

MMSE Based Block Diagonalization for Cognitive Radio MIMO Broadcast Channels

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Abstract—In this paper, we present a linear precoder design for cognitive radio (CR) multi-user multiple-input multiple-output (MU-MIMO) downlink systems where unlicensed secondary users (SUs) can simultaneously utilize the same spectrum used by a licensed primary user (PU). Although a zero-forcing block diagonalization (ZF-BD) precoder is extended to the CR network with the PU, a transmit power boost problem occurs. Therefore, we propose a regularized BD precoder method based on the minimum mean-squared error (MMSE) criteria subject to the interference power constraint under a predetermined threshold for the PU. As a result, the proposed CR-MMSE-BD scheme improves the signal-to-interference-plus-noise ratio at each SU's receiver, compared to the ZF-BD based method. The simulation results demonstrate that the proposed algorithm outperforms the ZF based technique for CR MU-MIMO downlink systems.

Index Terms—Cognitive radio, multi-user MIMO, minimum mean-squared error (MMSE), block diagonalization (BD).

I. INTRODUCTION

RECENTLY, cognitive radio (CR) technologies have been studied as a promising solution in tackling the spectrum deficiency problem caused by radio frequency limits [1]–[3]. When a primary user (PU) is licensed to use a specific spectrum for communication, the licensed frequency spectrum can be opportunistically or concurrently exploited by unlicensed secondary users (SUs) in spectrum sharing based CR networks [2] [3]. By employing multiple antennas, the concurrent spectrum sharing can be performed without any degradation in quality of service (QoS) for the PU [3]. For CR multiuser multiple-input multiple-output (MU-MIMO) downlink systems coexisting with a single PU, a dirty paper coding (DPC) [4] based algorithm was proposed in [3] that maximizes the sum-rate of the SUs subject to the interference power constraint at the PU. Although the sum-rate was successfully maximized in [3], the algorithm is difficult to implement in practical systems due to its high computational burden, since iterative nonlinear methods are required.

For conventional non-CR MU-MIMO downlink channels, in order to avoid the prohibitive complexity of the DPC based nonlinear schemes, many suboptimal linear methods have been developed [5]–[8]. Zero-forcing channel inversion (ZF-CI) [5] is one of the simplest precoding techniques used to remove MU interference (MUI). However, its performance is rather poor due to a transmit power boost issue which is analogous

to a noise enhancement in the ZF linear filter. Although a minimum mean-squared error channel inversion (MMSE-CI) scheme [5] overcomes this ZF-CI drawback, it remains confined to single receive antennas. When employing multiple receive antennas for each user, a ZF-based block diagonalization (ZF-BD) method was proposed as a generalization of the ZF-CI in [6]. More recently, an MMSE-based BD algorithm has been introduced which extends the MMSE-CI method to multiple-antenna users [7]–[9].

In this paper, we investigate non-iterative designs for linear precoders used in CR MU-MIMO downlink channels with K SUs. Unlike conventional MU-MIMO systems without the PU link, our CR MU-MIMO system considers the interference between the PU link and the SU link as well as the MUI among the SUs. To this end, we employ a whitening process that takes into account the interference from the PU link. Although the ZF-BD concept can be extended to CR MU-MIMO systems by nulling the interference channel from the SU transmitter, the CR-ZF-BD method still exhibits poor performance. Therefore, we propose an MMSE-based BD approach for CR MU-MIMO downlink channels. Unlike the ZF criterion, it is not easy to directly find the optimal filter based on the MMSE criteria in CR MU-MIMO systems.

We first derive a preprocessing solution that minimizes the mean-square error (MSE) for all of the SUs, while satisfying a predetermined interference power constraint at the PUs. Then, an orthonormal vector set for each SU's preprocessing matrix and a transmit combining matrix are employed for the proposed algorithm in order to support multiple receive antennas, as described in [7]. Consequently, the proposed CR-MMSE-BD scheme is able to increase the signal-to-interference-plus-noise ratio (SINR) in each SU, compared to the CR-ZF-BD method. Our simulation results show that the proposed MMSE scheme outperforms the ZF based method and achieves a comparable sum-rate performance to the sum capacity found for DPC-based CR MU-MIMO systems using the interference power constraint.

II. SYSTEM MODEL

We consider the CR system shown in Fig. 1, where a MIMO link for a PU coexists with an MU-MIMO downlink system consisting of K SUs. The PU's access point (PU-AP) is licensed to directly communicate with the PU. The unlicensed SU-AP then accesses the spectrum of the licensed link assigned to the PU to broadcast to the K SUs.¹ Let

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¹In our system model, the legacy PU link does not care about the performance of the CR SU link. We only consider a single PU as in [3], although the proposed algorithm can be extended to CR networks possessing multiple PUs.

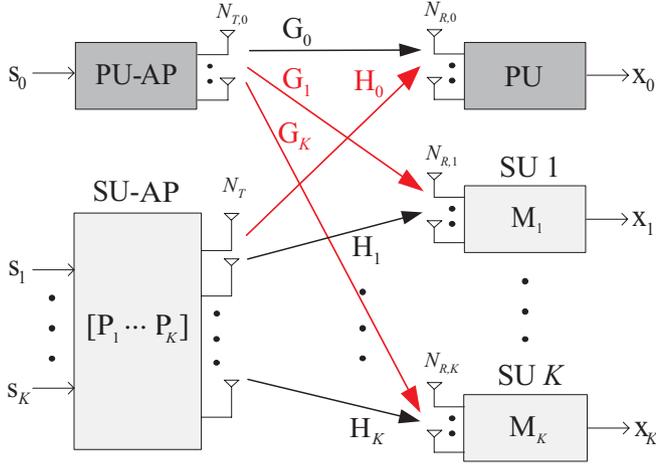


Fig. 1. System description of a CR MU-MIMO downlink system with K SUs coexisting with a PU MIMO link.

us denote $N_{T,0}$, $N_{R,0}$, N_T , and $N_{R,k}$ as the number of antennas for the PU-AP, the PU, the SU-AP, and the k th SU, respectively. The total number of SU receive antennas equals $N_R = \sum_{k=1}^K N_{R,k}$. In addition, the MIMO channels related to the SU-AP and the PU-AP are modeled by the $N_{R,j} \times N_T$ matrix \mathbf{H}_j and the $N_{R,j} \times N_{T,0}$ matrix \mathbf{G}_j for $j = 0, 1, \dots, K$, respectively, as illustrated in Fig. 1. Note that \mathbf{H}_0 represents the PU's interference (PUI) channel caused by the SU-AP and \mathbf{G}_k for $k = 1, \dots, K$ indicates the SU's interference (SUI) channel coming from the PU-AP.

