Degrees of Freedom for Mutually Interfering Broadcast Channels

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***Abstract*—In this paper, we study spatial degree-of-freedom (DOF) for two mutually interfering broadcast channels (IBC). As the demand for space-division multiple access (SDMA) increases, the IBC where each link has a single transmit node and multiple receive nodes becomes important. This paper presents the results of the lower and upper bounds on the DOF of the IBC. From the derived results, it is shown that for most cases, zero-forcing (ZF) beamforming can achieve the optimal DOF of the IBC except for some special cases. Also, we identify a condition that disabling receive cooperation in the multiple-input multiple-output interference channels causes no DOF loss. It is confirmed that we cannot expect a DOF improvement by enabling in-cell receive cooperation if in at least one of two BSs, the number of antennas is greater than or equal to that of users per cell. Furthermore, we observe a positive result that as the number of users goes to infinity, the total DOF of the IBC converges to the interference-free DOF, which is the maximum achievable DOF in the absence of the inter-cell interference.**

***Index Terms*—Beamforming, degrees of freedom, inter-cell interference, multi-user downlink, zero-forcing.**

I. Introduction

THERE have been many research activities to study the capacity of wireless multi-user networks. While the capacity analysis has been carried out mostly for systems where the transmitter or receiver can operate in a centralized mode such as multiple access channels (MAC) and broadcast channels (BC) [1]–[3], the capacity characterization of multi-point to multi-point communication systems is still an open problem [4].

Alternatively, degree-of-freedom (DOF)[1] has attracted a great deal of attention of many researchers, since the sum capacity of communication networks at high signal-to-noise ratio (SNR) regime is dominated by the DOF. The DOFs of various distributed wireless networks such as multiple-input multiple-output (MIMO) interference channels (IC) and MIMO X channels have been analyzed in [5]–[10]. Especially, the authors in [5] have derived the DOF of the MIMO IC for systems where each link has single transmit and receive node with both equipped with multiple antennas. However, in multi-cell[2] multi-user downlink channels where each link consists of a single transmit node and multiple receive nodes, each base station (BS) which supports multiple users within its cell suffers from inter-cell interference (ICI). As an initial step to identify the capacity in this scenario, the authors in [11] studied the performance of multicell processing and user cooperation for limited backhaul systems.

In this paper, we study the DOF for two mutually interfering broadcast channels (IBC) where the i-th BS equipped with M_i antennas $(i=1,2)$ transmits messages to its corresponding K_i single antenna users. We refer to the IBC with this configuration as (M_1, K_1, M_2, K_2) IBC. The IBC differs from the MIMO IC in a sense that K_i receive antennas in cell i are disconnected and cannot cooperate with each other. In a single-cell environment, whether receive cooperation exists or not, it does not affect the DOF, since the DOF of the point-to-point (PTP) MIMO channel with M transmit and K receive antennas is equal to that of the BC consisting of M base antennas and K single-antenna users, i.e., $\eta_{\rm PTP}(M,K) = \eta_{\rm BC}(M,K) = \min(M,K)$ [1], [2], [12], [13].

Similar to these results, we show that the DOF of the IBC is the same as that of the MIMO IC as long as $\max(M_1, M_2) \geq \min(K_1, K_2)$. In this case, it is interesting to see that the ZF beamforming can achive the optimal DOF. However, for the remaining case of $\max(M_1, M_2) < \min(K_1, K_2)$, the DOF with the ZF beamforming is strictly lower than that of the MIMO IC. Thus, we provide a tighter lower bound (LB) which can be achieved by the interference alignment scheme combined with the symbol extension[3] under the assumption of frequency-selective nature of the channel responses [14]–[16].

Also, by deriving the upper bound (UB) using the result of [14] and [15], it is shown that the IBC shows the DOF loss compared to MIMO IC as long as $K_{\min} > \frac{\max(2M, K_{\max})}{\max(2M, K_{\max})+1} 2M$ where $K_{\max}$ and $K_{\min}$ are defined as $K_{\max} = \max(K_1, K_2)$ and $K_{\min} = \min(K_1, K_2)$, respectively. Furthermore, we observe a positive result that as the number of users goes to infinity, the DOFs of both IBC and IC approach the interference-free DOF which is the maximum achievable DOF in the absence of inter-cell interference.

The remainder of this paper is organized as follows: In Section II, we introduce the IBC model and the earlier work in

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[1]In this paper, we refer to the multiplexing gain or capacity pre-log as degrees of freedom as in [5].

[2]It should be emphasized that it is straightforward to apply our discussion to multi-sector downlink channels by considering inter-sector interference as inter-cell interference.

[3]The symbol extension means a signaling over multiple frequency slots.

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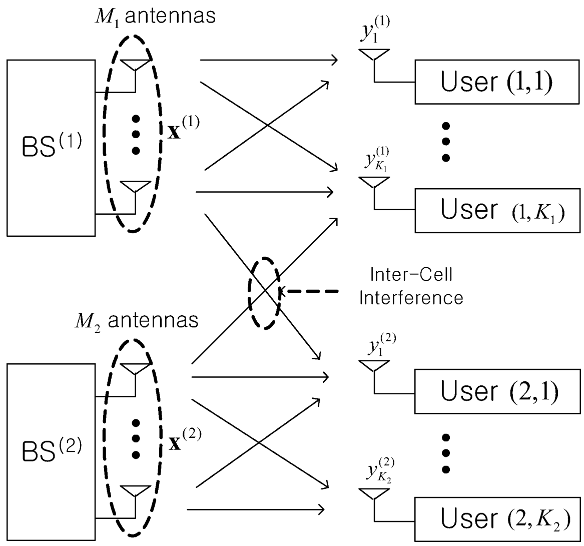

Fig. 1. (M_1, K_1, M_2, K_2) IBC channel.

[5], and review our results. Section III derives the achievable DOF with ZF beamforming and shows that the ZF beamforming attains the optimal DOF when $\max(M_1, M_2) \geq \min(K_1, K_2)$. In Section IV, we provide tighter lower and upper bounds for the case of $\max(M_1, M_2) < \min(K_1, K_2)$. In Section V, the IBC is compared to the MIMO IC in terms of the DOF and the achievable schemes. The paper is closed with conclusions in Section VI.

