Diversity Analysis for Space-Time Bit-Interleaved Coded Modulation Systems

Byoungchul Moon*, Carl-Erik W. Sundberg† Dong-Joon Shin‡ and Inkyu Lee*
*School of Electrical Eng., Korea University, Seoul, Korea
†SundComm, Chatham, NJ 07928, USA
‡Dept. of Eelectronics and Com. Eng., Hanyang University, Seoul, Korea
Email: bcmoon@wireless.korea.ac.kr, cews@ieee.or,
djshin@hanyang.ac.kr, inkyu@korea.ac.kr

***Abstract*— Transmission efficiency on fading channels can be considerably improved by using multiple transmit and receive antennas. Space-time bit-interleaved coded modulation schemes give spatial and temporal diversity gains on fading channels by combining binary convolutional codes, bit level interleaving and higher order signal constellations with multiple transmit and receive antennas. In this paper, we provide a diversity order analysis for space-time bit-interleaved coded modulation (ST-BICM) systems when punctured convolutional codes are employed. We first show that the ST-BICM systems with punctured codes can meet the Singleton bound, and present conditions on code construction for the Singleton bound. We give computer simulation results to support the analysis.**

I. Introduction

A desire for transmitting rich multimedia contents over wireless communication channels leads to a design of high-rate data transmission systems. Recently, it has been shown that enormous channel capacity can be achieved using multiple input multiple output (MIMO) systems [1]. These MIMO systems can provide either spatial multiplexing or diversity gain by utilizing multiple transmitter antennas.

In the meanwhile, overcoming the effects of fading [2] is one important area for reliable wireless communication. Diversity techniques are classic methods to improve the transmission reliability over fading channels. Coded modulation schemes are designed to utilize time diversity on antenna radio links [3]. A flexible signal design technique for code diversity is bit-interleaved coded modulation (BICM) [4], [5]. Also an iterative decoding is employed to utilize the turbo principle for better performance [6], [7].

In line with obtaining higher diversity gains, ST-BICM schemes were first proposed by Tonello in [8], and it was shown that the maximum diversity order $D_{\max} = N_t \cdot N_r$ is achieved where N_t and N_r are the number of transmit and receive antennas, respectively. The ST-BICM is extended for frequency selective channel where orthogonal frequency division multiplexing (OFDM) [9] is employed [10], [11]. Furthermore, with frequency hopping over successive time frames as in [12], additional diversity is also possible.

For the block fading channel case, it is shown in [8] that the maximum diversity gain is obtained for ST-BICM schemes when outgoing signal from every transmit antenna has a non-zero minimum Hamming distance. In [8], code design for ST-BICM was studied to guarantee the full diversity. Also it is proved in [13] that for any punctured convolutional code with rate $R > 1/N_t$, the ST-BICM fails to achieve full diversity.

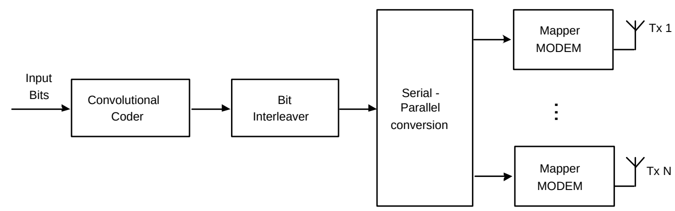

Fig. 1. ST-BICM transmitter structure

To complete the analysis attempted in [13], in this paper we provide an analytical theorem to determine the diversity order for punctured coded ST-BICM systems by applying the code construction criterion. According to this theorem, diversity order is determined by the convolutional code rate and the number of transmit antennas. We show that with properly designed punctured codes, the ST-BICM meets the Singleton bound, and provide conditions for code design to achieve the bound. The simulation results are presented to support our analysis. Among the results we can mention that increasing the number of transmit antennas yields systems with the same diversity order as systems with fewer transmit antennas but higher transmission rate and also higher receiver complexity.

