Blockwise Amplify-and-Forward Relaying Strategies for Multipoint-to-Multipoint MIMO Networks

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***Abstract*—In this letter, we consider a multipoint-to-multipoint transmission system which employs a single relay in wireless networks where all source, destination and relay nodes are equipped with multiple antennas. For amplify-and-forward relay systems, we propose new linear processing strategies for maximizing the sum rate performance by applying a blockwise relaying method combined with convex optimization techniques. To reduce a computational complexity, we first consider a zero-forcing based relay only optimization scheme, which generate a closed-form solution in a non-iterative fashion. In order to further improve the sum rate at low signal-to-noise ratio regimes, we additionally design an enhanced relay filter by exploiting the blockwise relaying method based on a minimum mean-square error criterion. Simulation results show that the proposed relay design strategies outperform the existing conventional schemes in terms of the sum rate.**

***Index Terms*—Multiple-input multiple-output (MIMO), amplify-and-forward (AF) relay, blockwise relaying, sum rate maximization.**

I. Introduction

VARIOUS wireless networks based on multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay have been extensively studied in recent years [1]–[11]. The optimal relay design of MIMO AF relay systems with single source, relay and destination was first investigated in [1] and [2], where it was shown that a singular value decomposition (SVD) method with power-loading achieves the capacity of the MIMO AF relay system. While only the relay filter was optimized in [1], authors in [2] introduced an iterative algorithm which jointly optimizes the source covariance matrix as well as the relay filter. For a multi-user two-hop MIMO AF relay system, the optimization frameworks for joint filter design of a source and a relay were presented in [4]. The main criterion of [4] is the maximization of the sum rate under the assumption of dirty paper coding (DPC) at the source. The work shown in [4] was generalized in [5] for a variety of the objective functions based on the DPC or successive interference cancelation.

In contrast, a MIMO AF relay communication for multipoint-to-multipoint networks was addressed in [6] and [7], where multiple MIMO source-destination pairs simultaneously communicate through multiple MIMO relays. Since this system model involves multiple source-destination pairs, inter-source (destination) interference is an inevitable cause for a sum rate impairment in the system. Thus, zero-forcing (ZF) and minimum mean-square error (MMSE) relay filters [6] and a group nulling scheme [7] were proposed to overcome the inter-source (destination) interference. Recently, with multiple single-input single-output (SISO) source-destination pairs, the problem of designing the multi-antenna AF relay filter in terms of minimizing the total transmit power has been well studied in [8].

In this letter, we consider a multipoint-to-multipoint MIMO network with a single multiple antenna relay by extending the work in [8] to MIMO source-destination pairs. Furthermore, instead of aiming at the transmit power minimization in [8], we here provide new linear processing strategies based on a blockwise relaying method associated with convex optimization techniques for maximizing the sum rate performance. We assume that the relay operates in an one-way half-duplex AF protocol and knows full channel state information (CSI) of all links.

Based on the blockwise relaying approach, we address two non-iterative relay filter designs by allocating equal power at each source antenna. We first develop a ZF based relay only optimization scheme which maximizes each individual pair rate under an assumption of uniform power distribution over all source-destination pairs at the relay. To further improve the performance for low signal-to-noise ratio (SNR), we design an enhanced relay filter which utilizes the blockwise relaying method based on the MMSE criterion. Simulation results show that the proposed relay schemes are superior to the existing schemes in terms of the sum rate.

This letter is organized as follows: Section II describes our system model. In Section III, we develop two design strategies employing the blockwise relaying method. Section IV presents the simulation results. Finally, the letter is terminated with conclusions in Section V.

The following notations are used throughout the letter. For a matrix $\mathbf{A}$, $\mathbf{A}^*$, $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ denote conjugate, transpose and conjugate transpose, respectively. $\mathrm{Tr}\,(\mathbf{A})$ indicates the trace, and $\det(\mathbf{A})$ represents the determinant of a matrix $\mathbf{A}$. For $m \times m$ matrices $\mathbf{A}_j$, diag$\{\mathbf{A}_1, \cdots, \mathbf{A}_n\}$ denotes an $mn \times mn$ block diagonal matrix. $E(\cdot)$, $\mathbf{I}_N$ and $\mathbf{0}_{M\times N}$ represent the expectation operator, an $N \times N$ identity matrix and an $M \times N$

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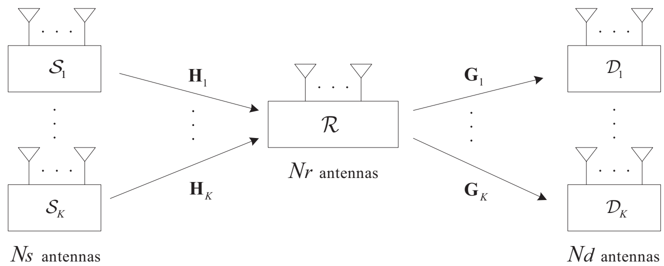

Fig. 1. Schematic diagram of the relay model with distributed sources and destinations.

zero matrix, respectively.