II. System Model

In this paper, the multi-cell multi-user downlink transmission is modeled as two mutually interfering BCs as illustrated in Fig. 1. There are two base stations BS$^{(1)}$ and BS$^{(2)}$, where BS$^{(i)}$ equipped with M_i antennas supports K_i single antenna users $(i = 1, 2)$. We refer to the l-th user in the i-th cell as user (i, l). Denoting $y_l^{(i)}(f)$ as the signal received by user (i, l) at the specific frequency slot f, the IBC is mathematically described as

$$y_l^{(i)}(f) = \mathbf{h}_l^{(i)}(f)\mathbf{x}^{(i)}(f) + \mathbf{z}_l^{(i)}(f)\mathbf{x}^{(\bar{i})}(f) \tag{1}$$

where $\mathbf{h}_l^{(i)}(f)$ denotes the channel vector of length M_i from BS$^{(i)}$ to user (i, l) and $\mathbf{z}_l^{(i)}(f)$ represents the inter-cell interference channel vector from BS$^{(\bar{i})}$ to user (i, l). Here we define $\bar{1} = 2$ and $\bar{2} = 1$. $\mathbf{x}^{(i)}(f)$ stands for the signal vector of length M_i transmitted from the BS$^{(i)}$ and $n_l^{(i)}(f)$ is the additive Gaussian noise for user (i, l) with variance N_0. It is assumed that the total channel matrix in (1) has full rank and varies at every channel use f. Also, all channel realizations are assumed to be perfectly known at all nodes. We notice that the channel use index f can equivalently be employed to indicate a time slot or a time-frequency tuple if coding is performed in both time and frequency [8]. Since each BS adopts its own amplifier, the per-BS power constraint $\frac{1}{N_c}\sum_{f=1}^{N_c} \|\mathbf{x}^{(i)}(f)\|^2 \leq P$ for $i = 1, 2$ is considered where N_c denotes the frame size over which the encoding is employed.

The spatial DOF is defined as

$$\eta \triangleq \lim_{\rho\to\infty} \frac{C_{\sum}(\rho)}{\log \rho} \tag{2}$$

where $C_{\sum}(\rho)$ is the sum capacity at SNR $\rho = P/N_0$ [5]. By definition, the DOF is equal to the multiplexing gain for a given system configuration. Throughout the paper, we denote the DOF of the IBC by $\eta_{\mathrm{IBC}}(M_1, K_1, M_2, K_2)$.

In [5], the DOF of the case where receive cooperation within each cell is available was proved to be

$$\eta_{\mathrm{IC}}(M_1, K_1, M_2, K_2) = \min\{M_1 + M_2, K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\}. \tag{3}$$

We refer to the MIMO IC with this configuration as the (M_1, K_1, M_2, K_2) IC. The authors in [5] showed the achievability with the ZF schemes and the converse was proved using the strong IC results. Since the DOF of the PTP MIMO system with M transmit and K receive antennas equals that of the BC consisting of a BS with M transmit antennas and K single antenna users, i.e., $\eta_{\mathrm{PTP}}(M, K) = \eta_{\mathrm{BC}}(M, K) = \min(M, K)$ [1], [2], [12], one might think that the DOF of the (M_1, K_1, M_2, K_2) IBC would be equal to (3).

In the following sections, we show that this is not the case for some special case. Our results are summarized in Table I. In the table, it is observed that η_{IBC} is strictly lower than η_{IC} whenever $K_{\min} > \frac{\max(2M, K_{\max})}{\max(2M, K_{\max})+1}2M$. Nonetheless, it is interesting to see that a DOF loss compared to the MIMO IC becomes negligible as $K_{\min}$ goes to infinity since both η_{IBC} and η_{IC} approach the interference-free DOF $2M$ which is the DOF achieved in the absence of inter-cell interference.

III. Achievable DOF With ZF Beamforming

In this section, we will verify that in the IBC described in Section II, the ZF beamforming can achieve the DOF of $\eta_{\mathrm{IBC-ZF}}(M_1, K_1, M_2, K_2)$ which is given as

$$\eta_{\mathrm{IBC-ZF}}(M_1, K_1, M_2, K_2) = \min\left\{\begin{array}{c}\max(M_1, M_2), K_1 + K_2,\\ \max(M_1, K_2), \max(M_2, K_1)\end{array}\right\}. \tag{4}$$

We notice that compared to $\eta_{\mathrm{IC}}(M_1, K_1, M_2, K_2)$ in (3), only the first argument in the minimum function is changed from $M_1 + M_2$ to $\max(M_1, M_2)$. Then, it will be shown that the ZF beamforming can achieve the optimal DOF in the (M_1, K_1, M_2, K_2) IBC as long as $\max(M_1, M_2) \geq \min(K_1, K_2)$.

A. DOF With ZF Beamforming

We first show that one can achieve the DOF of (4) with the ZF beamforming scheme in the (M_1, K_1, M_2, K_2) IBC. Through the following theorem, we provide the necessary and sufficient conditions for the achievable DOF pair (η_1, η_2) where η_i denotes the DOF associated with cell i. With some abuse of notation, we define $\bar{\eta}_i \triangleq \eta_{\bar{i}}$ to clarify presentation.

TABLE I
LOWER AND UPPER BOUNDS OF THE DOF FOR THE (M_1, K_1, M_2, K_2) IC AND IBC

	$\max(M_1, M_2) \geq \min(K_1, K_2)$	$\max(M_1, M_2) < \min(K_1, K_2)$
$\eta_{\rm IC}$ [5]	$\min\left\{\begin{array}{c} K_1+K_2, \\ \max(M_1,K_2), \\ \max(M_2,K_1)\end{array}\right\}$	$\min(K_{\min}, 2M)$ for $M_1 = M_2 = M$.
$\eta_{\rm IBC}$	$\min\left\{\begin{array}{c} K_1+K_2, \\ \max(M_1,K_2), \\ \max(M_2,K_1)\end{array}\right\}$	LB: $\frac{\lfloor \frac{K_{\min}}{M} \rfloor}{\lfloor \frac{K_{\min}}{M} \rfloor + 1} 2M$ UB: $\min\left\{\begin{array}{c} K_{\min}, \\ \frac{\max(2M, K_{\max})}{\max(2M, K_{\max})+1} 2M \end{array}\right\}$ for $M_1 = M_2 = M$.

Theorem 1: In the (M_1, K_1, M_2, K_2) IBC, the DOF pair (η_1, η_2) can be achieved with the ZF beamforming if and only if

$$0 \leq \eta_i \leq \eta_{{\rm UB},i}(\bar{\eta}_i), \quad \text{for } i = 1, 2 \tag{5}$$

where $\eta_{{\rm UB},i}(\bar{\eta}_i)$ is an upper bound on η_i for given $\bar{\eta}_i$ defined as

$$\begin{aligned}\eta_{{\rm UB},i}(\bar{\eta}_i) \triangleq\ & \min\{[M_i - \bar{\eta}_i]^+, K_i, [M_{\bar{i}} - \bar{\eta}_i]^+\} 1(\bar{\eta}_i \geq 1) \\ & + \min\{M_i, K_i\} 1(\bar{\eta}_i = 0).\end{aligned} \tag{6}$$

Here, $[a]^+ = \max(a, 0)$ and $1(\cdot)$ is the indicator function.