This paper is organized as follows: Section II contains a system model and introduces the transmitter and receiver structures. In section III, we summarize a pairwise error probability analysis for ST-BICM and the code design rules for the full diversity. The section IV provides analytical theorem with regards to the diversity order of punctured ST-BICM systems. The simulation results are presented in section V. Finally, the paper is terminated with a discussion and conclusion in section VI.

II. System Model

In this section, we review the ST-BICM system model introduced in [8]. Consider a multiple antenna system with N_t transmit and N_r receive antennas. In Figure 1, we show a

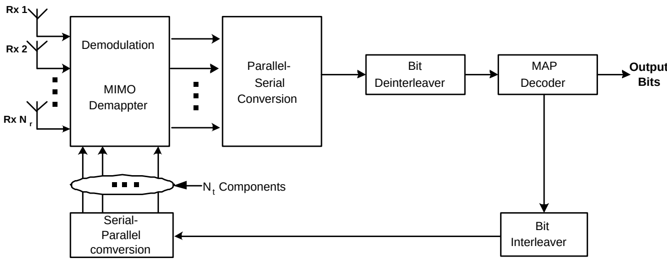

Fig. 2. ST-BICM receiver structure

transmitter structure of the ST-BICM system. We assume M-PSK or M-QAM with M being the constellation size. The spectral efficiency of the system is given as [8] $R_T = R_c \cdot N_t \cdot \log_2 M$ bits/s/Hz where R_c is the rate of the convolutional code.

In this work, we will focus on the narrowband block fading channel model [14] where fading coefficients are quasi-static over a block of transmitted symbols, and independent over blocks. Wideband channels could be considered by employing OFDM rather than single carrier modems.

Now we consider a multi-input multi-output (MIMO) channel model. The received signal at the kth time slot from the jth receive antenna is represented by

$$y_k^j = \sum_{i=1}^{N_t} h_k^{i,j} x_k^i + n_k^j \qquad \text{for } j = 1, 2, \cdots N_r \tag{1}$$

where x_k^i is the transmitted symbol with the symbol energy E_s at the ith transmit antenna at the kth time slot. The channel coefficient $h_k^{i,j}$ is the equivalent channel response of the link between the ith transmit antenna and jth receive antenna at the kth time slot, and is assumed to be complex Gaussian with zero mean and unit variance. Here, n_k^j is a sequence of independent identically distributed (*i.i.d.*) complex zero mean Gaussian random variable with variance $N_0/2$ per dimension.

Denoting N_p as the packet size in a vector format, (1) can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k \qquad \text{for } k = 1, 2, \cdots N_p$$

where we define $\mathbf{y}_k = [y_k^1 \cdots y_k^{N_r}]^T, \mathbf{x}_k = [x_k^1 \cdots x_k^{N_t}]^T, \mathbf{n}_k = [n_k^1 \cdots n_k^{N_r}]^T$. Here $[\ ]^T$ denotes the transpose operation. Then, the channel response matrix at the kth time is given as

$$\mathbf{H}_k = \begin{bmatrix} h_k^{1,1} & \cdots & h_k^{N_t,1} \\ \vdots & \ddots & \vdots \\ h_k^{1,N_r} & \cdots & h_k^{N_t,N_r} \end{bmatrix}.$$

Figure 2 shows the receiver structure with iterative decoding. The MIMO demapper and the maximum *a posteriori* (MAP) decoder for the convolutional code are the main components. For detailed illustration of the structures and explanation of the receiver operations, refer to [8].

