A New Channel Quantization Strategy for MIMO Interference Alignment with Limited Feedback

Jin-Sung Kim, *Student Member, IEEE,* Sung-Hyun Moon, *Student Member, IEEE,* Sang-Rim Lee, *Student Member, IEEE,* and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—In K-user multiple-input multiple-output (MIMO) interference channels, it was shown that interference alignment (IA) achieves a full spatial multiplexing gain when perfect channel state information (CSI) is available at each transmitter in the network. When the CSI is fed back from receivers using the limited number of feedback bits, a significant performance loss is inevitable in the IA due to quantized channel knowledge. In this paper, we propose a new channel quantization strategy to optimize the performance of the IA with limited feedback. In our proposed scheme, we introduce an additional receive filter to minimize the chordal distance which accounts for the quantization error on Grassmann manifold. Besides, we analyze a reduction in terms of the chordal distance in our scheme compared to conventional methods. Simulation results verify that the proposed scheme provides substantially better performance than the conventional method as the number of feedback bits is increased. We show that our scheme exhibits 30% and 40% sum rate gains compared to the conventional scheme when the numbers of the feedback bits are 10 and 15, respectively, with two antennas per node.**

***Index Terms*—Interference alignment, limited feedback, channel quantization.**

I. INTRODUCTION

THERE have been extensive researches on interference channels to characterize the capacity in the information theoretic aspect. Although several notable results have been obtained for certain cases [1] [2], characterization of the capacity region of the interference channel is still an open problem in general [3]. As an alternative means to specify the system performance, degrees of freedom (DOF) has been introduced to analyze various channels [3]–[5]. By definition, the DOF is analogous to the multiplexing gain, and the system capacity is dominated by the DOF at high signal to noise ratio (SNR).

Recently, an algorithm named interference alignment (IA) has demonstrated that the DOF of the interference channel can be substantially higher than previous wisdom [6] [7]. The IA is a linear precoding technique which attempts to align interference signals in the domain of time and frequency. Especially in multiple-input multiple-output (MIMO) systems, the spatial dimension provided by multiple antennas can be exploited for the alignment of the interference. At each receiver, the desired signals can be decoded without interference, since the signals from the other transmitters are precoded to be aligned using the IA. The authors in [6] have shown that the IA attains almost surely the maximum DOF of $K/2$ per dimension in the K-user interference channel. For general MIMO configurations, the feasibility conditions of the IA and the achievable DOFs have been studied in [8].

From a practical perspective, the IA can be adopted to enhance the cell-edge user throughput in a cellular network. In this case, global channel state information (CSI) should be obtained at the transmitters via feedback in order to manipulate the IA [6] [7]. Note that channel reciprocity is valid only for acquiring the local CSI. Thus, in the IA, time division duplex (TDD) does not provide savings in terms of the feedback overhead in comparison to frequency division duplex (FDD). For the case where the CSI is ideally known at each transmitter, the IA successfully achieves a theoretical bound on the DOF for interference channels [9]–[11]. However, the assumption of the perfect CSI is almost impossible to be realized at the transmitters, especially for quantized feedback systems using feedback links with finite bandwidth. Unlike point-to-point MIMO systems where the imperfect CSI causes only an SNR offset in the capacity vs. SNR curve, the accuracy of the CSI in the IA systems affects the slope of the curve, i.e., DOF [12]. This phenomenon is analogous to that of multiuser MIMO broadcast systems [13]–[16].

Consequently, recent studies addressed this issue of the IA with limited feedback. The paper in [17] examined the effect of the imperfect CSI with respect to the mutual information of the IA in a specific system configuration. Besides, the authors in [18] clarified the required number of feedback bits to achieve the maximum DOF of the IA for a frequency selective single-input single-output (SISO) system. An extended work was developed in [12] for the MIMO case, where an appropriate limited feedback scheme has also been proposed.

However, a large amount of feedback bits is still necessary to attain reasonable performance. To circumvent the feedback overhead issues in the IA, analog CSI feedback was considered with several restrictions in [19]. Instead of the IA, a leakage based scheme was studied in [20] for limited feedback cellular networks by assuming a single interference source for each cell. The papers in [21] [22] addressed the effect of channel feedback delay in cellular systems, and a scalable limited feedback design was proposed in [23] [24] for network MIMO systems.

In this paper, we propose a new channel quantization method to enhance the performance of MIMO IA systems with limited feedback. At each receiver, the proposed scheme introduces an additional receive filter before quantizing the channels. By judiciously designing the combined receive filter

Manuscript received May 2, 2011; revised September 11, 2011; accepted October 20, 2011. The associate editor coordinating the review of this paper and approving it for publication was D. Huang.

This work was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909), and in part by a Korea University Grant.

The authors are with the School of Electrical Engineering, Korea University, Seoul, Korea (e-mail: {jinsungkim, shmoon, sangrim78, inkyu}@korea.ac.kr).

Digital Object Identifier 10.1109/TWC.2011.111211.110810

which determines the effective channels, the quantization error can be substantially reduced in our scheme compared to the conventional one in [12]. The receive filter in the proposed scheme minimizes the chordal distance which is directly related to the quantization error on Grassmann manifold [25] [26]. Then at the transmitters, the IA is manipulated according to the feedback information of the effective channels.

To quantify a performance gain of the proposed scheme over the conventional one, we analyze the difference of the minimum chordal distance between these two methods with the fixed number of feedback bits. Simulation results show that our scheme exhibits significantly better performance than the conventional scheme for the whole SNR region regardless of the number of the feedback bits. We confirm that about 30% and 40% sum rate gains are obtained by utilizing the proposed method compared to the conventional scheme when the numbers of the feedback bits are 10 and 15, respectively, with two antennas per node at high SNR.

In multiuser MIMO systems, there also exist several methods adjusting the receive filters to improve the performance of systems with limited feedback [13]–[16]. However, these methods do not consider the interference channels when deriving the filters unlike our proposed scheme. Besides, the method in [16] assumes only one data stream per user by imposing the antenna combining vectors at the receivers, whereas the proposed scheme allows multiple data streams at each transmitter-receiver pair by properly designing the receive filter matrices.

The following notations are used for description throughout this paper. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. Also, $(\cdot)^*$, $(\cdot)^\dagger$ and $\mathbb{E}[\cdot]$ stand for conjugate, conjugate transpose and expectation, respectively. In addition, $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote $m\times n$ real and complex matrix spaces, respectively. An identity matrix with size $m\times m$ is represented as $\mathbf{I}_m$, and $\otimes$ indicates Kronecker product.

The organization of the paper is as follows: Section II presents an interference channel model, and briefly reviews the concept of the IA. In Section III, we introduce our new channel quantization algorithm. Section IV analyzes the difference of the minimum chordal distance between the conventional and the proposed scheme, and we compare the sum rate performance of our scheme with the conventional method in various system configurations in Section V. Finally, the paper is terminated with conclusions in Section VI.

II. System Descriptions and Background

Figure 1 illustrates K-user MIMO interference channels where each transmitter i communicates with its corresponding receiver i and interferes with all other receivers $j\neq i$. In this system, transmitter i is equipped with N_t transmit antennas to support d_i data streams, and receiver i has N_r receive antennas for all i $(d_i\leq\min(N_t,N_r))$. Although we consider the same numbers of antennas for every transmitter-receiver pair, the results can be generalized to a network with different numbers of antennas as long as the IA remains feasible [8]. In the discrete-time complex baseband MIMO case, the frequency-flat channel from transmitter i to receiver j is modeled by the matrix $\mathbf{H}_{j,i}=\left[\mathbf{h}_{j,i}^{(1)}\ \cdots\ \mathbf{h}_{j,i}^{(N_t)}\right]\in\mathbb{C}^{N_r\times N_t}$

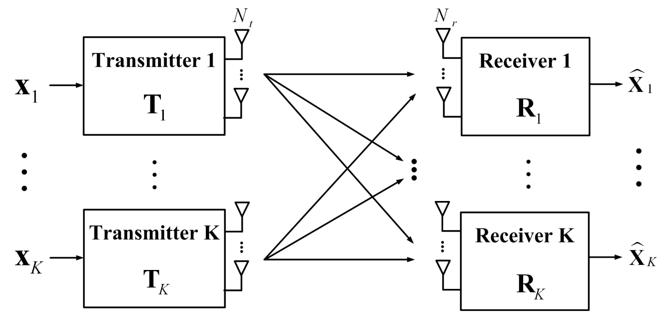

Fig. 1. Block diagram of K user MIMO interference channel systems.

where $\mathbf{h}_{j,i}^{(l)}\in\mathbb{C}^{N_r\times 1}$ represents the l-th column vector for $i,j=1,\cdots,K$. The entries of $\mathbf{H}_{j,i}$ are assumed as independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0,1)$.