Proof: Suppose $i = 1$ without loss of generality. For the case of $\eta_2 = 0$ where cell 2 is not operating, the BS$^{(1)}$ can support $\min(M_1, K_1)$ users with no interference from cell 2. We will show that when cell 2 is operating (i.e., $\eta_2 \geq 1$), the number of users which the BS$^{(1)}$ can serve is reduced to $\min\{[M_1 - \eta_2]^+, K_1, M_2 - \eta_2\}$. In order for the BS$^{(1)}$ to support η_1 users while nulling the ICI leakage to η_2 users in cell 2, M_1 should be greater than or equal to $\eta_1 + \eta_2$ which leads to $\eta_1 \leq [M_1 - \eta_2]^+$. Similarly, the ICI from the BS$^{(2)}$ to η_1 users in cell 1 can be nullified only if $\eta_1 \leq M_2 - \eta_2$. By additionally taking a trivial condition $\eta_1 \leq K_1$ into consideration, the BS$^{(1)}$ can serve up to $\min\{[M_1 - \eta_2]^+, K_1, M_2 - \eta_2\}$ users with the ZF beamforming. ■

We are now ready to formulate the problem for finding the maximum achievable DOF with the ZF beamforming $\eta_{\rm IBC-ZF}(M_1, K_1, M_2, K_2)$ as follows:

$$\begin{aligned}\eta_{\rm IBC-ZF}(M_1, K_1, M_2, K_2) = \\ \max_{(\eta_1,\eta_2) \in S(M_1,K_1,M_2,K_2)} \{\eta_1 + \eta_2\}\end{aligned} \tag{7}$$

where $S(M_1, K_1, M_2, K_2)$ is the set of (η_1, η_2) pairs satisfying (5) as

$$S(M_1, K_1, M_2, K_2) = \{(\eta_1, \eta_2) \,|\, 0 \leq \eta_i \leq \eta_{{\rm UB},i}(\bar{\eta}_i),\ i = 1, 2\}.$$

The problem (7) can be further simplified. Note that the inequality $\bar{\eta}_i \leq \min(M_{\bar{i}}, K_{\bar{i}})$ always holds. Therefore, by setting $\bar{\eta}_i = \min(M_{\bar{i}}, K_{\bar{i}})$, the BS$^{(i)}$ can support at least $\eta_{{\rm LB},i}$ users regardless of $\bar{\eta}_i$ where $\eta_{{\rm LB},i}$ is defined as

$$\begin{aligned}\eta_{{\rm LB},i} &\triangleq \eta_{{\rm UB},i}(\min(M_{\bar{i}}, K_{\bar{i}})) \\ &= \min\{[M_i - \min(M_{\bar{i}}, K_{\bar{i}})]^+, K_i, M_{\bar{i}} - \min(M_{\bar{i}}, K_{\bar{i}})\}.\end{aligned}$$

Thus, we can reduce the size of the set $S(M_1, K_1, M_2, K_2)$ without any loss to

$$\begin{aligned}&S(M_1, K_1, M_2, K_2) \\ &= \{(\eta_1, \eta_2) \,|\, \eta_{{\rm LB},i} \leq \eta_i \leq \eta_{{\rm UB},i}(\bar{\eta}_i),\ i = 1, 2\}.\end{aligned}$$

Theorem 2: The solution of the maximization problem (7) is expressed as

$$\begin{aligned}&\eta_{\rm IBC-ZF}(M_1, K_1, M_2, K_2) = \\ &\min\left\{\begin{array}{c}\max(M_1, M_2), K_1 + K_2, \\ \max(M_1, K_2), \max(M_2, K_1)\end{array}\right\}\end{aligned}$$

which matches with (4).

Proof: We will show that the solution of the maximization problem (7) is given as (4). To this end, all possible cases of M_1, K_1, M_2, K_2 are classified into the following three cases.

1) $\underline{\min(M_1, M_2) \geq K_1 + K_2}$

In this case, it is easy to show that $\min\{[M_i - \bar{\eta}_i]^+, K_i, M_{\bar{i}} - \bar{\eta}_i\} = K_i$. Plugging this into (6) leads to $\eta_{{\rm UB},i}(\bar{\eta}_i) = K_i 1(\bar{\eta}_i \geq 1) + K_i 1(\bar{\eta}_i = 0) = K_i$. Thus, the BS$^{(i)}$ can serve all of K_i users regardless of $\bar{\eta}_i$. As a result, all $K_1 + K_2$ users can be served with the ZF beamforming. This result coincides with (4) since

$$\begin{aligned}&\min\{\max(M_1, M_2), K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\} \\ &\quad = K_1 + K_2 \quad \text{if } \min(M_1, M_2) \geq K_1 + K_2.\end{aligned}$$

2) $\underline{\max(M_1, M_2) \geq K_1 + K_2 > \min(M_1, M_2)}$

Without loss of generality, suppose $M_1 > M_2$ (i.e., $M_1 \geq K_1 + K_2 > M_2$). The lower bound $\eta_{{\rm LB},2}$ then becomes $\eta_{{\rm LB},2} = [M_2 - K_1]^+$. If $M_2 > K_1$ (i.e., $\eta_{{\rm LB},2} \geq 1$), the upper bound $\eta_{{\rm UB},1}(\eta_2)$ is simplified to

$$\eta_{{\rm UB},1}(\eta_2) = \min\{[M_1 - \eta_2]^+, K_1, M_2 - \eta_2\} = M_2 - \eta_2$$

which leads to $\eta_1 + \eta_2 \leq M_2$. Therefore, the maximum achievable DOF is M_2.

For $M_2 \leq K_1$ (i.e., $\eta_{{\rm LB},2} \geq 0$), $\eta_{{\rm UB},1}(\eta_2)$ includes the case of $\eta_2 = 0$ as

$$\eta_{{\rm UB},1}(\eta_2) = (M_2 - \eta_2) 1(\eta_2 \geq 1) + K_1 1(\eta_2 = 0). \tag{8}$$

In this case, the ZF beamforming is capable of supporting up to K_1 users by choosing $\eta_2 = 0$. Here, $\eta_2 = 0$ implies BS$^{(2)}$ being turned off.

Consequently, the maximum achievable DOF is $\max(M_2, K_1)$ if $M_1 \geq K_1 + K_2 > M_2$. These results match with (4) as

$$\begin{aligned}&\min\{\max(M_1, M_2), K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\} \\ &\quad = \max(M_2, K_1) \quad \text{when } M_1 \geq K_1 + K_2 > M_2.\end{aligned}$$

TABLE II
POSSIBLE CASES OF M_1, K_1, M_2, K_2 FOR $K_1+K_2>\max(M_1,M_2)$

Case 3-1	$K_1+K_2>M_1\geq M_2\geq\max(K_1,K_2)$
Case 3-2	$K_1+K_2>M_1\geq\max(K_1,K_2)\geq\min(K_1,K_2)\geq M_2$
Case 3-3	$\max(K_1,K_2)\geq M_1\geq M_2\geq\min(K_1,K_2)$
Case 3-4	$K_1+K_2>M_1\geq\max(K_1,K_2)\geq M_2\geq\min(K_1,K_2)$
Case 3-5	$\max(K_1,K_2)\geq M_1\geq\min(K_1,K_2)\geq M_2$
Case 3-6	$\min(K_1,K_2)\geq M_1\geq M_2$

3) $\underline{K_1+K_2>\max(M_1,M_2)}$

Assuming $M_1\geq M_2$ without loss of generality, all possible cases of M_1,K_1,M_2,K_2 can be partitioned into six cases as summarized in Table II. We will evaluate all six cases in the following.