III. Performance Evaluation and Code Design

To derive the diversity order of ST-BICM system analytically, we briefly review the pairwise error probability (PEP) bound analysis. This work was also presented in [13]. For general space time coded systems in the block fading channel, we consider the average pairwise probability of error that the maximum likelihood (ML) decoder chooses the erroneous sequence $\hat{\mathbf{x}}$ over the transmitted correct sequence $\mathbf{x}$. Since we assume a quasi-static channel, the time index k is omitted in the channel matrix $\mathbf{H}_k$. Then, denoting the error vector as $\mathbf{e}_k = \mathbf{x}_k - \hat{\mathbf{x}}_k$, the PEP is expressed by [15]

$$P(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0} d^2(\mathbf{x}, \hat{\mathbf{x}})}\right)$$

where $d^2(\mathbf{x}, \hat{\mathbf{x}})$ is defined as $d^2(\mathbf{x}, \hat{\mathbf{x}}) = \sum_k \|\mathbf{H}\mathbf{e}_k\|^2$.

At high signal-to-noise ratios (SNRs), under the exact feedback assumption, the PEP is given by [13]

$$P(\mathbf{x} \to \hat{\mathbf{x}}) \leq (\prod_{i=1}^{N_t} \frac{E_s}{4N_0} d_E^i(\mathbf{x}, \hat{\mathbf{x}}))^{-N_r}$$

where we define $d_E^i(\mathbf{x}, \hat{\mathbf{x}})$ as the sum of the squared Euclidean distances computed on the sub-sequences transmitted over the ith antenna. We have a bound on $d_E^i(\mathbf{x}, \hat{\mathbf{x}})$ as $d_E^i(\mathbf{x}, \hat{\mathbf{x}}) \geq d_H^i(\mathbf{x}, \hat{\mathbf{x}}) \cdot d_{E,\min}^i$. Here the minimum squared Euclidean interdistance $d_{E,\min}^i$ is only influenced by the constellation mapping, hence code construction can affect the diversity by the maximum free Hamming distance over the ith antenna $d_H^i(\mathbf{x}, \hat{\mathbf{x}})$. Thus to achieve the full diversity $N_t \cdot N_r$ in the ST-BICM system, we need the following condition

$$d_H^i(\mathbf{x}, \hat{\mathbf{x}}) \neq 0 \qquad \text{for } i = 1, 2, \cdots N_t.$$

In other words, the diversity order of ST-BICM systems is determined by the number of transmit antennas with nonzero weight $d_H^i(\mathbf{x}, \hat{\mathbf{x}})$.

In [13], the optimal code construction for full diversity is presented. Note that adding additional receiver antennas always give improved diversity by a multiplicative factor of N_r and does not affect the spectral efficiency.

IV. Diversity Order in Punctured ST-BICM

Puncturing based on a rate $1/n$ convolutional code is a popular way of achieving good codes with higher code rates [16], [17]. The puncturing mechanism is also well suited to hybrid automatic repeat request (H-ARQ) of medium access control (MAC) operation as a form of rate compatible punctured code (RCPC). The rate compatible punctured codes are also conveniently used for unequal error protection [18], including ST-BICM systems [19]. It easily achieves the code rate k/n by changing the puncturing pattern. Although the optimum free distance $R_c = 1/N_t$ convolutional code will always yield the full diversity, it was shown in [13] that punctured codes with rate $R_c > 1/N_t$ can not achieve the full diversity in ST-BICM.

Now we will address the issue on determining diversity order for ST-BICM systems with various coding rates R_c in the following theorem.

Theorem 1: Denoting R_c as the code rate, the maximum achievable diversity is given by