The k th SU's data, noise vectors and the associated precoding matrix are given as $\mathbf{s}_k \in \mathbb{C}^{N_{R,k} \times 1}$, $\mathbf{n}_k \in \mathbb{C}^{N_{R,k} \times 1}$, and $\mathbf{P}_k \in \mathbb{C}^{N_T \times N_{R,k}}$, respectively. Then, for the overall SU MU-MIMO system, the transmitted data symbol vector $\underline{\mathbf{s}} \in \mathbb{C}^{N_R \times 1}$, the noise vector $\underline{\mathbf{n}} \in \mathbb{C}^{N_R \times 1}$, the precoding matrix $\underline{\mathbf{P}} \in \mathbb{C}^{N_T \times N_R}$, the broadcasting channel $\underline{\mathbf{H}} \in \mathbb{C}^{N_R \times N_T}$, and the SUI channel $\underline{\mathbf{G}} \in \mathbb{C}^{N_R \times N_0}$ are defined as $\underline{\mathbf{s}} = [\mathbf{s}_1^T \ \mathbf{s}_2^T \ \dots \ \mathbf{s}_K^T]^T$, $\underline{\mathbf{n}} = [\mathbf{n}_1^T \ \mathbf{n}_2^T \ \dots \ \mathbf{n}_K^T]^T$, $\underline{\mathbf{P}} = [\mathbf{P}_1 \ \mathbf{P}_2 \ \dots \ \mathbf{P}_K]$, $\underline{\mathbf{H}} = [\mathbf{H}_1^T \ \mathbf{H}_2^T \ \dots \ \mathbf{H}_K^T]^T$, and $\underline{\mathbf{G}} = [\mathbf{G}_1^T \ \mathbf{G}_2^T \ \dots \ \mathbf{G}_K^T]^T$, where $(\cdot)^T$ stands for the transpose of the matrix. Also, \mathbf{s}_0 , \mathbf{n}_0 , and \mathbf{P}_0 indicate the data symbol vector, the noise vector, and the precoding matrix for the PU MIMO link, respectively.

The total received signal of the SUs $\underline{\mathbf{y}} = [\mathbf{y}_1^T \ \mathbf{y}_2^T \ \dots \ \mathbf{y}_K^T]^T$ can therefore be expressed as

$$\underline{\mathbf{y}} = \underline{\mathbf{H}} \underline{\mathbf{P}} \underline{\mathbf{s}} + \underline{\mathbf{G}} \mathbf{P}_0 \mathbf{s}_0 + \underline{\mathbf{n}} \quad (1)$$

where $\mathbf{y}_k \in \mathbb{C}^{N_{R,k} \times 1}$ represents the received signal vector at the k th SU. We assume that the components of the noise vector \mathbf{n}_k have an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance σ_n^2 . Let $\underline{\tilde{\mathbf{s}}} = \underline{\mathbf{P}} \underline{\mathbf{s}}$ be the signal vector transmitted at the SU-AP, which satisfies $\mathbb{E}[\|\underline{\tilde{\mathbf{s}}}\|^2] \leq P_T$ where P_T equals the total downlink transmit power of the SU-AP, and $\mathbb{E}[\cdot]$ and $\|\cdot\|$ denote the expectation and the Euclidean norm, respectively. Since we assume that each data symbol in $\underline{\mathbf{s}}$ has unit variance, the total transmit power constraint is written as $\text{Tr}(\underline{\mathbf{P}}^\dagger \underline{\mathbf{P}}) \leq P_T$, where $(\cdot)^\dagger$ and $\text{Tr}(\cdot)$ indicate the conjugate transpose and trace, respectively.

Although the SUI signal $\underline{\mathbf{G}} \mathbf{P}_0 \mathbf{s}_0$ in (1) cannot be eliminated, the effect can be mitigated through the whitening process [8]. Assuming that the SUI is independent of the noise in (1), the SUI plus noise covariance matrix at the k th SU can be decomposed by Cholesky factorization as $\mathbf{G}_k \Phi_0 \mathbf{G}_k^\dagger + \sigma_n^2 \mathbf{I}_{N_{R,k}} = \mathbf{L}_k \mathbf{L}_k^\dagger$, where $\Phi_0 \triangleq \mathbf{P}_0 \mathbb{E}[\mathbf{s}_0 \mathbf{s}_0^\dagger] \mathbf{P}_0^\dagger$ satisfies the power constraint $\text{Tr}(\Phi_0) \leq P_0$ with the transmit power P_0 at the PU-AP, $\mathbf{L}_k \in \mathbb{C}^{N_{R,k} \times N_{R,k}}$ indicates a lower triangular matrix with positive diagonal entries, and \mathbf{I}_m represents an m dimensional identity matrix. Note that each SU can estimate the covariance matrix of the interference plus noise through various techniques, as described in [10] and [11].

Then, by defining the total receive whitening filter $\tilde{\mathbf{M}}$ as $\tilde{\mathbf{M}} = \text{diag}\{\mathbf{L}_1^{-1}, \mathbf{L}_2^{-1}, \dots, \mathbf{L}_K^{-1}\}$, the whitened receive signal vector at the SUs $\tilde{\mathbf{y}}$ can be taken from (1) as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}} \underline{\mathbf{P}} \underline{\mathbf{s}} + \underline{\mathbf{w}} \quad (2)$$

where we have $\tilde{\mathbf{H}} = [\tilde{\mathbf{H}}_1^T \ \tilde{\mathbf{H}}_2^T \ \dots \ \tilde{\mathbf{H}}_K^T]^T \triangleq \tilde{\mathbf{M}} \underline{\mathbf{H}}$ and $\underline{\mathbf{w}} = [\mathbf{w}_1^T \ \mathbf{w}_2^T \ \dots \ \mathbf{w}_K^T]^T \triangleq \tilde{\mathbf{M}} (\underline{\mathbf{G}} \mathbf{P}_0 \mathbf{s}_0 + \underline{\mathbf{n}})$. Note that $\underline{\mathbf{w}}$ becomes a whitened noise vector with zero mean and unit variance.

