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Characterization of effective capacity in AF relay systems

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Abstract: In this paper, effective capacity in amplify-and-forward (AF) relaying system is studied. For this purpose, a closed-form solution for the effective capacity of AF relays with a near optimal power allocation is extracted.

Keywords: effective capacity, power allocation, quality-of-service

Classification: Wireless circuits and devices

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1 Introduction

Cooperation is a promising approach which has attracted more interest in recent years. In this scheme, the intermediate terminals in the network, relay the received signals from the other nodes to their destinations. In the amplify-and-forward (AF) relays which is our concern here, the relay just amplifies its received signal with a variable gain, and transmits it to the destination [1].

Many applications such as video conferencing require low end-to-end delay. Once a delay requirement is violated, the corresponding data packet is discarded. In this regard, the interesting theory of effective capacity has been

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presented recently [2, 3]. Effective capacity is defined as the maximum constant arrival rate that a wireless channel can support in order to guarantee quality-of-service (QoS) requirements such as the queue length or the delay constraint.

Finding the effective capacity of the AF relays is the main contribution of this paper. By assuming the perfect channel state information (CSI) at the relay, an optimal power allocation is also suggested for the relay node. In this case, the relay adjusts the transmitted power in a way to maximize the effective capacity. However here, finding the optimal solution is difficult and time consuming, therefore, a near optimal power allocation is derived.

2 System model

A single-input single-output (SISO) AF relay system with a source node, a destination node and a relay node is considered. We ignore the weak direct source-destination link. Time division multiplexing is considered and at the source terminal, we further assume that an ideal adaptive modulation and coding (AMC) scheme is implemented at the physical layer. Therefore, data can leave the transmitter with the instantaneous capacity rate. A simple first-input first-output (FIFO) buffer is also assumed at the data link layer. Since the channel capacity is time varying, the service rate of the buffer is not constant. Hence, each frame needs to stay at the buffer for a while before transmission. For this reason, effective capacity is defined as the maximum constant arrival rate that a wireless channel can support in order to guarantee QoS requirements such as the delay constraint. The effective capacity concept is completely discussed in [2, 3]. Using the results of these papers, the effective capacity in the uncorrelated channel is written as

$$E_C(\theta) = -\frac{1}{\theta} \ln \left(E\left\{ e^{-\theta R} \right\} \right) \tag{1}$$

where R is the instantaneous channel capacity and θ denotes the QoS exponent. θ has an important role for the QoS guarantees. Larger θ corresponds to more strict QoS constraint, while smaller θ implies looser QoS guarantees.

An uncorrelated quasi-static Rayleigh fading channel with the average signal to noise ratio (SNR) γ_S and γ_R at the source-relay and relay-destination links is assumed. The total spectral bandwidth of the system is B and frame duration is denoted as T. Here, the overall instantaneous SNR at the destination node can be found as [1]

$$\gamma_{\mathrm{eq}} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1 + C} \tag{2}$$

where $\gamma_1 = \gamma_S |h|^2$, $\gamma_2 = \gamma_R |g|^2$, $C = 1$ is a constant, h and g denote the source-relay and relay-destination channel coefficients. To assist the tractability, we can assume $C = 0$ in (2), which is a proper assumption at the high SNR. For compressing, $\gamma_S = \gamma_R = \bar{\gamma}$ is also assumed. Finally, the cumulative distribution function (CDF) and the probability density function (PDF) of the SNR can be calculated as [1]

$$F_{\gamma_{\mathrm{eq}}}(x) = 1 - e^{-2x/\bar{\gamma}} \frac{2x}{\bar{\gamma}} K_1 \left(\frac{2x}{\bar{\gamma}} \right) \tag{3}$$

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$$f_{\gamma_{\mathrm{eq}}}(x) = e^{-2x/\bar{\gamma}} \frac{4x}{\bar{\gamma}^2} \left[K_1\left(\frac{2x}{\bar{\gamma}}\right) + K_0\left(\frac{2x}{\bar{\gamma}}\right) \right] \tag{4}$$

where $K_1(.)$ and $K_0(.)$ are the first and zero-order modified Bessel function of the first kind respectively [4, eq. 8.407].

