

Mapping Optimization for Space-Time Block Coded OFDM Systems with Iterative Decoding

Jinsoo Choi, Wookbong Lee* and Inkyu Lee

Dept. of Communication Engineering, Korea University, Seoul, Korea

*LG Electronics Inc., Seoul, Korea

Email: jinsoo@wireless.korea.ac.kr, wbong@lge.com, inkyu@korea.ac.kr

Abstract—An iterative detection scheme enhances the performance of bit-interleaved coded modulation (BICM) by exploiting frequency diversity gains. Additionally, space-time block coding (STBC) scheme can significantly improve the system performance in fading channels by proper coordinating of the signaling over multiple antennas. In this paper, STBC technique based on BICM is applied to orthogonal frequency division multiplexing (OFDM) system over frequency selective channels. When iterative decoding (ID) is applied, the overall performance of the STBC BICM-ID OFDM is affected by the chosen mapping. We classify the mapping selection and search for the optimal mapping for the proposed system. Simulation results show that up to 3dB gain is achieved for 8PSK and 16QAM in various conditions.

I. INTRODUCTION

Next generation wireless communication systems are expected to provide high data rate transmission over a wide-band channel, which is characterized by frequency selective nature. As a result, inter-symbol interference (ISI) occurs and this prevents reliable transmission. In general, a complex equalization is needed to mitigate ISI effect for single carrier system, whereas orthogonal frequency division multiplexing (OFDM) technique can easily eliminate the ISI by utilizing parallel transmission and cyclic prefix [1]. OFDM has been proposed for a wide range of radio channels such as the wireless local area network (WLAN) system defined by IEEE 802.11a standard in packet-based communications [2].

When we apply OFDM systems in frequency selective fading channels, several subcarriers may experience deep fadings. Thus, without a channel coding, the performance of the OFDM system degrades considerably because the overall bit error rate (BER) is largely dominated by a few subcarriers with small fading amplitudes. To avoid such impairment, forward error correcting (FEC) coding is essential for OFDM [2]. By applying the channel coding across subcarriers, errors occurred in attenuated subcarriers can be corrected.

For combined coded modulation schemes, bit-interleaved coded modulation (BICM) scheme is a good candidate for fading channels [3]. Additionally, the distance property of BICM can be enhanced by applying iterative decoding, which improves the performance of BICM [4]. This is referred to as BICM with iterative decoding (BICM-ID).

To further improve the overall performance, additional diversity is needed for the reliable transmission. An antenna diversity can improve the performance for multiple-input multiple-output (MIMO) systems. Space-time coding (STC)

is one method to provide diversity gain over wireless MIMO systems. From the system performance aspect, space-time block coding (STBC) is an effective means to provide transmit diversity by utilizing a simple block code to multiple antennas [5], [6]. Especially, orthogonal space-time block codes achieve maximum diversity order with linear decoding. Also, with iterative decoding, the STBC system shows the improved error performance. In [7], space-time block coding with iterative decoding (STBC-ID) for independent channels is presented, and the performance of the STBC-ID in the case of flat fading channels is exhibited with Gray mapping in [8]. In this paper, we extend an STBC scheme based on BICM-ID to OFDM systems for frequency selective channels.

With iterative decoding, the choice of the mapping is crucial to achieve a high performance gain. Thus, we investigate the optimum mapping with respect to the system requirement. Also, the effects of the code rate and the channel condition on mapping optimization are analyzed.

This paper is organized as follows: In section II, we present a proposed system model. In section III, the detection and demodulation process are evaluated. The mapping optimization is investigated in section IV. Then in section V, simulation results are provided with various conditions. Finally, we conclude this paper in section VI.

II. SYSTEM MODEL

Consider a MIMO system with N_t transmit and N_r receive antennas. Frequency selective channel between the n th transmit antenna and the m th receive antenna can be expressed as

$$h^{n,m}(t, \tau) = \sum_{i=1}^K \bar{h}^{n,m}(i; t) \delta(\tau - \tau_n) \quad (1)$$

where the time domain channel coefficients $\bar{h}^{n,m}$ are complex Gaussian with zero mean (Rayleigh fading), $\delta(\cdot)$ is the Dirac delta function, and K denotes the number of channel taps.