3-1) $\underline{K_1+K_2>M_1\geq M_2\geq\max(K_1,K_2)}$

In this case, we have $\eta_{\text{LB},2}=M_2-K_1$ and the upper bound $\eta_{\text{UB},1}(\eta_2)$ becomes

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+K_1 1(\eta_2=0).$$

It is possible to achieve the DOF of up to M_2 by turning on the BS$^{(2)}$ (i.e., $\eta_2\geq 1$). This result coincides with (4) when $K_1+K_2>M_1\geq M_2\geq\max(K_1,K_2)$.

3-2) $\underline{K_1+K_2>M_1\geq\max(K_1,K_2)\geq\min(K_1,K_2)\geq M_2}$

For the case of 3-2, the upper bound $\eta_{\text{UB},1}(\eta_2)$ is simplified to

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+K_1 1(\eta_2=0).$$

By choosing $\eta_2=0$, we can obtain the maximum DOF of up to K_1. This accords with (4) when $K_1+K_2>M_1\geq\max(K_1,K_2)\geq\min(K_1,K_2)\geq M_2$.

3-3) $\underline{\max(K_1,K_2)\geq M_1\geq M_2\geq\min(K_1,K_2)}$

All possible cases of 3-3 can be classified into the following cases:

$$K_1\ \geq\ M_1\geq M_2\ \geq\ K_2 \tag{9}$$
$$K_2\ \geq\ M_1\geq M_2\ \geq\ K_1. \tag{10}$$

For the case of (9), the upper bound $\eta_{\text{UB},1}(\eta_2)$ becomes

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+M_1 1(\eta_2=0),$$

which gives the maximum achievable DOF of M_1 with $\eta_2=0$.

In contrast, for (10), we have

$$\eta_{\text{UB},2}(\eta_1)=[M_2-\eta_1]^+1(\eta_1\geq 1)+M_2 1(\eta_1=0).$$

Since $\eta_1\leq\min(M_1,K_1)=K_1\leq M_2$, $[M_2-\eta_1]^+=M_2-\eta_1$. Regardless of η_1, we can achieve the DOF of up to M_2.

For all cases of 3-3, we can express the maximum achievable DOF as $\min\{\max(M_1,K_2),\max(M_2,K_1)\}$. This result agrees with (4) when $\max(K_1,K_2)\geq M_1\geq M_2\geq\min(K_1,K_2)$.

3-4) $\underline{K_1+K_2>M_1\geq\max(K_1,K_2)\geq M_2\geq\min(K_1,K_2)}$

We partition all possible cases of 3-4 into the following cases:

$$K_1+K_2\ >\ M_1\ \geq\ K_1\ \geq\ M_2\ \geq\ K_2 \tag{11}$$
$$K_1+K_2\ >M_1\ \geq\ K_2\ \geq\ M_2\ \geq\ K_1. \tag{12}$$

In the case of (11), the upper bound $\eta_{\text{UB},1}(\eta_2)$ becomes

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+K_1 1(\eta_2=0)$$

which indicates that K_1 $(\geq\ M_2)$ users can be served with $\eta_2=0$.

Now consider the case of (12). Since we have the lower bound $\eta_{\text{LB},2}=M_2-K_1$, we get the upper bound $\eta_{\text{UB},1}(\eta_2)$ as

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+K_1 1(\eta_2=0).$$

Thus, by setting $\eta_2\geq 1$, we are able to achieve the DOF of up to M_2.

For all cases in (11) and (12), we can express the maximum achievable DOF with the ZF beamforming as $\min(\max(M_1,K_2),\max(M_2,K_1))$. This matches with (4) when $K_1+K_2>M_1\geq\max(K_1,K_2)\geq M_2\geq\min(K_1,K_2)$.

3-5) $\underline{\max(K_1,K_2)\geq M_1\geq\min(K_1,K_2)\geq M_2}$

All possible cases can be classified into the following cases:

$$K_1\ \geq\ M_1\ \geq\ K_2\ \geq\ M_2 \tag{13}$$
$$K_2\ \geq\ M_1\ \geq\ K_1\ \geq\ M_2. \tag{14}$$

For (13), the upper bound $\eta_{\text{UB},1}(\eta_2)$ is equal to

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+M_1 1(\eta_2=0)$$

which leads to the maximum achievable DOF of M_1 with $\eta_2=0$.

Similarly, in (14), the upper bound $\eta_{\text{UB},1}(\eta_2)$ reduces to

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+K_1 1(\eta_2=0).$$

Thus, we can achieve the DOF of up to K_1 with $\eta_2=0$.

For all cases of 3-5, the maximum achievable DOF is given by $\min(\max(M_1,K_2),\max(M_2,K_1))$ coinciding with (4) when $\max(K_1,K_2)\geq M_1\geq\min(K_1,K_2)\geq M_2$.

3-6) $\underline{\min(K_1,K_2)\geq M_1\geq M_2}$

The upper bound $\eta_{\text{UB},1}(\eta_2)$ is given by

$$\eta_{\text{UB},1}(\eta_2)=(M_2-\eta_2)1(\eta_2\geq 1)+M_1 1(\eta_2=0),$$

which yields the maximum achievable DOF of M_1. This result accords with (4) when $\min(K_1,K_2)\geq M_1\geq M_2$. ■

The proof for achievability of (4) with the ZF beamforming is now completed for all system configurations.

B. Achieving the Optimal DOF With ZF Beamforming

In this subsection, we show that the ZF beamforming achieves the optimal DOF of the IBC for most cases except for

$\max(M_1, M_2) < \min(K_1, K_2)$. Theorem 3 illustrates our key result in this section.

Theorem 3: The ZF beamforming achieves the optimal DOF in the (M_1, K_1, M_2, K_2) IBC, i.e.,

$$\eta_{\rm IBC}(M_1, K_1, M_2, K_2) = \eta_{\rm IBC-ZF}(M_1, K_1, M_2, K_2)$$

as long as $\max(M_1, M_2) \geq \min(K_1, K_2)$.

Proof: Consider the following inequalities:

$$\eta_{\rm IC} \geq \eta_{\rm IBC} \geq \eta_{\rm IBC-ZF},$$

where the first inequality comes from disabling receive cooperation and the second one is due to the restriction to the ZF beamforming. Thus, for the case of $\eta_{\rm IC} = \eta_{\rm IBC-ZF}$, we can say that the ZF beamforming achieves the optimal DOF in the (M_1, K_1, M_2, K_2) IBC. This means that the proof is completed by showing that if $\max(M_1, M_2) \geq \min(K_1, K_2)$, then $\eta_{\rm IC} = \eta_{\rm IBC}$.

To this end, we prove the contraposition which is described as follows:

$$\text{If } \eta_{\rm IC} > \eta_{\rm IBC-ZF}, \text{ then } \max(M_1, M_2) < \min(K_1, K_2). \tag{15}$$

We start from the fact that if

$$\max(M_1, M_2) \geq \min\{K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\}$$

then we have $\eta_{\rm IC} = \eta_{\rm IBC-ZF}$. Therefore, the inequality $\eta_{\rm IC} > \eta_{\rm IBC-ZF}$ implies that

$$\eta_{\rm IBC-ZF} = \max(M_1, M_2) < \min\{K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\} \tag{16}$$

which is true only when $\max(M_1, K_2) = K_2$. This is because

$$\max(M_1, M_2) \geq M_1 \geq \min\{K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\}$$

if $\max(M_1, K_2) = M_1$. Thus, (16) means $\max(M_1, K_2) = K_2$. Similarly, $\max(M_2, K_1) = K_1$. Then, we know that if (16) is true, the following statement is also true:

$$\max(M_1, M_2) < \min\{K_1 + K_2, K_2, K_1\} = \min(K_1, K_2).$$

Now, the contraposition (15) is proved.