3 The effective capacity in the AF relays

The instantaneous SNR and its related CDF and PDF without optimal power allocation are expressed in section 2 in equations (2), (3) and (4) respectively. By substituting (2) into the channel capacity $R = \frac{BT}{2}\log_2(1+\gamma_{\mathrm{eq}})$ and using the PDF in (4), the statistical average at the effective capacity expression is given by

$$\begin{aligned} E\ &\left\{(1+\gamma_{\mathrm{eq}})^{-\tilde{\theta}}\right\} \approx E\left\{(\gamma_{\mathrm{eq}})^{-\tilde{\theta}}\right\} = \int_0^\infty x^{-\tilde{\theta}} f_{\gamma_{\mathrm{eq}}}(x)dx \\ =\ & \frac{\sqrt{\pi}}{\bar{\gamma}}\left(\frac{4}{\bar{\gamma}}\right)^{\tilde{\theta}-1} \frac{1}{\Gamma\left(\frac{5}{2}-\tilde{\theta}\right)} \left[\Gamma(3-\tilde{\theta})\Gamma(2-\tilde{\theta})F\left(3-\tilde{\theta},\frac{3}{2};\frac{5}{2}-\tilde{\theta};0\right)\right. \\ +\ & \left.\Gamma(2-\tilde{\theta})\Gamma(2-\tilde{\theta})F\left(2-\tilde{\theta},\frac{1}{2};\frac{5}{2}-\tilde{\theta};0\right)\right] \end{aligned} \tag{5}$$

where $F(.,.;.;.)$ represents Gauss hypergeometric function [4, eq. 9.10], $\Gamma(.)$ denotes the Gamma function, $\tilde{\theta} = \theta BT/(2\ln 2)$ and the integral in (5) has a solution using [4, eq. 6.621-3]. After that, the effective capacity can be found using (1).

Now the optimal power allocation is considered in order to maximize the effective capacity. The instantaneous SNR without power allocation is defined in (2). When the optimal power allocation is the goal, it can be written as

$$\tilde{\gamma}_{\mathrm{eq}} = \frac{\gamma_1\gamma_2\mu_0}{\gamma_2\mu_0+\gamma_1+C} \tag{6}$$

where $\mu_0 \geq 0$ is the power allocation coefficient at the relay node and $E\{\mu_0\} = 1$ is assumed for the constant average transmitted power from the relay. An optimization problem can be arranged to find μ_0 as follows:

$$\begin{aligned} \mu_0 &= \arg \max_{\mu_0\geq 0, E\{\mu_0\}=1} \left(-\frac{1}{\theta}\ln E\left\{(1+\tilde{\gamma}_{\mathrm{eq}})^{-\tilde{\theta}}\right\}\right) \\ &= \arg \min_{\mu_0\geq 0, E\{\mu_0\}=1} E\left\{\left(1+\frac{\gamma_1\gamma_2\mu_0}{\gamma_2\mu_0+\gamma_1+C}\right)^{-\tilde{\theta}}\right\}. \end{aligned} \tag{7}$$

Using the Lagrangian optimization method, μ_0 is the solution of

$$\left(\frac{1}{\gamma_0}\right)^{\frac{1}{1+\tilde{\theta}}} \left(\frac{\gamma_S\gamma_R|h|^2|g|^2\left(\gamma_S|h|^2+1\right)}{\left(\gamma_S|h|^2+\gamma_R|g|^2\mu_0+1\right)^2}\right)^{\frac{1}{1+\tilde{\theta}}} - \frac{\gamma_S\gamma_R|h|^2|g|^2\mu_0}{\gamma_S|h|^2+\gamma_R|g|^2\mu_0+1} = 1,\ \ \mu_0 \geq 0 \tag{8}$$

where γ_0 is a cutoff SNR threshold which can be obtained from the mean power constraint $E\{\mu_0\} = 1$. Since μ_0 depends implicitly on the cutoff threshold γ_0, and γ_0 also depends on the distribution of μ_0 and its average, therefore, finding the optimal solution from (8) is difficult and time consuming, even numerically. A similar optimization problem is defined in [5] to