Fig. 1 shows the transmitter structure of our proposed system. Coded bits are bitwise interleaved by an interleaver π and serial-to-parallel (S/P) converted. Each substream is then mapped into transmitted symbols. For each subcarrier, symbols are encoded by the STBC encoder over several time epochs, then these are subsequently modulated by OFDM. After IFFT operation, the proper cyclic prefix is added to prevent intercarrier interference (ICI).

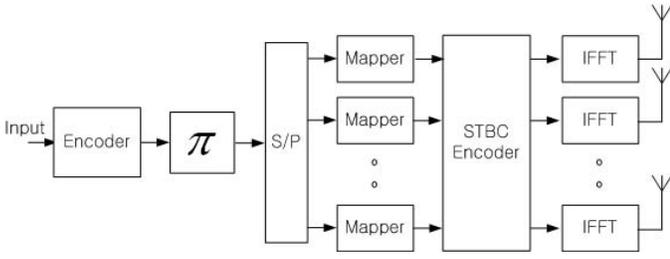


Fig. 1. Transmitter structure

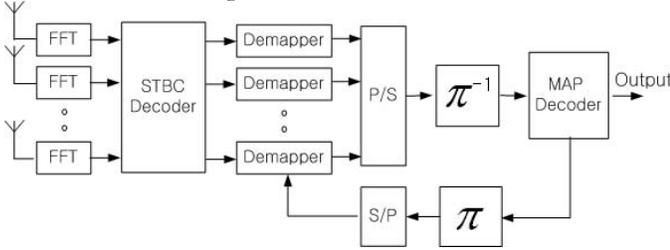


Fig. 2. Receiver structure

When we use the M -ary constellation, the spectral efficiency of the system is given as $R_T \cdot R_C \cdot \log_2 M$ bps/Hz where R_T and R_C are the STBC code rate and the convolutional code rate, respectively.

III. DETECTION AND DEMODULATION

Fig. 2 exhibits the receiver structure. The receiver basically performs inverse operations of the transmitter. After proper cyclic prefix removal operation and fast Fourier transform, the received signal at the k th subcarrier at the l th time from the m th receive antenna is written as

$$r_{k,l}^m = \sum_{n=1}^{N_t} H_{k,l}^{n,m} c_{k,l}^n + n_{k,l}^m \quad (2)$$

where $c_{k,l}^n$ is the (l,n) component of the STBC code matrix \mathbf{C} with the STBC block periods L , $H_{k,l}^{n,m}$ indicates the channel frequency response, and $n_{k,l}^m$ represents an additive white complex Gaussian noise with zero-mean and variance $N_0/2$ per dimension. Assuming quasi-static channels, we will omit the time index l in $H_{k,l}^{n,m}$ for simplicity. Denoting F and T_s as the number of subcarriers and the OFDM sampling period, respectively, the channel frequency response can be expressed by

$$H_k^{n,m} = \sum_{i=1}^K \bar{h}^{n,m}(i) e^{-j2\pi k \tau_n / FT_s}$$

where $\bar{h}^{n,m}(i)$ is converted from $\bar{h}^{n,m}(i;t)$ in (1) by assuming quasi-static channels.

By combining (2) in a vector format, we have

$$\mathbf{R}_k = \mathbf{C}\mathbf{H}_k^T + \mathbf{N}_k \quad \text{for } k = 1, 2, \dots, F.$$

where $(\cdot)^T$ denotes transpose, \mathbf{H}_k is the N_r by N_t channel matrix with the (m,n) element $H_k^{n,m}$, and \mathbf{C} represents a L by N_t STBC code matrix with the (l,n) element $c_{k,l}^n$.

It has been shown in [5] that STBC systems with more two transmit antennas can not achieve full transmit rate and full diversity simultaneously. Thus, it is necessary to compromise diversity order and multiplexing gain. In this paper, we consider an orthogonal code scheme with four Tx antennas because of its low decoding complexity. We choose a rate 3/4 orthogonal STBC scheme proposed in [6], which achieves a full diversity of $4 \cdot N_r$. The STBC code matrix is given as

$$\mathbf{C} = \begin{bmatrix} z_1 & z_2 & z_3 & 0 \\ -z_2^* & z_1^* & 0 & -z_3 \\ -z_3^* & 0 & z_1^* & z_2 \\ 0 & z_3^* & -z_2^* & z_1 \end{bmatrix}$$

where three symbols are transmitted during four time slots.