At the i-th receiver, the received signal vector $\mathbf{y}_i\in\mathbb{C}^{N_r\times 1}$ is given as

$$\mathbf{y}_i=\mathbf{H}_{i,i}\mathbf{T}_i\mathbf{x}_i+\sum_{j=1,j\neq i}^{K}\mathbf{H}_{i,j}\mathbf{T}_j\mathbf{x}_j+\mathbf{n}_i \tag{1}$$

where $\mathbf{T}_i\in\mathbb{C}^{N_t\times d_i}$ indicates the transmit precoder at transmitter i with unit-norm columns, $\mathbf{x}_i\in\mathbb{C}^{d_i\times 1}$ denotes the transmit symbol vector from transmitter i, and $\mathbf{n}_i\in\mathbb{C}^{N_r\times 1}$ is the additive white Gaussian noise vector observed at receiver i. Here the symbols in $\mathbf{x}_i$ are assumed to be independently generated with unit variance and the entries of $\mathbf{n}_i$ are i.i.d. with zero mean and variance N_0. Defining $\mathbf{R}_i\in\mathbb{C}^{N_r\times d_i}$ as the receive combining matrix for the i-th receiver, the received signal vector $\hat{\mathbf{x}}_i$ after the receiver combining is expressed as

$$\hat{\mathbf{x}}_i=\mathbf{R}_i^\dagger\mathbf{y}_i=\mathbf{R}_i^\dagger\mathbf{H}_{i,i}\mathbf{T}_i\mathbf{x}_i+\mathbf{R}_i^\dagger\sum_{j=1,j\neq i}^{K}\mathbf{H}_{i,j}\mathbf{T}_j\mathbf{x}_j+\mathbf{R}_i^\dagger\mathbf{n}_i. \tag{2}$$

We assume that each receiver i knows its MIMO channels $\mathbf{H}_{i,1},\cdots,\mathbf{H}_{i,K}$ perfectly based on separate pilot signals transmitted by each of K transmitters. Also, error-free dedicated broadcast links are assumed from each receiver to other transmitters j $(\forall j\neq i)$ in the network. During the channel feedback phase, receiver i broadcasts its CSI using B bits. This is suitable for the FDD systems, where each transmitter must rely on the quantized feedback from the receivers to obtain the CSI. Note that channel reciprocity is useful only for having its local CSI in the TDD systems.

Under the assumption of the perfect CSI at the transmitters, one can achieve the maximum multiplexing gain, or the maximum DOF by utilizing the IA techniques. This implies that the transmit precoding matrix $\mathbf{T}_i$ is chosen in the null space of $\mathbf{R}_j^\dagger\mathbf{H}_{j,i}$ such that $\mathbf{R}_j^\dagger\mathbf{H}_{j,i}\mathbf{T}_i=0$ $(\forall j\neq i)$ [8]. Consequently, $\mathbf{T}_i$ causes no interference to receiver j by completely removing the interference term in (2). Also, the total number of the transmitted data streams $\sum d_i$ is set to attain a full spatial multiplexing gain, i.e., $\sum d_i=KN_t/2$ is achieved in case of $N_t=N_r$ [6]. However, when the CSI is fed back through the limited feedback channel, it is difficult to obtain such optimized performance which satisfies zero interference. That is, a significant loss of performance is inevitable due to the imperfect CSI. To reduce the interference caused by the limited feedback, we propose a new channel quantization scheme suitable for the limited feedback MIMO IA in the following section.

III. Proposed Channel Quantization Scheme

In this section, we describe our proposed channel quantization method for limited feedback IA systems. Before starting, we briefly review the previous work in [12] to study how each receiver quantizes its respective channels $\mathbf{H}_{i,j}$ $(\forall j \neq i)$ for implementing the MIMO IA. According to [12], the aggregated channel matrix $\mathbf{W}_i \in \mathbb{C}^{N_t N_r \times (K-1)}$ fed back from the i-th receiver is expressed as

$$\mathbf{W}_i = \left[\tilde{\mathbf{h}}_{i,1} \cdots \tilde{\mathbf{h}}_{i,i-1} \ \tilde{\mathbf{h}}_{i,i+1} \cdots \tilde{\mathbf{h}}_{i,K}\right] \tag{3}$$

where a unit-norm vector $\tilde{\mathbf{h}}_{i,j} \in \mathbb{C}^{N_t N_r \times 1}$ is obtained by stacking the columns of $\mathbf{H}_{i,j}$ as

$$\tilde{\mathbf{h}}_{i,j} = \frac{\left[\mathbf{h}_{i,j}^{(1)\dagger} \cdots \mathbf{h}_{i,j}^{(N_t)\dagger}\right]^{\dagger}}{\left\|\left[\mathbf{h}_{i,j}^{(1)\dagger} \cdots \mathbf{h}_{i,j}^{(N_t)\dagger}\right]\right\|}. \tag{4}$$

Note that the vector $\tilde{\mathbf{h}}_{i,i}$ corresponding to $\mathbf{H}_{i,i}$ is excluded in (3), because it is not mandatory for the transmitters to manipulate the IA [6]–[8]. Also, we do not consider symbol extension in time slots or frequency slots, since alignment in spatial dimension is found to be more robust to practical limitations than in time or frequency dimension [8] [27] [28].

Using the concept of the composite Grassmann manifold [12], the matrix $\mathbf{W}_i = [\mathbf{w}_i^{(1)} \ \cdots \ \mathbf{w}_i^{(K-1)}]$ can be quantized with a codebook $\mathcal{C} = \{\mathbf{C}_1, \cdots, \mathbf{C}_{2^B}\}$ where each codeword $\mathbf{C}_j = [\mathbf{c}_j^{(1)} \ \cdots \ \mathbf{c}_j^{(K-1)}] \in \mathbb{C}^{N_t N_r \times (K-1)}$ with $\|\mathbf{c}_j^{(m)}\| = 1$ for $\forall j, m$ has the same size of $\mathbf{W}_i$. Specifically, we can present the chordal distance between these two matrices as

$$\mathcal{D}(\mathbf{W}_i, \mathbf{C}_j) = \sum_{l=1}^{K-1} \left(1 - \left|\mathbf{w}_i^{(l)\dagger}\mathbf{c}_j^{(l)}\right|^2\right) \tag{5}$$

which is commonly used for a distance metric on the composite Grassmann manifold [12].

Then, receiver i computes the chordal distance from $\mathbf{W}_i$ to each codeword in $\mathcal{C}$, and feeds back the index of the codeword which shows the minimum chordal distance. This is because the chordal distance accounts for the quantization error on the Grassmann manifold. Based on these indices fed back from all receivers, each transmitter can obtain the CSI for $\mathbf{H}_{i,j}$ $(i, j = 1, \cdots, K, \ i \neq j)$ from the corresponding codewords and the IA becomes feasible. However, unless the feedback bit B is large enough, the performance of the IA may be significantly degraded due to the imperfect CSI. To minimize a performance loss caused by the quantized feedback, we propose our scheme in the following.