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maximize the effective capacity based on the allocated power at the source and relay nodes simultaneously. In this paper, the optimal allocated power at the relay is our concern. Our assumptions and method for solving the optimization problem are different from [5]. Consequently, the obtained results are also different.

Since finding the optimal solution is difficult, finding a suboptimal solution is advantageous. Hence, the optimization problem is redefined here as

$$\begin{aligned}\mu_0 &= \arg \min_{\mu_0 \geq 0, E\{\mu_0\}=1} E\left\{\left(1+\frac{\gamma_1\gamma_2\mu_0}{\gamma_2\mu_0+\gamma_1+C}\right)^{-\tilde{\theta}}\right\} \\ &\approx \arg \min_{\mu_0 \geq 0, E\{\mu_0\}=1} E\left\{\left(1+\frac{\gamma_1\gamma_2\mu_0}{\gamma_2\times 1+\gamma_1+C}\right)^{-\tilde{\theta}}\right\} \\ &= \arg \min_{\mu_0 \geq 0, E\{\mu_0\}=1} E\left\{(1+\mu_0\gamma_{\mathrm{eq}})^{-\tilde{\theta}}\right\} \end{aligned} \tag{9}$$

where μ_0 is replaced by its average value, one, at the denominator. The simulation results show that the standard deviation of μ_0 around its average value is not large. Therefore, approximation of μ_0 by its average $E\{\mu_0\} = 1$ is accurate. This is also presented at Fig. 1. Using the Lagrangian optimization method again, this new problem has a closed-form solution which is expressed as

$$\mu_0 = \begin{cases} 0 & , \gamma_{\mathrm{eq}} < \gamma_0 \\ \left(\frac{1}{\gamma_0}\right)^{\frac{1}{1+\tilde{\theta}}}\left(\frac{1}{\gamma_{\mathrm{eq}}}\right)^{\frac{\tilde{\theta}}{1+\tilde{\theta}}} - \frac{1}{\gamma_{\mathrm{eq}}} & , \gamma_{\mathrm{eq}} \geq \gamma_0 \end{cases} \tag{10}$$

where γ_0 is a cutoff SNR threshold which can be obtained from the mean power constraint $E\{\mu_0\} = 1$. Now, it is possible to find a closed-form solution for the average of μ_0. Fortunately, in this case we have

$$\begin{aligned} E\{\mu_0\} = \int_{\gamma_0}^{\infty}\left[\left(\frac{1}{\gamma_0}\right)^{\frac{1}{1+\tilde{\theta}}}\left(\frac{1}{x}\right)^{\frac{\tilde{\theta}}{1+\tilde{\theta}}} - \frac{1}{x}\right] f_{\gamma_{\mathrm{eq}}}(x)dx = 1 \\ = \frac{\sqrt{\pi}}{\bar{\gamma}}\left(\frac{4\gamma_0}{\bar{\gamma}}\right)^{\frac{\tilde{\theta}}{1+\tilde{\theta}}}\left\{G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix} 0, \frac{1}{2}+\frac{1}{1+\tilde{\theta}} \\ -1, 1+\frac{1}{1+\tilde{\theta}}, -1+\frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right. \\ \left. + G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix} 0, \frac{1}{2}+\frac{1}{1+\tilde{\theta}} \\ -1, \frac{1}{1+\tilde{\theta}}, \frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right\} - \frac{\sqrt{\pi}}{\bar{\gamma}}\left(\frac{4\gamma_0}{\bar{\gamma}}\right) \\ \times \left\{G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix} 0, \frac{1}{2} \\ -1, 1, -1\end{matrix}\right) + G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix} 0, \frac{1}{2} \\ -1, 0, 0\end{matrix}\right)\right\} = 1 \end{aligned} \tag{11}$$