Using real lattice representation in [9], we can represent the above equation to a linear superposition form. Denoting $(\cdot)^R$ and $(\cdot)^I$ as real and imaginary part, respectively, the final equation is given as

$$\tilde{\mathbf{r}}_k = \tilde{\mathbf{H}}_k \tilde{\mathbf{z}}_k + \tilde{\mathbf{n}}_k$$

where

$$\tilde{\mathbf{r}}_k = [\mathbf{r}_{k,1}^R \ \mathbf{r}_{k,2}^R \ \mathbf{r}_{k,3}^R \ \mathbf{r}_{k,4}^R \ \mathbf{r}_{k,1}^I \ \mathbf{r}_{k,2}^I \ \mathbf{r}_{k,3}^I \ \mathbf{r}_{k,4}^I]^T,$$

$$\tilde{\mathbf{z}}_k = [z_{k,1}^R \ z_{k,2}^R \ z_{k,3}^R \ z_{k,1}^I \ z_{k,2}^I \ z_{k,3}^I]^T,$$

$$\tilde{\mathbf{n}}_k = [\mathbf{n}_{k,1}^R \ \mathbf{n}_{k,2}^R \ \mathbf{n}_{k,3}^R \ \mathbf{n}_{k,4}^R \ \mathbf{n}_{k,1}^I \ \mathbf{n}_{k,2}^I \ \mathbf{n}_{k,3}^I \ \mathbf{n}_{k,4}^I]^T,$$

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{h}_{k,1}^R & \mathbf{h}_{k,2}^R & \mathbf{h}_{k,3}^R & -\mathbf{h}_{k,1}^I & -\mathbf{h}_{k,2}^I & -\mathbf{h}_{k,3}^I \\ \mathbf{h}_{k,2}^R & -\mathbf{h}_{k,1}^R & -\mathbf{h}_{k,4}^R & \mathbf{h}_{k,2}^I & -\mathbf{h}_{k,1}^I & \mathbf{h}_{k,4}^I \\ \mathbf{h}_{k,3}^R & \mathbf{h}_{k,4}^R & -\mathbf{h}_{k,1}^R & \mathbf{h}_{k,3}^I & -\mathbf{h}_{k,4}^I & -\mathbf{h}_{k,1}^I \\ \mathbf{h}_{k,4}^R & -\mathbf{h}_{k,3}^R & \mathbf{h}_{k,2}^R & -\mathbf{h}_{k,4}^I & -\mathbf{h}_{k,3}^I & \mathbf{h}_{k,2}^I \\ \mathbf{h}_{k,1}^I & \mathbf{h}_{k,2}^I & \mathbf{h}_{k,3}^I & \mathbf{h}_{k,1}^R & \mathbf{h}_{k,2}^R & \mathbf{h}_{k,3}^R \\ \mathbf{h}_{k,2}^I & -\mathbf{h}_{k,1}^I & -\mathbf{h}_{k,4}^I & -\mathbf{h}_{k,2}^R & \mathbf{h}_{k,1}^R & -\mathbf{h}_{k,4}^R \\ \mathbf{h}_{k,3}^I & \mathbf{h}_{k,4}^I & -\mathbf{h}_{k,1}^I & -\mathbf{h}_{k,3}^R & \mathbf{h}_{k,4}^R & \mathbf{h}_{k,1}^R \\ \mathbf{h}_{k,4}^I & -\mathbf{h}_{k,3}^I & \mathbf{h}_{k,2}^I & \mathbf{h}_{k,4}^R & \mathbf{h}_{k,3}^R & -\mathbf{h}_{k,2}^R \end{bmatrix}$$

Here $\mathbf{r}_{k,l}$, $\mathbf{n}_{k,l}$, and $\mathbf{h}_{k,n}$ are defined as $[r_{k,l}^1, r_{k,l}^2, \dots, r_{k,l}^{N_r}]^T$, $[n_{k,l}^1, n_{k,l}^2, \dots, n_{k,l}^{N_r}]^T$, and $[H_k^{n,1}, H_k^{n,2}, \dots, H_k^{n,N_r}]^T$, respectively.