II. System Model

In this letter, we consider a multipoint-to-multipoint MIMO relaying system with the half-duplex AF relay protocol where there are K source-destination terminal pairs and a single relay node as shown in Fig. 1. Here, the source, destination and relay nodes are equipped with N_s, N_d and N_r antennas, respectively. In this system, the k-th source terminal node $\mathcal{S}_k$ $(k = 1, 2, \cdots, K)$ wants to send information to the k-th destination terminal node $\mathcal{D}_k$, and the relay node $\mathcal{R}$ helps the communication between each pair. We here assume that there is no direct link between the source and destination terminal nodes due to a relatively larger path loss compared to the links via relay. As we consider a spatial multiplexing system which transmits $M = \min(N_s, N_d)$ data streams at each source simultaneously, it is assumed that the number of the total transmitted data streams is $K \cdot M$. Throughout the letter, we assume $N_s = N_d$ for compact presentation. It is straightforward to describe the case of $N_s \neq N_d$.

We first define the transmitted data symbol vector $\mathbf{s}$, and the channel matrices $\mathbf{H}$ and $\mathbf{G}$ as $\mathbf{s} = [\mathbf{s}_1^T\ \mathbf{s}_2^T\ \cdots\ \mathbf{s}_K^T]^T$, $\mathbf{H} = [\mathbf{H}_1\ \mathbf{H}_2\ \cdots\ \mathbf{H}_K]$ and $\mathbf{G} = [\mathbf{G}_1^T\ \mathbf{G}_2^T\ \cdots\ \mathbf{G}_K^T]^T$ where $\mathbf{s}_k$ is the M dimensional transmitted signal vector, $\mathbf{H}_k$ denotes the $N_r \times M$ complex channel matrix from $\mathcal{S}_k$ to $\mathcal{R}$, and $\mathbf{G}_k$ represents the $M \times N_r$ complex channel matrix from $\mathcal{R}$ to $\mathcal{D}_k$. In the first time slot, the signal vector $\mathbf{s}_k$ is precoded by the $M \times M$ source precoder $\mathbf{P}_k$, and K source terminal nodes $\{\mathcal{S}_k\}$ for $k = 1, 2, \cdots, K$ transmit their precoded signals $\mathbf{P}_k\mathbf{s}_k$ to the relay node $\mathcal{R}$ simultaneously. Hence, the transmission from the sources to the relay can be modeled as a MIMO multiple access channel (MAC).

The N_r dimensional received signal vector $\mathbf{r}$ at the relay node is given by $\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k\mathbf{P}_k\mathbf{s}_k + \mathbf{n}$ where $\mathbf{n}$ denotes the additive complex Gaussian noise vector with zero mean and $E(\mathbf{n}\mathbf{n}^\dagger) = \sigma_n^2\mathbf{I}_{N_r}$. Here, the signal after the linear precoding should satisfy the individual power constraint $E\|\mathbf{P}_k\mathbf{s}_k\|^2 \leq P_s$. Assuming that the signal vector is normalized to have $E(\mathbf{s}_k\mathbf{s}_k^\dagger) = \mathbf{I}_M$, the transmit power constraint at the k-th source can be written as $\mathrm{Tr}\{\mathbf{P}_k\mathbf{P}_k^\dagger\} \leq P_s$.

In the second time slot, the transmission from the relay to the destinations can be modeled as a MIMO broadcast channel (BC). Assuming that a linear processing is employed at the relay node, the received signal $\mathbf{r}$ is multiplied by the $N_r \times N_r$ relay filter $\mathbf{F}$. Then, the signal vector $\mathbf{x}$ transmitted from the relay node is represented as $\mathbf{x} = \mathbf{F}\mathbf{r} = \sum_{k=1}^{K} \mathbf{F}\mathbf{H}_k\mathbf{P}_k\mathbf{s}_k + \mathbf{F}\mathbf{n}$ where the transmit power constraint at the relay is expressed as $E\|\mathbf{x}\|^2 \leq P_r$, which can be written explicitly with respect to $\mathbf{P}$ and $\mathbf{F}$ as

$$\mathrm{Tr}\{\mathbf{F}(\mathbf{H}\mathbf{P}\mathbf{P}^\dagger\mathbf{H}^\dagger + \sigma_n^2\mathbf{I}_{N_r})\mathbf{F}^\dagger\} \leq P_r \tag{1}$$

where $\mathbf{P} = [\mathbf{P}_1^T\ \mathbf{P}_2^T\ \cdots\ \mathbf{P}_K^T]^T$.

Finally, the received signal vector at the k-th destination node $\mathcal{D}_k$ can be expressed as

$$\mathbf{y}_k = \mathbf{G}_k\mathbf{F}\mathbf{H}_k\mathbf{P}_k\mathbf{s}_k + \sum_{\substack{j=1\\ j\neq k}}^{K} \mathbf{G}_k\mathbf{F}\mathbf{H}_j\mathbf{P}_j\mathbf{s}_j + \mathbf{G}_k\mathbf{F}\mathbf{n} + \mathbf{z}_k \tag{2}$$

where $\mathbf{z}_k$ is the additive complex Gaussian noise vector with zero mean and $E(\mathbf{z}_k\mathbf{z}_k^\dagger) = \sigma_z^2\mathbf{I}_M$. Here, the first term of the right hand side in (2) includes the signal vector originally transmitted from $\mathcal{S}_k$ to $\mathcal{D}_k$, and the second term denotes the interference from the other source nodes. Also, the third term indicates the amplified and forwarded noise from the relay node. Defining the interference-plus-noise term in (2) and its covariance matrix as $\mathbf{v}_\mathrm{k} = \sum_{\substack{j=1\\ j\neq k}}^{K} \mathbf{G}_k\mathbf{F}\mathbf{H}_j\mathbf{P}_j\mathbf{s}_j + \mathbf{G}_k\mathbf{F}\mathbf{n} + \mathbf{z}_k$ and $\mathbf{R}_k = E(\mathbf{v}_k\mathbf{v}_k^\dagger)$, respectively, the individual rate of the k-th source-destination pair R_k is derived as [12]