Thus, the ZF beamforming evidently achieves the optimal DOF in the IBC as long as $\max(M_1, M_2) \geq \min(K_1, K_2)$. ■

As can be seen in the proof of Theorem 3, when we have $\max(M_1, M_2) \geq \min(K_1, K_2)$, the ZF scheme can achieve the optimal DOF of the IBC as well as the DOF of the (M_1, K_1, M_2, K_2) MIMO IC. For the remaining case of $\max(M_1, M_2) < \min(K_1, K_2)$, the DOF with the ZF beamforming is strictly lower than the DOF of MIMO IC since $\eta_{\rm IBC-ZF}$ and $\eta_{\rm IC}$ are given as

$$\eta_{\rm IBC-ZF} = \max(M_1, M_2) < \eta_{\rm IC} = \min\{M_1 + M_2, K_1, K_2\}.$$

In Section IV, we will derive tighter lower and upper bounds for the case of $\max(M_1, M_2) < \min(K_1, K_2)$.

C. Sum Rate Maximizing Beamforming

In this subsection, we present numerical results to confirm that in the IBC, the ZF beamforming can achieve the DOF identical to that of the IC only when $\max(M_1, M_2) \geq \min(K_1, K_2)$. To this end, a sum rate maximizing beamforming scheme is proposed under the restriction of linear beamforming combined with a single-user Gaussian code. Since the proposed scheme attempts to identify the beamforming vectors which maximize the sum rate performance by employing the zero-gradient-based linear beamforming schemes of [17] and [18] in the IBC environments, the DOF attained by the proposed beamforming should be better than or equal to that of the ZF beamforming.

For linear beamforming systems, the transmitted signal vector $\mathbf{x}^{(i)}$ in (1) is precoded as

$$\mathbf{x}^{(i)} = \sum_{l=1}^{K_i} \mathbf{w}_l^{(i)} u_l^{(i)} \tag{17}$$

where data symbol $u_l^{(i)} \sim CN(0,1)$ intended for user (i, l) is multiplied by the beamforming vector $\mathbf{w}_l^{(i)}$. The total power constraint which is looser than the per-BS constraint can be written as

$$\sum_{i=1}^{2} E\left\|\mathbf{x}^{(i)}\right\|^2 = \sum_{i=1}^{2}\sum_{l=1}^{K_i}\left\|\mathbf{w}_l^{(i)}\right\|^2 \leq P_t \tag{18}$$

where $P_t = 2P$. Note that relaxing the power constraint does not hurt the DOF.

With the linear beamforming in (17), the received signal of user (i, l) is expressed as

$$y_l^{(i)} = \mathbf{h}_l^{(i)}\mathbf{w}_l^{(i)}u_l^{(i)} + \mathbf{h}_l^{(i)}\sum_{k=1,k\neq l}^{K_i}\mathbf{w}_k^{(i)}u_k^{(i)} + \mathbf{z}_l^{(i)}\sum_{k=1}^{K_{\bar{i}}}\mathbf{w}_k^{(\bar{i})}u_k^{(\bar{i})} + n_l^{(i)}.$$

Notice that the third term indicates the ICI which was not considered in [17] and [18]. To solve the sum rate maximization problem in an unconstrained manner, we express each beamforming vector as $\mathbf{w}_l^{(i)} = \frac{\tilde{\mathbf{w}}_l^{(i)}}{\sqrt{\gamma}}$ where $\gamma = \left(\sum_{i=1}^{2}\sum_{l=1}^{K_i}\|\tilde{\mathbf{w}}_l^{(i)}\|^2\right)/P_t$ is introduced to satisfy (18).

For given $\tilde{\mathbf{w}}_l^{(i)}$'s, the individual rate of user (i, l), denoted by $R_l^{(i)}$, is given as shown in (19), at the bottom of the page. Our

$$R_l^{(i)} = \log\left(\frac{\gamma N_0 + \sum_{k=1}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2}{\gamma N_0 + \sum_{k=1,k\neq l}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2}\right). \tag{19}$$

cost function for maximization on the total sum rate is obtained by summing (19) over all users as $R_{\rm sum} = \sum_{i=1}^{2}\sum_{l=1}^{K_i} R_l^{(i)}$. Then, our problem can be formulated as shown in (20), at the bottom of the page. To solve the problem (20), we apply a zero-gradient condition introduced in [17] and [18]. If $\tilde{\mathbf{w}}_l^{(i)}$'s are solutions of (20), they should satisfy the zero-gradient condition as

$$\frac{\partial R_{\rm sum}}{\partial \tilde{\mathbf{w}}_m^{(j)}} = \mathbf{0} \quad \text{for all } j \text{ and } m. \tag{21}$$

Now, we show that the gradient $\frac{\partial R_{\rm sum}}{\partial \tilde{\mathbf{w}}_m^{(j)}}$ can be computed as a simple form derived in [18]. The gradient is obtained as

$$\begin{aligned}\frac{\partial R_{\rm sum}}{\partial \tilde{\mathbf{w}}_m^{(j)}} =& \sum_{(i,l)\in\pi} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{\gamma N_0 + \sum_{k=1}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2} \\ &- \sum_{(i,l)\neq(j,m)} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{\gamma N_0 + \sum_{k\neq l}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2} \\ &- \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)}}{\gamma N_0 + \sum_{k\neq m}\left|\mathbf{h}_m^{(j)}\tilde{\mathbf{w}}_k^{(j)}\right|^2 + \sum_{k=1}^{K_{\bar{j}}}\left|\mathbf{z}_m^{(j)}\tilde{\mathbf{w}}_k^{(\bar{j})}\right|^2}\end{aligned} \tag{22}$$

where $\pi = \{(1,1),\cdots,(1,K_1),(2,1),\cdots,(2,K_2)\}$ and $\mathbf{h}_l^{ij} = \mathbf{h}_l^{(i)}1(i=j) + \mathbf{z}_l^{(i)}1(i\neq j)$.

