# A Decoupling Approach for Low-Complexity Vector Perturbation in Multiuser Downlink Systems

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***Abstract*—In this letter, we propose an efficient algorithm which reduces the complexity of conventional vector perturbation schemes by searching the real and imaginary components of a perturbation vector individually. To minimize a performance loss induced from the decoupled joint search, we apply diagonal precoding at the transmitter whose parameters are iteratively optimized to maximize the chordal distance between subspaces spanned by the real and imaginary components. We also propose a simple non-iterative method with a slight performance loss which can achieve a significant complexity reduction compared to the conventional vector perturbation schemes.**

***Index Terms*—Multiuser downlink, vector perturbation.**

## I. INTRODUCTION

IN a multiple-input multiple-output (MIMO) broadcast channel (BC) [1], co-channel interference (CCI) is inevitable when communicating to several users. A simple channel inversion (CI) technique was proposed to eliminate the CCI and allow independent signals to be directed to the intended users [2]. However, the performance of the CI is quite poor due to a power boosting effect for ill-conditioned channels. To enhance the performance of the CI, vector perturbation was introduced in [3], which adopts a modulo operator at both transmitter and receiver. With the modulo operation, the transmitter gains a degree of freedom to choose an element in the multiple of constellation to be transmitted as a desired value for the modulo operator input. In the vector perturbation technique where the data is perturbed such that the transmit power is minimized, finding a perturbation vector is a lattice closest-point problem which can efficiently be solved via sphere encoding (SE) [4] or LLL-aided lattice reduction (LR) [5]. Also, the vector perturbation scheme with limited lattice search was proposed in [6]. However, complexity of those algorithms is still problematic for a large number of users.

In this letter, we introduce a new algorithm which further reduces the complexity of the conventional vector perturbation schemes such as the SE and LR algorithms by searching for the real and imaginary components of the perturbation vectors individually [7]. This provides us significant complexity savings since the complexity of the search problem is determined by its dimension. To minimize a performance loss induced by the decoupled joint search, diagonal precoding is applied at the transmitter. We first describe a method which iteratively optimizes parameters of the diagonal precoder. Since our main objective is to reduce the system cost, we also present a simple method of choosing parameters which can provide the performance almost identical to the optimized ones. It is confirmed that the proposed scheme can achieve a significant complexity reduction compared to the conventional vector perturbation algorithms.

## II. CONVENTIONAL VECTOR PERTURBATION SYSTEMS

Throughout this letter, normal letters represent scalar quantities, boldface letters indicate vectors, and boldface uppercase letters designate matrices. With a bar accounting for complex variables, for any complex notation $\bar{c}$, we denote the real and imaginary part of $\bar{c}$ by $\Re[\bar{c}]$ and $\Im[\bar{c}]$, respectively. We use $(\cdot)^T$, $(\cdot)^H$, $||\cdot||$ and $||\cdot||_F$ for transpose, complex conjugate transpose, 2-norm and Frobenius norm, respectively. We define span($\mathbf{X}$) as the column space of a matrix $\mathbf{X}$.

We consider a multiuser downlink system where the base station with $N_T$ antennas transmits independent data streams to $K$ single-antenna users. Let us define the $N_T$-dimensional complex transmitted signal vector $\overline{\mathbf{x}}$, and the $K$-dimensional complex received signal vector $\overline{\mathbf{y}} = [\bar{y}_1 \cdots \bar{y}_K]^T$ where $\bar{y}_k$ is the received signal at user $k$. Then, the corresponding complex vector equation can be written as

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\overline{\mathbf{x}} + \overline{\mathbf{w}} \quad (1)$$

where $\overline{\mathbf{w}} \in \mathbb{C}^{K\times 1}$ is the white Gaussian noise vector at the users with zero mean and the covariance matrix $\sigma_w^2\mathbf{I}_K$, and the channel matrix $\overline{\mathbf{H}}$ consists of $K \times N_T$ independent and identically distributed (i.i.d.) complex Gaussian coefficients with zero mean and unit variance.