First, in our method, we introduce an additional receive filter $\mathbf{G}_i \in \mathbb{C}^{N_r \times N_r}$ at the i-th receiver before quantizing the channels. We assume that $\mathbf{G}_i$ is a unitary matrix so that the noise remains uncorrelated, i.e., $\mathbb{E}[\mathbf{G}_i\mathbf{n}_i\mathbf{n}_i^\dagger\mathbf{G}_i^\dagger] = N_0\mathbf{I}_{N_r}$. Denoting $\overline{\mathbf{H}}_{i,j} = [\overline{\mathbf{h}}_{i,j}^{(1)} \cdots \overline{\mathbf{h}}_{i,j}^{(N_t)}] = \mathbf{G}_i\mathbf{H}_{i,j}$ $(j = 1, \cdots, K)$ as the effective channels, we can feed back $\overline{\mathbf{H}}_{i,j}$ as the actual channel matrix instead of $\mathbf{H}_{i,j}$. That is, by considering $\overline{\mathbf{h}}_{i,j}^{(m)}$ as $\mathbf{h}_{i,j}^{(m)}$ in (4) for all m, each column of the aggregated channel matrix $\overline{\mathbf{W}}_i = \left[\overline{\mathbf{h}}_{i,1} \cdots \overline{\mathbf{h}}_{i,i-1} \ \overline{\mathbf{h}}_{i,i+1} \cdots \overline{\mathbf{h}}_{i,K}\right]$ in (3) for the CSI quantization of our scheme is presented as

$$\begin{aligned} \overline{\mathbf{h}}_{i,j} &= \frac{(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\left[\mathbf{h}_{i,j}^{(1)\dagger} \cdots \mathbf{h}_{i,j}^{(N_t)\dagger}\right]^{\dagger}}{\left\|(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\left[\mathbf{h}_{i,j}^{(1)\dagger} \cdots \mathbf{h}_{i,j}^{(N_t)\dagger}\right]\right\|} \\ &= \frac{(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\tilde{\mathbf{h}}_{i,j}}{\left\|(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\tilde{\mathbf{h}}_{i,j}\right\|} = (\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\tilde{\mathbf{h}}_{i,j} \end{aligned} \tag{6}$$

where the last equality holds since $\left\|(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\tilde{\mathbf{h}}_{i,j}\right\| = 1$ with the unitary matrix $\mathbf{G}_i$ and the unit-norm vector $\tilde{\mathbf{h}}_{i,j}$. By judiciously designing $\mathbf{G}_i$ in (6) which determines $\overline{\mathbf{W}}_i$, the minimum chordal distance for the given codebook $\mathcal{C}$ becomes smaller than that from quantizing the original $\mathbf{W}_i$. Consequently, an overall performance degradation incurred by the imperfect CSI is mitigated without increasing the codebook size in our method. Note that our filter $\mathbf{G}_i$ optimizes the quantization on the composite Grassmann manifold unlike the filters used in point-to-point MIMO systems.

Now we describe how to compute the filter $\mathbf{G}_i$ at the i-th receiver to reduce the minimum chordal distance in detail. Using the relation in (6), the chordal distance between $\overline{\mathbf{W}}_i$ and a given arbitrary codeword $\mathbf{C}_m \in \mathcal{C}$ is developed as

$$\begin{aligned} \mathcal{D}(\overline{\mathbf{W}}_i, \mathbf{C}_m) &= \sum_{l=1}^{K-1} \left(1 - \left|\mathbf{c}_m^{(l)\dagger}(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}\right|^2\right) \\ &= \sum_{l=1}^{K-1} \left(1 - \left|\mathrm{tr}\big((\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}\mathbf{c}_m^{(l)\dagger}\big)\right|^2\right) \qquad (7) \\ &= \sum_{l=1}^{K-1} \left(1 - \mathrm{tr}\big(e^{-j\theta_l}(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}\mathbf{c}_m^{(l)\dagger}\big)^2\right) \\ &\geq K - 1 - \mathrm{tr}\left((\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\sum_{l=1}^{K-1} e^{-j\theta_l}\mathbf{w}_i^{(l)}\mathbf{c}_m^{(l)\dagger}\right)^2 \\ &= K - 1 - \mathrm{tr}(\mathbf{G}_i\mathbf{A})^2 \qquad (8) \end{aligned}$$

where we have $\mathbf{A} \triangleq \sum_{l=1}^{K-1}\sum_{n=1}^{N_t} e^{-j\theta_l}\bar{\mathbf{w}}_{i,n}^{(l)}\bar{\mathbf{c}}_{m,n}^{(l)\dagger}$, $j = \sqrt{-1}$, θ_l indicates the phase of $\mathbf{c}_m^{(l)\dagger}(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}$, and $\bar{\mathbf{w}}_{i,n}^{(l)} \in \mathbb{C}^{N_r \times 1}$ and $\bar{\mathbf{c}}_{m,n}^{(l)} \in \mathbb{C}^{N_r \times 1}$ denote the n-th block of $\mathbf{w}_i^{(l)}$ and $\mathbf{c}_m^{(l)}$, respectively $\big($i.e., $\mathbf{w}_i^{(l)} = \left[\bar{\mathbf{w}}_{i,1}^{(l)\dagger} \cdots \bar{\mathbf{w}}_{i,N_t}^{(l)\dagger}\right]^\dagger$ and $\mathbf{c}_m^{(l)} = \left[\bar{\mathbf{c}}_{m,1}^{(l)\dagger} \cdots \bar{\mathbf{c}}_{m,N_t}^{(l)\dagger}\right]^\dagger\big)$. To reduce the chordal distance in (7), the lower bound in (8) is minimized by considering the following lemma.

Lemma 1: Consider the singular value decomposition (SVD) of an $m \times m$ matrix $\mathbf{Q}$ as $\mathbf{Q} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$ where $\mathbf{U} \in \mathbb{C}^{m \times m}$ and $\mathbf{V} \in \mathbb{C}^{m \times m}$ represent unitary matrices composed of the left and right singular vectors of $\mathbf{Q}$, respectively, and $\mathbf{\Sigma} \in \mathbb{C}^{m \times m}$ equals a diagonal matrix with the corresponding singular values. For a unitary matrix $\mathbf{P}$, the real part of $\mathrm{tr}\big(\mathbf{PQ}\big)$ is maximized by setting $\mathbf{P} = \mathbf{V}\mathbf{U}^\dagger$.

Proof: Without loss of generality, a unitary matrix $\mathbf{P}$ can be expressed as $\mathbf{P} = \mathbf{V}\mathbf{L}\mathbf{U}^\dagger$ with a unitary matrix $\mathbf{L} = \mathbf{V}^\dagger\mathbf{P}\mathbf{U}$ of size $m \times m$. Then, $\mathrm{tr}\big(\mathbf{PQ}\big) = \mathrm{tr}\big(\mathbf{L}\mathbf{\Sigma}\big) = l_1\sigma_1 + \cdots + l_m\sigma_m$ where l_n and σ_n denote the n-th diagonal entry of $\mathbf{L}$ and $\mathbf{\Sigma}$, respectively. Since $\sigma_n \geq 0$ for all n, $\mathbf{L}$ which maximizes the real part of $\mathrm{tr}\big(\mathbf{PQ}\big)$ is equal to $\mathbf{I}_m$. ■

After applying the SVD to $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$ in (8), $\mathbf{G}_i$ is set to $\mathbf{G}_i = \mathbf{V}\mathbf{U}^\dagger$ following Lemma 1. Then, the phases $\theta_1, \cdots, \theta_{K-1}$ which determine $\mathbf{A}$ in (8) need to be computed after obtaining $\mathbf{G}_i$. Therefore, we exploit an iterative process to update $\mathbf{G}_i$ and θ_l $(l = 1, \cdots, K-1)$ one after another. In this process, we first initialize $\mathbf{G}_i = \mathbf{I}_{N_r}$. Then $\theta_1, \cdots, \theta_{K-1}$ are calculated with the assumption that $\mathbf{G}_i$ is fixed and vice versa, until convergence is reached. Our proposed scheme is summarized as below.

Algorithm 1 Proposed process for computing $\mathbf{G}_i$

1) Initialize $\mathbf{G}_i = \mathbf{I}_{N_r}$
2) Set θ_l to the phase of $\mathbf{c}_m^{(l)\dagger}(\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}$ for all l
3) Apply SVD to $\sum_{l=1}^{K-1}\sum_{n=1}^{N_t} e^{-j\theta_l}\bar{\mathbf{w}}_{i,n}^{(l)}\bar{\mathbf{c}}_{m,n}^{(l)\dagger} = \mathbf{U\Sigma V}^\dagger$
4) $\mathbf{G}_i = \mathbf{VU}^\dagger$
5) Go back to 2) until convergence

Note that the performance of our scheme becomes identical to that of the conventional method when the filter $\mathbf{G}_i$ is fixed to $\mathbf{G}_i = \mathbf{I}_{N_r}$. By computing $\mathbf{G}_i$ which minimizes (8) in our method, we can reduce the chordal distance between $\overline{\mathbf{W}}_i$ and the given codeword $\mathbf{C}_m$. The reduced chordal distance $\mathcal{D}(\overline{\mathbf{W}}_i, \mathbf{C}_m)$ is computed in (7) after obtaining $\mathbf{G}_i$ through the proposed method. Now the chordal distance from $\overline{\mathbf{W}}_i$ to each codeword in $\mathcal{C}$ can be derived by utilizing the above process. Finally, the index of the codeword and the corresponding $\mathbf{G}_i$ are identified which minimize the chordal distance in our method.