$$D = \Big(N_t - \lceil R_c \cdot N_t \rceil + 1\Big) \cdot N_r.$$

Proof: Let N_r be 1 for simplicity. Consider a convolutional code of rate R_c as its equivalent block code by terminating codeword sequences. Then neglecting the number of the terminating bits, the convolutional code with input word length K can be viewed as a $(\frac{K}{R_c}, K)$ block code, which consists of 2^K codewords. Without loss of generality, consider a linear systematic block code where a codeword is divided into the information part and the parity part. Now if we divide a codeword of length $\frac{K}{R_c}$ into N_t sub-codewords of length $K_s = \frac{K}{R_c \cdot N_t}$, then the number of sub-codewords to cover the information part is $\lceil \frac{K}{K_s} \rceil = \lceil R_c \cdot N_t \rceil$. In the worst case, the information part may have only one non-zero weight sub-codeword. Defining $N_{\max}$ as the maximum number of sub-codewords which have zero weight in the information part, we have $N_{\max} = \lceil R_c \cdot N_t \rceil - 1$. Thus, there are N_{max} subcodewords with zero weights among N_t subcodewords. Then, the maximum achievable diversity with $N_r = 1$ is obtained as

$$D = N_t - N_{max} = N_t - \lceil R_c \cdot N_t \rceil + 1. \tag{2}$$

■

Note that equation (2) is equal to $\lfloor N_t(1 - R_c) \rfloor + 1$, and it agrees with the Singleton bound in [12], [14]. In other words, the ST-BICM system with code rate R_c meets the Singleton bound. To illustrate this, we plot the maximum achievable diversity order in various N_t, R_c and $N_r = 1$ in Figure 3.

Now we investigate conditions on code design to satisfy the Singleton bound. We will give examples for the diversity order analysis. Assume that a codeword is specified by a K by N generator matrix $\mathbf{G} = [\mathbf{I}_K \ \mathbf{P}]$ where $\mathbf{I}_K$ denotes a K by K identity matrix and $\mathbf{P}$ represents the K by $N-K$ parity check matrix.

Example 1: $N_t = 3$ and $R_c = 1/3$

The codeword is given as

$$\mathbf{x} = \mathbf{uG} = \mathbf{u}\begin{bmatrix} \mathbf{I}_K & \mathbf{P}_1 & \mathbf{P}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{u} & \mathbf{uP}_1 & \mathbf{uP}_2 \end{bmatrix}$$

where $\mathbf{u}$ indicates the information row vector of length K, and $\mathbf{P}_1$ and $\mathbf{P}_2$ denote parity check matrices of size K by K. Figure 4 (a) illustrates the diversity analysis, where 'x' means non-zero sub-codewords. The sub-codewords $\mathbf{u}$, $\mathbf{uP}_1$ and $\mathbf{uP}_2$ are positioned at each transmit antenna, respectively. For any $\mathbf{u}$ with non-zero weight, all sub-codewords have non-zero Hamming distance as long as $\mathbf{P}_1$ and $\mathbf{P}_2$ are invertible. Thus, this case achieves full diversity.

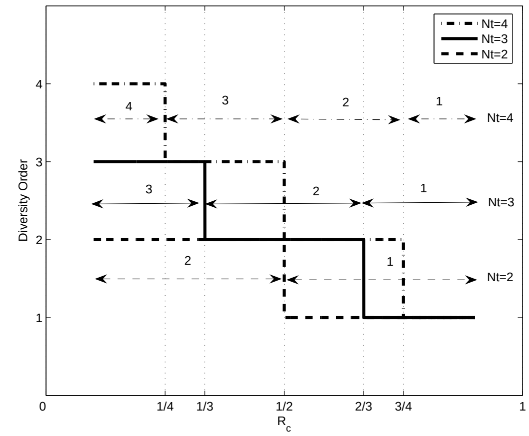

Fig. 3. The maximum achievable diversity order of ST-BICM with various R_c and N_t

1st sub-codeword	2nd sub-codeword	3rd sub-codeword
u	uP_1	uP_2
x	uP_1	uP_2

(a) $R_c = 1/3$

1st sub-codeword	2nd sub-codeword	3rd sub-codeword
u_1	u_2	uP
x	0	u_1P_1
0	x	u_2P_2

(b) $R_c = 2/3$

1st sub-codeword		2nd sub-codeword		3rd sub-codeword	
u_1		u_2	u_3		uP
x		0	0		u_1P_1
0		x	0		u_2P_2
0		0	x	0	u_3P_3
0		0	0	x	u_3P_3

