Quasi-Orthogonal STBC with Iterative Decoding in Bit Interleaved Coded Modulation

Chang-Kyung Sung, Jihoon Kim, and Inkyu Lee
Dept. of Communication Engineering
Korea University
Seoul, Korea
cksung@korea.ac.kr, realidku@wireless.korea.ac.kr, inkyu@korea.ac.kr

***Abstract*— In this paper, we present a method to improve the performance of the four transmit antenna quasi-orthogonal space-time block code (STBC) in the coded system. For the four transmit antenna case, the quasi-orthogonal STBC consists of two symbol groups which are orthogonal to each other, but intra group symbols are not. In uncoded system with the matched filter detection, constellation rotation can improve the performance. However, in coded systems, its gain is absorbed by the coding gain especially for lower rate code. We propose an iterative decoding method to improve the performance of quasi-orthogonal codes in coded systems. With conventional quasi-orthogonal STBC detection, the joint ML detection can be improved by iterative processing between the demapper and the decoder. Simulation results shows that the performance improvement is about 2dB at 1% frame error rate.**

I. Introduction

For next generation wireless communications, wideband channel environments are commonly expected. In such a channel, several impairments caused by fading channels make it hard to transmit user data reliably with desired throughput. Multiple antenna system is one of the most attractive solutions for such an environment. It has been shown [1] that MIMO antenna systems provide multiple independent channel, and thus, the channel capacity increases linearly with the number of antennas.

There are two approaches for multiple antenna systems. One approach is to achieve the transmit diversity through space time coding (STC) [2] and the other is to increase the throughput using spatial division multiplexing (SDM). The STC mitigates fading through the spatial diversity by using multiple transmit and receive antennas combined with matching modulation and coding whereas the SDM increases link capacity by transmitting independent information streams. In practical wireless communication systems where the number of the receive antenna is more restricted than that of the transmit antenna, STC techniques can be directly adopted.

In this paper, we present a method to improve the performance of the quasi-orthogonal space-time block code (STBC) with four transmit antennas in a coded system. The quasi-orthogonal STBC is first introduced by Jafarkhani [3]. When employing four transmit antennas, the hybrid STBC scheme proposed in [4] outperforms the quasi-orthogonal STBC at 1% FER with relative low complexity if the number of the receive antenna is more than one. However in one receive antenna case, the hybrid STBC fails to work since the equivalent channel matrix does not have full rank.

Our design goal is to improve the quasi-orthogonal STBC in single antenna case although our solution also works for multiple receive antennas. The quasi-orthogonal STBC consists of two symbol groups which are orthogonal to each other, but intra group symbols have no orthogonality. In uncoded systems, the constellation rotation method proposed at [5] can improve the system performance for detecting symbols. Increased minimum Euclidean distances within inflated constellation by constellation rotation reduces the symbol error probability [5]. However, its gain is diminished in coded system because of error correcting capabilities of channel codes.

In this paper, we propose an iterative decoding method for improving the quasi-orthogonal STBC in coded systems. Among various ways to employing channel codes to combat impairments caused by fading and additive noises, we adopt bit-interleaved coded modulation (BICM) [6] for our scheme.

This paper is organized as follows: In section II, we describe the system model of the quasi-orthogonal STBC and combine with BICM. In section III, we present two detection methods for a quasi-orthogonal STBC. Our proposed iterative receiver is presented in section IV. Finally, we exhibit the simulation results and the conclusion in section V and VI, respectively.

II. System Model

In this section, we review the quasi-orthogonal STBC and present basic configurations for our proposed system. Our system assumes that information sequences are mapped by the BICM and transmitted over block fading channels.

The quasi-orthogonal STBC for four transmit antennas is constructed as [3], [5], [7]

$$\mathbf{G}(\underline{\mathbf{s}}) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix} \tag{1}$$

where $\underline{s} = [s_1 \ s_2 \ s_3 \ s_4]^T$.