where $G_{p,q}^{m,n}(.)$ is the Meijer's G function defined in [4, eq. 9.301] which is easily evaluated using the popular numerical softwares. The integral in (11) can be found with the help of [4, eq. 6.625-7]. Note that, although γ_0 does not have a closed-form solution using (11), but, obtaining γ_0 numerically from (11), is much simpler than finding it directly from its general integral equation $E\{\mu_0\} = 1$.

Now, in the suboptimal case, we have $R = \frac{BT}{2}\log_2(1+\mu_0\gamma_{\mathrm{eq}})$ for the channel capacity. Substituting it into (1), the effective capacity can be determined. Using (4) for the PDF of γ_{eq}, the statistical average of the effective

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capacity is evaluated as

$$E\left\{(1+\mu_0\gamma_{\rm eq})^{-\tilde{\theta}}\right\} = \underbrace{\int_0^{\gamma_0} f_{\gamma_{\rm eq}}(x)dx}_{I_1} + \underbrace{\int_{\gamma_0}^{\infty} (x/\gamma_0)^{-\tilde{\theta}/(1+\tilde{\theta})} f_{\gamma_{\rm eq}}(x)dx}_{I_2}. \quad (12)$$

For solving I_1, it can be changed to two infinite integrals as

$$\begin{aligned} I_1 &= \underbrace{\int_0^{\infty} f_{\gamma_{\rm eq}}(x)dx}_{I_3} - \underbrace{\int_{\gamma_0}^{\infty} f_{\gamma_{\rm eq}}(x)dx}_{I_4} \\ &= \frac{\sqrt{\pi}}{2\Gamma(5/2)}\left[F(3,3/2,5/2;0) + \frac{1}{2}F(2,1/2,5/2;0)\right] \\ &- \frac{\sqrt{\pi}\gamma_0}{\bar{\gamma}}\left[G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,3/2\\-1,2,0\end{matrix}\right) + G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,3/2\\-1,1,1\end{matrix}\right)\right] \quad (13) \end{aligned}$$

where the closed-form solution of I_3 and I_4 is possible using [4, eq. 6.621-3] and [4, eq. 6.625-7], respectively. Once again [4, eq. 6.625-7] can be used for finding I_2. Therefore, we have

$$\begin{aligned} I_2 &= \frac{\sqrt{\pi}}{4}\left(\frac{4\gamma_0}{\bar{\gamma}}\right)^{\frac{1+2\tilde{\theta}}{1+\tilde{\theta}}}\left[G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,\frac{1}{2}+\frac{1}{1+\tilde{\theta}}\\-1,1+\frac{1}{1+\tilde{\theta}},-1+\frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right. \\ &+ \left. G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,\frac{1}{2}+\frac{1}{1+\tilde{\theta}}\\-1,\frac{1}{1+\tilde{\theta}},\frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right]. \quad (14) \end{aligned}$$

Now, connecting the obtained results for I_1 and I_2, we have

$$\begin{aligned} E_C(\theta) &= -\frac{1}{\theta}\ln\left\{\frac{\sqrt{\pi}}{2\Gamma(5/2)}\left[F(3,3/2,5/2;0) + \frac{1}{2}F(2,1/2,5/2;0)\right]\right. \\ &- \frac{\sqrt{\pi}\gamma_0}{\bar{\gamma}}\left[G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,3/2\\-1,2,0\end{matrix}\right) + G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,3/2\\-1,1,1\end{matrix}\right)\right] \\ &+ \frac{\sqrt{\pi}}{4}\left(\frac{4\gamma_0}{\bar{\gamma}}\right)^{\frac{1+2\tilde{\theta}}{1+\tilde{\theta}}}\left[G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,\frac{1}{2}+\frac{1}{1+\tilde{\theta}}\\-1,1+\frac{1}{1+\tilde{\theta}},-1+\frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right. \\ &+ \left.\left. G_{23}^{30}\left(\frac{4\gamma_0}{\bar{\gamma}}\middle|\begin{matrix}0,\frac{1}{2}+\frac{1}{1+\tilde{\theta}}\\-1,\frac{1}{1+\tilde{\theta}},\frac{1}{1+\tilde{\theta}}\end{matrix}\right)\right]\right\}. \quad (15) \end{aligned}$$