(c) $R_c = 3/4$

Fig. 4. Code structure for $N_t = 3$

Example 2: $N_t = 3$ and $R_c = 2/3$

For this case, we have

$$\mathbf{x} = [\mathbf{u_1} \ \mathbf{u_2}]\begin{bmatrix} \mathbf{I}_{K/2} & \mathbf{0} & \mathbf{P}_1 \\ \mathbf{0} & \mathbf{I}_{K/2} & \mathbf{P}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \mathbf{u_1}\mathbf{P}_1 + \mathbf{u_2}\mathbf{P}_2 \end{bmatrix}$$

where $\mathbf{u_1}$ and $\mathbf{u_2}$ represent the information row vector of length $K/2$, and $\mathbf{P}_1$ and $\mathbf{P}_2$ denote parity check matrices of size $K/2$ by $K/2$. The sub-codewords $\mathbf{u_1}$, $\mathbf{u_2}$ and $\mathbf{u_1}\mathbf{P}_1 + \mathbf{u_2}\mathbf{P}_2$ are partitioned into each transmit antenna, respectively as illustrated in Figure 4 (b). If either $\mathbf{u}_1$ or $\mathbf{u}_2$ has non-zero weight, then the parity part becomes either $\mathbf{u_1}\mathbf{P}_1$ or $\mathbf{u_2}\mathbf{P}_2$. Thus as long as both $\mathbf{P}_1$ and $\mathbf{P}_2$ are invertible, those parity parts are nonzero. As a result, the minimum number of nonzero sub-codewords is two. If R_c is higher than 2/3, there exists one sub-codeword which has both the information part

1st sub-codeword	2nd sub-codeword	3rd sub-codeword	4th sub-codeword
u_1	u_2	uP	uP
x	0	u_1P_1	u_1P_2
0	x	u_2P_3	u_2P_4

Fig. 5. Code structure for $N_t = 4$ and $R_c = 1/2$

and the parity part. This makes the diversity order less than two, as illustrated in the following example.

Example 3: $N_t = 3$ and $R_c = 3/4$

This is an example for $R_c > 2/3$. The codeword is represented by

$$\mathbf{x} = [\mathbf{u_1}\ \mathbf{u_2}\ \mathbf{u_3}] \begin{bmatrix} \mathbf{I}_{K/3} & \mathbf{0} & \mathbf{0} & \mathbf{P}_1 \\ \mathbf{0} & \mathbf{I}_{K/3} & \mathbf{0} & \mathbf{P}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{K/3} & \mathbf{P}_3 \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \mathbf{u_3} & \mathbf{u_1}\mathbf{P}_1 + \mathbf{u_2}\mathbf{P}_2 + \mathbf{u_3}\mathbf{P}_3 \end{bmatrix}$$

where $\mathbf{u_1}$, $\mathbf{u_2}$ and $\mathbf{u_3}$ denote the information row vector of length $K/3$ and $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ represent parity check matrices of size $K/3$ by $K/3$. The worst case is illustrated in the last row in Figure 4 (c). If $\mathbf{u_1} = \mathbf{u_2} = 0$ and a part of $\mathbf{u_3}$ which belongs to the second sub-codeword is also 0, the first and second sub-codeword have zero weight. Thus, this limits the order of diversity to one.

For $N_t = 4$, we can show codes with $R_c = 1/4$ and 3/4 have diversity orders four and two, similarly shown in Example 1 and Example 2, respectively.