For the orthogonal code matrix, the transpose of the real lattice channel matrix corresponds to the matched filter (MF) [9]. After the matched filtering, the modified received symbol at the j th symbol position is given as [9]

$$r_j = \alpha z_j + \hat{n}_j = \sum_{m=1}^{N_r} \sum_{n=1}^4 |H_j^{n,m}|^2 z_j + \hat{n}_j \quad \text{for } j = 1, 2, 3 \quad (3)$$

where z_j denotes the transmitted symbol, α equals $\sum_{m=1}^{N_r} \sum_{n=1}^4 |H_j^{n,m}|^2$, and \hat{n}_k is also an additive noise term with zero mean and variance $\alpha \cdot \frac{N_0}{2}$ per dimension.

The maximum *a posteriori* (MAP) demapper is used for generating reliability values. Let d_j^i be the bit that is mapped at the i th bit position into the j th symbol. Then the log likelihood ratio (LLR) values are given by

$$L(d_j^i) = \log \frac{p(d_j^i = +1)}{p(d_j^i = -1)}.$$

Denote S_d^i as a set of all symbol vectors with a +1 or -1 value of bit d_j^i . Then the LLR conditioned on the channel state information is computed by

$$\log \frac{p(d_j^i = +1|r_j, H_j)}{p(d_j^i = -1|r_j, H_j)} = \log \frac{\sum_{z_j \in S_{+1}^i} p(z_j, r_j, H_j)}{\sum_{z_j \in S_{-1}^i} p(z_j, r_j, H_j)}.$$

When iterative decoding is applied, the joint probability density function is related to

$$p(z_j, r_j, H_j) \sim \exp \left(-\frac{1}{N_0} |r_j - \alpha z_j|^2 + \frac{1}{2} \sum_{i=1}^{\log_2 M} d_j^i L(d_j^i) \right).$$

where M denotes the constellation size and $L(d_j^i)$ indicates the priori information from the MAP decoder. For the first iteration, $L(d_j^i)$ is set to zero. By exchanging of extrinsic information between the MIMO demapper and MAP decoder iteratively, the reliability of the bit sequence increases and the error performance improves.

IV. MAPPING OPTIMIZATION WITH ITERATIVE DECODING

In this section, we optimize our system by choosing a proper mapping with respect to the system requirement. When we apply BICM-ID structure, the performance of the system is affected by two factors: diversity order and coding gain. The equation (3) indicates that the proposed system is asymptotically equivalent to BICM-ID OFDM system with diversity order of $4 \cdot N_r$. Thus, it can be shown that for the error floor region, the pairwise error probability of our system is given as [10]

$$P(\mathbf{z} \rightarrow \hat{\mathbf{z}}) \leq \prod_{n=1}^D \prod_{m=1}^{4N_r} \frac{1}{1 + \frac{E_s}{4N_0} \lambda_n}.$$

where \mathbf{z} and $\hat{\mathbf{z}}$ are the transmitted and erroneously detected sequence, respectively, E_s denotes the symbol energy, and λ_n is a random variable which is determined by mapping of the constellation.

Here D represents the diversity order and equals $\min(d_H, K)$ where d_H denotes the minimum Hamming distance of the convolutional code sequence. This indicates that the diversity order is determined by the convolutional code, where the coding gain is affected by the mapping selection. Thus, the mapping selection is a dominant factor for the system performance in this case.

In general, performance curves with iterative decoding are divided into three regions: the non-convergence region, the water fall region, and the error floor region. The Gray mapping is shown to be optimum for the non-convergence region [11]. For the error floor region, the optimal mappings are examined in [12]. In [13], the optimal mapping for space-time BICM is presented by means of classifying mapping group where each group exhibits a distinctive BER curve. It was shown that one mapping group reaches an error floor at a low signal-to-noise ratio (SNR), while the other mapping group results in a lower error floor at high SNR. Thus, in accordance with the system requirement, the optimal mapping selection is crucial.