The desired signal vector for $K$ users is denoted by $\overline{\mathbf{u}} = [\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_K]^T$ whose elements are chosen from an $M_c$-ary QAM with unit energy. In the CI, the transmit signal vector is formed as

$$\overline{\mathbf{x}} = \frac{1}{\sqrt{\gamma}}\overline{\mathbf{P}}\overline{\mathbf{u}} \quad (2)$$

where $\overline{\mathbf{P}}$ is the right pseudoinverse of $\overline{\mathbf{H}}$ as $\overline{\mathbf{P}} = \overline{\mathbf{H}}^H\left(\overline{\mathbf{H}}\,\overline{\mathbf{H}}^H\right)^{-1}$ and $\gamma = \left|\left|\overline{\mathbf{P}}\overline{\mathbf{u}}\right|\right|^2$ denotes the normalization

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factor to satisfy the transmit power constraint. After passing through the channel, the received signals are interference-free, i.e.,

$$\overline{y}_k = \frac{1}{\sqrt{\gamma}}\overline{u}_k + \overline{w}_k$$

where $\overline{w}_k$ represents the additive noise at user $k$. We notice that the effective channel gain is given by $1/\sqrt{\gamma}$ and is assumed to be known at all users. For ill-conditioned channels, the system performance is degraded due to boosting of $\gamma$.

The vector perturbation technique proposed in [3] attempts to reduce $\gamma$ by adopting

$$\overline{\mathbf{x}} = \frac{1}{\sqrt{\gamma}}\overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}}) \tag{3}$$

where $\overline{\mathbf{l}}$ is a complex integer vector of length $K$, $\tau$ denotes a positive real number, and $\gamma = \left|\left|\overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}})\right|\right|^2$ represents the new normalization factor for the vector perturbation. The integer vector $\overline{\mathbf{l}}$ which minimizes $\gamma$ can be found as

$$\overline{\mathbf{l}} = \arg\min_{\overline{\mathbf{l}}' \in \mathbb{Z}^K + \mathrm{j}\mathbb{Z}^K} \left|\left|\overline{\mathbf{P}}(\overline{\mathbf{u}} + \tau\overline{\mathbf{l}}')\right|\right|^2. \tag{4}$$

To solve (4), we can apply some efficient methods such as the SE [4] and LR [5] algorithms. Especially, the LR-based vector perturbation schemes have advantage in slow fading channels since the lattice basis reduction has to be performed only once for each channel realization. However, the complexity of these algorithms is problematic for a large number of users.

## III. Decoupled Vector Perturbation Scheme

In this section, we propose a new algorithm which further reduces the complexity of the conventional vector perturbation techniques. For systems with $K$ users, the lattice closest-point problem (4) becomes a $2K$-dimensional problem in real-valued representation which requires prohibitive complexity for large $K$. We address this problem by computing the real and imaginary part of integer vectors separately so that the closest-point problem (4) changes to two $K$-dimensional problems. The real-valued representation of $\overline{\mathbf{P}}$ is given as

$$\mathbf{P} = \begin{bmatrix} \Re[\overline{\mathbf{P}}] & -\Im[\overline{\mathbf{P}}] \\ \Im[\overline{\mathbf{P}}] & \Re[\overline{\mathbf{P}}] \end{bmatrix} = \begin{bmatrix} \mathbf{P}_\mathrm{e} & \dot{\mathbf{P}}_\mathrm{e} \end{bmatrix}$$

where $\mathbf{P}_\mathrm{e} = [\Re[\overline{\mathbf{P}}]^T \ \Im[\overline{\mathbf{P}}]^T]^T$ and $\dot{\mathbf{P}}_\mathrm{e} = [-\Im[\overline{\mathbf{P}}]^T \ \Re[\overline{\mathbf{P}}]^T]^T$. Then, the $2K$-dimensional problem in (4) is equivalent to

$$\begin{aligned}&\left(\Re[\overline{\mathbf{l}}], \Im[\overline{\mathbf{l}}]\right) \\ &\quad = \arg\min_{\mathbf{l}'_R, \mathbf{l}'_I \in \mathbb{Z}^{2K}} \left\|\mathbf{P}_\mathrm{e}(\mathbf{u}_R + \tau\mathbf{l}'_R) + \dot{\mathbf{P}}_\mathrm{e}(\mathbf{u}_I + \tau\mathbf{l}'_I)\right\|^2 \end{aligned} \tag{5}$$

where $\mathbf{u}_R = \Re[\overline{\mathbf{u}}]$, $\mathbf{u}_I = \Im[\overline{\mathbf{u}}]$, $\mathbf{l}'_R = \Re[\overline{\mathbf{l}}']$, and $\mathbf{l}'_I = \Im[\overline{\mathbf{l}}']$. We will call the vector perturbation schemes using the SE and LR algorithms to solve (5) as the *joint SE* and *joint LR*, respectively.