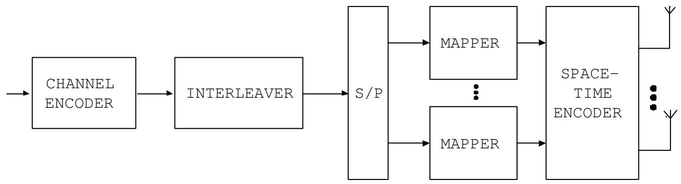

Fig. 1. Transmitter structure for the 4 antenna quasi-orthogonal STBC

This code can be divided into two orthogonal group $G(s_1, 0, s_3, 0)$ and $G(0, s_2, 0, s_4)$, and we have $G^H(s_1, 0, s_3, 0) \cdot G(0, s_2, 0, s_4) + G^H(0, s_2, 0, s_4) \cdot G(s_1, 0, s_3, 0) = 0$ where $(.)^H$ denotes the Hermitian transpose. We assume the quasi-static channel model in which channel frequency responses stay constant during the STBC block period T_{STBC}. For the STBC represented in (1), T_{STBC} equals 4. With one receive antenna, the received signal can be expressed by

$$\underline{\mathbf{y}} = \mathbf{H}_1 \begin{bmatrix} s_1 \\ s_3 \end{bmatrix} + \mathbf{H}_2 \begin{bmatrix} s_2 \\ s_4 \end{bmatrix} + \underline{\mathbf{n}} \tag{2}$$

where

$$\mathbf{H}_1 = \begin{bmatrix} h_1 & h_3 \\ -h_2^* & -h_4^* \\ -h_3 & h_1 \\ -h_4^* & h_2^* \end{bmatrix} \quad \mathbf{H}_2 = \begin{bmatrix} h_2 & h_4 \\ h_1^* & h_3^* \\ h_4 & -h_2 \\ -h_3^* & h_1^* \end{bmatrix}$$

$$\begin{aligned} \underline{\mathbf{y}} &= \begin{bmatrix} y_1 & y_2^* & y_3 & y_4^* \end{bmatrix}^T \\ \underline{\mathbf{n}} &= \begin{bmatrix} n_1 & n_2^* & n_3 & n_4^* \end{bmatrix}^T. \end{aligned}$$

Here h_i denotes the channel frequency response from the i th transmit antenna to the receive antenna and the noise n_i are mutually independent zero mean complex Gaussian variables of variance $N_0/2$ per dimension. Using above equations, each symbol pair $[s_1 \;\, s_3]^T$ and $[s_2 \;\, s_4]^T$ should be detected jointly. We refer to these symbol pairs as intra block symbols.

In this paper, we will concentrate on the block fading model describing wireless local area network system with slow movement. Denoting $h_{i,k}$ as the channel frequency response from the i th transmit antennas at time k, it is assumed to stay invariant during the block period $T_{STBC} \cdot F$ where F denotes the number of code words. With this assumption, the time index k is omitted in (2) for simplicity. The channel coefficients h_i are independent complex Gaussian with zero mean (Rayleigh fading).

Figure 1 shows the transmitter structure for the four transmit antenna quasi-orthogonal STBC system. Information bit sequences are encoded by convolutional encoder and interleaved by a bit level random interleaver with the size of $F \cdot N_t \cdot \log_2 M$ where M is the constellation size. Then interleaved sequences are space-time encoded after mapping. Correlations between adjacent coded bits are eliminated by the interleaver.

III. Detection of the Quasi-Orthogonal STBC

In this section, we will present two detection algorithms for detecting the transmitted signal through the quasi-orthogonal space time coding. One is to apply the channel matched filter [7] and the other is to employ the joint maximum likelihood detection [8].

In [7], the detection of the quasi-orthogonal STBC utilizes the channel matched filter $\mathbf{H_1}^H$ and $\mathbf{H_2}^H$. By multiplying $\mathbf{H_1}^H$ and $\mathbf{H_2}^H$ to the received signal, two orthogonal blocks are represented as,

$$\begin{aligned} \begin{bmatrix} r_{MF,1} \\ r_{MF,3} \end{bmatrix} &= \begin{bmatrix} \gamma & \alpha \\ -\alpha & \gamma \end{bmatrix} \begin{bmatrix} s_1 \\ s_3 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_3 \end{bmatrix} \\ \begin{bmatrix} r_{MF,4} \\ r_{MF,2} \end{bmatrix} &= \begin{bmatrix} \gamma & \alpha \\ -\alpha & \gamma \end{bmatrix} \begin{bmatrix} s_4 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_4 \\ \tilde{n}_2 \end{bmatrix} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \gamma &= \sum_{i=1}^{4} \|h_i\|^2 \\ \alpha &= 2j\Im\{h_1^* h_3 + h_4^* h_2\} \end{aligned}$$

and $\Im(.)$ denotes the imaginary part of a complex value. Here, α represents the interference caused by the non-orthogonality in the intra STBC block and the filtered noise $\tilde{n}_i$ becomes colored.