The k th whitened receive signal $\tilde{\mathbf{y}}_k$ is rewritten as

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{H}}_k \mathbf{P}_k \mathbf{s}_k + \tilde{\mathbf{H}}_k \sum_{l=1, l \neq k}^K \mathbf{P}_l \mathbf{s}_l + \mathbf{w}_k. \quad (3)$$

In addition, the received signal at the PU is given by

$$\mathbf{y}_0 = \mathbf{G}_0 \mathbf{P}_0 \mathbf{s}_0 + \mathbf{H}_0 \underline{\mathbf{P}} \underline{\mathbf{s}} + \mathbf{n}_0. \quad (4)$$

The channel matrices $\mathbf{H}_0, \tilde{\mathbf{H}}_1, \dots, \tilde{\mathbf{H}}_K$ are assumed to be available at the SU-AP via feedback channels or through uplink-downlink reciprocity. Although the DPC based non-linear method found in [3] can achieve the sum capacity of the CR MU-MIMO link in (3) subject to a power constraint of the PUI in (4), this paper focuses on linear processing based on non-iterative methods in order to reduce the complexity.

III. CR MU-MIMO LINEAR PROCESSING

In this section, we present an MU-MIMO precoder design for CR systems. First, the CR-ZF-BD method is briefly described as an extension of the conventional ZF-BD. We then propose the CR-MMSE-BD precoding algorithm with the PUI power constraint.

A. CR-ZF-BD Precoder

The key idea of the conventional ZF-BD method for MU-MIMO systems in [6] is in the complete removal of the MUI by employing the null space matrix of the interference channels. In order to eliminate the PUI in (4) as well as the MUI in (3) in the CR MU-MIMO systems, the following constraints are imposed:

$$\mathbf{H}_0 \mathbf{P}_k = \mathbf{0}_{N_{R,0} \times N_{R,k}} \quad \text{for } k = 1, 2, \dots, K, \quad (5)$$

$$\tilde{\mathbf{H}}_l \mathbf{P}_k = \mathbf{0}_{N_{R,l} \times N_{R,k}} \quad \text{for all } k \neq l \text{ and } 1 \leq k, l \leq K, \quad (6)$$

where $\mathbf{0}_{m \times n}$ indicates an $m \times n$ zero matrix. From this, the k th SU's precoder matrix \mathbf{P}_k should lie in the null space of $\mathbf{\Pi}_k$ defined as $\mathbf{\Pi}_k = [\mathbf{H}_0^T \ \tilde{\mathbf{H}}_1^T \ \dots \ \tilde{\mathbf{H}}_{k-1}^T \ \tilde{\mathbf{H}}_{k+1}^T \ \dots \ \tilde{\mathbf{H}}_K^T]^T$. Note

that $\mathbf{\Pi}_k$ includes the PUI channel \mathbf{H}_0 due to constraint (5), which is not considered for non-CR channels in [6]. Denoting $\mathbf{\Pi}_k^\perp$ as a matrix which contains the orthogonal bases for the null space of $\mathbf{\Pi}_k$, the precoder with $\mathbf{\Pi}_k^\perp$ satisfies the zero-interference constraints (5) and (6), and so makes each block channel $\tilde{\mathbf{H}}_k \mathbf{\Pi}_k^\perp$ free from any interference.

Now, in order to decouple this block channel into $N_{R,k}$ parallel subchannels, a singular value decomposition (SVD) for the effective channel $\tilde{\mathbf{H}}_k \mathbf{\Pi}_k^\perp = \bar{\mathbf{U}}_k \bar{\mathbf{\Lambda}}_k \bar{\mathbf{V}}_k^\dagger$ is applied. Then, employing the precoder $\mathbf{\Pi}_k^\perp \bar{\mathbf{V}}_k \Phi_k^{\frac{1}{2}}$ with the power loading diagonal matrix Φ_k and the receive filter $\bar{\mathbf{U}}_k^\dagger$ from (3), the k th SU's filtered output signal \mathbf{x}_k becomes

$$\mathbf{x}_k = \bar{\mathbf{\Lambda}}_k \Phi_k^{\frac{1}{2}} \mathbf{s}_k + \tilde{\mathbf{w}}_k \quad (7)$$

where we define $\tilde{\mathbf{w}}_k = \bar{\mathbf{U}}_k^\dagger \mathbf{w}_k$, and Φ_k can be optimized in terms of the sum-rate by using the water-filling (WF) method as in [6].

Finally, the total transmit and receive matrices of the CR-ZF-BD scheme are obtained through

$$\mathbf{P}_k^{\text{ZB}} = \mathbf{\Pi}_k^\perp \bar{\mathbf{V}}_k \Phi_k^{\frac{1}{2}} \quad \text{and} \quad \mathbf{M}_k^{\text{ZB}} = \bar{\mathbf{U}}_k^\dagger \mathbf{L}_k^{-1}. \quad (8)$$

This modified ZF technique for the CR MU-MIMO systems completely nullifies all interference without considering the noise term. Therefore, the CR-ZF-BD scheme suffers from a transmit power boosting problem especially in the low SNR region, which also depends on the number of transmit and receive antennas [5] [6]. In this paper, we focus on the case when $N_T \geq N_R + N_{R,0}$ due to a dimensional constraint. In order to improve the performance of the ZF based scheme, a precoder design based on the MMSE criteria is proposed in the following subsection.

B. CR-MMSE-BD Precoder

We develop an MMSE based linear algorithm for CR MU-MIMO systems. First a linear filter is optimized according to the MMSE criterion in order to suppress the MUI among the SUs subject to the PUI power constraint. Then, to exploit a cooperation gain among the multiple receive antennas per user, the orthonormal bases of the MMSE preprocessing matrix are employed.