$$R_k = \frac{1}{2}\log_2\det\left(\mathbf{I}_M + \mathbf{G}_k\mathbf{F}\mathbf{H}_k\mathbf{P}_k\mathbf{P}_k^\dagger\mathbf{H}_k^\dagger\mathbf{F}^\dagger\mathbf{G}_k^\dagger\mathbf{R}_k^{-1}\right)$$

where the factor of $\frac{1}{2}$ comes from the half-duplex operation of the system. Thus, the sum rate of the multipoint-to-multipoint MIMO relaying system, denoted by R_Σ, can be expressed as $R_\Sigma = \sum_{k=1}^{K} R_k$.

III. Blockwise Relaying Strategies under ZF and MMSE criteria

To obtain the optimum source and relay signaling matrices in terms of maximizing the sum rate, our problem can be formulated as

$$\{\mathbf{P}_{\text{opt}}, \mathbf{F}_{\text{opt}}\} = \arg\max_{\mathbf{P},\mathbf{F}} R_\Sigma. \tag{3}$$

However, since R_Σ is not concave with respect to the matrices $\mathbf{P}$ and $\mathbf{F}$ in general, the maximization problem in (3) is not trivial to solve analytically and requires high computational cost for even finding a local optimum solution. Furthermore, the joint source and relay design may lead to the additional complexity as well as the global CSI burden at all source nodes. Therefore, in this section, we consider the scenario where the optimization is only performed at the relay node $\mathcal{R}$ by adopting an equal power allocation at all source antennas (i.e., $\mathbf{P}_k = \sqrt{\frac{P_s}{M}}\mathbf{I}_M$). In what follows, we propose two new relaying strategies with non-iterative processing. A relay only optimization scheme with the ZF based blockwise relaying method is first devised, and then we design an enhanced relay filter based on the MMSE criterion so as to further improve the sum rate.

A. ZF Based Relay Filter Design

In this subsection, we first introduce a ZF based blockwise relaying method. For simplicity, we develop the relay filter which maximizes each individual pair rate by allocating equal power to each source-destination pair at the relay. We will refer to this as the ZF based blockwise relay only optimization (ZBR) scheme.

In the proposed multipoint-to-multipoint MIMO relay system, we eliminate multiple source/destination interferences by placing all data streams of the other sources/destinations at null space of the intended source/destination's channel. From this idea, we first define the relay filter $\mathbf{F}$ as

$$\mathbf{F} = \mathbf{Z}_G \mathbf{W} \mathbf{Z}_H \tag{4}$$

where $\mathbf{Z}_H$ and $\mathbf{Z}_G$ are the blockwise backward channel combining matrix and the blockwise forward channel splitting matrix, respectively, and $\mathbf{W}$ denotes the weighting matrix for the power allocation, which will be defined later.

First, we derive the blockwise combining matrix $\mathbf{Z}_H$ with respect to the source-relay channel $\mathbf{H}$. Note that this ZF based blockwise relaying method has a dimension constraint, i.e., $N_r \geq KM$. By exploiting the symmetry between the BC and the MAC, we define $\overline{\mathbf{H}}$ as

$$\overline{\mathbf{H}} = [\overline{\mathbf{H}}_1^\dagger \ \overline{\mathbf{H}}_2^\dagger \ \cdots \ \overline{\mathbf{H}}_K^\dagger]^\dagger = \left(\mathbf{H}^\dagger \mathbf{H}\right)^{-1} \mathbf{H}^\dagger \tag{5}$$

where $\overline{\mathbf{H}}_j^\dagger$ is an $N_r \times M$ matrix which represents the j-th subblock of the ZF channel inversion matrix $\overline{\mathbf{H}}$. For identifying the orthonormal basis of column space of $\overline{\mathbf{H}}_j^\dagger$, we consider the QR decomposition of $\overline{\mathbf{H}}_j^\dagger$ as $\overline{\mathbf{H}}_j^\dagger = \mathbf{Q}_{H,j}\mathbf{R}_{H,j}$ where $\mathbf{Q}_{H,j}$ is an $N_r \times M$ matrix whose columns form an orthonormal basis for $\overline{\mathbf{H}}_j^\dagger$ and $\mathbf{R}_{H,j}$ represents an $M \times M$ upper triangular matrix [13]. Finally, $\mathbf{Z}_H$ can be expressed as

$$\mathbf{Z}_H = [\mathbf{Q}_{H,1} \ \mathbf{Q}_{H,2} \ \cdots \ \mathbf{Q}_{H,K}]^\dagger. \tag{6}$$

Similarly, we can also compute the blockwise splitting matrix $\mathbf{Z}_G$ for the relay-destination channel $\mathbf{G}$. Defining $\overline{\mathbf{G}}$ as

$$\overline{\mathbf{G}} = [\overline{\mathbf{G}}_1 \ \overline{\mathbf{G}}_2 \ \cdots \ \overline{\mathbf{G}}_K] = \mathbf{G}^\dagger \left(\mathbf{G}\mathbf{G}^\dagger\right)^{-1}, \tag{7}$$