Consider the singular value decomposition (SVD) of $\mathbf{H}_i = U_i S_i W_i^H$ where U_i and H_i are unitary matrices. Due to the unitary property, employing U_i^H as matched filter leaves the distribution of noise unchanged and the equivalent channel as $S_i W_i^H$. Multiplying U_1^H to (2) yields two sets of equations for i=1 and 2 as [5]

$$r_i = \sqrt{\rho/4}\sqrt{\frac{\gamma + (-1)^{i-1}\alpha/j}{2}}(s_1 + j(-1)^{i-1}s_3) + \tilde{n}_i$$

where ρ denotes the received SNR and $\tilde{n}_i$ represents the filtered noise. Note that both the s_1 and s_3 are scaled by the same gain irrespective of the channel. Then, each signal s_1 and s_3 can be detected by applying joint ML detection.

If the symbols s_1 and s_2 are drawn from constellation χ_1 and χ_2 of size M, then the inflated constellations $\mathcal{B}_i$ of the received symbol $\tilde{r}_i = s_1 + j(-1)^{i-1}s_3$ will be of maximum size M^2. The probability of error for the received symbol $\tilde{r}_i$ is determined by the minimum Euclidean distance in $\mathcal{B}_i$. When the same constellation is employed for each χ_j, the size of B_i can not be M^2 since overlapped constellation points are introduced. The overlapped constellation points cause the minimum distance of the constellation to equal zero. To prevent this situation , [5] proposed the constellation rotation method. All constellation points in $\mathcal{B}_i$ become distinct by rotating one of χ_i in the same orthogonal block. The rotation results in the size of $\mathcal{B}_i$ to be M^2. Thus the probability of symbol error is reduced by having nonzero minimum Euclidean distance.

Another detection method for quasi-orthogonal STBC is a joint ML detection. In (2), it is obvious that $\mathbf{H_1}$ and $\mathbf{H_2}$ are orthogonal to each other. Because of the orthogonality between $\mathbf{H}_1$, $\mathbf{H}_2$ ($\mathbf{H_1}^H\mathbf{H_2} = \mathbf{H_2}^H\mathbf{H_1} = 0$), the ML metric for detecting $\underline{s}$ is

$$\min_{\underline{s}\in\chi^4} \|\underline{\mathbf{y}} - \mathbf{H_1}\begin{bmatrix} s_1 \\ s_3 \end{bmatrix} - \mathbf{H_2}\begin{bmatrix} s_2 \\ s_4 \end{bmatrix}\|^2 \quad (4)$$

where χ is the signal constellation alphabet. Then, also (4) is equivalent to minimize the following two metrics [8], we can show that

$$\min_{s_1,s_3} \|\underline{\mathbf{y}} - \mathbf{H_1}\begin{bmatrix} s_1 \\ s_3 \end{bmatrix}\|^2$$
$$\min_{s_2,s_4} \|\underline{\mathbf{y}} - \mathbf{H_2}\begin{bmatrix} s_2 \\ s_4 \end{bmatrix}\|^2 \ .$$

Unlike the matched filter detection in [5], each signal is not scaled by the same amount in the ML detection. In the case of different channel gains for each signal, the constellation rotation has little effect on the performance since each channel frequency response for the signal introduces phase shift which has the uniform distribution between 0 and 2π. Therefore, the optimal phase shift value which maximizes the Euclidean distance among two constellation set is difficult to determine because the relative phase shift is time varying with respect to fading coefficients.

IV. Quasi-Orthogonal Code with Iterative Decoding

In the previous section, we have introduced the constellation rotation proposed in [5]. This achieves full diversity in uncoded system, and the probability of symbol error is reduced due to the increased minimum Euclidean distance. However, if we apply channel coding to information sequences, the gain obtained by the constellation rotation may be absorbed by coding gains.

In coded systems, maximizing the coding gain is important to improve the system performance. Iterative decoding is one of common approaches to improve the decoder performance. In this section, we will propose an iterative detection method for the quasi-orthogonal STBC. The receiver structure of our proposed system is depicted in Figure 2. As shown in Figure 2, it can be considered as the serial concatenated code with the quasi-orthogonal STBC as an inner code and the convolutional code as an outer code.

The ML detection in (4) can be improved by iterative processing between the demapper and the decoder in the coded system. After each iteration, reliability value of soft decision bits are increased. Since the joint ML detection is performed at the demapper, soft decision bit information from the previous stage decoder are used for updating the intrinsic probability of the current stage demapper [9].