1) *MMSE Preprocessing*: We need to determine a preprocessing matrix which minimizes the MSE for the received signal in the whitened channel model from (2) and also controls the power of the PUI $\mathbf{H}_0 \mathbf{P} \mathbf{s}$ from (4) under the predetermined threshold I_{th} . This problem is formulated with the transmit power constraint by

$$\begin{aligned} \min_{\gamma, \mathbf{P}} \quad & \mathbb{E} \left[\|\mathbf{s} - \gamma^{-1} \tilde{\mathbf{y}}\|^2 \right] \\ \text{s.t.} \quad & \mathbb{E} \left[\|\mathbf{H}_0 \mathbf{P} \mathbf{s}\|^2 \right] \leq I_{th} \\ & \mathbb{E} \left[\|\mathbf{P} \mathbf{s}\|^2 \right] \leq P_T \end{aligned} \quad (9)$$

where γ indicates a scaling factor for the received signal. The solution of this problem can mitigate the power boost issue by considering the noise as well as the interference and maintains a certain sum-rate of the PU link by adjusting the average sum power for the PUI signals, derived in the following theorem.

Theorem 1: The preprocessing solution $\hat{\mathbf{P}}$ for the MMSE problem under the PUI power threshold and the transmit power constraint from (9) is written as

$$\begin{aligned} \hat{\mathbf{P}} &= \gamma \bar{\mathbf{P}}(\hat{\mu}) \\ &\triangleq \gamma \left(\tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} + \hat{\mu} \mathbf{H}_0^\dagger \mathbf{H}_0 + \frac{N_R - \hat{\mu} I_{th}}{P_T} \mathbf{I}_{N_T} \right)^{-1} \tilde{\mathbf{H}}^\dagger \end{aligned} \quad (10)$$

where γ is given by $\gamma = \sqrt{P_T / \text{Tr}(\bar{\mathbf{P}}(\hat{\mu}) \bar{\mathbf{P}}(\hat{\mu})^\dagger)}$. Let us define $f(x) = \text{Tr}\{(P_T \mathbf{H}_0^\dagger \mathbf{H}_0 - I_{th} \mathbf{I}_{N_T}) \bar{\mathbf{P}}(x) \bar{\mathbf{P}}(x)^\dagger\}$. If $f(0) \leq 0$, then $\hat{\mu}$ is determined as $\hat{\mu} = 0$, otherwise a unique positive value $\hat{\mu}$ satisfying $f(\hat{\mu}) = 0$ for $0 < \hat{\mu} \leq N_R / I_{th}$ can be found by using a bisection line search algorithm [12].

Proof: By substituting (2) into problem (9), the associated Lagrange equation is expressed as

$$\begin{aligned} \mathcal{L} = & \text{Tr}\left\{ \mathbf{I}_{N_R} - \gamma^{-1} \tilde{\mathbf{H}} \mathbf{P} - \gamma^{-1} \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger + \gamma^{-2} \tilde{\mathbf{H}} \mathbf{P} \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger + \gamma^{-2} \mathbf{I}_{N_R} \right\} \\ & + \lambda_1 \left\{ \text{Tr}(\mathbf{H}_0 \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_0^\dagger) - I_{th} \right\} + \lambda_2 \left\{ \text{Tr}(\mathbf{P} \mathbf{P}^\dagger) - P_T \right\} \end{aligned}$$

where λ_1 and λ_2 indicate the Lagrangian multipliers. The Karush-Kuhn-Tucker (KKT) conditions are written by

$$-\gamma^{-1} \tilde{\mathbf{H}}^\dagger + \gamma^{-2} \tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} \mathbf{P} + \lambda_1 \mathbf{H}_0^\dagger \mathbf{H}_0 \mathbf{P} + \lambda_2 \mathbf{P} = \mathbf{0}, \quad (11)$$

$$\gamma^{-2} \text{Tr}(\tilde{\mathbf{H}} \mathbf{P} + \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger) - 2\gamma^{-3} \text{Tr}(\tilde{\mathbf{H}} \mathbf{P} \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger + \mathbf{I}_{N_R}) = 0, \quad (12)$$

$$\lambda_1 \geq 0, \quad \text{Tr}(\mathbf{H}_0 \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_0^\dagger) - I_{th} \leq 0, \quad (13)$$

$$\lambda_1 \left\{ \text{Tr}(\mathbf{H}_0 \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_0^\dagger) - I_{th} \right\} = 0, \quad (14)$$

$$\lambda_2 \geq 0, \quad \text{Tr}(\mathbf{P} \mathbf{P}^\dagger) - P_T \leq 0, \quad (15)$$

$$\lambda_2 \left\{ \text{Tr}(\mathbf{P} \mathbf{P}^\dagger) - P_T \right\} = 0, \quad (16)$$

where (11) and (12) are derived from the zero gradient conditions by using some rules of differentiation [13].

From (11), we have

$$\mathbf{P} = \gamma \left(\tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} + \mu_1 \mathbf{H}_0^\dagger \mathbf{H}_0 + \mu_2 \mathbf{I}_{N_T} \right)^{-1} \tilde{\mathbf{H}}^\dagger \quad (17)$$

where constant values μ_1 and μ_2 are defined as $\mu_1 = \lambda_1 \gamma^2$ and $\mu_2 = \lambda_2 \gamma^2$, respectively, to avoid inter-connected variables as in [5]. In addition, since $\tilde{\mathbf{H}} \mathbf{P}$ is Hermitian, (12) yields

$$\gamma \text{Tr}(\tilde{\mathbf{H}} \mathbf{P}) = \text{Tr}(\tilde{\mathbf{H}} \mathbf{P} \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger + \mathbf{I}_{N_R}). \quad (18)$$

By exploiting (14), (16), and (17), the left hand side of (18) is manipulated to

$$\begin{aligned} & \text{Tr}(\gamma \tilde{\mathbf{H}} \mathbf{P}) \\ &= \text{Tr} \left\{ \mathbf{P}^\dagger \left(\tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} + \mu_1 \mathbf{H}_0^\dagger \mathbf{H}_0 + \mu_2 \mathbf{I}_{N_T} \right) \mathbf{P} \right\} \\ &= \text{Tr}(\mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} \mathbf{P}) + \mu_1 \text{Tr}(\mathbf{P}^\dagger \mathbf{H}_0^\dagger \mathbf{H}_0 \mathbf{P}) + \mu_2 \text{Tr}(\mathbf{P}^\dagger \mathbf{P}) \\ &= \text{Tr}(\tilde{\mathbf{H}} \mathbf{P} \mathbf{P}^\dagger \tilde{\mathbf{H}}^\dagger) + \mu_1 I_{th} + \mu_2 P_T. \end{aligned} \quad (19)$$

Now, comparing (19) with the right hand side of (18), we can show that $\mu_2 = \frac{N_R - \mu_1 I_{th}}{P_T}$.