Example 4: $N_t = 4$ and $R_c = 1/2$

With $R_c = 1/2$, the codeword is obtained by

$$\mathbf{x} = [\mathbf{u_1}\ \mathbf{u_2}] \begin{bmatrix} \mathbf{I}_{K/2} & \mathbf{0} & \mathbf{P}_1 & \mathbf{P}_2 \\ \mathbf{0} & \mathbf{I}_{K/2} & \mathbf{P}_3 & \mathbf{P}_4 \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \mathbf{u_1}\mathbf{P}_1 + \mathbf{u_2}\mathbf{P}_3 & \mathbf{u_1}\mathbf{P}_2 + \mathbf{u_2}\mathbf{P}_4 \end{bmatrix}$$

where $\mathbf{u_1}$ and $\mathbf{u_2}$ denote the information row vector of length $K/2$ and $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ and $\mathbf{P}_4$ represent parity check matrices of size $K/2$ by $K/2$. Each sub-codeword $\mathbf{u_1}$, $\mathbf{u_2}$, $\mathbf{u_1}\mathbf{P}_1 + \mathbf{u_2}\mathbf{P}_3$ and $\mathbf{u_1}\mathbf{P}_2 + \mathbf{u_2}\mathbf{P}_4$ is represented as in Figure 5. In order for the parity part to have non-zero weight, all parity check matrices must be invertible. Then, the achievable diversity is equal to three. Also we can show that for $R_c > 3/4$, the diversity order becomes one, similarly shown in Example 3.

From the above examples, we can conclude that all parity check submatrices $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ and $\mathbf{P}_4$ must have full rank to achieve the diversity order described in Theorem 1 for the ST-BICM coded system with a punctured code. Note that the diversity analysis derived in this section can be applied to systems with arbitrary number of transmit antennas.

V. Simulation Results

In this section, we present extensive computer simulations to support Theorem 1. The above theoretical results are obtained for systematic binary block codes. In the simulation results however, we use the codes that are more suitable in a practical system, i.e. nonsystematic convolutional codes. Since terminated systematic and nonsystematic convolutional codes are block codes, we hypothesize that the above results are also applicable to convolutional codes. The simulation results below indicate that this is the case.

	$N_t = 2$, $R_C = 1/2$	$N_t = 3$, $R_C = 1/3$	$N_t = 4$, $R_C = 1/4$
$\nu = 4$	(23,35)	(25,33,37)	(25,27,33,37)

TABLE I
Code polynomials for simulations.

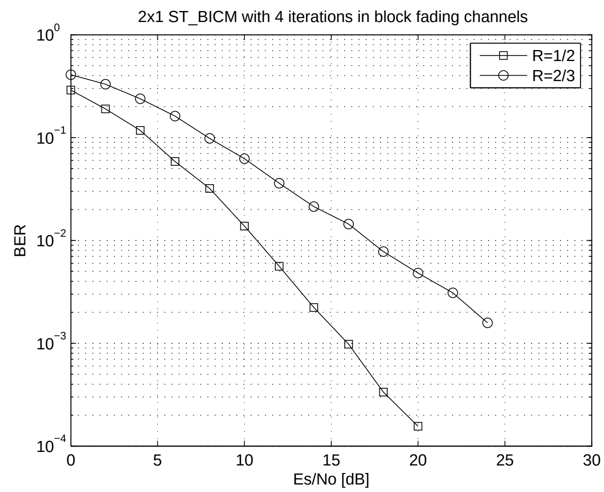

Fig. 6. BER with $N_t = 2$, $N_r = 1$

For the simulations, we use an optimal code polynomial of $\nu = 4$ in Table I where ν denotes the code memory length. For puncturing codes, we adopt the puncturing patterns in [20]. Simulations assume the quasi-static block fading channels. The x axis represents the received SNR per antenna in decibels and the y axis indicates the bit error rate (BER). Four iterations are performed between the MIMO demapper and the MAP decoder. The modulation type used is BPSK for simplicity.

First, we show the performance of the two transmit and one receive antenna ST-BICM systems in Figure 3. The system with puncturing adopts a rate $2/3$ punctured convolutional code, while the no puncturing employs a rate $1/2$ code. In this plot, it is clear that the diversity orders for each system are 1 and 2, respectively.