$\mathbf{Z}_G$ is calculated as

$$\mathbf{Z}_G = [\mathbf{Q}_{G,1} \ \mathbf{Q}_{G,2} \ \cdots \ \mathbf{Q}_{G,K}] \tag{8}$$

where $\mathbf{Q}_{G,j}$ is an $N_r \times M$ matrix whose columns form an orthonormal basis for $\overline{\mathbf{G}}_j$ obtained from the QR decomposition $\overline{\mathbf{G}}_j = \mathbf{Q}_{G,j}\mathbf{R}_{G,j}$. In the calculation of $\mathbf{Z}_H\mathbf{H}$ and $\mathbf{G}\mathbf{Z}_G$, we can see that the zero-interference condition $\mathbf{Q}_{H,j}^\dagger \mathbf{H}_k = \mathbf{0}_{M\times M}$ and $\mathbf{G}_k\mathbf{Q}_{G,j} = \mathbf{0}_{M\times M}$ for all $j \neq k$ is satisfied.

In order to exploit the property of our blockwise relaying approach, we define the relay weighting matrix $\mathbf{W}$ as

$$\mathbf{W} \triangleq \mathrm{diag}\{\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_K\} \tag{9}$$

where $\mathbf{W}_k$ indicates the subblock matrix which allocates the relay power of the k-th source-destination pair. Now, applying (4), (6), (8) and (9) to (2), the received signal vector at the k-th destination can be explicitly expressed as

$$\mathbf{y}_k = \widehat{\mathbf{G}}_k \mathbf{W}_k \widehat{\mathbf{H}}_k \mathbf{P}_k \mathbf{s}_k + \widehat{\mathbf{G}}_k \mathbf{W}_k \mathbf{Q}_{H,k}^\dagger \mathbf{n} + \mathbf{z}_k \tag{10}$$

where $\widehat{\mathbf{H}}_k = \mathbf{Q}_{H,k}^\dagger \mathbf{H}_k$ denotes the $M \times M$ effective channel for the k-th source-relay link and $\widehat{\mathbf{G}}_k = \mathbf{G}_k\mathbf{Q}_{G,k}$ indicates the $M \times M$ effective channel for the k-th relay-destination link. Thus, each subblock of the resulting block-diagonalized channel matrices becomes the effective channel for the corresponding source-destination pair. For this reason, we can consider the proposed multipoint-to-multipoint MIMO relay system as K parallel MIMO relay systems which have independent relay channels.

In (10), the SVDs of the effective channel matrices $\widehat{\mathbf{H}}_k$ and $\widehat{\mathbf{G}}_k$ are given as $\widehat{\mathbf{H}}_k = \mathbf{U}_{H,k}\mathbf{\Sigma}_{H,k}\mathbf{V}_{H,k}^\dagger$ and $\widehat{\mathbf{G}}_k = \mathbf{U}_{G,k}\mathbf{\Sigma}_{G,k}\mathbf{V}_{G,k}^\dagger$ where $\mathbf{U}_{H,k}$, $\mathbf{V}_{H,k}$, $\mathbf{U}_{G,k}$ and $\mathbf{V}_{G,k}$ are unitary matrices, and we have $\mathbf{\Sigma}_{H,k} = \mathrm{diag}\{\sqrt{\alpha_{k1}}, \sqrt{\alpha_{k2}}, \cdots, \sqrt{\alpha_{kM}}\}$ and $\mathbf{\Sigma}_{G,k} = \mathrm{diag}\{\sqrt{\beta_{k1}}, \sqrt{\beta_{k2}}, \cdots, \sqrt{\beta_{kM}}\}$. The optimal source and relay matrices jointly diagonalize the MIMO relay system into a set of parallel SISO channels [1] [2]. Motivated by the work, the optimum $\mathbf{W}_k$ is represented as

$$\mathbf{W}_k = \mathbf{V}_{G,k}\mathbf{\Lambda}_{W,k}\mathbf{U}_{H,k}^\dagger \tag{11}$$

where $\mathbf{\Lambda}_{W,k} = \mathrm{diag}\{\sqrt{w_{k1}}, \sqrt{w_{k2}}, \cdots, \sqrt{w_{kM}}\}$ denote the diagonal matrices for allocating the power. Substituting (11) into (10) and assuming the source precoder $\mathbf{P}_k = \sqrt{\frac{P_s}{M}}\mathbf{I}_M$, the received signal vector for the k-th destination can be rewritten as

$$\begin{aligned}\mathbf{y}_k = & \sqrt{\frac{P_s}{M}}\mathbf{U}_{G,k}\mathbf{\Sigma}_{G,k}\mathbf{\Lambda}_{W,k}\mathbf{\Sigma}_{H,k}\mathbf{V}_{H,k}^\dagger\mathbf{s}_k \\ & + \mathbf{U}_{G,k}\mathbf{\Sigma}_{G,k}\mathbf{\Lambda}_{W,k}\mathbf{U}_{H,k}^\dagger\mathbf{Q}_{H,k}^\dagger\mathbf{n} + \mathbf{z}_k. \end{aligned}\tag{12}$$