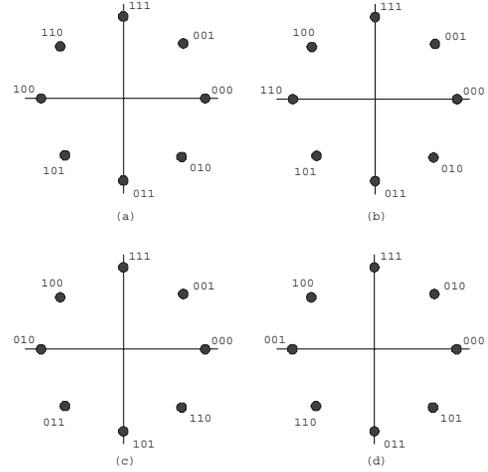


Fig. 3. 8PSK Mappings (a) $G_{2.5}$ (b) $G_{3.0}$ (c) $G_{3.5}$ (d) $G_{4.0}$

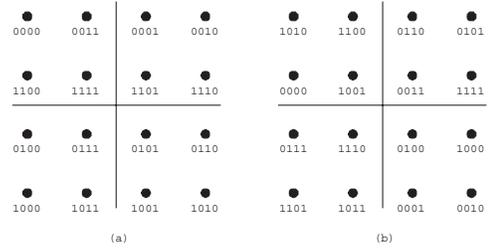


Fig. 4. 16QAM Mappings (a) Mixed Labeling (b) Random Labeling

In the water fall region, the overall performance is dominated by the demapper performance, which can be approximated as $P_{demap} \approx N_b P_e$ [14] where P_e denotes the symbol error probability as a function of the minimum Euclidean distance of the constellation and SNR. The number of the average total bit errors N_b is defined as

$$N_b = \sum_{i=0}^{M-1} p_x(i) \sum_{j=1}^{N_i} n_b(i, j).$$

where $p_x(i)$ denotes the probability of the i th constellation symbol $x(i)$, N_i is the number of neighboring constellation points of $x(i)$, and $n_b(i, j)$ represents the number of bit errors when $x(i)$ is erroneously detected as $x(j)$. N_b can be computed for a given mapping.

In this paper, following the optimization process addressed in [13], we adopt the same approach to the STBC case. For 8-PSK, the Gray mapping has $N_b = 2$ and shows the best first-pass performance [11], while the semi-set partitioning (SSP) mapping has $N_b = 4.5$ and exhibits the best error floor performance [4]. We refer to G_{N_b} as a group of mappings with respect to N_b . Mappings with various N_b are presented in Fig. 3.

For 16-QAM, the Gray mapping has $N_b = 3$ and also shows the best first-pass performance [11]. When iterative decoding is applied, the mixed labeling, the modified set partitioning (MSP) labeling, the random labeling [15], the