To reduce the system cost, we propose to find $\Re[\overline{\mathbf{l}}]$ and $\Im[\overline{\mathbf{l}}]$ individually as

$$\begin{aligned}\Re[\overline{\mathbf{l}}] &= \arg\min_{\mathbf{l}'_R \in \mathbb{Z}^K} \|\mathbf{P}_\mathrm{e}(\mathbf{u}_R + \tau\mathbf{l}'_R)\|^2, \\ \Im[\overline{\mathbf{l}}] &= \arg\min_{\mathbf{l}'_I \in \mathbb{Z}^K} \|\dot{\mathbf{P}}_\mathrm{e}(\mathbf{u}_I + \tau\mathbf{l}'_I)\|^2. \end{aligned} \tag{6}$$

The proposed vector perturbation schemes which use the SE and LR algorithms to solve (6) will be referred to as the *decoupled SE* and *decoupled LR*, respectively. It is natural that solving (6) instead of (5) results in a significant performance loss since the joint search over $2K$ integers is transformed into the two $K$-dimensional searches. Thus, we attempt to minimize this performance loss by applying a diagonal precoding at the transmitter.

Notice that (6) is equivalent to (5) if and only if the column space of $\mathbf{P}_\mathrm{e}$ is orthogonal to that of $\dot{\mathbf{P}}_\mathrm{e}$. Thus, we try to make two subspaces orthogonal by applying a diagonal precoding $\overline{\mathbf{F}}_\theta$ at the transmitter as

$$\overline{\mathbf{x}} = \frac{1}{\sqrt{\gamma}}\overline{\mathbf{P}} \ \overline{\mathbf{F}}_\theta(\overline{\mathbf{u}} + \tau\bar{l})$$

where $\overline{\mathbf{F}}_\theta = \mathrm{diag}(1, e^{j\theta_2}, \cdots, e^{j\theta_K})$. Here, we use a diagonal matrix since the effect of non-diagonal precoding cannot be compensated at the receiver side due to the lack of receive cooperation. It is noted that the base station should report the information about $\theta_i$ to user $i$ through feedforward links ($i = 2, \cdots, K$).

With the diagonal precoding, $\mathbf{P}_e$ and $\dot{\mathbf{P}}_e$ are replaced with $\mathbf{P}_{e,\theta}$ and $\dot{\mathbf{P}}_{e,\theta}$ where $\mathbf{P}_{e,\theta} = [\Re[\overline{\mathbf{P}}_\theta]^T \ \Im[\overline{\mathbf{P}}_\theta]^T]^T$, $\dot{\mathbf{P}}_{e,\theta} = [-\Im[\overline{\mathbf{P}}_\theta]^T \ \Re[\overline{\mathbf{P}}_\theta]^T]^T$. Here $\overline{\mathbf{P}}_\theta$ is defined as $\overline{\mathbf{P}}_\theta = \overline{\mathbf{P}} \ \overline{\mathbf{F}}_\theta$. Let us define $\mathbf{p}_{i,\theta}$ and $\dot{\mathbf{p}}_{i,\theta}$ as the $i$-th column of $\mathbf{P}_{e,\theta}$ and $\dot{\mathbf{P}}_{e,\theta}$, respectively. Since we have $\mathbf{p}_{i,\theta}^T\dot{\mathbf{p}}_{i,\theta} = 0$ and $\mathbf{p}_{i,\theta}^T\dot{\mathbf{p}}_{j,\theta} = -\mathbf{p}_{j,\theta}^T\dot{\mathbf{p}}_{i,\theta}$, $\forall i, j$, we achieve $\mathrm{span}(\mathbf{P}_{e,\theta}) \perp \mathrm{span}(\dot{\mathbf{P}}_{e,\theta})$ if and only if $\mathbf{p}_{i,\theta}^T\dot{\mathbf{p}}_{j,\theta} = 0$ for $i = 1, \cdots, K-1$ and $j = i+1, \cdots, K$. Thus, $\binom{K}{2}$ requirements should be satisfied by using $(K-1)$ variables, which is possible only when $K = 2$.