Finally, the preprocessing matrix is calculated as (10) by defining $\hat{\mu}$ as the optimal μ_1 . Note that $\hat{\mu}$ should be a positive value $0 \leq \hat{\mu} \leq \frac{N_R}{I_{th}}$ because $\mu_1, \mu_2 \geq 0$. Substituting (10) into both the inequality constraints, we

obtain $\gamma^2 \leq \frac{I_{th}}{\text{Tr}\{\mathbf{H}_0 \hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger \mathbf{H}_0^\dagger\}}$ and $\gamma^2 \leq \frac{P_T}{\text{Tr}\{\hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger\}}$. If $f(\hat{\mu}) \triangleq \text{Tr}\{(P_T \mathbf{H}_0^\dagger \mathbf{H}_0 - I_{th} \mathbf{I}_{N_T}) \hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger\} \leq 0$, γ is given by $\gamma^2 = \frac{P_T}{\text{Tr}\{\hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger\}}$ since the condition $f(\hat{\mu}) \leq 0$ results in $\frac{I_{th}}{\text{Tr}\{\mathbf{H}_0 \hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger \mathbf{H}_0^\dagger\}} \geq \frac{P_T}{\text{Tr}\{\hat{\mathbf{P}}(\hat{\mu}) \hat{\mathbf{P}}(\hat{\mu})^\dagger\}}$. In this case, $\hat{\mu}$ should satisfy $f(0) < 0$ at $\hat{\mu} = 0$ or $f(\hat{\mu}) = 0$ according to the three conditions in (13) and (14). On the other hand, for $f(\hat{\mu}) > 0$, we obtain $\mu_2 = 0$ from (15) and (16), and thus $\hat{\mu}$ should be given by $\hat{\mu} = \frac{N_R}{I_{th}}$. However, it can be shown that $f(\frac{N_R}{I_{th}}) \leq 0$ at $\hat{\mu} = \frac{N_R}{I_{th}}$ by utilizing $\mathbf{H}_0 \hat{\mathbf{P}}(\frac{N_R}{I_{th}}) = \mathbf{0}_{N_R, 0 \times N_R}$ for $N_T \geq N_R + N_{R,0}$. Therefore, we consider only the case of $f(\hat{\mu}) \leq 0$. Since $f(x)$ is a monotonic decreasing function and $f(\frac{N_R}{I_{th}}) \leq 0$, there exists a unique solution $\hat{\mu}$ of $f(\hat{\mu}) = 0$ unless $f(0) < 0$. ■

In this theorem, it is worth noting that if $f(0) = \text{Tr}\{(P_T \mathbf{H}_0^\dagger \mathbf{H}_0 - I_{th} \mathbf{I}_{N_T}) \hat{\mathbf{P}}(0) \hat{\mathbf{P}}(0)^\dagger\} \leq 0$, the preprocessing matrix with $\hat{\mu} = 0$ in (10) equals $\hat{\mathbf{P}} = \gamma \left(\tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} + \frac{N_R}{P_T} \mathbf{I}_{N_T} \right)^{-1} \tilde{\mathbf{H}}^\dagger$. In other words, if the PUI power threshold I_{th} is set to be a large value, our solution (10) becomes equivalent to the conventional MMSE-CI precoder [5] without considering the PUI constraint. The preprocessing matrix for the case with $I_{th} = 0$ is computed as follows.

Corollary 1: With the zero PUI power constraint ($I_{th} = 0$), the preprocessing matrix in (10) is rewritten by

$$\hat{\mathbf{P}} = \gamma \mathbf{H}_0^\dagger \tilde{\mathbf{H}}^\dagger \left\{ \tilde{\mathbf{H}} \mathbf{H}_0^\dagger \tilde{\mathbf{H}}^\dagger + \frac{N_R}{P_T} \mathbf{I}_{N_R} \right\}^{-1} \quad (20)$$

where $\mathbf{H}_0^\dagger = \mathbf{I}_{N_T} - \mathbf{H}_0^\dagger (\mathbf{H}_0 \mathbf{H}_0^\dagger)^{-1} \mathbf{H}_0$.

Proof: In the proof of Theorem 1, if $I_{th} = 0$, the conditions in (13) and (14) are changed to $\text{Tr}(\mathbf{H}_0 \hat{\mathbf{P}} \hat{\mathbf{P}}^\dagger \mathbf{H}_0^\dagger) = 0$ and we obtain $\mu_2 = \frac{N_R}{P_T}$ from (18) and (19). Then, in order to guarantee $\text{Tr}(\mathbf{H}_0 \hat{\mathbf{P}} \hat{\mathbf{P}}^\dagger \mathbf{H}_0^\dagger) = 0$, we find $\hat{\mu}$ which satisfies $\mathbf{H}_0 \hat{\mathbf{P}}(\hat{\mu}) = \mathbf{0}$. To this end, the preprocessing matrix with zero I_{th} in (10) is represented as

$$\begin{aligned} \hat{\mathbf{P}}(\hat{\mu}) &= \left\{ \tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} + \frac{N_R}{P_T} \left(\frac{\hat{\mu} P_T}{N_R} \mathbf{H}_0^\dagger \mathbf{H}_0 + \mathbf{I}_{N_T} \right) \right\}^{-1} \tilde{\mathbf{H}}^\dagger \\ &= \left(\frac{\hat{\mu} P_T}{N_R} \mathbf{H}_0^\dagger \mathbf{H}_0 + \mathbf{I}_{N_T} \right)^{-1} \tilde{\mathbf{H}}^\dagger \times \\ &\quad \left\{ \tilde{\mathbf{H}} \left(\frac{\hat{\mu} P_T}{N_R} \mathbf{H}_0^\dagger \mathbf{H}_0 + \mathbf{I}_{N_T} \right)^{-1} \tilde{\mathbf{H}}^\dagger + \frac{N_R}{P_T} \mathbf{I}_{N_R} \right\}^{-1} \\ &= \left(\mathbf{I}_{N_T} - \mathbf{H}_0^\dagger \left(\mathbf{H}_0 \mathbf{H}_0^\dagger + \frac{N_R}{\hat{\mu} P_T} \mathbf{I}_{N_{R,0}} \right)^{-1} \mathbf{H}_0 \right) \tilde{\mathbf{H}}^\dagger \times \\ &\quad \left\{ \tilde{\mathbf{H}} \left(\mathbf{I}_{N_T} - \mathbf{H}_0^\dagger \left(\mathbf{H}_0 \mathbf{H}_0^\dagger + \frac{N_R}{\hat{\mu} P_T} \mathbf{I}_{N_{R,0}} \right)^{-1} \mathbf{H}_0 \right) \tilde{\mathbf{H}}^\dagger + \frac{N_R}{P_T} \mathbf{I}_{N_R} \right\}^{-1} \end{aligned} \quad (21)$$