The transmit precoding matrix $\mathbf{T}_i$ and the receive combining matrix $\mathbf{R}_i$ in (2) are computed based on the newly obtained effective channels $\overline{\mathbf{H}}_{i,j}$. At the i-th transmitter, the quantized feedback information corresponding to $\overline{\mathbf{H}}_{i,j}$ is utilized to yield a solution for $\mathbf{T}_i$. In contrast, at receiver i, the filter $\mathbf{R}_i$ is derived by using the actual (unquantized) values of $\overline{\mathbf{H}}_{i,j}$ since $\mathbf{G}_i$ and $\mathbf{H}_{i,j}$ are both known exactly at the receiver. In the next section, we compare the minimum chordal distance between our proposed scheme and the conventional method through mathematical derivations.

IV. Minimum Chordal Distance Analysis

The performance of limited feedback systems depends on the minimum chordal distance which characterizes the rate-distortion tradeoff in the quantization on the Grassmann manifold [25] [26]. First, we will investigate the difference of the minimum chordal distance between the conventional and the proposed scheme when the codebook $\mathcal{C}$ is generated through random vector quantization (RVQ) with the feedback bits B. This analysis will quantify a reduction of the CSI quantization error in our scheme compared to the conventional one. Using this analysis, we will compare the numbers of required bits for both schemes later.

A. Difference of Minimum Chordal Distance

First, we apply the following assumption in quantizing the aggregated channel $\mathbf{W}_i$ for simple analysis. Note that the aggregated channel for the conventional method in (3) is quantized on the domain of the $(K-1)$-composite Grassmann manifold using B bits [12]. Considering this fact, we assume that each Grassmann manifold is quantized by utilizing $U = B/(K-1)$ bits in our analysis. This assumption is asymptotically true as B increases, and the total number of the required bits to quantize $\mathbf{W}_i$ is invariant. Under this assumption, the l-th column $\mathbf{w}_i^{(l)}$ in $\mathbf{W}_i$ is quantized to one of unit norm codewords $\widetilde{\mathbf{c}}_j^{(l)} \in \mathbb{C}^{N_tN_r\times 1}$ in $\widetilde{\mathcal{C}}_l = \{\widetilde{\mathbf{c}}_1^{(l)}, \cdots, \widetilde{\mathbf{c}}_{2^U}^{(l)}\}$ based on the minimum chordal distance criterion.

Likewise, we apply the same assumption to quantize $\overline{\mathbf{W}}_i = \left[\overline{\mathbf{w}}_i^{(1)} \cdots \overline{\mathbf{w}}_i^{(K-1)}\right]$ in our scheme, and the difference of the minimum chordal distance between the conventional and the proposed scheme can be computed with given B as

$$\Delta\mathcal{D} = \sum_{l=1}^{K-1}\min_{j=1,\cdots,2^U}\left(1-\left|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2\right) - \sum_{l=1}^{K-1}\min_{j=1,\cdots,2^U}\left(1-\left|\overline{\mathbf{w}}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2\right) \quad (9)$$

$$= \sum_{l=1}^{K-1}\max_{j=1,\cdots,2^U}\left|\overline{\mathbf{w}}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2 - \sum_{l=1}^{K-1}\max_{j=1,\cdots,2^U}\left|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2. \quad (10)$$

Now, we will prove that the expectation of $\Delta\mathcal{D}$ in (10) is greater than 0 for all B, i.e., $\mathbb{E}[\Delta\mathcal{D}] > 0$. This result will demonstrate that our proposed scheme exhibits smaller quantization errors than the conventional method regardless of the codebook size.

Let us consider the distribution of $\alpha \triangleq \left|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2$ and $\beta \triangleq \left|\overline{\mathbf{w}}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2$. The cumulative density function (CDF) of α in (10) with two independent and uniformly distributed vectors $\mathbf{w}_i^{(l)}$ and $\widetilde{\mathbf{c}}_j^{(l)}$ is obtained as [29]

$$F_\alpha(x) = 1-(1-x)^{N_tN_r-1} \qquad \text{for } 0 \le x \le 1. \quad (11)$$

To derive the distribution of β in (10), we first clarify the relation between $\overline{\mathbf{w}}_i^{(l)}$ and $\widetilde{\mathbf{c}}_j^{(l)}$. In our scheme, $\overline{\mathbf{w}}_i^{(l)} = (\mathbf{I}_{N_t} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)}$ is designed to maximize β, which in turn minimizes the chordal distance. Although the optimal β is achieved when $\overline{\mathbf{w}}_i^{(l)} = \widetilde{\mathbf{c}}_j^{(l)}$, it is impossible to attain in general due to the finite size of the unitary matrix $\mathbf{G}_i \in \mathbb{C}^{N_r\times N_r}$. Since it is difficult to derive an exact distribution of β because of joint relations among each process in our method, we approximate it by considering the following two cases regarding the size of $\mathbf{G}_i$.

As one special case, suppose that $\mathbf{G}_i$ reduces to a scalar g_i with unit amplitude, and the corresponding $\overline{\mathbf{w}}_i^{(l)}$ is defined as $\overline{\mathbf{w}}_i^{(l)} = (\mathbf{I}_{N_tN_r} \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)} = g_i\mathbf{w}_i^{(l)}$. In this case, the distribution of $\beta = \left|g_i^*\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2 = \left|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\right|^2$ is determined by [29]

$$\beta \sim \frac{\gamma_1}{\gamma_1+\gamma_2} \quad (12)$$

where $\sim$ represents equivalence in distribution, and γ_1 and γ_2 indicate two independent Chi-square random variables with 2 and $2(N_tN_r-1)$ degrees of freedom, respectively (i.e., $\gamma_1 \sim \chi_2^2$ and $\gamma_2 \sim \chi_{2(N_tN_r-1)}^2$).

Then, as the second special case, suppose that the size of the unitary matrix $\mathbf{G}_i$ is as large as $N_tN_r \times N_tN_r$ and the corresponding $\overline{\mathbf{w}}_i^{(l)}$ is presented as $\overline{\mathbf{w}}_i^{(l)} = (\mathbf{I}_1 \otimes \mathbf{G}_i)\mathbf{w}_i^{(l)} = \mathbf{G}_i\mathbf{w}_i^{(l)}$. Obviously, there always exists $\mathbf{G}_i$ which maximizes $\beta = \left|\widetilde{\mathbf{c}}_j^{(l)\dagger}\mathbf{G}_i\mathbf{w}_i^{(l)}\right|^2 = 1$ in this case, and the distribution of β follows (12) with the degrees of freedoms $2N_tN_r$ and 0 for γ_1 and γ_2, respectively (i.e., $\gamma_1 \sim \chi_{2N_tN_r}^2$ and $\gamma_2 \sim \chi_0^2$).