Defining $d_l^{(i)}$ as

$$d_l^{(i)} \triangleq \gamma N_0 + \sum_{k=1,k\neq l}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2$$

(22) can be written as

$$\begin{aligned}\frac{\partial R_{\rm sum}}{\partial \tilde{\mathbf{w}}_m^{(j)}} =& \sum_{(i,l)\in\pi} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{d_l^{(i)} + \left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_l^{(i)}\right|^2} \\ &- \sum_{(i,l)\neq(j,m)} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{d_l^{(i)}} - \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)}}{d_m^{(j)}}.\end{aligned} \tag{23}$$

Subtracting and adding $\frac{1}{d_m^{(j)}}\mathbf{h}_m^{jj\,\dagger}\mathbf{h}_m^{jj}\tilde{\mathbf{w}}_m^{(j)}$ to (23) results in

$$\begin{aligned}\frac{\partial R_{\rm sum}}{\partial \tilde{\mathbf{w}}_m^{(j)}} =& \sum_{(i,l)\in\pi} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{d_l^{(i)} + \left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_l^{(i)}\right|^2} \\ &- \sum_{(i,l)\in\pi} \frac{\frac{N_0}{P_t}\tilde{\mathbf{w}}_m^{(j)} + \mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}}{d_l^{(i)}} + \frac{1}{d_m^{(j)}}\mathbf{h}_m^{jj\,\dagger}\mathbf{h}_m^{jj}\tilde{\mathbf{w}}_m^{(j)} \\ =& \frac{1}{d_m^{(j)}}\mathbf{h}_m^{(j)\,\dagger}\mathbf{h}_m^{(j)}\tilde{\mathbf{w}}_m^{(j)} - \frac{N_0}{P_t}\sum_{(i,l)\in\pi}\tilde{d}_l^{(i)}\tilde{\mathbf{w}}_m^{(j)} \\ &- \sum_{(i,l)\in\pi}\tilde{d}_l^{(i)}\mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\tilde{\mathbf{w}}_m^{(j)}\end{aligned} \tag{24}$$

where $\tilde{d}_l^{(i)}$ is defined as

$$\tilde{d}_l^{(i)} \triangleq \frac{\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_l^{(i)}\right|^2}{d_l^{(i)}\left(d_l^{(i)} + \left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_l^{(i)}\right|^2\right)}.$$

Substituting (24) into the zero-gradient condition of (21) yields

$$\left(\frac{N_0}{P_t}\left(\sum_{(i,l)\in\pi}\tilde{d}_l^{(i)}\right)\mathbf{I} + \sum_{(i,l)\in\pi}\tilde{d}_l^{(i)}\mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\right)\tilde{\mathbf{w}}_m^{(j)} = \frac{1}{d_m^{(j)}}\mathbf{h}_m^{(j)\,\dagger}\mathbf{h}_m^{(j)}\tilde{\mathbf{w}}_m^{(j)}. \tag{25}$$

Since it is complicated to obtain the closed-form solution for (25) over all j and m, we propose the following iterative algorithm:

1) Initialize $\tilde{\mathbf{w}}_l^{(i)}$'s as arbitrary beamforming vectors.
2) Compute $d_l^{(i)}$ and $\tilde{d}_l^{(i)}$ for all i and l.
3) For all j and m, update $\tilde{\mathbf{w}}_m^{(j)}$'s as

$$\frac{1}{d_m^{(j)}}\left(\frac{N_0}{P_t}\left(\sum\nolimits_{(i,l)\in\pi}\tilde{d}_l^{(i)}\right)\mathbf{I} + \sum\nolimits_{(i,l)\in\pi}\tilde{d}_l^{(i)}\mathbf{h}_l^{ij\,\dagger}\mathbf{h}_l^{ij}\right)^{-1} \times \mathbf{h}_m^{(j)\,\dagger}\mathbf{h}_m^{(j)}\tilde{\mathbf{w}}_m^{(j)}.$$

4) Repeat until the sum rate converges.

With the proposed scheme, we have a non-decreasing objective value with respect to the number of iterations. The proof of this result is not described here, but can be directly obtained from Appendix II in [18]. Due to non-convexivity of the maximization problem in (20), the above algorithm cannot guarantee a global optimal solution. However, the probability of finding the global optimal solution can be increased by performing the proposed algorithm over multiple initial points and selecting the one corresponding to the maximum sum rate.

Notice that the proposed algorithm starts from initial multiplexing gain of $K_1 + K_2$ and the rank adaptation is automatically performed during iterations. Thus, we conjecture that the DOF achieved by the proposed sum rate maximizing beamforming guarantees the optimal DOF under the restriction of the

$$\max_{\tilde{\mathbf{w}}_l^{(i)}\text{'s}} \sum_{i=1}^{2}\sum_{l=1}^{K_i}\log\left(\frac{\gamma N_0 + \sum_{k=1}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2}{\gamma N_0 + \sum_{k=1,k\neq l}^{K_i}\left|\mathbf{h}_l^{(i)}\tilde{\mathbf{w}}_k^{(i)}\right|^2 + \sum_{k=1}^{K_{\bar{i}}}\left|\mathbf{z}_l^{(i)}\tilde{\mathbf{w}}_k^{(\bar{i})}\right|^2}\right) \tag{20}$$

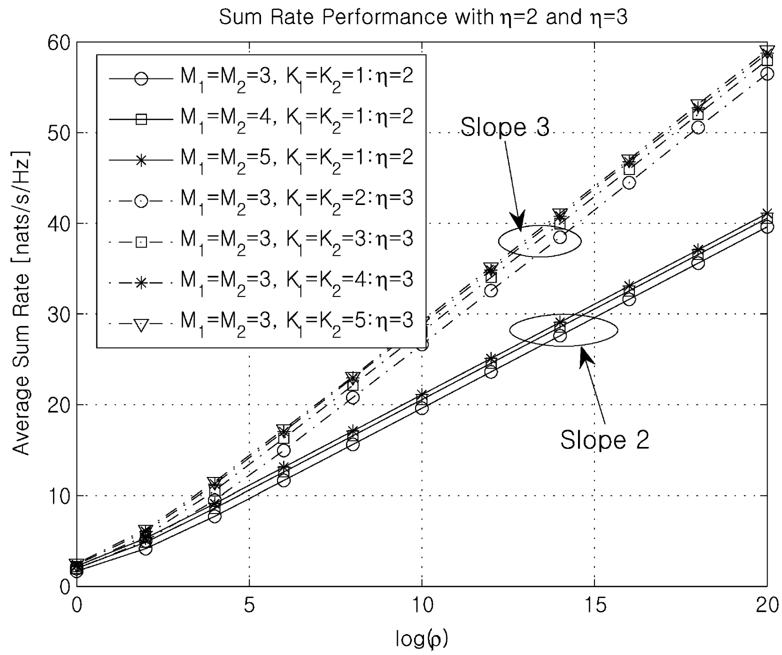

Fig. 2. Average sum rate performance with $\eta = 2$ and $\eta = 3$.

