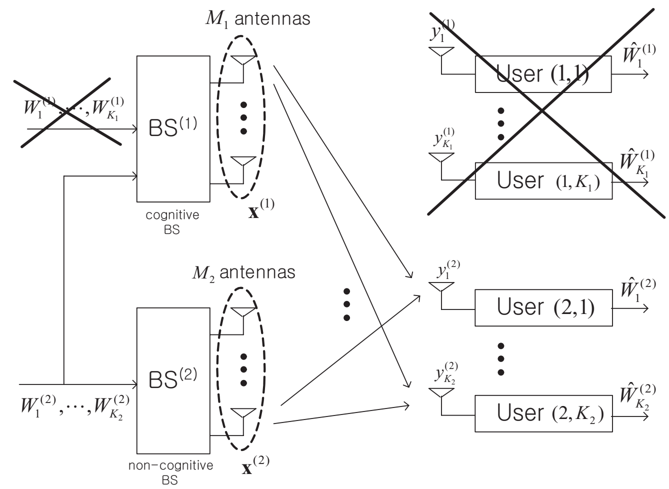

Fig. 6. Establishing a single-cell $(M_1 + M_2) \times K_2$ BC for the case of $M_1 < K_2$.

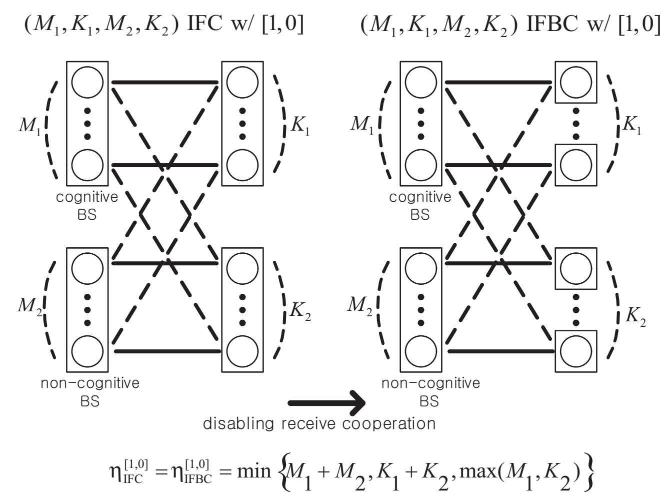

Fig. 7. The interference networks with partial cognitive setting.

which is the same as the DOF of (M_1, K_1, M_2, K_2) IFC with $[1_{T_1}, 1_{T_2}] = [1, 0]$ in (2). This implies that if at least one of two BSs is a cognitive BS[3], whether in-cell receive cooperation exists or not, the DOF is not affected.

Since the converse argument $\eta_{\text{IFBC}}^{[1,0]} \leq \eta_{\text{IFC}}^{[1,0]}$ is quite straightforward, we provide only the achievability proof by showing that a ZF beamforming is able to achieve the DOF of (13) without an aid of symbol extension. To this end, we divide all possible cases of M_1, K_2 into the following two cases.

1) $\underline{M_1 \geq K_2}$

For the case of $M_1 \geq K_2$, the expression (13) is simplified to $\min(M_1, K_1 + K_2)$. Then, we need to show that the DOF of $\min(M_1, K_1 + K_2)$ can be achieved. Note that by shutting off BS 2 as shown in Figure 5, we can obtain a single-cell $M_1 \times (K_1 + K_2)$ BC where a BS equipped with M_1 transmit antennas serves $K_1 + K_2$ users. This is due to the fact that BS 1 has messages intended for users in both cell 1 and cell 2. In this scenario, the DOF of $\min(M_1, K_1 + K_2)$ can be achieved by applying a ZF beamforming at BS 1.

2) $\underline{M_1 < K_2}$

Now, consider the case of $M_1 < K_2$ where the expression (13) is given as $\min(M_1 + M_2, K_2)$. If we eliminate the users in cell 1, this results in a $(M_1 + M_2) \times K_2$ BC as illustrated in Figure 6. Thus, we can obtain the DOF of $\min(M_1 + M_2, K_2)$ by employing a ZF beamforming.

From the results in the above two cases, the achievability proof for (13) is completed for the cognitive IFBC. It is confirmed that there is no DOF loss compared to the IFC equipped with in-cell receiver cooperation as shown in Figure 7. This means that the DOF loss induced by disabling receive cooperation of the MIMO IFC can be completely recovered by the cognitive BS.

B. DOF Comparison between the Non-Cognitive and Cognitive IFBC

Now, we briefly observe the DOF gain obtained from the cognitive message sharing. To this end, we compare the DOFs of the non-cognitive setting ($1_{T_1} = 1_{T_2} = 0$) and the partial cognitive setting ($1_{T_1} = 1$ and $1_{T_2} = 0$) for the two-cell IFBC with $M_1 = M_2 = M$ and $K_1 = K_2 = K$. Assuming $\beta = \frac{\max(M,K)}{\min(M,K)}$ is an integer number for simplicity, the DOF comparison is summarized in Table IV. It is observed that the DOF performance is improved by a cognitive BS when K exceeds $2M$. This is reasonable since the effect of cognitive message sharings at the transmit side would be significant when the ratio of M to K is small. More interestingly, as K goes to infinity, the DOF gain of the cognitive setting becomes minor since

$$\lim_{K\to\infty} \eta_{\text{IFBC}}^{[0,0]} = \lim_{K\to\infty} \eta_{\text{IFBC}}^{[1,0]} = 2M.$$

However, it should be emphasized that achievable schemes of the non-cognitive IFBC are quite different from those of the cognitive scenario. In the cognitive IFBC, the derived DOF can be achieved with the ZF scheme, whereas for the non-cognitive IFBC, the IA precoding combined with symbol extension of infinite length should be applied to obtain the optimal DOF.