As we are interested in the general cases of $K > 2$, we try to make the column space of $\mathbf{P}_{\mathrm{e},\theta}$ roughly orthogonal to the column space of $\dot{\mathbf{P}}_{\mathrm{e},\theta}$ by adjusting $\theta_2, \cdots, \theta_K$. To this end, the *chordal distance* [8] is adopted as the metric for assessing the orthogonality of two subspaces defined as

$$\mathrm{dist}(\mathbf{P}_{\mathrm{e},\theta}, \dot{\mathbf{P}}_{\mathrm{e},\theta}) \triangleq K - \|\mathbf{Q}^T\dot{\mathbf{Q}}\|_F^2 \tag{7}$$

where $\mathbf{Q}$ and $\dot{\mathbf{Q}}$ are orthonormal bases of the column spaces of $\mathbf{P}_{\mathrm{e},\theta}$ and $\dot{\mathbf{P}}_{\mathrm{e},\theta}$, respectively. If two subspaces are fully orthogonal, then the chordal distance will be $K$, whereas the distance becomes zero if they are the same. Thus, the optimization of $\theta_2, \cdots, \theta_K$ can be formulated as

$$\max_{\theta_2, \cdots, \theta_K} \mathrm{dist}(\mathbf{P}_{\mathrm{e},\theta}, \dot{\mathbf{P}}_{\mathrm{e},\theta}) \tag{8}$$

which is generally non-convex. To obtain the suboptimal parameters, we propose two methods: an iterative algorithm which guarantees a local optimal solution, and a non-iterative solution with a small performance loss.

### A. Iterative Optimization of $\theta_2, \cdots, \theta_K$

To solve the problem (8) efficiently, we propose an iterative algorithm which updates $K-1$ phase angles in an alternating fashion. Thus, we now discuss the optimization of $\theta_i$ with other angles $\theta_j$ fixed ($j \neq i$). Since only $\mathbf{p}_{i,\theta}$ and $\dot{\mathbf{p}}_{i,\theta}$ depend on $\theta_i$, we compute $\mathbf{Q}$ and $\dot{\mathbf{Q}}$ by performing the Gram-Schmidt process in the following order.

$$\mathbf{p}_{1,\theta} \to \cdots \to \mathbf{p}_{i-1,\theta} \to \mathbf{p}_{i+1,\theta} \to \cdots \to \mathbf{p}_{K,\theta} \to \ \mathbf{p}_{i,\theta},$$
$$\dot{\mathbf{p}}_{1,\theta} \to \cdots \to \dot{\mathbf{p}}_{i-1,\theta} \to \dot{\mathbf{p}}_{i+1,\theta} \to \cdots \to \dot{\mathbf{p}}_{K,\theta} \to \ \dot{\mathbf{p}}_{i,\theta}.$$

Following the above ordering, $\mathbf{Q}$ and $\dot{\mathbf{Q}}$ are given by

$$\mathbf{Q} = \left[ \begin{array}{cc} \widetilde{\mathbf{Q}} & \mathbf{q} \end{array} \right], \dot{\mathbf{Q}} = \left[ \begin{array}{cc} \dot{\widetilde{\mathbf{Q}}} & \dot{\mathbf{q}} \end{array} \right] \tag{9}$$

where the columns of $\widetilde{\mathbf{Q}}$ and $\dot{\widetilde{\mathbf{Q}}}$ are the orthonormal bases of $\operatorname{span}\left([\mathbf{p}_{1,\theta} \cdots \mathbf{p}_{i-1,\theta}\, \mathbf{p}_{i+1,\theta} \cdots \mathbf{p}_{K,\theta}]\right)$ and $\operatorname{span}\left([\dot{\mathbf{p}}_{1,\theta} \cdots \dot{\mathbf{p}}_{i-1,\theta}\, \dot{\mathbf{p}}_{i+1,\theta} \cdots \dot{\mathbf{p}}_{K,\theta}]\right)$, respectively, and $\mathbf{q}$ and $\dot{\mathbf{q}}$ are equal to

$$\mathbf{q} = \frac{\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}}{\left\|\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}\right\|} \quad \text{and} \quad \dot{\mathbf{q}} = \frac{\dot{\mathbf{p}}_{i,\theta} - \dot{\widetilde{\mathbf{Q}}}\dot{\widetilde{\mathbf{Q}}}^T \dot{\mathbf{p}}_{i,\theta}}{\left\|\dot{\mathbf{p}}_{i,\theta} - \dot{\widetilde{\mathbf{Q}}}\dot{\widetilde{\mathbf{Q}}}^T \dot{\mathbf{p}}_{i,\theta}\right\|}. \tag{10}$$

Notice that only $\mathbf{q}$ and $\dot{\mathbf{q}}$ are dependent on $\theta_i$.