We assume that the channel state information is perfectly known to the receiver. Soft information computed at the demapper is passed to the decoder to estimate the transmitted signal. Before passing soft information to the decoder, deinterleaving is performed to ensure that sequences used by the demapper and the decoder to be independent.

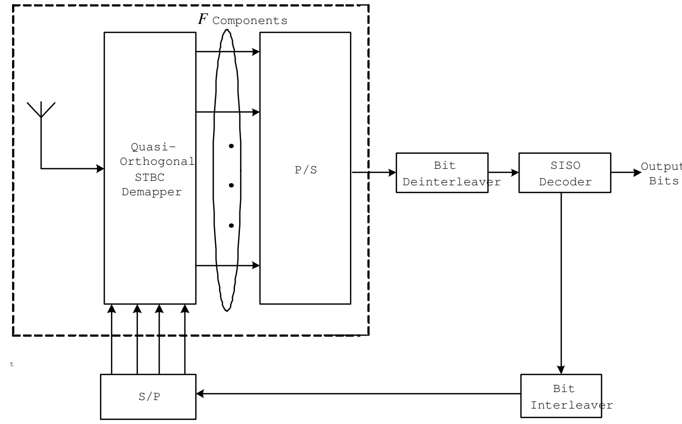

Fig. 2. Receiver structure for the quasi-Orthogonal STBC with iterative decoding

Denoting the bit-interleaved encoded symbol as $\underline{\mathbf{d}}_\mathbf{i} = [d_i^1 \dots d_i^{\log_2 M}\ d_i^{\log_2 M+1} \dots d_i^{2\log_2 M}]^T$, the modulated signal s_i,s_{i+2} are mapped as $s_i = \mu(d_i^1 \dots d_i^{\log_2 M})$ and $s_{i+2} = \mu(d_i^{\log_2 M+1} \dots d_i^{2\log_2 M})$ for $i = 1, 2$. Here, $\mu(.)$ denotes a mapping function of the constellation. Let the set $S_d^{i,m}$,$d = +1$ or $d = -1$ be the set of all symbol vectors with a +1 or -1 value of bit d_i^m respectively. The size of such a set is $2^{2\log_2 M}$. The soft output bit $L^e(d_i^m)$ of the demapper for the bit d_i^m is defined as

$$L^e(d_i^m) \triangleq \log \frac{\sum_{s_i\in S_{+1}^{i,m}} P(s_i \in S_{+1}^{i,m}|\tilde{\mathbf{x}}, \mathbf{H_1}, \mathbf{H_2})}{\sum_{s_i\in S_{-1}^{i,m}} P(s_i \in S_{-1}^{i,m}|\tilde{\mathbf{x}}, \mathbf{H_1}, \mathbf{H_2})} \ . \quad (5)$$

Using the Bayes' theorem, we have

$$P(s_i|\tilde{\mathbf{x}}, \mathbf{H_1}, \mathbf{H_2}) = \frac{P(s_i)P(\tilde{\mathbf{x}}, \mathbf{H_1}, \mathbf{H_2}|s_i)}{P(\tilde{\mathbf{x}}, \mathbf{H_1}, \mathbf{H_2})} \ . \quad (6)$$

where $P(s_i)$ is the intrinsic probability which is passed from the soft-in/soft-out (SISO) decoder of the previous stage. This can be derived as

$$P(s_i) \propto \exp(\mathbf{d}_i^T \mathbf{L}_i/2) \quad (7)$$

where $\mathbf{L_i}$ is a column vector of length $2\log_2 M$ comprised of the soft bit d_i^m passed from the decoder. Initially, $\mathbf{L_i}$ is set to all zero column vector. This information helps decoupling the overlapped signal in the same orthogonal group, and thus can improve the overall performance for the quasi-orthogonal STBC detection.

The extrinsic probability $P(\tilde{x}|\tilde{s}_i)$ is obtained by

$$P(\tilde{\mathbf{x}}|s_i) \propto \exp(-\frac{1}{N_0}\|\tilde{\mathbf{x}} - \mathbf{H_i}s_i\|^2) \ . \quad (8)$$

This extrinsic probability is a metric for selecting the symbol which has the minimum squared Euclidean distance among all possible constellation points. By using (5)-(8), the soft decision value for demapper is obtained and improve the reliability of the soft decision by subsequently updating the intrinsic probability throughout iterations.