[3]It is trivial to show that if both of two BSs are cognitive BSs, the degree of cooperation at the receivers does not influence the DOF.

TABLE IV
DOF GAIN FROM THE COGNITIVE BS

	$K \leq \frac{M}{2}$	$K = M$	$K \geq 2M$
$\eta_{\text{IFBC}}^{[0,0]}(M,K,M,K)$	$2K$	M	UB: $\frac{K}{K+1}2M$ LB: $\frac{K}{K+M}2M$
$\eta_{\text{IFBC}}^{[1,0]}(M,K,M,K)$	$2K$	M	$2M$

V. Conclusion

In this paper, we have studied the DOF of multiple interfering BCs and provided the upper and lower bounds on the DOF which are tight under some conditions. The lower bound is derived using the IA schemes and the upper bound is obtained by allowing cooperation among some nodes or increasing the number of receive antennas. We have also compared the derived result with the DOF of the MIMO IFC and observed that the IFBC achieves the same DOF as the MIMO IFC in most cases, even if receive cooperation is not allowed. Additionally, the exact DOF for two-cell BCs in the presence of the partial cognitive message sharing is derived. The achievability is shown with the ZF beamforming and the converse is confirmed by comparing with the DOF of MIMO IFC. We have arrived at an interesting conclusion that if one of two BSs is a cognitive one, the DOF of the IFBC shows no degradation compared to the MIMO IFC. As a future work, an extension of the DOF study for cognitive scenarios to multiple BCs would be meaningful. This paper also motivates a research on the effect of imperfect synchronization and CSI estimation errors. Since our derived lower and upper bounds meet with each other if and only if $(M = 1)$ or $(B \geq \lfloor \beta \rfloor + 1$ and β is integer) or $(B \leq \lfloor \beta \rfloor$ and $M \geq K)$, we leave the derivation of the DOF for the remaining cases as the future work.

VI. Acknowledgement

The authors would like to thank Prof. Jindal and the reviewers for helpful remarks.

References

[1] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, pp. 5534-5562, Dec. 2008.

[2] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proc. IEEE ISIT*, pp. 2065-2069, Sep. 2005.

[3] S. A. Jafar and M. J. Fakhereddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 53, pp. 2637-2642, July 2007.

[4] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Trans. Inf. Theory*, vol. 55, pp. 3893-3908, Sep. 2009.

[5] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, pp. 3425-3441, Aug. 2008.

[6] T. Gou and S. A. Jafar, "Degrees of freedom of the K user $M \times N$ MIMO interference channel," submitted to *IEEE Trans. Inf. Theory*, Aug. 2008.

[7] C. Huang and S. A. Jafar, "Degrees of freedom of the MIMO interference channel with cooperation and cognition," in *Proc. IEEE ISIT*, pp. 1348-1352, July 2008.

[8] C. Suh and D. Tse, "Interference alignment for cellular networks," in *Proc. IEEE Allerton Conf.*, pp. 1037-1044, Sep. 2008.

[9] S.-H. Park and I. Lee, "Degrees of freedom and sum-rate maximization for two mutually interfering broadcast channels," in *Proc. IEEE ICC*, June 2009.

[10] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, "Feasibility conditions for interference alignment," *Proc. IEEE Globecom*, Nov. 2009.

[11] J. Kim, S.-H. Park, H. Sung, and I. Lee, "Spatial multiplexing gain for two interfering MIMO broadcast channels based on linear transceiver," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3012-3018, Oct. 2010.

[12] H. Sung, S.-H. Park, K.-J. Lee, and I. Lee, "Linear precoder designs for K-user interference channels," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 291-301, Jan. 2010.

[13] S.-H. Park, H. Park, and I. Lee, "Distributed beamforming techniques for weighted sum-rate maximization in MISO interference channels," *IEEE Commun. Lett.*, vol. 14, pp. 1131-1133, Dec. 2010.

Seok-Hwan Park received the B.S., M.S. and Ph.D. degrees in electrical engineering from Korea University, Seoul, Korea, in 2005, 2007 and 2011, respectively. Since January 2011, he has been a research engineer at the Agency for Defense Development, Daejeon, Korea. His research topics include signal processing techniques for wireless communication systems with an emphasis on multiple antenna techniques for throughput optimization in multi-cell multi-user downlink networks. He is currently interested in convex optimization, random matrix theory and stochastic geometry.

Inkyu Lee received the B.S. degree (Hon.) in control and instrumentation engineering from Seoul National University, Seoul, Korea, in 1990, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1992 and 1995, respectively. From 1995 to 2001, he was a Member of Technical Staff at Bell Laboratories, Lucent Technologies, where he conducted research on the high-speed wireless system design. He later worked for Agere Systems (formerly Microelectronics Group of Lucent Technologies), Murray Hill, NJ, as a Distinguished Member of Technical Staff from 2001 to 2002. In September 2002, he joined the faculty of Korea University, Seoul, Korea, where he is currently a Professor in the School of Electrical Engineering. During 2009, he visited University of Southern California, LA, USA, as a visiting Professor. He has published over 60 journal papers in IEEE, and has 30 U.S. patents granted or pending. His research interests include digital communications, signal processing, and coding techniques applied for next generation wireless systems. Dr. Lee currently serves as an Associate Editor for IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. Also, he has been a Chief Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (Special Issue on 4G Wireless Systems). He received the IT Young Engineer Award as the IEEE/IEEK joint award in 2006. Also he received the Best Paper Award at APCC in 2006 and IEEE VTC in 2009.