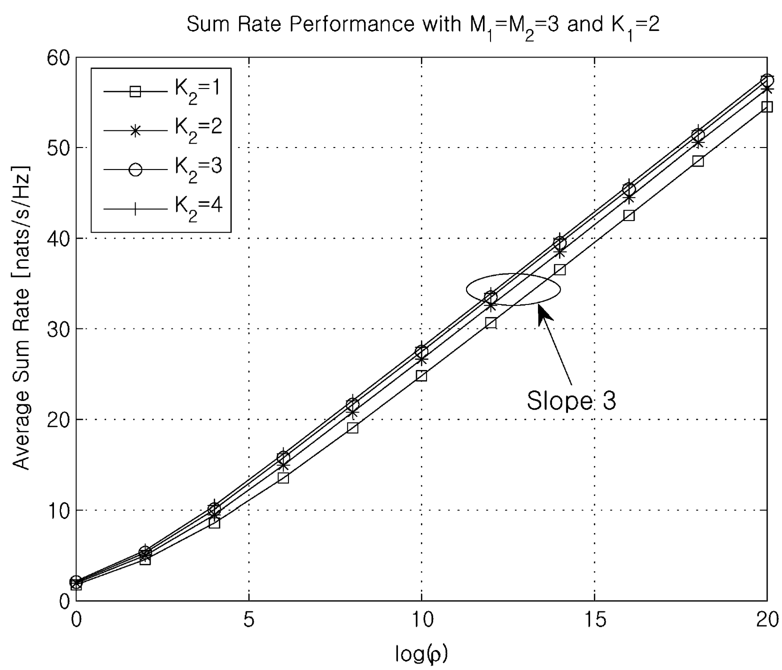

Fig. 3. Average sum rate performance with $M_1 = M_2 = 2$ and $K_1 = 2$.

linear beamforming combined with the single-user Gaussian coding if sufficiently many initial points are employed. As a result, the achievable DOF of the ZF beamforming cannot be greater than that of the proposed sum-rate maximizing scheme.

From now on, we provide numerical results evaluating the sum rate performance of the proposed beamforming scheme. Then, we will demonstrate that, for all simulated configurations, the DOF of the ZF beamforming is limited to $\eta_{\mathrm{IBC-ZF}}$ in (4). Fig. 2 plots the average sum rate curve as a function of $\log(\rho)$. According to (4), the solid lines correspond to the configurations with $\eta_{\mathrm{IBC}} = 2$ because $\eta_{\mathrm{IBC}}(n, 1, n, 1) = \min\{n, 2, n, n\} = 2$ for $n \geq 2$. As expected, these curves exhibit the slope of 2. Similarly, dashed lines corresponding to $\eta_{\mathrm{IBC}} = 3$ also grow with the slope of 3. For the case of $M_1 = M_2 = 3$ and $K_1 = K_2 > 3$, the DOF of 3 is observed while we can achieve the DOF of $K_1 = K_2$ in the MIMO IC systems.

Fig. 3 illustrates the sum rate performance for unsymmetric system settings of $M_1 = M_2 = 3$, $K_1 = 2$ and various K_2's. From the derived result, the DOF of the ZF beamforming should be $\eta_{\mathrm{IBC-ZF}} = \min(3, K_2 + 2) = 3$ for any $K_2 \geq 1$. It can be seen that the DOF of the proposed beamforming well matches with the derived DOF for the ZF beamforming.

IV. Tighter Bounds

From the result of the previous section, we get lower and upper bounds on $\eta_{\rm IBC}$ as

$$\eta_{\rm IBC-ZF} \le \eta_{\rm IBC} \le \eta_{\rm IC}.$$

Since these bounds meet with each other only when $\max(M_1, M_2) \ge \min(K_1, K_2)$, in this section, we derive tighter bounds for the case of $\max(M_1, M_2) < \min(K_1, K_2)$. For simplicity, we consider the case of $M_1 = M_2 = M$.

A. Lower Bound

In this subsection, we provide a lower bound on the DOF of the IBC which is tighter than $\eta_{\rm IBC-ZF} = M$ for $M < \min(K_1, K_2)$ in Theorem 4.

Theorem 4: For $M < \min(K_1, K_2)$, the DOF of the (M, K_1, M, K_2) IBC is lower bounded as

$$\eta_{\rm IBC}(M, K_1, M, K_2) \ge \frac{\lfloor \frac{K_{\min}}{M} \rfloor}{\lfloor \frac{K_{\min}}{M} \rfloor + 1} 2M.$$

Note that the above lower bound is always larger than or equal to $\eta_{\rm IBC-ZF} = M$.

Proof: We start with the following inequality:

$$\begin{aligned}\eta_{\rm IBC}(M, K_1, M, K_2) &\overset{(a)}{\ge} \eta_{\rm IBC}(M, K_{\min}, M, K_{\min}) \\ &\overset{(b)}{\ge} \eta_{\rm IBC}\left(M, \left\lfloor \frac{K_{\min}}{M} \right\rfloor M, M, \left\lfloor \frac{K_{\min}}{M} \right\rfloor M\right)\end{aligned}$$

where (a) comes from reducing the number of users and (b) is true since $K_{\min} = \frac{K_{\min}}{M} M \ge \lfloor \frac{K_{\min}}{M} \rfloor M$. Defining $\eta_{\rm IBC}((M \times K)^B)$ as the DOF of the IBC where there are B BCs interfering with each other and each BS equipped with M antennas supports its corresponding K single antenna users. Then, we have the inequality as

$$\begin{aligned}&\eta_{\rm IBC}\left(M, \lfloor \tfrac{K_{\min}}{M} \rfloor M, M, \lfloor \tfrac{K_{\min}}{M} \rfloor M\right) \\ &\ge \eta_{\rm IBC}\left(\left(1 \times \lfloor \tfrac{K_{\min}}{M} \rfloor\right)^{2M}\right)\end{aligned} \tag{26}$$

which is obtained by separating each BS with M antennas into M distributed antennas.

The right-hand side (RHS) of (26) is derived in [14], [15] as

$$\eta_{\rm IBC}\left(\left(1 \times \left\lfloor \frac{K_{\min}}{M} \right\rfloor\right)^{2M}\right) = \frac{\lfloor \frac{K_{\min}}{M} \rfloor}{\lfloor \frac{K_{\min}}{M} \rfloor + 1} 2M$$

which can be achieved by applying the interference alignment technique combined with symbol extension in frequency selective (or time-varying) channels. As a result, we get the lower bound of Theorem 4 and the proof is completed. ■

B. Upper Bound

For the case of $M < \min(K_1, K_2)$, a straightforward upper bound is $\eta_{\rm IC}(M, K_1, M, K_2) = \min\{2M, K_{\min}\}$. Now, we present tighter upper bound as summarized in Theorem 5.

Theorem 5: For $M < \min(K_1, K_2)$, the DOF of the (M, K_1, M, K_2) IBC is upper bounded as

$$\eta_{\rm IBC}(M, K_1, M, K_2) \le \min\left\{\frac{\max(2M, K_{\max})}{\max(2M, K_{\max}) + 1} 2M, K_{\min}\right\}.$$

The above upper bound is always less than or equal to $\eta_{\rm IC} = \min\{2M, K_{\min}\}$.

Proof: We first consider the case of $K_{\max} \ge 2M$. It always holds that

$$\eta_{\rm IBC}(M, K_1, M, K_2) \le \eta_{\rm IBC}(M, K_{\max}, M, K_{\max}).$$

Defining $\eta_{\rm IBC}((M \times (K, N))^B)$ as the DOF of the IBC where each BS with M transmit antennas serves its corresponding K users each with N receive antennas, $\eta_{\rm IBC}(M, K_{\max}, M, K_{\max})$ can be upper bounded as

$$\begin{aligned}\eta_{\rm IBC}(M, K_{\max}, M, K_{\max}) &= \eta_{\rm IBC}((M \times (K_{\max}, 1))^2) \\ &\le \eta_{\rm IBC}((M \times (K_{\max}, M))^2).\end{aligned} \tag{27}$$

From the results of [14] and [15], the RHS of (27) has an upper bound as

$$\eta_{\rm IBC}((M \times (K_{\max}, M))^2) \le \frac{K_{\max}}{K_{\max} + 1} 2M.$$

Thus, we can see that the DOF of the (M, K_1, M, K_2) IBC with $M < K_{\min}$ and $K_{\max} \ge 2M$ is upper bounded by $\frac{K_{\max}}{K_{\max}+1} 2M$.