Now, observing the distribution of these two special cases, we approximate the distribution of β for practical cases where $\mathbf{G}_i$ has the size of $m \times m$ $(1 < m < N_tN_r)$ by applying linear interpolation to the degrees of freedoms on $\gamma_1 \sim \chi^2_{2m}$ and $\gamma_2 \sim \chi^2_{2(N_tN_r-m)}$ in (12). Since the probability density function (PDF) of a Chi-square random variable with n degrees of freedom is expressed as

$$f_{\chi^2_n}(x) = \frac{x^{\frac{n-2}{2}} e^{-x}}{\Gamma\left(\frac{n}{2}\right)}\ , \tag{13}$$

we can derive the PDF of β by utilizing (12) and (13) with $m = N_r$ as

$$\begin{aligned} f_\beta(x) &= \int_0^\infty f_\beta(x|\gamma_2) f_{\gamma_2}(\omega) d\omega \\ &= \int_0^\infty \frac{\omega}{(1-x)^2} f_{\gamma_1}\Big(\frac{x\omega}{1-x}\Big) f_{\gamma_2}(\omega) d\omega \qquad (14) \\ &= \frac{x^{N_r-1}}{\Gamma(N_r)\Gamma(N_tN_r-N_r)(1-x)^{N_r+1}} \\ &\qquad \times \int_0^\infty \exp\Big(-\frac{\omega}{1-x}\Big)\omega^{N_tN_r-1} d\omega \\ &= \frac{x^{N_r-1}(1-x)^{N_tN_r-N_r-1}}{\Psi(N_r, N_tN_r-N_r)} \quad \text{for } 0 \le x \le 1 \qquad (15) \end{aligned}$$

where $\Gamma(\cdot)$ and $\Psi(\cdot,\cdot)$ denote the gamma and the beta function, respectively [30]. Here the second equality in (14) is derived using the fact $F_\beta(x|\gamma_2) = F_{\gamma_1}\Big(\frac{x\gamma_2}{1-x}\Big)$ obtained from (12). Then the CDF of β in (10) is obtained from the PDF in (15) as [30]

$$\begin{aligned} F_\beta(x) &= I_x(N_r, N_tN_r - N_r) \\ &= \sum_{p=N_r}^{N_tN_r-1} \binom{N_tN_r-1}{p} x^p(1-x)^{N_tN_r-1-p} \end{aligned} \tag{16}$$

where $I_x(\cdot,\cdot)$ indicates the regularized incomplete beta function.

Now using the above CDFs the expectation of $\Delta\mathcal{D}$ in (10) is given as

$$\begin{aligned} \mathbb{E}[\Delta\mathcal{D}] &= (K-1)\bigg(\mathbb{E}\Big[\max_{j=1,\cdots,2^U} \big|\overline{\mathbf{w}}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\big|^2\Big] \\ &\qquad - \mathbb{E}\Big[\max_{j=1,\cdots,2^U} \big|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\big|^2\Big]\bigg) \qquad (17) \\ &= (K-1)2^U\bigg(\int_0^1 x\big(F_\beta(x)\big)^{2^U-1} f_\beta(x) dx \\ &\qquad - \int_0^1 x\big(F_\alpha(x)\big)^{2^U-1} f_\alpha(x) dx\bigg) \\ &= (K-1)\bigg(\int_0^1 \Big(\big(F_\alpha(x)\big)^{2^U} - \big(F_\beta(x)\big)^{2^U}\Big) dx\bigg). \qquad (18) \end{aligned}$$

By utilizing (11) and (16), we have

$$\begin{aligned} F_\alpha(x) - F_\beta(x) &= 1 - (1-x)^{N_tN_r-1} \\ &\quad - \sum_{p=N_r}^{N_tN_r-1} \binom{N_tN_r-1}{p} x^p(1-x)^{N_tN_r-1-p} \qquad (19) \\ &= \big(x + (1-x)\big)^{N_tN_r-1} - (1-x)^{N_tN_r-1} \\ &\quad - \sum_{p=N_r}^{N_tN_r-1} \binom{N_tN_r-1}{p} x^p(1-x)^{N_tN_r-1-p} \\ &= \sum_{p=0}^{N_r-1} \binom{N_tN_r-1}{p} x^p(1-x)^{N_tN_r-1-p} \\ &\quad - (1-x)^{N_tN_r-1} \\ &= \sum_{p=1}^{N_r-1} \binom{N_tN_r-1}{p} x^p(1-x)^{N_tN_r-1-p}. \end{aligned}$$

Thus, we can see that $F_\alpha(x) - F_\beta(x) > 0$ for all $0 < x < 1$, which leads to $\big(F_\alpha(x)\big)^{2^U} > \big(F_\beta(x)\big)^{2^U}$ in (18). As a result, it follows $\mathbb{E}[\Delta\mathcal{D}] > 0$. This indicates that the minimum chordal distance which accounts for the channel quantization error is reduced by exploiting our proposed method in the limited feedback IA systems with a general configuration of antennas and feedback bits. This result is important to explain a performance gain of the proposed scheme over the conventional method, and will be verified through simulations later.

B. Comparison on the Number of Required Bits

In the following, we study the number of bits required for the proposed scheme compared to the conventional method. To this end, we compute the actual value of $\mathbb{E}[\Delta\mathcal{D}]$ in (17) first. The second expectation term in (17) is derived as [29]

$$\begin{aligned} \mathbb{E}\Big[\max_{j=1,\cdots,2^U} \big|\mathbf{w}_i^{(l)\dagger}\widetilde{\mathbf{c}}_j^{(l)}\big|^2\Big] &= 1 - \sum_{p=0}^{2^U} \frac{\binom{2^U}{p}(-1)^p}{p(N_tN_r-1)+1} \\ &= 1 - 2^U\Psi\Big(2^U, \frac{N_tN_r}{N_tN_r-1}\Big). \end{aligned} \tag{20}$$

Although a closed-form expression of the first expectation term in (17) is difficult to obtain in general, we can develop it for a simple $N_t = N_r = 2$ case with the distributions in (15) and (16).

For $N_t = N_r = 2$, $\mathbb{E}[\Delta\mathcal{D}]$ is expressed with the result in (20) as

$$\begin{aligned} \mathbb{E}[\Delta\mathcal{D}] = (K-1)\bigg\{ & 2^{U+1}3^{2^U} \sum_{p=0}^{2^U-1} \binom{2^U-1}{p} \Big(\frac{1}{2^{U+1}+p+1} \\ & - \frac{1}{2^{U+1}+p+2}\Big)\Big(-\frac{2}{3}\Big)^p + 2^U\Psi\Big(2^U, \frac{4}{3}\Big) - 1\bigg\}. \end{aligned} \tag{21}$$

Figure 2 exhibits the expectation of $\Delta\mathcal{D}$ using both the analysis in (21) and the simulation result with three users. To obtain the simulation curve in this figure, Monte Carlo simulations are carried out with the IA configurations in [6]. As can be seen, our analysis provides a good approximation in the wide range of the codebook bits B. A small mismatch is incurred as we simplify the distribution of β using (12).

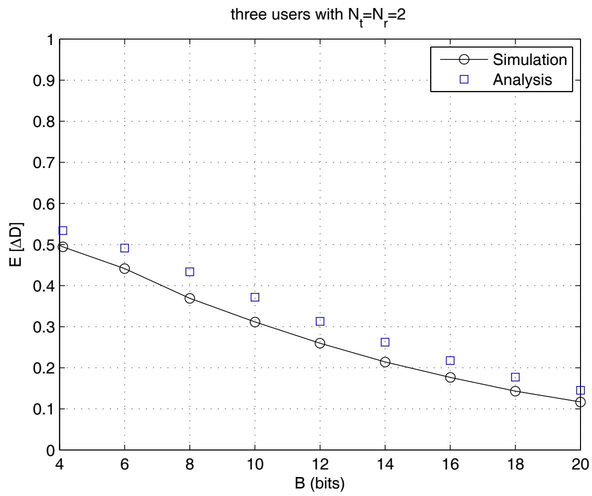

Fig. 2. $\mathbb{E}[\Delta\mathcal{D}]$ in 2×2 systems with $K=3$.

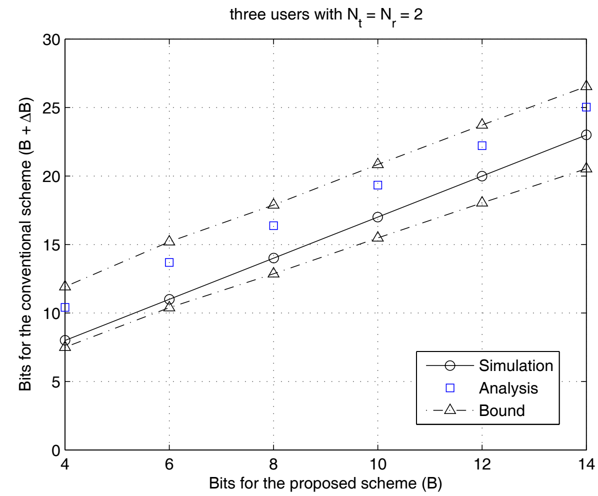

Fig. 3. Required number of bits in 2×2 systems with $K=3$.