Next, we compare the ST-BICM systems with three transmit and one receive antenna in Figure 7. We can check from this plot that codes with $R_c = 3/4$, $2/3$ and $1/3$ have diversity orders of 1, 2 and 3, respectively, as expected from the analysis.

Finally, Figure 8 presents the simulation results for the 4×1 ST-BICM system. The plot obviously shows distinct performance slopes at each code rate. Curves with code rates of $4/5$, $3/4$, $1/2$ and $1/4$ correspond to the diversity order of 1, 2, 3 and 4.

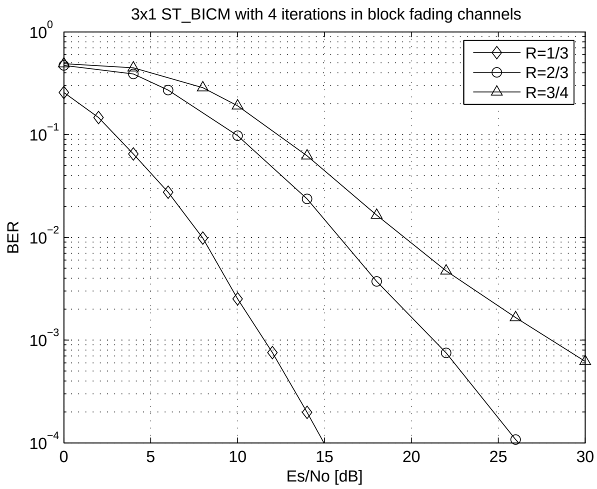

Fig. 7. BER with $N_t = 3$, $N_r = 1$

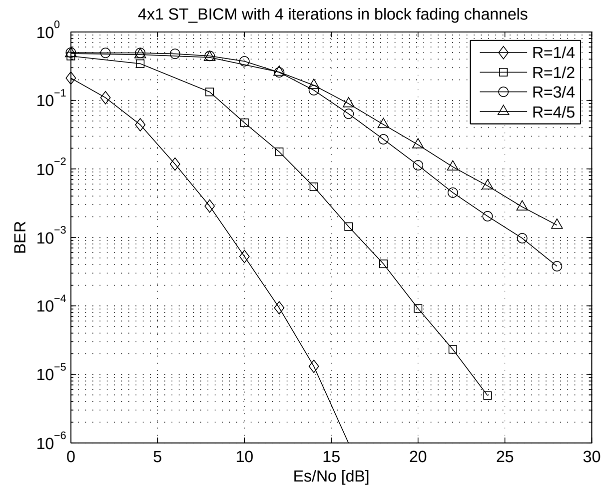

Fig. 8. BER with $N_t = 4$, $N_r = 1$

The above simulation results confirm that the diversity analysis made in the previous section is accurate.

VI. Discussion and Conclusions

In this paper, we have studied a flexible class of space-time codes based on bit-interleaved coded modulation systems. We have derived the maximum achievable diversity for punctured ST-BICM systems with rate higher than $1/N_t$. Based on the derivation, we provide a general theorem describing the diversity order. This theorem indicates that the properly designed ST-BICM can achieve the Singleton bound. Also we have presented conditions on code construction to meet the Singleton bound. Computer simulation results demonstrate the validity of the theorem in quasi-static channels. we make the interesting observation that high rate ST-BICM systems can be obtained with large number of transmit antennas at reduced degree of diversity. For example, with $N_t = 3$, $N_r = 1$ and 16QAM the diversity is 3 with a rate 1/3 code at transmission rate $R_T = 4$. With $N_t = 4$ and all else unchanged the diversity degree is still 3 but the transmission rate is 5.33, i.e. 33% higher.

Many open issues remain to look into, especially the choice of constellation mapper and interleaver for the best engineered systems, because the code design rules for ST-BICM and the degree of diversity in the system are independent of the mapper rule.

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