Now we will analyze the performance for the iterative decoder. The union bound of the bit error probability for BICM using convolutional codes with rate $R_c = k_c/n_c$ is given by [10]

$$P_b \leq \frac{1}{k_c} \sum_{d=d_H}^{\infty} W_I(d) f(d, \mu, \chi) \tag{9}$$

where $W_I(d)$ represents the information error weight of error events with Hamming distance d, d_H stands for the free Hamming distance of the code and $f(d, \mu, \chi)$ denotes the pairwise error probability (PEP) of the BICM with Hamming distance d. This PEP assumes a code sequence S is transmitted but a code sequence $\hat{S}$ is selected at the decoder.

Now, we assume the block fading channel. Then the pairwise error probability can be written as

$$P(S \to \hat{S}) = E[Q(\frac{\|H\Delta\|}{\sqrt{2N_0}})] \tag{10}$$

where $S = [\underline{s}_1 \dots \underline{s}_F]$ is the received signal, Δ is defined as $S - \hat{S}$ and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{t^2/2} dt$. As in (9), the bit error probability is a function of information error weight and the pairwise error probability. The information weight distribution depends only on channel codes selected [10]. $f(d, \mu, \chi)$ is related to the channel condition and the detection scheme. As shown in (10), to improve the performance, the PEP should be decreased.

The PEP can be further calculated by the complex integration of its moment generation function (MGF) [11]. Thus $f(d, \mu, \chi)$ can be obtained by

$$\begin{aligned} f(d, \mu, \chi) &= \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{\Phi_{\Delta}(s, d)}{2s} ds \\ &= \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{\Phi_{\xi}(s, d)}{2s} (1-2s)^{1/2} ds \end{aligned} \tag{11}$$

where $\Phi_{\Delta}(s, d)$ and $\Phi_{\xi}(s, d)$ are the MGF of Δ and $\xi = \frac{\|H\Delta\|^2}{2N_0}$ with Hamming distance d, respectively. The MGF of ξ can be computed as [11]

$$\begin{aligned} \Phi_{\xi}(s, d) &= E_{\xi}[e^{-s\xi}] \\ &= \frac{1}{\det[I_{N_t} + s\Delta\Delta^H/(2N_0)]} \end{aligned} \tag{12}$$

where $E_{\xi}[.]$ denotes the expectation with respect to ξ and I_{N_t} is the N_t by N_t identity matrix.

Minimization of the PEP is equivalent to minimizing the MGF in (12). The dominant term in (12) is $\Delta\Delta^H$ which denotes the squared Euclidean distance between two code words. Without iterations, it is proportional to the minimum squared distance for given constellation. However, the iterative process increases the reliability of soft values and makes it possible to approach error-free feedback. This ideal feedback results in M-QAM channel being transformed into $\log_2 M$ independent BPSK channels [12]. With this assumption, (12) can be simplified to

$$\Phi_{\xi}(s, d) \approx E_d[\prod_{t=1}^{2} \frac{1}{1 + \frac{1}{2N_0} \sum_{i=1}^{d_t} \|\tilde{x}_i^t - \hat{\tilde{x}}_i^t\|^2}] \tag{13}$$

where $d = d_1 + d_2$, $\tilde{x}^t = [s_t \quad s_{t+2}]^T$ and $E_d[.]$ denotes the expectation with respect to d. The ideal feedback means that the demapper has the perfect knowledge of other bits except the bit to detect. In that situation, the squared Euclidean distance which affect the pairwise error probability is not the minimum squared Euclidean of the combined two signal constellations but the distance between two points in the constellation which differs only one bit to be detected. Thus iterative process improves the system performance by decreasing the PEP. This is equivalent to maximize the term $\|\tilde{x}_i^t - \hat{\tilde{x}}_i^t\|^2$ in (13). Derivation of $f(d, \mu, \chi)$ for fast fading channels is straightforward.

V. Simulation Results

In this section, we present simulation results to compare the proposed system with the conventional quasi-orthogonal STBC system. The spectral efficiency of the system is given as [9]

$$R_T = \frac{R_c \cdot N_t}{T_{STBC}} \cdot \log_2 M \quad \text{bps/Hz}$$

where R_c denotes the convolutional code rate and T_{STBC} represents the block period for the STBC. We assume quasi-static channels such that the channel frequency response stays invariant during T_{STBC}. Throughout simulations, the symbol length of one frame F is set to 256 which results in the size of interleaver to be 1024.

In Figure 3, we show the effect of the constellation rotation. We will compare coded system with uncoded system in [5] as a function of signal-to-noise ratios (SNR) and bit error rates (BER). Here we assume that channel frequency responses are uncorrelated between STBC blocks but stay invariant during the block period T_{STBC} as in [5].