Using (9), the chordal distance (7) is computed as

$$\begin{aligned} & K - \left\| \left[ \begin{array}{c} \widetilde{\mathbf{Q}}^T \\ \mathbf{q}^T \end{array} \right] \left[ \begin{array}{cc} \dot{\widetilde{\mathbf{Q}}} & \dot{\mathbf{q}} \end{array} \right] \right\|_F^2 \\ = \;& K - \left\|\widetilde{\mathbf{Q}}^T \dot{\widetilde{\mathbf{Q}}}\right\|_F^2 - \left\|\widetilde{\mathbf{Q}}^T \dot{\mathbf{q}}\right\|^2 - \left\|\dot{\widetilde{\mathbf{Q}}}^T \mathbf{q}\right\|^2 - \left|\mathbf{q}^T \dot{\mathbf{q}}\right|^2 \\ = \;& K - 2\left\|\dot{\widetilde{\mathbf{Q}}}^T \mathbf{q}\right\|^2 \end{aligned} \tag{11}$$

where the last equality comes from $\left\|\tilde{\mathbf{Q}}^T \dot{\tilde{\mathbf{Q}}}\right\|_F^2 = |\mathbf{q}^T \dot{\mathbf{q}}| = 0$ and $\left\|\tilde{\mathbf{Q}}^T \dot{\mathbf{q}}\right\| = \left\|\dot{\tilde{\mathbf{Q}}}^T \mathbf{q}\right\|$. In (11), the last term is the only one which is dependent on $\theta_i$, and is written as

$$\left\|\dot{\widetilde{\mathbf{Q}}}^T \mathbf{q}\right\|^2 = \left\|\dot{\widetilde{\mathbf{Q}}}^T \frac{\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}}{\|\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}\|}\right\|^2$$

where $\mathbf{p}_{i,\theta} = \mathbf{p}_i \cos\theta_i + \dot{\mathbf{p}}_i \sin\theta_i$.

Then, $\theta_{i,\text{opt}}$ is given as

$$\begin{aligned} \theta_{i,\text{opt}} &= \arg\min_{\theta_i} \left\|\dot{\widetilde{\mathbf{Q}}}^T \frac{\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}}{\|\mathbf{p}_{i,\theta} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T \mathbf{p}_{i,\theta}\|}\right\|^2 \\ &= \arg\max_{\theta_i} \frac{\|\mathbf{A}\mathbf{x}_i\|^2}{\|\mathbf{B}\mathbf{x}_i\|^2} \end{aligned} \tag{12}$$

where $\mathbf{A} = (\mathbf{I} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T)[\mathbf{p}_i\ \dot{\mathbf{p}}_i]$, $\mathbf{B} = \dot{\widetilde{\mathbf{Q}}}^T (\mathbf{I} - \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^T)[\mathbf{p}_i\ \dot{\mathbf{p}}_i]$, and $\mathbf{x}_i = [\cos\theta_i\ \ \sin\theta_i]^T$. It can be shown from the Rayleigh-Ritz quotient result [9] that the problem (12) is convex with respect to $\theta_i$ for the given $\theta_j$ $(j \neq i)$, and the solution is obtained as

$$\mathbf{x}_{i,\text{opt}} = [x_1\ x_2]^T = \text{maximum eigenvector}\left((\mathbf{B}^T\mathbf{B})^{-1}\mathbf{A}^T\mathbf{A}\right).$$

Finally, the optimal $\theta_{i,\text{opt}}$ with other angles $\theta_j$ fixed $(j \neq i)$ is computed as

$$\theta_{i,\text{opt}} = \tan^{-1}\frac{x_1}{x_2}. \tag{13}$$

The whole algorithm for iterative optimization of $\theta_i$'s is summarized as follows:

**Step 1.** Initialize $\theta_2, \cdots \theta_K$.
**Step 2.** Update $\theta_2, \cdots, \theta_K$ with the following loop:
For $i = 2 : K$
Construct $\widetilde{\mathbf{Q}}$ and $\dot{\widetilde{\mathbf{Q}}}$ as
$\widetilde{\mathbf{Q}} \leftarrow$ orthonormal basis of
$\operatorname{span}\left([\mathbf{p}_{1,\theta} \cdots \mathbf{p}_{i-1,\theta}\, \mathbf{p}_{i+1,\theta} \cdots \mathbf{p}_{K,\theta}]\right)$,
$\dot{\widetilde{\mathbf{Q}}} \leftarrow$ orthonormal basis of
$\operatorname{span}\left([\dot{\mathbf{p}}_{1,\theta} \cdots \dot{\mathbf{p}}_{i-1,\theta}\, \dot{\mathbf{p}}_{i+1,\theta} \cdots \dot{\mathbf{p}}_{K,\theta}]\right)$.
Update $\theta_i$ according to (13).
End
**Step 3.** Compute the chordal distance dist($\mathbf{P}_{\text{e},\theta}, \dot{\mathbf{P}}_{\text{e},\theta}$) using (7).
**Step 4.** Go back to **Step 2** until the convergence of the chordal distance.

It should be emphasized that due to the convexity of (12) over $\theta_i$ for the fixed $\theta_j$ $(j \neq i)$, we have a non-decreasing chordal distance with respect to the number of iterations. Since the chordal distance dist($\mathbf{P}_{\text{e},\theta}, \dot{\mathbf{P}}_{\text{e},\theta}$) is upper bounded by $K$, the sequence of the chordal distance generated by the above algorithm converges to a local optimal point [10]. We notice that if we apply the above algorithm to determine $\theta_i$'s, then our main objective which is complexity reduction cannot be achieved. Thus, a simple non-iterative method is proposed in the following subsection.

TABLE I
AVERAGE AND MAXIMUM NUMBER OF THE VISITED NODES FOR THE SE-BASED VECTOR PERTURBATION SYSTEMS

| | | Average | Maximum |
|---|---|---|---|
| $N_T$=$K$=4 | Joint SE | 64 | 5553 |
| | Decoupled SE | 28 | 535 |
| $N_T$=$K$=6 | Joint SE | 353 | 108861 |
| | Decoupled SE | 63 | 1096 |

### B. Non-Iterative Choice of $\theta_2, \cdots, \theta_K$

In this subsection, we propose a simple non-iterative choice of $\theta_2, \cdots, \theta_K$ by relaxing the orthogonal condition as

$$\begin{aligned} \mathbf{p}_{1,\theta} &\perp \operatorname{span}\left([\dot{\mathbf{p}}_{1,\theta}\ \cdots\ \dot{\mathbf{p}}_{K,\theta}]\right), && (14) \\ \dot{\mathbf{p}}_{1,\theta} &\perp \operatorname{span}\left([\mathbf{p}_{1,\theta}\ \cdots\ \mathbf{p}_{K,\theta}]\right). && (15) \end{aligned}$$

We select $\mathbf{p}_{1,\theta}$ as a representative basis vector for $\operatorname{span}\left([\mathbf{p}_{1,\theta}\ \cdots\ \mathbf{p}_{K,\theta}]\right)$ and make $\mathbf{p}_{1,\theta}$ orthogonal to $\operatorname{span}\left([\dot{\mathbf{p}}_{1,\theta}\ \cdots\ \dot{\mathbf{p}}_{K,\theta}]\right)$. Similarly, $\dot{\mathbf{p}}_{1,\theta}$ which is selected as a representative basis for $\operatorname{span}\left([\dot{\mathbf{p}}_{1,\theta}\ \cdots\ \dot{\mathbf{p}}_{K,\theta}]\right)$ is orthogonalized to $\operatorname{span}\left([\mathbf{p}_{1,\theta}\ \cdots\ \mathbf{p}_{K,\theta}]\right)$.