where we use the matrix inversion lemma. Since the first part of the last expression in (21) obviously approaches a null space matrix of \mathbf{H}_0 , i.e., \mathbf{H}_0^\dagger as $\frac{N_R}{\hat{\mu} P_T}$ becomes zero, $\hat{\mu}$ is determined as $\hat{\mu} \rightarrow \infty$ in (21). Finally, we get (20) which satisfies all the KKT conditions in the proof of Theorem 1. ■

This corollary shows that although the bisection procedure is required for the nonzero PUI constraint in (10), the case with a zero PUI has the closed-form solution represented in (20).

Since problem (9) does not consider cooperation among the multiple receive antennas, the preprocessing matrix $\hat{\mathbf{P}}$ in (10) or (20) not only mitigates other user's interference, but also attempts to suppress its own signal from other antennas. Therefore, if we use only this preprocessing matrix, there should be a performance loss similar to the MMSE-CI when each user has multiple receive antennas. Consequently, the following process is additionally required.

2) *Multiple Receive Antenna Extension:* To support multiple receive antennas for each SU, we compensate the suppression of each antenna signal of the k th SU by employing the orthonormal bases of the k th preprocessing matrix $\hat{\mathbf{P}}_k$ in (10), as described in [7]. For finding the bases, we apply the QR decomposition as

$$\hat{\mathbf{P}}_k = \mathbf{Q}_k \mathbf{R}_k \quad \text{for } k = 1, \dots, K \quad (22)$$

where $\mathbf{R}_k \in \mathbb{C}^{N_{R,k} \times N_{R,k}}$ is an upper triangular matrix and $\mathbf{Q}_k \in \mathbb{C}^{N_T \times N_{R,k}}$ is composed of the $N_{R,k}$ orthonormal basis vectors from $\hat{\mathbf{P}}_k$. It should be noted that the operation of \mathbf{Q}_k is equivalent to that of $\hat{\mathbf{P}}_k$ in terms of suppressing the interference from other users, whereas \mathbf{Q}_k does not affect its own effective channel $\tilde{\mathbf{H}}_k \mathbf{Q}_k$ in terms of the MSE thanks to the orthonormal columns.

Unlike the ZF-BD based technique, the proposed MMSE method constructed by \mathbf{Q}_k generates residual interferences, and so a conventional WF is not feasible since the residual interference may vary according to the power allocation. Therefore, as a counterpart of the WF solution for the CR-ZF-BD scheme, we apply an MMSE combining matrix \mathbf{T}_k for each block channel, which minimizes the sum-MSE under the total transmit power constraint as in [7]. The combining matrix is given as $\mathbf{T}_k = \beta \tilde{\mathbf{T}}_k$ with the sum power normalization parameter $\beta = \sqrt{P_T / \sum_{k=1}^K \text{Tr}(\tilde{\mathbf{T}}_k^\dagger \tilde{\mathbf{T}}_k)}$ where $\tilde{\mathbf{T}}_k$ equals [7]

$$\tilde{\mathbf{T}}_k = \left(\mathbf{Q}_k^\dagger \sum_{l=1}^K \tilde{\mathbf{H}}_l^\dagger \tilde{\mathbf{H}}_l \mathbf{Q}_k + \frac{N_R}{P_T} \mathbf{I}_{N_k} \right)^{-1} \mathbf{Q}_k^\dagger \tilde{\mathbf{H}}_k^\dagger \tilde{\mathbf{H}}_k \mathbf{Q}_k. \quad (23)$$

After finding \mathbf{Q}_k and \mathbf{T}_k through (10), (22) and (23), in order to make each SU's received signal vector single symbol decodable, the block channel of the k th SU $\tilde{\mathbf{H}}_k \mathbf{Q}_k \mathbf{T}_k$ can be decomposed by applying the SVD as $\tilde{\mathbf{H}}_k \mathbf{Q}_k \mathbf{T}_k = \tilde{\mathbf{U}}_k \tilde{\mathbf{\Lambda}}_k \tilde{\mathbf{V}}_k^\dagger$. Note that the block channel decomposing matrices $\tilde{\mathbf{V}}_k$ and $\tilde{\mathbf{U}}_k^\dagger$ do not affect the total MSE since they are unitary. As a result, by applying the proposed precoder $\mathbf{Q}_k \mathbf{T}_k \tilde{\mathbf{V}}_k$ and the receive filter $\tilde{\mathbf{U}}_k^\dagger$ to (3), the output signal vector at the k th SU can be written as

$$\mathbf{x}_k = \tilde{\mathbf{\Lambda}}_k \mathbf{s}_k + \tilde{\mathbf{U}}_k^\dagger \tilde{\mathbf{H}}_k \sum_{l=1, l \neq k}^K \mathbf{P}_l \mathbf{s}_l + \tilde{\mathbf{w}}_k. \quad (24)$$

Consequently, the precoder and the receiver matrices of the proposed CR-MMSE-BD technique are given as

$$\mathbf{P}_k^{\text{MB}} = \mathbf{Q}_k \mathbf{T}_k \tilde{\mathbf{V}}_k \quad \text{and} \quad \mathbf{M}_k^{\text{MB}} = \tilde{\mathbf{U}}_k^\dagger \tilde{\mathbf{L}}_k^{-1}. \quad (25)$$