Hence, as a function of w_{km}'s, the individual rate R_k for the k-th source-destination pair is given as

$$\begin{aligned} & R_k(\mathcal{W}) \\ & = \frac{1}{2}\log_2 \frac{\det\left(\frac{P_s}{M}\mathbf{\Sigma}_{G,k}^2\mathbf{\Lambda}_{W,k}^2\mathbf{\Sigma}_{H,k}^2 + \sigma_n^2\mathbf{\Sigma}_{G,k}^2\mathbf{\Lambda}_{W,k}^2 + \sigma_z^2\mathbf{I}_M\right)}{\det\left(\sigma_n^2\mathbf{\Sigma}_{G,k}^2\mathbf{\Lambda}_{W,k}^2 + \sigma_z^2\mathbf{I}_M\right)} \\ & = \frac{1}{2}\sum_{m=1}^{M}\log_2\left(1 + \rho_s\alpha_{km} - \frac{\rho_s\alpha_{km}}{1 + \frac{\sigma_n^2}{\sigma_z^2}\beta_{km}w_{km}}\right) \end{aligned}$$

where $\mathcal{W}$ and ρ_s are defined as $\mathcal{W} = \{w_{km} | w_{km} \geq 0 \ \forall k, m\}$ and $\rho_s = \frac{P_s}{M\sigma_n^2}$, respectively..

Also, the relay transmit power constraint for each pair can be computed by

$$\begin{aligned} & E\left(\left\|\mathbf{W}_k\widehat{\mathbf{H}}_k\mathbf{P}_k\mathbf{s}_k + \mathbf{W}_k\mathbf{n}\right\|^2\right) \\ & \quad = \sum_{m=1}^{M}\left(\frac{P_s}{M}\alpha_{km} + \sigma_n^2\right)w_{km} \leq \frac{P_r}{K}. \end{aligned}\tag{13}$$

Therefore, the problem of maximizing R_k under the relay transmit power constraint can be written as

$$\begin{aligned} \max_{\mathcal{W}} \quad & \frac{1}{2}\sum_{m=1}^{M}\log_2\left(1 + \rho_s\alpha_{km} - \frac{\rho_s\alpha_{km}}{1 + \frac{\sigma_n^2}{\sigma_z^2}\beta_{km}w_{km}}\right) \\ \text{subject to} \quad & \sum_{m=1}^{M}\left(\left(1 + \rho_s\alpha_{km}\right)w_{km}\right) \leq \frac{P_r}{K\sigma_n^2}. \end{aligned}$$

$$w_{km} = \left[\frac{\sigma_z^2}{\sigma_n^2}\left(\frac{-2-\rho_s\alpha_{km}+\sqrt{\left(\rho_s\alpha_{km}\right)^2+2\frac{\log_2 e}{\lambda}\rho_s\alpha_{km}\beta_{km}\frac{\sigma_n^2}{\sigma_z^2}}}{2\left(1+\rho_s\alpha_{km}\right)\beta_{km}}\right)\right]^+ \tag{14}$$

$$\begin{aligned}\nabla R_\Sigma(\overline{\mathbf{W}}) &= \frac{1}{\ln 2}\sum_{k=1}^{K}\left(\rho_s\widetilde{\mathbf{G}}_k^\dagger\left(\mathbf{\Pi}_k-\mathbf{\Omega}_k\right)\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\sum_{j=1}^{K}\widetilde{\mathbf{H}}_j\widetilde{\mathbf{H}}_j^\dagger+\rho_s\widetilde{\mathbf{G}}_k^\dagger\mathbf{\Omega}_k\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\widetilde{\mathbf{H}}_k\widetilde{\mathbf{H}}_k^\dagger\right)\\ &\quad+\frac{1}{\ln 2}\sum_{k=1}^{K}\left(\widetilde{\mathbf{G}}_k^\dagger\left(\mathbf{\Pi}_k-\mathbf{\Omega}_k\right)\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}+\frac{1}{N_r\rho_r}\mathrm{Tr}\left(\mathbf{\Pi}_k-\mathbf{\Omega}_k\right)\overline{\mathbf{W}}(\rho_s\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^\dagger+\mathbf{I}_{N_r})\right)\end{aligned} \tag{15}$$

$$\mathbf{\Pi}_k = \left(\rho_s\sum_{j=1}^{K}\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\widetilde{\mathbf{H}}_j\widetilde{\mathbf{H}}_j^\dagger\overline{\mathbf{W}}_1^\dagger\widetilde{\mathbf{G}}_k^\dagger+\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\ \overline{\mathbf{W}}^\dagger\widetilde{\mathbf{G}}_k^\dagger+\frac{\sigma_z^2}{\eta^2\sigma_n^2}\mathbf{I}_M\right)^{-1} \tag{16}$$

$$\mathbf{\Omega}_k = \left(\rho_s\sum_{\substack{j=1\\ j\neq k}}^{K}\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\widetilde{\mathbf{H}}_j\widetilde{\mathbf{H}}_j^\dagger\overline{\mathbf{W}}^\dagger\widetilde{\mathbf{G}}_k^\dagger+\widetilde{\mathbf{G}}_k\overline{\mathbf{W}}\ \overline{\mathbf{W}}^\dagger\widetilde{\mathbf{G}}_k^\dagger+\frac{\sigma_z^2}{\eta^2\sigma_n^2}\mathbf{I}_M\right)^{-1} \tag{17}$$