According to the property $\mathbf{p}_{i,\theta}^T \dot{\mathbf{p}}_{j,\theta} = -\mathbf{p}_{j,\theta}^T \dot{\mathbf{p}}_{i,\theta}$, both (14) and (15) are satisfied when

$$-\mathbf{p}_1^T \mathbf{p}_i \sin\theta_i + \mathbf{p}_1^T \dot{\mathbf{p}}_i \cos\theta_i = 0, \quad \text{for } i = 2, \cdots, K.$$

The solution $\theta_i$ of the above equation is given by

$$\theta_i = \tan^{-1}\left(\frac{\mathbf{p}_1^T \dot{\mathbf{p}}_i}{\mathbf{p}_1^T \mathbf{p}_i}\right), \quad \text{for } i = 2, \cdots, K. \tag{16}$$

In Figure 1, the cumulative distribution functions (CDF) of the effective channel gain $1/\sqrt{\gamma}$ with non-iterative and

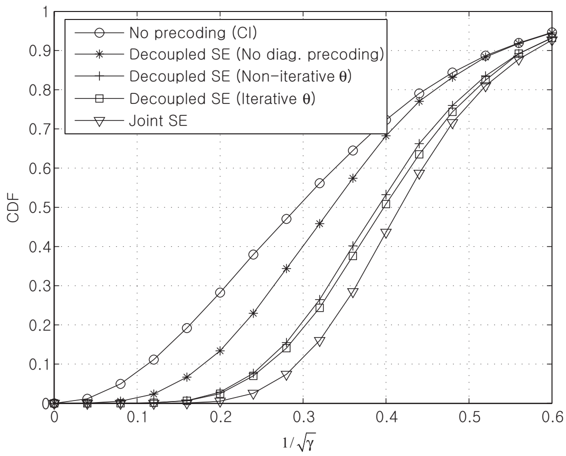


Fig. 1. CDF of $1/\sqrt{\gamma}$ for the SE-based vector perturbation system with $N_T = K = 4$ and 4-QAM.

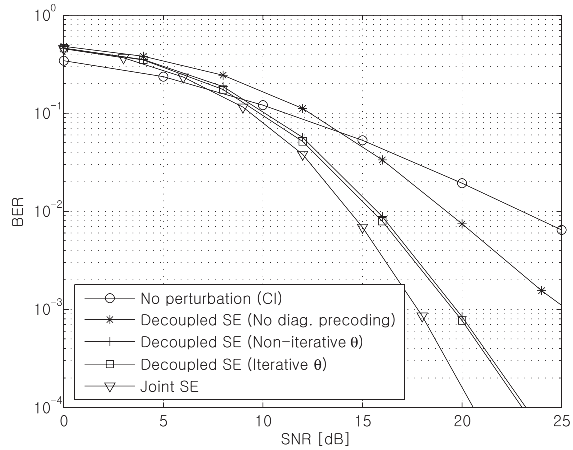


Fig. 2. Performance comparison with sphere encoding for $N_T = K = 4$ and 4-QAM.

TABLE II
AVERAGE NUMBER OF THE FLOATING POINT MULTIPLICATIONS FOR THE LR-BASED VECTOR PERTURBATION SYSTEMS

| | $N_T = K = 4$ | $N_T = K = 6$ |
|---|---|---|
| Joint LR | 1166.9 | 3568.5 |
| Decoupled LR | 804.8 | 2540.5 |

iterative setting of $\theta_i$'s are presented for SE-based vector perturbation systems with $N_T = K = 4$. It is observed that the proposed non-iterative method achieves the performance almost identical to the iterative scheme with much reduced complexity. Also, we can see that the diagonal precoding is essential for the proposed decoupled scheme since the decoupled SE without it shows significantly degraded performance.

We briefly investigate the complexity of the conventional vector perturbation schemes based on joint search and the proposed decoupled approach. Table I shows the average number of the nodes visited by the SE algorithm to find the perturbation vector $\bar{\mathbf{l}}$ for various antenna configurations with 4-QAM modulation. The results in Table I are obtained through simulations over 10000 channel realizations. For the implementation of the SE-based vector perturbation, we have used a sphere decoding algorithm proposed in [11]. The proposed decoupled approach is simulated with the non-iterative setting of $\theta_2, \cdots, \theta_K$. It is observed that for 4-by-4 system, the proposed decoupled approach achieves a complexity reduction of about $56\%$ and $90\%$ in terms of the average and maximum number of the visited nodes, respectively.