To compare the role of the preprocessing in the CR-MMSE-BD scheme with the CR-ZF-BD scheme, let us consider a zero PUI threshold ($I_{th} = 0$) in the high SNR regime ($P_T \rightarrow \infty$). Since $\hat{\mathbf{P}}$ from (20) makes the total effective channel $\tilde{\mathbf{H}} \hat{\mathbf{P}}$ an

identity matrix at high SNRs, \mathbf{Q}_k approaches $\mathbf{\Pi}_k^\perp$ of the CR-ZF-BD which guarantees the zero interference conditions of (5) and (6). Now, since $\mathbf{Q}_k^\dagger \sum_{l=1, l \neq k}^K \tilde{\mathbf{H}}_l^\dagger \tilde{\mathbf{H}}_l \mathbf{Q}_k = \mathbf{0}$ in (23) due to (6), the combining matrix \mathbf{T}_k in (23) becomes a scaled identity matrix like the WF matrix $\Phi_k^{\frac{1}{2}}$ for the CR-ZF-BD as the SNR increases. As a result, at high SNRs, we can observe that the CR-ZF-BD precoder (8) becomes equivalent to the CR-MMSE-BD precoder (25).

IV. SIMULATION RESULTS

In this section, we provide simulation results that demonstrate the efficiency of the proposed CR-MMSE-BD scheme in terms of the sum-rate. In our simulations, the elements of the signaling channel matrices $\mathbf{G}_0, \mathbf{H}_1, \dots, \mathbf{H}_K$ are assumed to be i.i.d. complex Gaussian variables with zero mean and unit variance. In addition, assuming a path loss exponent of 4, the elements of the interfering channel matrices $\mathbf{H}_0, \mathbf{G}_1, \dots, \mathbf{G}_K$ are generated with the variance of $(d_S/d_I)^4$, where d_S denotes the distance between the PU-AP and the PU or the SU-AP and all of the SUs, and d_I indicates the distance between the PU-AP and all of the SUs or the SU-AP and the PU.² For the PU licensed link, we employ a capacity optimal SVD precoder combined with the water-filling (WF) technique and calculate the rate performance. The sum-rates of the SUs for the CR-ZF-BD and the CR-MMSE-BD schemes are computed from (7) and (24) as

$$R_{ZB} = \sum_{k=1}^K \sum_{i=1}^{N_{R,k}} \log_2 (1 + \bar{\lambda}_{k,i}^2 \phi_{k,i}) \quad \text{and}$$

$$R_{MB} = \sum_{k=1}^K \sum_{i=1}^{N_{R,k}} \log_2 \left\{ 1 + \frac{\tilde{\lambda}_{k,i}^2}{1 + \sum_{l=1, l \neq k}^K \|\mathbf{m}_{k,i} \mathbf{H}_k \mathbf{P}_l\|^2} \right\}$$

where $\bar{\lambda}_{k,i}$, $\tilde{\lambda}_{k,i}$ and $\phi_{k,i}$ equal the i th diagonal element of $\bar{\Lambda}_k$, $\tilde{\Lambda}_k$ and Φ_k , respectively, and $\mathbf{m}_{k,i}$ is the i th row vector of \mathbf{M}_k . We define the SNR of the unlicensed MU-MIMO channel for the SUs as $\rho_T = P_T/\sigma_n^2$ and the SNR for the PU link as $\rho_0 = P_0/\sigma_n^2$, with $\sigma_n^2 = 1$. In our simulations, we denote $N_T \times \{N_{R,1}, N_{R,2}, \dots, N_{R,K}\}$ for the MU-MIMO antenna configurations.

In Fig. 2, we present the sum-rate of the proposed CR linear schemes for $12 \times \{4, 4\}$ and $12 \times \{2, 2, 2, 2\}$ MU-MIMO channels in the presence of a 4×4 PU MIMO link with $\rho_0 = 15$ dB, $I_{th} = 0$ and $d_S = d_I$. The DPC based sum-rate maximization method is also compared as a benchmark. This figure shows that the sum-rate of the proposed CR-MMSE-BD scheme is greater than that of the CR-ZF-BD scheme for all antenna configurations. In addition, as the number of receive antennas per user increases, the sum-rates of the proposed schemes are enhanced through the antenna coordination at the receiver and the performance gap from the DPC capacity decreases. It is important to note that due to the fixed ρ_0 , the rate of the licensed PU link is unchanged for all of the simulated configurations even when the SU SNR ρ_T increases. This result confirms that the CR methods do not interrupt

²Although all of the simulations assume a complete channel knowledge of $\mathbf{H}_0, \mathbf{H}_1, \dots, \mathbf{H}_K$ at the PU-AP, a substantial performance gain can be obtained even for the case with imperfect channel information, as described in [7].

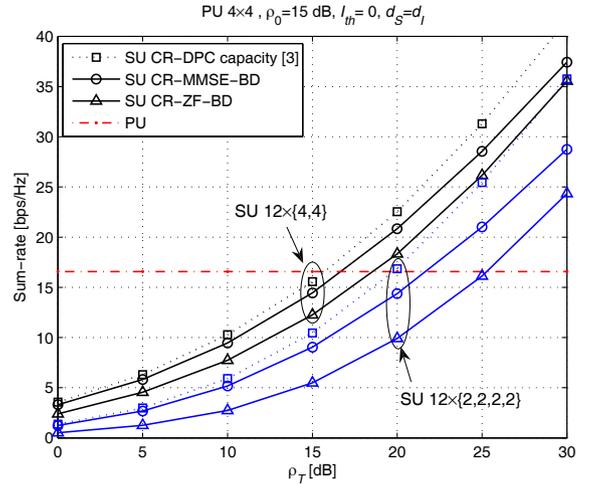


Fig. 2. Sum-rate comparison for different numbers of receive antennas in CR MU-MIMO channels.

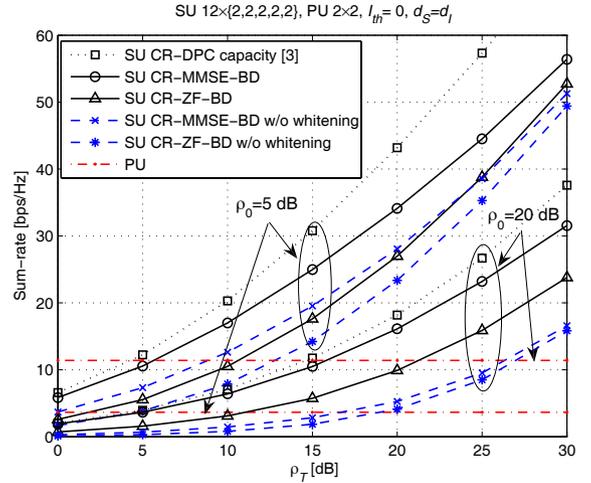


Fig. 3. Sum-rate comparison for various PU SNR ρ_0 with or without whitening in CR MU-MIMO channels.

the licensed PU communication at all, due to the zero PUI constraint ($I_{th} = 0$).