By utilizing the result in (21), we can compute the number of bits ΔB additionally required for the conventional scheme to attain the minimum chordal distance of the proposed scheme when B is fixed. In other words, ΔB satisfies

$$\mathbb{E}\left[\sum_{l=1}^{K-1}\min_{j=1,\cdots,2^{\widehat{U}}}\left(1-\left|\mathbf{w}_i^{(l)\dagger}\tilde{\mathbf{c}}_j^{(l)}\right|^2\right)\right]= \mathbb{E}\left[\sum_{l=1}^{K-1}\min_{j=1,\cdots,2^{U}}\left(1-\left|\overline{\mathbf{w}}_i^{(l)\dagger}\tilde{\mathbf{c}}_j^{(l)}\right|^2\right)\right] \quad (22)$$

where $\widehat{U}=(B+\Delta B)/(K-1)$.

Denoting

$$\mathcal{E}(X)=\inf_{\widetilde{\mathcal{C}}_l}\mathbb{E}\left[\min_{j=1,\cdots,2^X}\left(1-\left|\mathbf{w}_i^{(l)\dagger}\tilde{\mathbf{c}}_j^{(l)}\right|^2\right)\right] \quad (23)$$

as the distortion rate function [26] for a uniformly distributed $\mathbf{w}_i^{(l)}$, we can obtain the relation

$$\mathcal{E}(U)-\frac{\mathbb{E}[\Delta\mathcal{D}]}{K-1}=\mathcal{E}(\widehat{U}) \quad (24)$$

from (9) and (22) with a sufficiently large B.

Since $\mathcal{E}(X)$ in (23) can be approximated as [25]

$$\mathcal{E}(X)\approx\left(\frac{N_tN_r-1}{N_tN_r}\right)2^{-\frac{X}{N_tN_r-1}}, \quad (25)$$

we can yield ΔB with some manipulations after applying (25) to (24) as

$$\Delta B \approx (K-1)(1-N_tN_r)\log_2\left(2^{-\frac{U}{N_tN_r-1}} -\frac{N_tN_r\mathbb{E}[\Delta\mathcal{D}]}{(N_tN_r-1)(K-1)}\right)-B. \quad (26)$$

Specifically, by employing the bounds of $\mathcal{E}(X)$ [26]

$$\frac{N_tN_r-1}{N_tN_r}\,2^{-\frac{X}{N_tN_r-1}}(1+o(1))\le\mathcal{E}(X) \le\frac{\Gamma\left(\frac{1}{N_tN_r-1}\right)}{N_tN_r-1}2^{-\frac{X}{N_tN_r-1}}(1+o(1)), \quad (27)$$

the bound of ΔB is given as

$$(K-1)(1-N_tN_r)\log_2\left(\frac{N_tN_r}{N_tN_r-1}\left(\frac{\Gamma\left(\frac{1}{N_tN_r-1}\right)}{N_tN_r-1}\times 2^{-\frac{U}{N_tN_r-1}}-\frac{\mathbb{E}[\Delta\mathcal{D}]}{K-1}\right)\right)-B\le\Delta B\le(K-1)(1-N_tN_r) \times\log_2\left(\frac{N_tN_r-1}{\Gamma\left(\frac{1}{N_tN_r-1}\right)}\left(\frac{N_tN_r-1}{N_tN_r}2^{-\frac{U}{N_tN_r-1}}-\frac{\mathbb{E}[\Delta\mathcal{D}]}{K-1}\right)\right)-B. \quad (28)$$

Here the residual terms $o(1)$ in (27) can be neglected, since the main order terms are usually sufficiently accurate to characterize the distortion rate function [26].

Figure 3 plots the total number of bits $B+\Delta B$ required for the conventional scheme with respect to that of our scheme B with $N_t=N_r=2$ and $K=3$. Both the analysis in (26) and the bounds in (28) well estimate the simulation curve. From this figure, we can see that the proposed method reduces a substantial number of feedback bits for various B. For instance, approximately 40% of the number of bits is saved by utilizing our scheme when $B=14$. In the next section, we will exhibit the sum rate enhancement of our scheme compared to the conventional method.

V. Simulation Results

In this section, we present the sum rate performance of the proposed scheme comparing with the conventional one for limited feedback IA systems through the Monte Carlo simulations. In simulations, we utilize a closed-form IA solution in [6] for the $K=3$ case, whereas an iterative IA scheme in [31] is used for the case of $K>3$. The number of the transmitted data streams is set to attain the maximum DOF [8]. That is, the data stream d_i for the i-th transmitter is set to satisfy $N_t+N_r-(K+1)d_i=0$ for all i. Also, the same transmission power constraint P is assumed for each transmitter, i.e., $\mathbb{E}[\mathbf{x}_i^\dagger\mathbf{x}_i]=P$ for all i, and the SNR is defined as P/N_0. The codewords in $\mathcal{C}$ are generated through RVQ, and flat Rayleigh fading channels are considered in simulations.

Now we verify a performance gain of our scheme over the conventional method with various numbers of B. Figure 4

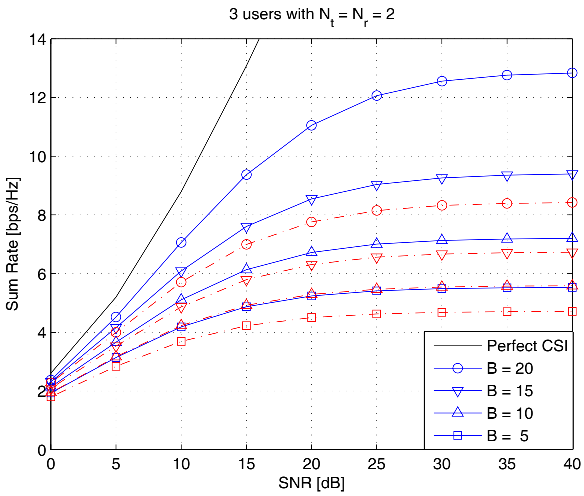

Fig. 4. Sum rate comparison in 2×2 IA systems with $K = 3$.

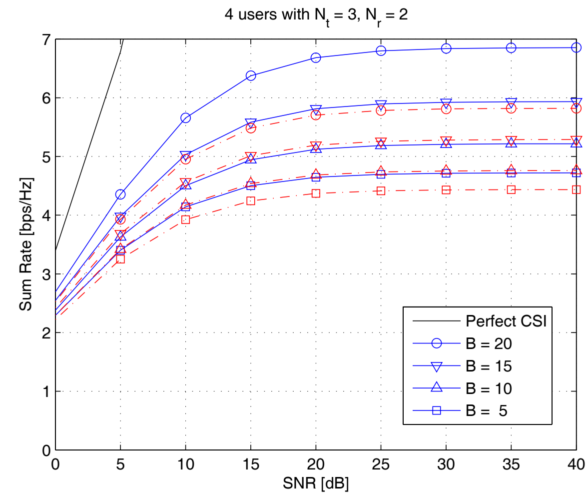

Fig. 6. Sum rate comparison in 3×2 IA systems with $K = 4$.

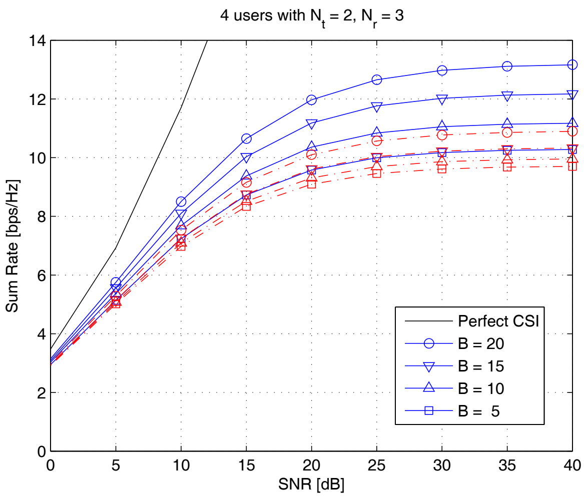

Fig. 5. Sum rate comparison in 2×3 IA systems with $K = 4$.