Now, the case of $K_{\max} < 2M$ is investigated. Since both K_1 and K_2 are less than $2M$, the inequality $\eta_{\rm IBC}(M, K_1, M, K_2) \le \eta_{\rm IBC}((M \times (2M, M))^2)$ is satisfied. From [14] and [15], we have an inequality $\eta_{\rm IBC}((M \times (2M, M))^2) \le \frac{2M}{2M+1} 2M$. Thus, the DOF of the (M, K_1, M, K_2) IBC cannot be larger than $\frac{2M}{2M+1} 2M$ if $K_{\max} < 2M$.

In conclusion, the DOF of (M_1, K_1, M_2, K_2) IBC is upper bounded by $\frac{\max(2M, K_{\max})}{\max(2M, K_{\max})+1} 2M$. Combining the derived bound with the straightforward upper bound $\eta_{\rm IC} = K_{\min}$, we complete the proof. ■

V. Comparison to MIMO Interference Channels

In this section, we compare the derived result with the DOF of the (M_1, K_1, M_2, K_2) IC derived in [5]. To this end, we classify all possible cases of M_1, M_2, K_1, K_2 into the following two cases.

1) $\underline{\max(M_1, M_2) \ge \min(K_1, K_2)}$

In this case, as clarified in Section III, disabling receive cooperation in the (M_1, K_1, M_2, K_2) IC incurs no DOF loss since both $\eta_{\rm IC}$ and $\eta_{\rm IBC}$ are computed as $\min\{K_1 + K_2, \max(M_1, K_2), \max(M_2, K_1)\}$. Also, we can achieve the DOF by applying the ZF beamforming (possibly combined with the ZF receive processing) for both IC and IBC systems.

2) $\underline{\max(M_1, M_2) < \min(K_1, K_2)}$

Assuming $M_1 = M_2 = M$, the DOF of the (M, K_1, M, K_2) IC is computed as $\min(2M, K_{\min})$, while the upper and lower bound expressions for the IBC are given as

$$\begin{aligned}&\frac{\lfloor \frac{K_{\min}}{M} \rfloor}{\lfloor \frac{K_{\min}}{M} \rfloor + 1} 2M \le \eta_{\rm IBC}(M, K_1, M, K_2) \\ &\le \min\left\{\frac{\max(2M, K_{\max})}{\max(2M, K_{\max}) + 1} 2M, K_{\min}\right\}.\end{aligned}$$

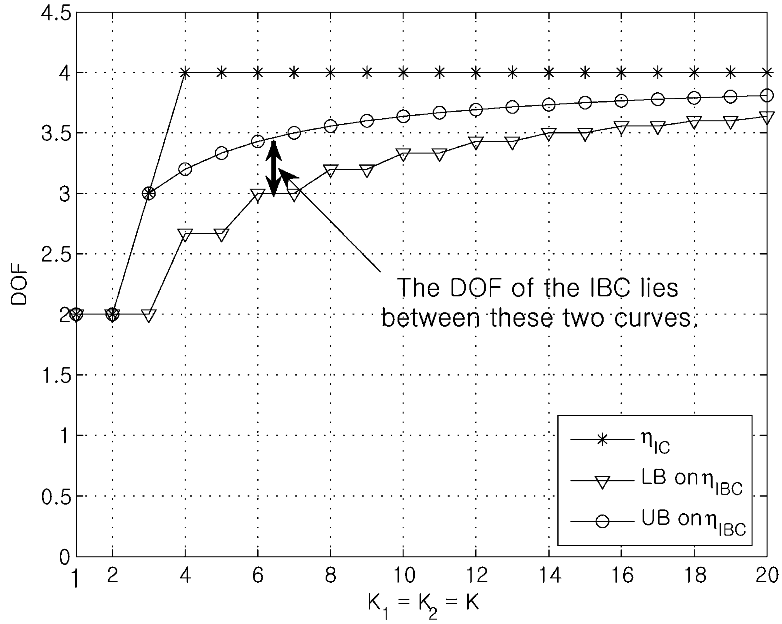

Fig. 4. DOF comparison between the (M, K_1, M, K_2) IC and IBC for $M = 2$ and $K_1 = K_2 = K$.

Thus, it is obvious that the DOF of the IBC is strictly lower than the (M, K_1, M, K_2) IC whenever both K_1 and K_2 are larger than $\frac{\max(2M, K_{\max})}{\max(2M, K_{\max})+1} 2M$. Nonetheless, surprisingly, it is observed that this DOF loss becomes negligible as $K_{\min}$ increases, since both $\eta_{\rm IBC}$ and $\eta_{\rm IC}$ converge to the interference-free DOF $2M$ which is the DOF in the absence of the inter-cell interference.

This pattern can be confirmed in Fig. 4 where the DOFs of the (M, K_1, M, K_2) IC and IBC are plotted as a function of $K_1 = K_2 = K$ for $M = 2$. The IBC and IC show the same DOF when $M \geq K$. As the number of users increases, the IBC exhibits a DOF loss compared to the IC. However, the DOF loss of the IBC becomes negligible as K further increases. Also, the achievable schemes of the IBC are different from those of the IC. For all combinations of M_1, K_2, M_2, K_2, we can achieve the optimal DOF with the ZF scheme in the (M_1, K_2, M_2, K_2) IC [5]. On the other hand, in the (M_1, K_2, M_2, K_2) IBC, the ZF achieves the optimal DOF only when $\max(M_1, M_2) \geq \min(K_1, K_2)$. This is because for fixed M_1 and M_2, the optimal DOF of the IBC continues to increase as K_1 and K_2 grows until it saturates to the interference-free DOF $2M$, while the interference mitigating capability of the ZF beamforming is limited by $\max(M_1, M_2)$. When there are users more than the number of base antennas, extra users do not lead to the increased DOF of the ZF beamforming since all interference control is performed at the transmit side. Thus, for a large number of users, we should apply more advanced schemes such as the interference alignment combined with the symbol extension to achieve the optimal DOF.

VI. Conclusion

Multiuser downlink channels in multi-cell environments can be modeled as two mutually interfering BCs. In this paper, we have analyzed the DOF of the IBC. From the derived results, it is shown that for most cases, ZF beamforming can achieve the optimal DOF of the IBC as well as the DOF of the MIMO IC which can be obtained by enabling in-cell receive cooperation in the IBC. Only for a special case where the number of users is larger than that of the base antennas, advanced techniques may be required to achieve the optimal DOF. It would be an interesting topic to derive the exact DOF for the case where the lower and upper bounds are not tight. Also, since this paper is limited to the single antenna receiver case, determining how we can express the achievable DOF in the IBC with multiple-antenna receivers would be challenging and meaningful.

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