4 Simulation results

We assume $B = 100\,\mathrm{KHz}$, $T = 2\,\mathrm{msec}$, and in each average SNR, $\gamma_S = \gamma_R = \bar{\gamma}$ and the Mont-Carlo simulation is repeated 1,000,000 times. Note that for a simple representation, the normalized effective capacity $E_C(\theta)/(BT/2)$ is plotted. In a special case where $\tilde{\theta} = 0.01BT/2$, the effective capacity of a relay with the optimal allocated power is calculated numerically and compared with the near optimal solution in Fig. 1. It can be observed that the near optimal solution follows the optimal one and the gap between solutions is very small. The effective capacity with the near optimal allocated power of [5] is also plotted in this figure. An improvement in our results can be

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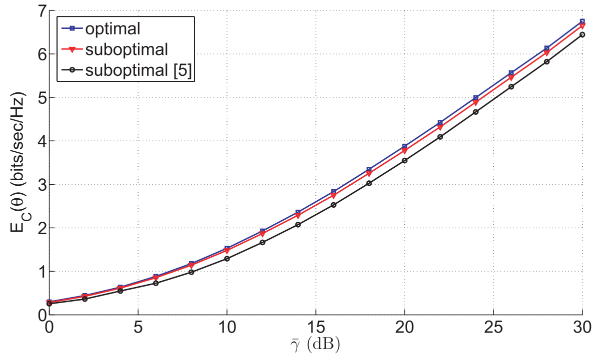

Fig. 1. The effective capacity with the optimal and near optimal power allocation.

seen when it compares with the results of [5]. The effective capacity with the suboptimal power allocation is plotted in Fig. 2 versus the QoS exponent θ in two different average SNRs, 10 dB and 20 dB. For comparison, the effective capacity without the optimal power allocation is also presented. In Fig. 2, the advantage of the power allocation can be observed. We have gains of 1.68 bits/sec/Hz and 0.4 bits/sec/Hz at $\bar{\gamma} = 20$ dB and $\bar{\gamma} = 10$ dB where $\theta = 0.1$. Therefore, power allocation is specially recommended when more strict QoS is required where it can extensively improve the entire performance. The gap between suboptimal and constant power assignment $\mu_0 = 1$ becomes larger when the average SNR increases.

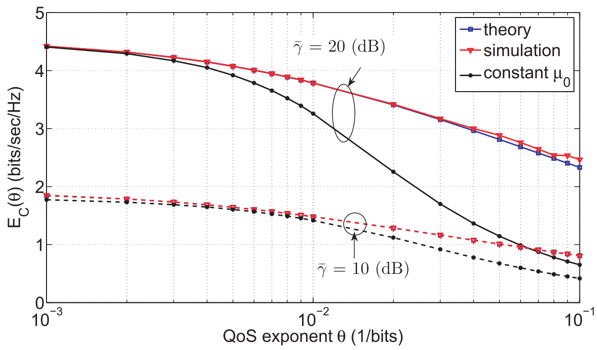

Fig. 2. The effective capacity with the suboptimal power allocation.

5 Conclusion

In this study, the effective capacity in the AF relay systems is proposed. Effective capacity is a cross-layer subject depends on the physical and data link layer which specifies a channel rate with the QoS guarantees. For performance enhancement, a near optimal power allocation is suggested.

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