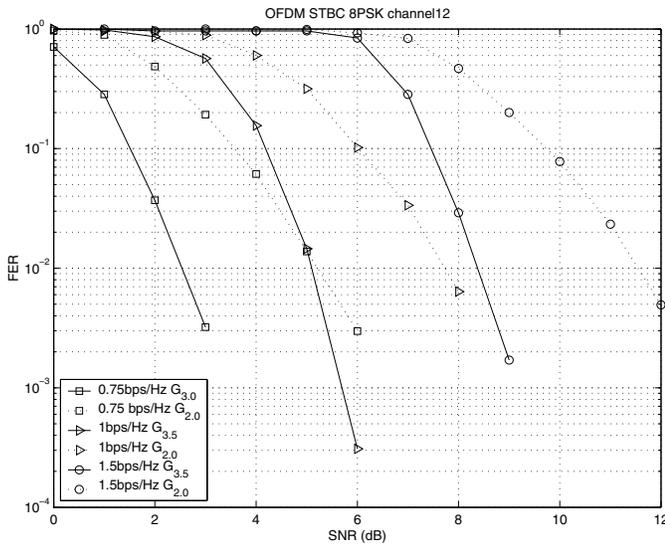


Fig. 5. 8-PSK modulation with different code rates

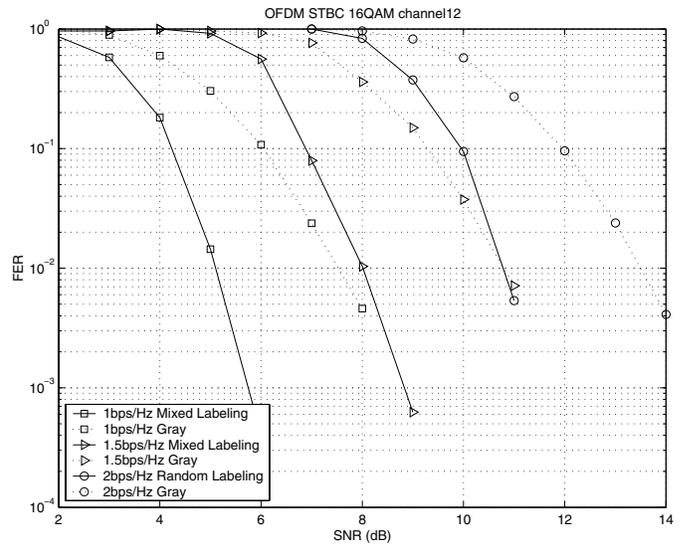


Fig. 7. 16-QAM modulation with different code rates

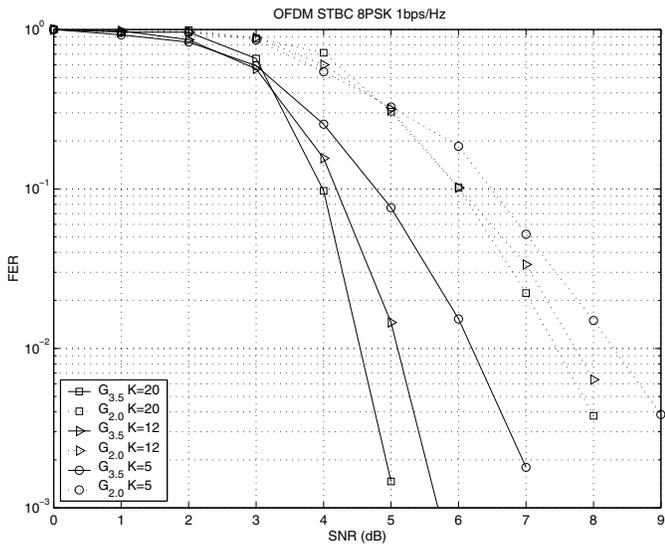


Fig. 6. 8-PSK modulation with different channel conditions

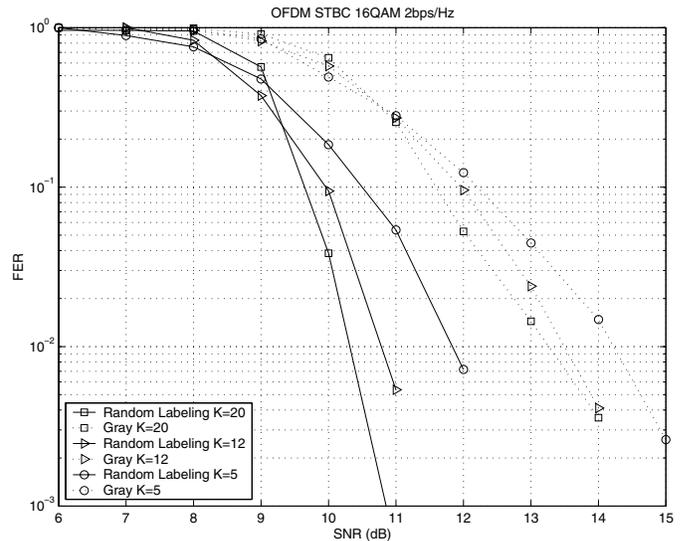


Fig. 8. 16-QAM modulation with different channel conditions

$M16^r$ labeling [12], and the maximum squared Euclidean weight mapping [16] presents a better error floor performance, and each labeling has $N_b = 5, 6.5, 6.5, 7,$ and $9,$ respectively.