Also, the complexity of the LR-based vector perturbation system is summarized in Table II in terms of the average number of the floating point multiplications. In order to focus on the dominant part in the system cost, the number of multiplications is counted during the LLL algorithm, the phase computation in (16) and the construction of the rotated inverse matrix $\overline{\mathbf{P}}_\theta = \overline{\mathbf{P}}\,\overline{\mathbf{F}}_\theta$. With the proposed decoupled approach, we can achieve a complexity saving of about $30\%$ which is smaller compared to the SE-based system. However, it should be emphasized that a performance loss caused by decoupling

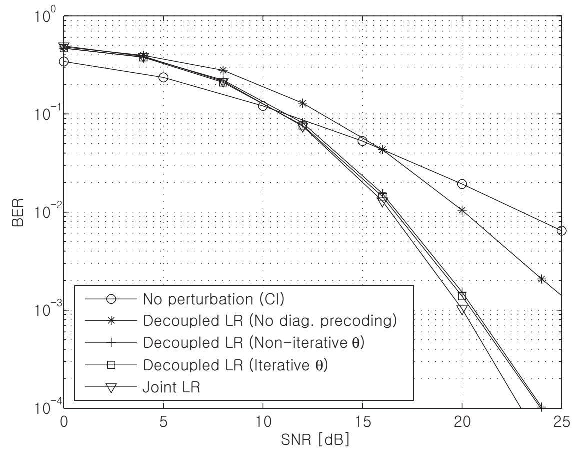


Fig. 3. Performance comparison with lattice-reduction for $N_T = K = 4$ and 4-QAM.

joint search is minor in the LR-based vector perturbation system.

## IV. SIMULATION RESULTS

In this section, we evaluate the bit-error rate (BER) performance of several vector perturbation systems. Throughout the simulations, the signal-to-noise ratio (SNR) is defined as $1/\sigma_w^2$ and we set $\tau = 2(\Delta/2 + |c|_{\max})$ in (4) where $\Delta$ and $|c|_{\max}$ are defined in [3]. Figure 2 illustrates the results for the SE-based vector perturbation systems. As expected, the proposed decoupled SE significantly outperforms the CI scheme while achieving a significant complexity savings compared to the conventional joint SE scheme. Also, we can confirm the importance of the diagonal precoding since the decoupled SE without it suffers from a diversity loss. Our simple choice of $\theta_i$'s provides the performance almost identical to that of iteratively optimized parameters.

The BER performance of the vector perturbation systems employing LR is evaluated in Figure 3. The performance gap between the joint LR and the proposed decoupled LR reduces

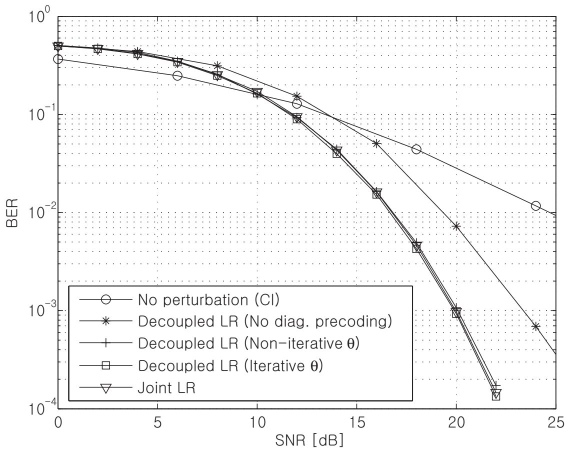


Fig. 4. Performance comparison with lattice-reduction for $N_T = K = 6$ and 4-QAM.

significantly in comparison to the SE-based systems. Also, as in the SE, the simple non-iterative method provides the performance almost identical to the iterative method.

One might think that a performance loss of the proposed scheme would be significant for a large number of users. We show that this is not the case in Figure 4 where the vector perturbation with the LR is evaluated for $N_T = K = 6$. The performance of the proposed decoupled scheme is shown to be within a few tenth of a dB away from that of the conventional joint LR system.

## V. Conclusion

In this letter, we have proposed a low-complexity vector perturbation scheme based on decoupled search of perturbation vectors. To minimize a performance loss compared to the conventional perturbation scheme, we have adopted the diagonal precoding to divide the column space into two orthogonal subspaces efficiently. It is shown that the proposed decoupled scheme achieves a significant complexity reduction at the expense of a slight performance loss. We also verify that our decoupling approach can be efficiently combined with any conventional algorithms to further reduce the complexity.

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