Fig. 3 illustrates the ergodic sum-rates of the proposed linear method either with or without the whitening process for the $12 \times \{2, 2, 2, 2\}$ MU-MIMO and the 2×2 MIMO channels at both $\rho_0 = 5$ dB and 20 dB. Here, the curves that do not employ the whitening process are plotted by using $\mathbf{I}_{N_{R,k}}$ instead of \mathbf{L}_k^{-1} in the receive filter \mathbf{M} from (8) and (25). It is observed from this figure that the sum-rates of both the proposed CR-MMSE-BD and the CR-ZF-BD schemes are remarkably enhanced by the whitening process, and the gain for the CR-MMSE-BD scheme is much greater. As the PU SNR ρ_0 is changed from 5 dB to 20 dB, the sum-rate of the SUs is degraded because the interference increases, however the performance gap between the DPC capacity and the CR-MMSE-BD sum-rate is reduced. In addition, the proposed CR-MMSE-BD achieves a SNR gain of more than 5 dB at the spectral efficiency of 10bps/Hz over the CR-ZF-BD method for both $\rho_0 = 5$ dB and 20 dB.

In Fig. 4, the proposed CR-MMSE-BD scheme is compared to the conventional MMSE-BD method for non-CR

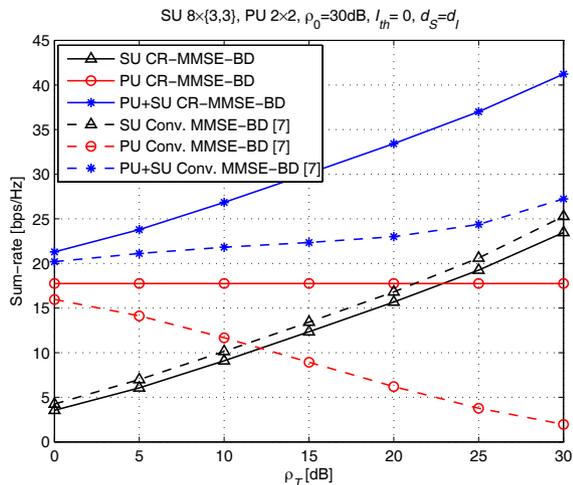


Fig. 4. Sum-rate comparison of the CR-MMSE-BD and conventional MMSE-BD schemes in CR MU-MIMO channels.

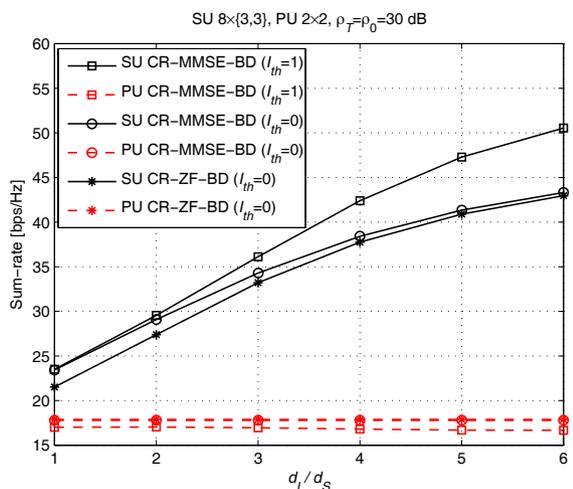


Fig. 5. Effect of PUI power thresholds for various distances in CR MU-MIMO channels.

MU-MIMO systems from [7] and [8], where we consider PU 2×2 MIMO channels and SU $8 \times \{3, 3\}$ MU-MIMO channels employing the whitening process at $\rho_0 = 30$ dB. The proposed CR-MMSE-BD with the additional zero PUI constraint exhibits only a sum-rate loss of 1 bps/Hz for the SUs at $\rho_T = 20$ dB compared to the conventional MMSE-BD scheme. We can see that unlike the proposed CR method, the PU rate for the conventional MMSE-BD is severely degraded since it does not consider the PUI. As a result, in terms of the total sum-rate of the SUs and the PU, denoted as PU+SU in the legend, the proposed CR technique outperforms the conventional MMSE-BD scheme by more than 10 bps/Hz at $\rho_T = 20$ dB.

Finally, Fig. 5 presents the effect of different PUI power thresholds in regard to the proposed CR-MMSE-BD scheme. For various distance ratios for the signal and interference links, the sum-rate performance is illustrated with two PUI thresholds $I_{th} = 0$ and $I_{th} = 1$. It should be noted that the CR-ZF-BD scheme has no solution with nonzero PUI constraints.

We can see that the rate of the PU with $I_{th} = 1$ is within 1 bps/Hz compared to the no PUI case ($I_{th} = 0$). If the distance of the interference links d_I increases, the sum-rate of the SUs becomes higher especially for $I_{th} = 1$ rather than $I_{th} = 0$, since the interference nulling requirement at the SU-AP is relatively relaxed according to the increased path loss. These results confirm the efficiency of the CR-MMSE-BD method with the PUI power constraint.

V. CONCLUSION

In this paper, non-iterative algorithms for linear precoding designs have been investigated for CR MU-MIMO downlink systems where unlicensed SUs attempt to use the spectrum of the licensed PU MIMO link. First, it is shown that the ZF-BD concept can be extended to the CR MU-MIMO systems. Then, in order to enhance the performance, we presented an MMSE based algorithm, which identifies a preprocessing matrix in the MMSE criteria, which permits residual interferences and satisfies a predetermined threshold. In order to support the receive antenna cooperation at each SU, the orthonormal bases of the preprocessing matrix and the combining matrix are employed. Our simulations have verified that the proposed MMSE scheme can provide an improved sum-rate performance over the ZF based method for CR MU-MIMO downlink channels, while successfully guaranteeing the PUI power constraint.

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