Since the objective function of the above maximization problem is concave with respect to w_{km}, the optimization of w_{km} is easily carried out by using a Lagrange multiplier [14]. A new cost function $\mathcal{C}(\mathcal{W},\lambda)$ to maximize R_k can be expressed as

$$\begin{aligned}\mathcal{C}(\mathcal{W},\lambda) &= \frac{1}{2}\sum_{m=1}^{M}\log_2\left(1+\rho_s\alpha_{km}-\frac{\rho_s\alpha_{km}}{1+\frac{\sigma_n^2}{\sigma_z^2}\beta_{km}w_{km}}\right)\\ &\quad+\lambda\left(\sum_{k=1}^{M}\left(\left(\rho_s\alpha_{km}+1\right)w_{km}\right)-\frac{P_r}{K\sigma_n^2}\right).\end{aligned}$$

Now, we take a derivative of the above cost function with respect to w_{km} and set it to zero. Then, we can obtain a quadratic equation. By solving the quadratic equation, w_{km} is derived as (14) at the top of the page. Here $[A]^+ = \max(0, A)$ and λ is computed to meet the relay power constraint (13). With this closed-form solution, we will illustrate in the simulation section that the proposed ZBR scheme outperforms the conventional schemes in terms of the sum rate.

B. MMSE Based Relay Filter Design

Recognizing that the ZBR scheme can be considered as an extension of the conventional ZF scheme in [6], an alternative method based on the MMSE criterion is also established as in [13]. We will refer to this as the MMSE based blockwise relay only optimization (MBR) scheme. As in the ZBR scheme, the k-th source precoder and the relay filter of the MBR are expressed as $\mathbf{P}_k = \sqrt{\frac{P_s}{M}}\mathbf{I}_M$ and $\mathbf{F} = \mathbf{M}_G\mathbf{B}\mathbf{M}_H$, respectively. In order to calculate the MMSE based blockwise backward channel combining matrix $\mathbf{M}_H$ and the MMSE based blockwise forward channel splitting matrix $\mathbf{M}_G$, $\overline{\mathbf{H}}$ and $\overline{\mathbf{G}}$ in (5) and (7) are redefined as

$$\begin{aligned}\overline{\mathbf{H}} &= \left(\mathbf{H}^\dagger\mathbf{H}+\frac{M\sigma_n^2}{P_s}\mathbf{I}_{KM}\right)^{-1}\mathbf{H}^\dagger \quad \text{and}\\ \overline{\mathbf{G}} &= \mathbf{G}^\dagger\left(\mathbf{G}\mathbf{G}^\dagger+\frac{N_r\sigma_z^2}{P_r}\mathbf{I}_{KM}\right)^{-1},\end{aligned}$$

respectively. Then, by fulfilling the same procedure made in Section III-A, $\mathbf{M}_H = [\mathbf{Q}_{H,1}\ \mathbf{Q}_{H,2}\ \cdots\ \mathbf{Q}_{H,K}]^\dagger$ and $\mathbf{M}_G = [\mathbf{Q}_{G,1}\ \mathbf{Q}_{G,2}\ \cdots\ \mathbf{Q}_{G,K}]$ can be obtained from the above MMSE channel inversion matrices $\overline{\mathbf{H}}$ and $\overline{\mathbf{G}}$ which take the noise at both relay and destination into account. In comparison to (5) and (7), it is clear that the $\mathbf{Z}_H$ and $\mathbf{Z}_G$ matrices approach the $\mathbf{M}_H$ and $\mathbf{M}_G$ matrices for high SNR regime.

However, unlike the ZBR scheme, the relay filter of the MBR constructed by a linear combination of $\mathbf{Q}_{H,j}^\dagger$ and $\mathbf{Q}_{G,j}$ generates residual interference, i.e., $\mathbf{Q}_{H,j}^\dagger\mathbf{H}_k \neq \mathbf{0}_{M\times M}$ and $\mathbf{G}_k\mathbf{Q}_{G,j} \neq \mathbf{0}_{M\times M}$ for all $j \neq k$ and $1 \leq j,k \leq K$. Hence, for identifying the weighting matrix $\mathbf{B}$, it is not easy to perform the power allocation similar to the ZBR due to the residual interference term. Thus, we do not consider the power allocation algorithm for the MBR. Instead, we need to improve the sum rate performance by introducing a proper residual interference suppression. In the following, we address a way of computing the weighting matrix $\mathbf{B}$ given $\mathbf{M}_H$ and $\mathbf{M}_G$ by utilizing the idea of the gradient descent algorithm.