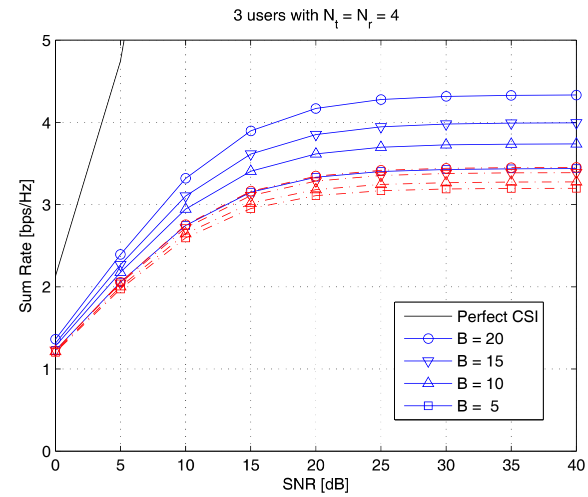

Fig. 7. Sum rate comparison in 4×4 IA systems with $K = 3$.

depicts the sum rate curves of the proposed scheme using solid lines when $N_t = N_r = 2$ and $K = 3$, while the curves of the conventional scheme are plotted with dash-dot lines. In this figure, we see that our method exhibits significantly enhanced performance compared to the conventional one for overall SNR regardless of the number of feedback bits. Especially, the performance gap between the proposed and the conventional scheme becomes larger as B increases. For instance, when $B = 15$ and 20, we can obtain a performance gain of 40% and 52%, respectively, for the proposed scheme at high SNR.

In Figure 5, we plot the sum rate curves when the number of users is increased to $K = 4$ with $N_t = 2$ and $N_r = 3$. In this case, unlike our proposed scheme, only a marginal gain is observed in the conventional method as B increases. This is because a reduction in the channel quantization error for the conventional scheme becomes less significant as the numbers of antennas and users increase. The proposed method shows a performance gain of 20% when $B = 20$ compared to the conventional scheme at high SNR.

For the comparison with a different system configuration, we depict the performance curves when the numbers of the antennas are set to $N_t = 3$ and $N_r = 2$ in Figure 6. The performance of both schemes under limited feedback is degraded in this figure compared to Figure 5, since incorrect alignment of interference caused by the imperfect CSI becomes severe as N_t increases with a fixed B. Nevertheless, our method exhibits a sum rate gain of 18% over the conventional scheme with $B = 20$ at high SNR.

Figure 7 illustrates the performance when the number of antennas is set to $N_t = N_r = 4$ with $K = 3$. When the number of the feedback bits is fixed, the performance in this case is lower than that with $N_t = N_r = 2$ and $K = 3$ shown in Figure 4. This is because more feedback bits are required with additional antennas in order to maintain the performance, according to the analysis in [12]. Still, our method exhibits a sum rate gain of 26% over the conventional scheme with $B = 20$ at high SNR. Again the performance gap between the conventional scheme and the proposed method becomes larger as the number of B increases. From these simulation results, we confirm that our scheme shows an improved performance compared to the conventional method regardless of the system parameters such as the numbers of antennas, feedback bits and the SNR value.

To further enhance the performance in both the conven-

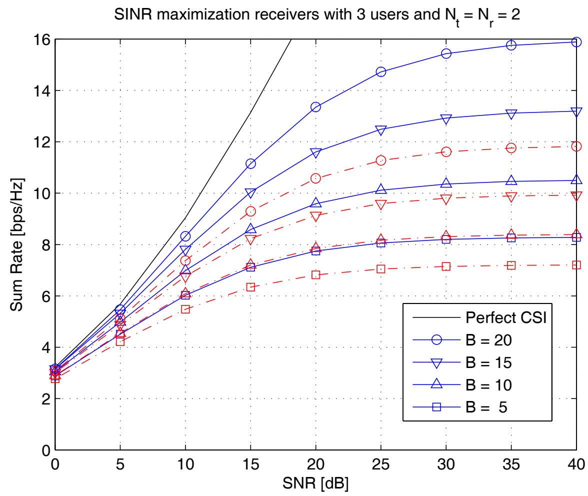

Fig. 8. Sum rate comparison in 2×2 IA systems with the SINR maximizing receivers and $K = 3$.

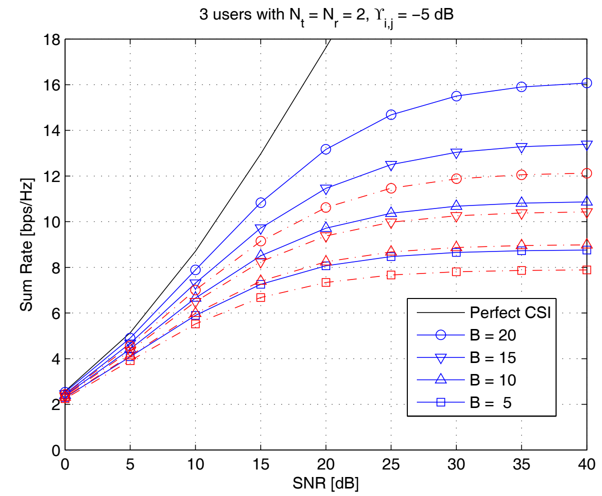

Fig. 10. Sum rate comparison in 2×2 IA systems for heterogeneous channels $\Upsilon_{i,j} = -5$ dB with $K = 3$.

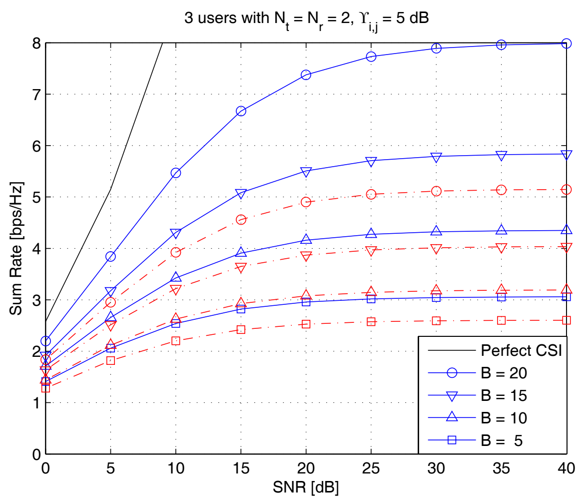

Fig. 9. Sum rate comparison in 2×2 IA systems for heterogeneous channels $\Upsilon_{i,j} = 5$ dB with $K = 3$.

tional and the proposed scheme, the receive combining matrix $\mathbf{R}_i$ in (2) at each receiver i can be designed to maximize signal-to-interference plus noise ratio (SINR) by considering interferences from other non-corresponding transmitters. A solution of $\mathbf{R}_i$ for this case is found by utilizing the SINR maximization results in [32]. In Figure 8, we plot the improved performance curves compared to those in Figure 4 when the SINR maximizing $\mathbf{R}_i$ is employed with $N_t = N_r = 2$ and $K = 3$. As can be seen, the performance gap between the conventional and the proposed scheme is still maintained similar to Figure 4. Our proposed method outperforms the conventional method by 33% at high SNR when $B = 15$. A similar trend is observed in various user and antenna configurations.

Finally, we consider the scenario where the channel gains from different transmitters are heterogeneous due to the presence of pathloss and shadowing. Denoting $\Upsilon_{i,j}$ as the average channel gain regarding $\mathbf{H}_{i,j}$ (i.e., $\mathbb{E}[\mathbf{h}_{i,j}^{(n)}\mathbf{h}_{i,j}^{(n)\dagger}] = \Upsilon_{i,j}\mathbf{I}_{N_r}, \ \forall n$) [33], we first suppose that at each receiver i the channel power $\Upsilon_{i,j}$ $(j \neq i)$ corresponding to the j-th transmitter is greater than that of its respective channel $\Upsilon_{i,i}$ as $\Upsilon_{i,j} = 5$ dB and $\Upsilon_{i,i} = 0$ dB. Figure 9 exhibits the performance curves of this case with $N_t = N_r = 2$ and $K = 3$. Our proposed scheme shows 45% and 55% gains of sum rates with $B = 15$ and 20, respectively, compared to the conventional method at high SNR in this figure. Also, Figure 10 plots the performance when the channel gains $\Upsilon_{i,j}$ are reduced to $\Upsilon_{i,j} = -5$ dB. In this figure, the proposed scheme still outperforms the conventional method by 28% and 33% with $B = 15$ and 20, respectively. These simulation results manifest that our scheme maintains improved performance compared to the conventional method even in heterogeneous channel environments.