In this paper, we evaluate the performance of various mappings in STBC-OFDM systems. In the following section, the mappings which show the best performance for given configurations are presented.

V. SIMULATION RESULTS

In this section, we employ 8PSK and 16QAM modulations using punctured convolutional codes with constraint length 5 [17], and exponentially decaying K tap channels are assumed. The number of OFDM subcarriers and guard symbols are set to 1024 and 100, respectively. We apply all the mappings presented in section IV to find the optimal mapping at 1% frame error rate (FER) with five iteration. In each simulation,

only one mapping which exhibits the best performance is shown compared to the Gray mapping in plots.

Figures 5 and 6 show the performance of the system using 8-PSK. In Fig. 5, different code rates are applied to the proposed system with an exponentially decaying channel ($K = 12$). The spectral efficiencies are equal to 0.75, 1, and 1.5 bps/Hz with code rates $1/3, 4/9,$ and $2/3,$ respectively. In the 0.75bps/Hz case, the mapping $G_{3,0}$ exhibits the best performance among other mappings, and achieves 2.5dB gain over the Gray mapping at 1% FER. In contrast, in the 1 and 1.5bps/Hz case, the mapping $G_{3,5}$ shows the best performance with 2.6dB and 3.0dB gain, respectively.

In Fig. 6, different channel conditions are employed to the system with code rate $4/9$. Exponentially decaying channels with $K = 5, 12,$ and 20 are assumed. For all $K,$ the performance

of the mapping $G_{3.5}$ is the best compared to the Gray mapping. The plot shows that the performance gain of the mapping $G_{3.5}$ over the Gray mapping increases up to 3dB, as K grows. This demonstrates that the proposed system with the optimized mapping is capable of exploiting frequency diversity better as frequency selectivity in fading channels increases.

Fig. 7 and Fig. 8 present the performance of the system using 16-QAM modulation. Fig. 7 compares the performance of the Gray mapping and the optimized mapping over an exponentially decaying 12 tap channel. The spectral efficiencies are set to 1, 1.5, and 2 bps/Hz with code rates 1/3, 1/2, and 2/3, respectively. In the 1 and 1.5bps/Hz cases, the mixed labeling mapping shows the best performance with 2.3dB and 2.6dB gain at 1% FER, respectively. In the 2bps/Hz case, the random labeling mapping exhibits 2.8dB gain at 1% FER.

Fig. 8 presents the performance results with a rate 2/3 convolutional code over various channel conditions. In this simulation, exponentially decaying channels with $K = 5, 12,$ and 20 are assumed. Among various mappings, the random labeling mapping shows the best performance, and the performance gains over the Gray mapping are 2.5dB, 2.7dB, and 3dB, respectively. Similarly with 8-PSK modulation, the advantage of the mapping optimization increases with larger channel taps. Note that unlike 8PSK, the optimization for 16QAM is not complete. Thus, more performance gain is expected with further optimization of the 16QAM mapping.

In simulation results, it is confirmed that as the code rate increases, the performance gain of the optimized mapping grows. An interesting point is that once the system parameters such as code rates and modulation levels are determined, the optimal mapping is not affected by different channel conditions. In other words, only one optimal mapping selection exists for a certain spectral efficiency and for various channel conditions, the selected mapping performs the best. Thus, we can expect that the proposed system with the mapping optimization is robust to various channel changes. These results are desirable from a system implementation point of view, as the mapping can be optimized without considering the channel effect much.

VI. CONCLUSION

We have proposed an iterative decoding scheme which enhances the performance of STBC-OFDM systems over wide-band frequency selective channels. Space-time block coding scheme based on bit-interleaved coded modulation applied to OFDM systems are investigated. The detection and demodulation process of the system using the orthogonal code matrix have been evaluated with lattice representation. Also, the mapping optimization with iterative decoding has been studied with various system conditions. Simulation results have demonstrated that the mapping selection plays an important role in the error performance. With a proper mapping selection, the proposed system has shown the performance gain up to 3dB over the Gray mapping. Also, the effect of code rates and channel taps on the mapping optimization has been investigated. It is straightforward to expand the proposed mappings to more general STBC schemes.

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