The gradient descent algorithm is a well-known way of solving unconstrained optimization problems [15]. Denoting an initial matrix of $\mathbf{B}$ as $\overline{\mathbf{W}}$, we exploit the fact that the sum rate R_Σ increases the fastest from $\overline{\mathbf{W}}$ if $\overline{\mathbf{W}}$ moves in the direction of the gradient of $R_\Sigma(\overline{\mathbf{W}})$. From this idea, we construct the weighting matrix $\mathbf{B}$ in the MBR scheme as

$$\mathbf{B} = \overline{\mathbf{W}} + \mathbf{D}$$

where the $N_r \times N_r$ matrix $\mathbf{D}$ is defined as $\mathbf{D} = \delta\cdot\nabla R_\Sigma(\overline{\mathbf{W}})$. Here, δ indicates a step size, and the gradient of the sum rate with respect to $\overline{\mathbf{W}}$, denoted by $\nabla R_\Sigma(\overline{\mathbf{W}})$, is given as (15) at the top of this page where $\widetilde{\mathbf{H}}_k$, $\widetilde{\mathbf{G}}_k$ and ρ_r are denoted as $\widetilde{\mathbf{H}}_k = \mathbf{M}_H\mathbf{H}_k$, $\widetilde{\mathbf{G}}_k = \mathbf{G}_k\mathbf{M}_G$ and $\rho_r = \frac{P_r}{N_r\sigma_z^2}$, respectively. Also, $\mathbf{\Pi}_k$ and $\mathbf{\Omega}_k$ are defined as (16) and (17), respectively, at the top of this page where η indicates a normalizing factor

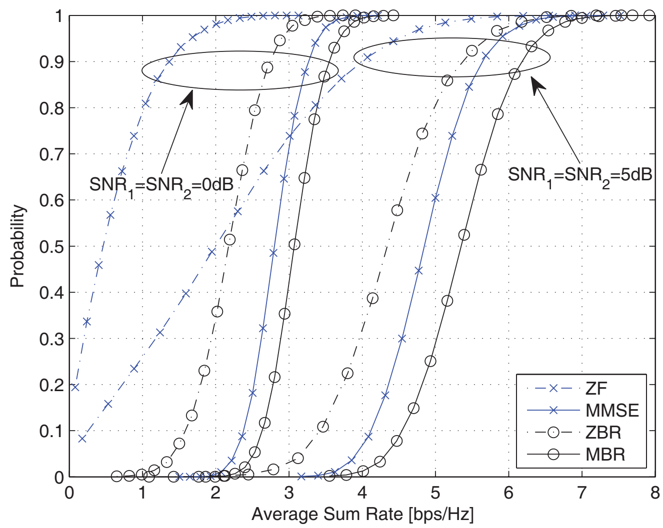

Fig. 2. CDF of the average sum rate at low SNRs when $N_s = N_d = 2$, $K = 3$ and $N_r = 6$.

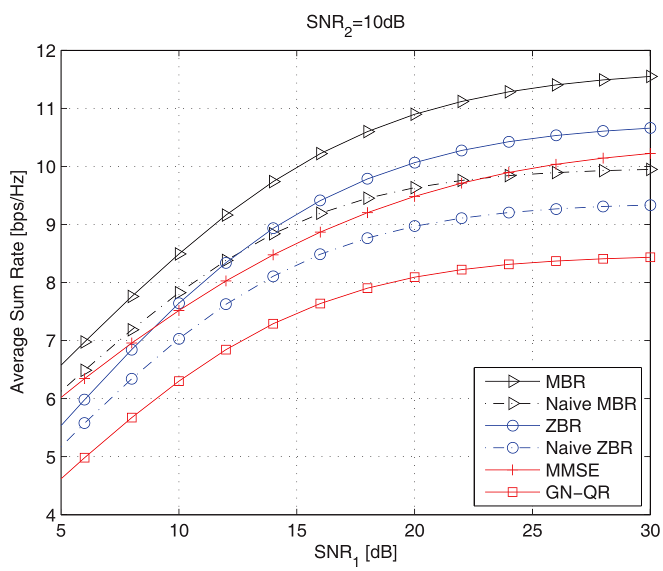

Fig. 3. Average sum rate as a function of SNR_1 with fixed SNR_2 when $N_s = N_d = 2$, $K = 3$ and $N_r = 6$.

expressed as

$$\eta = \sqrt{\frac{P_r}{\mathrm{Tr}\left(\mathbf{M}_G \overline{\mathbf{W}} (\frac{P_s}{M} \widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^\dagger + \sigma_n^2 \mathbf{I}_{N_r}) \overline{\mathbf{W}}^\dagger \mathbf{M}_G^\dagger \right)}}. \quad (18)$$

A more detailed derivation of (15) is omitted due to the page limitation. Note that the initial matrix $\overline{\mathbf{W}}$ is set to $\mathbf{W}$ of the ZBR scheme computed with $\mathbf{M}_H$ and $\mathbf{M}_G$ instead of $\mathbf{Z}_H$ and $\mathbf{Z}_G$. Compared to the ZBR scheme, this MBR scheme gives sufficient gains in terms of the sum rate, especially at low SNR region, at the expense of the increased complexity, which will be shown in simulation results.