VI. Conclusions

In this paper, we have proposed a new channel quantization algorithm for the MIMO IA with limited feedback. By introducing an additional receive filter at each receiver before quantizing the channels, we can minimize the chordal distance to reduce the imperfect CSI error. Unlike the filters for point-to-point MIMO systems, our proposed filter optimizes the quantization on the composite Grassmann manifold. Besides, we have analyzed the difference of the minimum chordal distance between the conventional scheme and the proposed method. From the analysis, we have verified that the quantization error is reduced in our scheme. Also, we have shown that the number of required bits to attain a reasonable performance can be substantially saved by utilizing our proposed method. As a result, the sum rate performance is enhanced about 50% by utilizing the proposed method compared to the conventional scheme at high SNR regime with 2×2 systems and $B = 20$. We confirm that the performance gap is increased as the number of feedback bits becomes larger.

References

[1] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, pp. 49–60, Jan. 1981.

[2] I. Sason, "On the achievable rate regions for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 50, pp. 1345–1356, Jan. 2004.

[3] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, pp. 5534–5562, Dec. 2008.

[4] S. A. Jafar and M. J. Fakhereddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 53, pp. 2637–2642, July 2007.

[5] S.-H. Park and I. Lee, "Degrees of freedom of multiple broadcast channels in the presence of inter-cell interference," *IEEE Trans. Commun.*, vol. 59, pp. 1481–1487, May 2011.

[6] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, pp. 3425–3441, Aug. 2008.

[7] H. Sung, S.-H. Park, K.-J. Lee, and I. Lee, "Linear precoder designs for K-user interference channels," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 291–301, Jan. 2010.

[8] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "On feasibility of interference alignment in MIMO interference networks," *IEEE Trans. Signal Process.*, vol. 58, pp. 4771–4782, Sept. 2010.

[9] M. Guillaud, "Receive diversity and ergodic performance of interference alignment on the MIMO Gaussian interference channel," arXiv:1006.4509, June 2010.

[10] G. Caire, S. A. Ramprashad, H. C. Papadopoulos, C. Pepin, and C. E. W. Sundberg, "Multiuser MIMO downlink with limited inter-cell cooperation: approximate interference alignment in time, frequency and space," in *Proc. 2008 IEEE Allerton Conference on Communication, Control, and Computing*, pp. 730–737.

[11] C. Suh, M. Ho, and D. Tse, "Downlink interference alignment," arXiv:1003.3707, May 2010.

[12] R. T. Krishnamachari and M. K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," arXiv:0911.5509, Nov. 2009.

[13] N. Ravindran and N. Jindal, "Limited feedback-based block diagonalization for the MIMO broadcast channel," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1473–1482, Oct. 2008.

[14] M. Trivellato, F. Boccardi, and H. Huang, "On transceiver design and channel quantization for downlink multiuser MIMO systems with limited feedback," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1494–1504, Oct. 2008.

[15] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, pp. 5045–5060, Nov. 2006.

[16] N. Jindal, "Antenna combining for the MIMO downlink channel," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 3834–3844, Oct. 2008.

[17] R. Tresch and M. Guillaud, "Cellular interference alignment with imperfect channel knowledge," in *Proc. IEEE ICC '09*, pp. 1–5.

[18] H. Bolcskei and I. J. Thukral, "Interference alignment with limited feedback," in *Proc. IEEE ISIT '09*, pp. 1759–1763.

[19] O. E. Ayach and R. W. Heath, "Interference alignment with analog channel state feedback," arXiv:1010.2787, Oct. 2010.

[20] R. Bhagavatula and R. W. Heath, "Adaptive limited feedback for sum-rate maximizing beamforming in cooperative multicell systems," arXiv:0912.0962, Dec. 2009.

[21] S. Akoum and R. W. Heath, "Limited feedback for temporally correlated MIMO channels with other cell interference," *IEEE Trans. Signal Process.*, vol. 58, pp. 5219–5232, Oct. 2010.

[22] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSI," arXiv:1101.5809, Jan. 2011.

[23] R. Chen, R. W. Heath, and J. G. Andrews, "A scalable limited feedback design for network MIMO using per-cell product codebook," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3093–3099, Oct. 2010.

[24] S. Zhou, J. Gong, and Z. Niu, "Distributed adaptation of quantized feedback for downlink network MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 61–67, Jan. 2011.

[25] B. Mondal, S. Dutta, and R. W. Heath, "Quantization on the Grassmann manifold," *IEEE Trans. Signal Process.*, vol. 55, pp. 4208–4216, Aug. 2007.

[26] W. Dai, Y. Liu, and B. Rider, "Quantization bounds on Grassmann manifolds and applications to MIMO communications," *IEEE Trans. Inf. Theory*, vol. 54, pp. 1108–1123, Mar. 2008.

[27] S. Gollakota, S. D. Perli, and D. Katabi, "Interference alignment and cancellation," in *Proc. ACM SIGCOMM 2009*, pp. 159–170, Aug. 2009.

[28] J. Kim, S.-H. Park, H. Sung, and I. Lee, "Spatial multiplexing gain for two interfering MIMO broadcast channels based on linear transceiver," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3012–3018, Oct. 2010.

[29] C. K. Au-Yeung and D. J. Love, "On the performance of random vector quantization limited feedback beamforming in a MISO system," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 458–462, Feb. 2007.

[30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition. Academic Press, 2007.

[31] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, pp. 3309–3322, June 2011.

[32] S.-J. Kim, A. Magnani, A. Mutapcic, S. P. Boyd, and Z.-Q. Luo, "Robust beamforming via worst-case SINR maximization," *IEEE Trans. Signal Process.*, vol. 56, pp. 1539–1547, Apr. 2008.

[33] F. Héliot, R. Hoshyar, and R. Tafazolli, "An accurate closed-form approximation of the distributed MIMO outage probability," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 5–11, Jan. 2011.

Jin-Sung Kim (S'09) received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea in 2006 and 2008, respectively. He is currently working toward the Ph.D. degree at Korea University, Seoul, Korea. During the summer in 2009, he visited University of Southern California, Los Angeles, CA, to conduct collaborative research under the Brain Korea 21 (BK21) Program. In April 2011, he has won the best graduate student award from BK21 Information Technology Center at Korea University. His research interests are signal processing techniques and interference mitigation algorithms for MIMO-OFDM systems.

Sung-Hyun Moon (S'05) received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea, in 2005 and 2007, where he is currently working toward the Ph.D. degree in the School of Electrical Engineering. During the spring in 2009, he visited University of Southern California, Los Angeles, CA, to conduct collaborative research under the Brain Korea 21 Program. His research interests include communication theory and signal processing techniques for multi-user MIMO wireless networks and interference management.

Sang-Rim Lee (S'06) received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea, in 2005 and 2007. From 2007 to 2010, he was a research engineer in the Samsung Electronics, where he conducted research on WiMAX system. Currently, he is working toward the Ph.D. degree at Korea University, Seoul, Korea. His research topics include communication theory and signal processing techniques for multi-user MIMO wireless networks and distributed antenna systems. He is currently interested in convex optimization, random matrix theory and stochastic geometry.

Inkyu Lee (S'92-M'95-SM'01) received the B.S. degree (Hon.) in control and instrumentation engineering from Seoul National University, Seoul, Korea, in 1990, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1992 and 1995, respectively. From 1995 to 2001, he was a Member of Technical Staff at Bell Laboratories, Lucent Technologies, where he conducted research on high-speed wireless system designs. He later worked for Agere Systems (formerly Microelectronics Group of Lucent Technologies), Murray Hill, NJ, as a Distinguished Member of Technical Staff from 2001 to 2002. In September 2002, he joined the faculty of Korea University, Seoul, Korea, where he is currently a Professor in the School of Electrical Engineering. During 2009, he visited University of Southern California, LA, USA, as a visiting Professor. He has published around 70 journal papers in IEEE, and has 30 U.S. patents granted or pending. His research interests include digital communications and signal processing techniques applied for next generation wireless systems. Dr. Lee currently serves as an Associate Editor for IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. Also, he has been a Chief Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (Special Issue on 4G Wireless Systems). He received the IT Young Engineer Award as the IEEE/IEEK joint award in 2006. Also he received the Best Paper Award at APCC in 2006 and IEEE VTC in 2009.