In coded systems, we use 16-QAM constellation with gray mapping, and a convolutional encoder with $K = 7$. For spectral efficiencies 2bps/Hz and 3bps/Hz in coded systems, we employ the convolutional code with rate 1/2 and the punctured convolutional code with rate 3/4 defined in [13], respectively. To make 2bps/Hz and 3bps/Hz in uncoded systems, we utilize QPSK and 8-PSK constellation with gray mapping. Optimal rotation angles are chosen as 0.53 radian for QPSK constellation and 0.3 radian for 8-PSK constellation [5].

As shown in Figure 3, the constellation rotation outperforms the quasi-orthogonal STBC with no rotation in uncoded systems as reported at [5]. However, its gain is almost completely diminished in coded systems.

Now, we will exhibit results in coded systems by applying an iterative process. In coded systems, information sequences are encoded by convolutional code and one physical frame

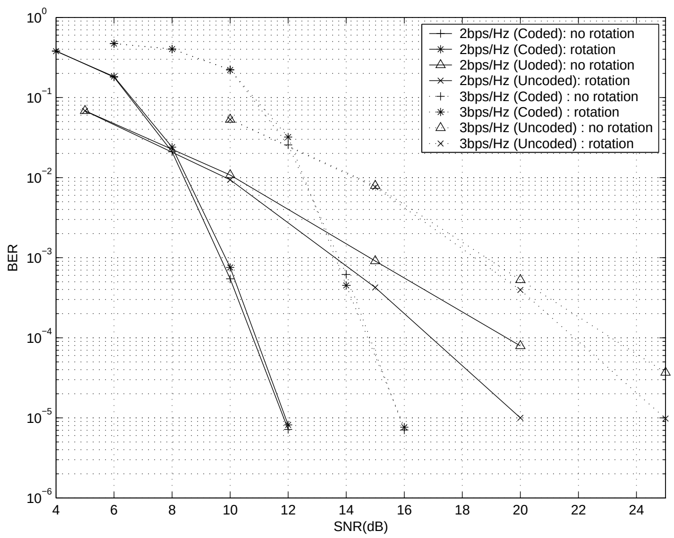

Fig. 3. Performance comparison of a quasi-orthogonal STBC iterative decoding with and without rotated constellation at 2bps/Hz in uncorrelated channel

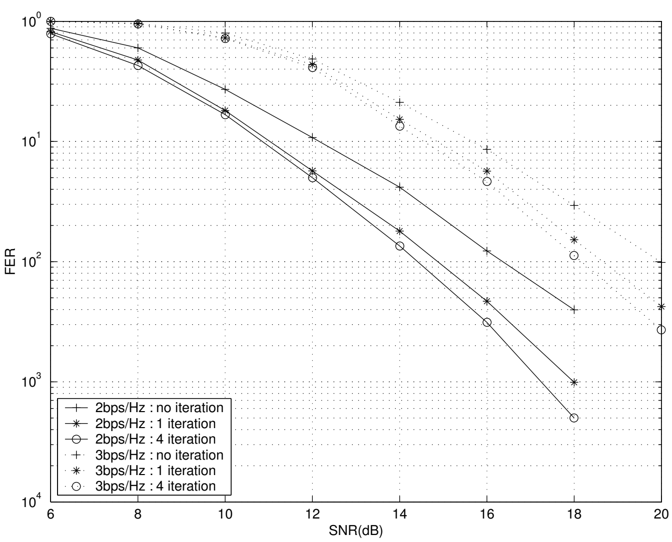

Fig. 4. Performance of quasi-orthogonal STBC with iterative decoding at 2bps/Hz and 3bps/Hz in block fading channel

size is set to be equal to the interleaver size. We assume block fading channels that channel frequency responses stay invariant during transmitting one physical frame. Since channel coding and interleaving are performed in a frame base, we plot simulation results for coded systems in terms of frame error rates (FER).

In Figure 4, we plot performance results with various iterations at 2bps/Hz and 3bps/Hz. As shown in the plot, the performance improvement is about 2dB at FER = 10^{-2} and saturated at more than 2 iterations. This indicates that the proposed iterative decoding scheme is effective in improving the decoder performance.

VI. Conclusion

In this paper, we propose a method for improving the quasi-orthogonal STBC performance with iterative decoding. In general, the iterative decoding can improve overall performance whenever there exists signal processing loss in detection mechanism. We have shown that our proposed detection scheme outperforms the conventional quasi-orthogonal system by 2dB and 2 iterations would be enough for the iterative detection scheme to converge in coded system.

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