IV. Simulation Results

In this section, we present numerical results of the proposed schemes in terms of the sum rate. For all simulations, the

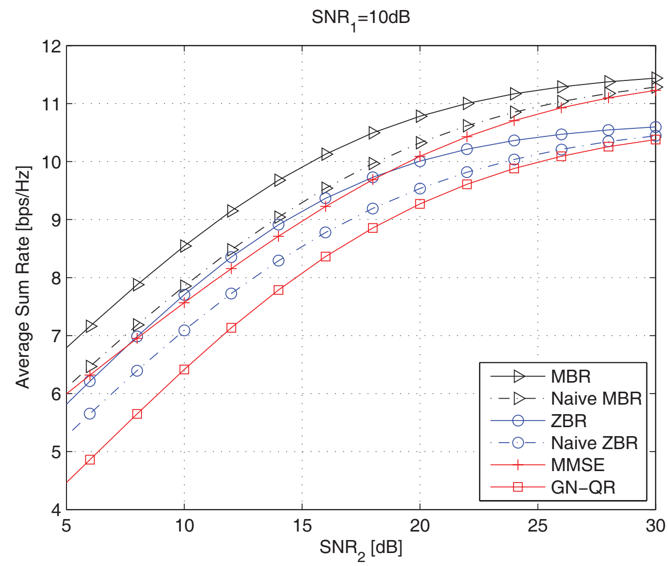

Fig. 4. Average sum rate as a function of SNR_2 with fixed SNR_1 when $N_s = N_d = 2$, $K = 3$ and $N_r = 6$.

source power and the relay power are given as $P_s = M\sigma_n^2 \cdot \text{SNR}_1$ and $P_r = N_r \sigma_z^2 \cdot \text{SNR}_2$, respectively, where SNR_1 and SNR_2 denote the average received SNR per antenna at the source-relay link and at the relay-destination link, respectively. Also, we use spatially uncorrelated MIMO channels with the elements generated by independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. In the plots, we will compare the following existing schemes:

- ZF and MMSE: As in multi-user MIMO systems with single antenna users, the conventional ZF and MMSE filters are applied to obtain the relay matrices [6].
- Group nulling based QR decomposition (GN-QR): Based on a group nulling approach, the QR decomposition is applied to each of the decomposed MIMO relay links [7].

In addition, the naive ZBR (or MBR) represents the ZBR (or MBR) scheme with the weighting matrix $\mathbf{W}$ (or $\mathbf{B}$) $= \gamma \mathbf{I}_{KM}$ simply normalized by a coefficient γ which meets the relay power constraint (1) instead of the optimization of $\mathbf{W}$ (or $\mathbf{B}$). In the simulation, we have employed Armijo's rule in [15] to find a proper step size δ for the MBR scheme.

Figure 2 shows the cumulative distribution functions (CDFs) of the sum rate for various schemes with the ZF and MMSE criteria at low SNR region. The graph is plotted with $N_s = N_d = 2$, $K = 3$ and $N_r = 6$ when the $\text{SNR}_1 = \text{SNR}_2$ is set to 0dB and 5dB. From this plot, we can observe that our proposed MBR outperforms the ZBR as well as the conventional ZF and MMSE schemes. With the same antenna configuration, we compare the performance of the MBR and other schemes in Fig. 3 according to SNR_1 with $\text{SNR}_2 = 10\text{dB}$, and vice versa in Fig. 4. Clearly, the MBR scheme performs best in all SNR ranges, and especially achieves a 1bps/Hz sum rate gain over the ZBR scheme by balancing the interference and noise for identifying the channel inversion matrices. This gap increases compared with other existing schemes. Also, the

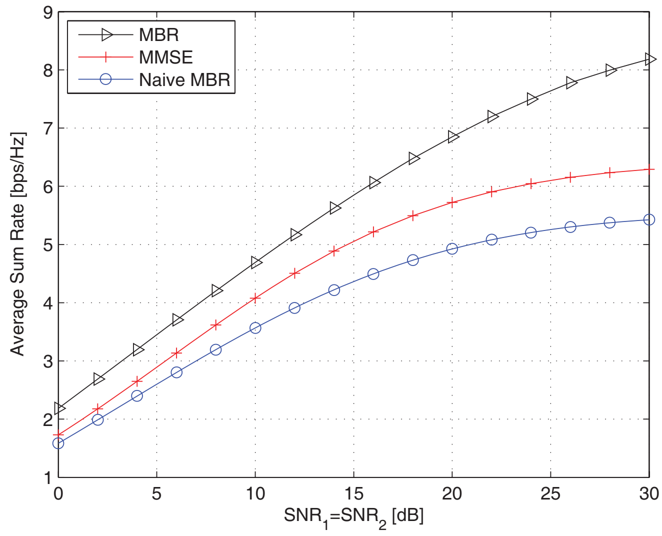

Fig. 5. Average sum rate as a function of SNR_1=SNR_2 when $N_s = N_d = 3$, $K = 2$ and $N_r = 4$.

performance of the ZBR becomes inferior to the conventional MMSE scheme at low SNR due to the noise enhancement issue.

In order to manifest the performance of the MBR scheme, we plot the average sum rate curves with respect to the $\text{SNR}_1 = \text{SNR}_2$ in Fig. 5 when the number of the relay antennas is smaller than that of the total source or destination antennas. From this figure, we can check that the naive MBR performs poor for the case of $N_s = N_d = 3$, $K = 2$ and $N_r = 4$ compared to the MMSE scheme. However, it is observed that in such a case, the sum rate of the proposed MBR scheme is substantially superior to that of the conventional MMSE scheme at all SNR region. Note that the GN-QR scheme is not included in this figure since the scheme cannot be applied to this antenna constraint.

V. Conclusion

In this letter, we have developed a new design framework by adopting the ZF and MMSE based blockwise relaying method and convex optimization techniques for the multipoint-to-multipoint relay system. As the proposed schemes generate a closed-form solution in a non-iterative fashion, it is more feasible than the jointly designed system. Simulation results show that the proposed schemes outperform the existing schemes in terms of the sum rate.

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