Coordinated Spatial Multiplexing with Orthogonalized Channels for Multiuser MIMO Downlink Systems

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***Abstract*—In this paper, we propose a new coordinated spatial multiplexing algorithm for multiuser multiple-input multiple-output (MIMO) downlink systems. Unlike other conventional coordination methods, our scheme exploits orthogonalized effective channels to yield the transmit precoding and receive combining matrices without requiring an iterative operation. In order to determine each row of the effective channels, a greedy type successive process is utilized. Simulation results show that the performance of our proposed scheme is very close to the conventional iterative scheme with significantly reduced complexity. Also, our method achieves a 3 dB performance gain over the conventional non-iterative scheme.**

***Index Terms*—MIMO, multiuser downlink, coordinated spatial multiplexing.**

I. Introduction

MULTIUSER multiple-input multiple-output (MIMO) broadcast channels (BC) have attracted much attention in the area of wireless communication research. Although several nonlinear precoding methods including dirty paper coding (DPC) have been developed to achieve the capacity region for MIMO BC [1], those are still too complex to implement. Consequently, an intensive study on linear processing techniques has been carried out to avoid complex nonlinear processing. One of the most intuitive schemes among linear processing algorithms is block diagonalization (BD) [2] [3] when the receivers have multiple antennas. However, to support all the receive antennas in the network without multiuser interference, the BD may require a large number of transmit antennas.

Unlike the BD, coordination techniques [4] [5] which jointly design the transmit and receive filters have been proposed to transmit fewer data streams than the number of receive antennas. The coordination methods in [4]–[7] obtain the transmit precoding and the receive combining vectors through an iterative procedure. However, the convergence of these iterative algorithms cannot be guaranteed [8]. In order to circumvent the iterative nature of conventional coordination schemes, a suboptimal method was introduced in [9]. Also the author in [10] proposed a zero-forcing solution valid for two transmit antenna systems. Recently, generalized beamforming vectors were derived for the two user case in [8], although the solution is restricted only for a single data stream per user. The authors in [11] extended the result in [8] to multiple data streams with restricted system configurations. Also, considering channel estimation errors with ill-conditioned channels [6], several methods based on minimum mean square error (MMSE) criterion were proposed in [6] [12] to maintain the performance. Similar approaches based on signal-to-leakage-and-noise ratio (SLNR) maximizing criterion have been studied in [13] [14]. Another coordination design in terms of quality of service (QoS) was presented in [15] with the sum power constraint.

In this paper, we propose a new low-complexity coordinated spatial multiplexing (CSM) scheme. The proposed algorithm first makes the rows of the effective channels near-orthogonal through a 'greedy' type successive process. After that, the transmit precoding matrices and the corresponding receive combining matrices are obtained from the effective channels without requiring an iterative fashion unlike the conventional schemes in [4]–[7]. Our scheme supports more than two users with arbitrary antenna configurations, whereas the schemes in [8] [10] [11] can be employed only for two users. Also, unlike the scheme in [8], our method is applicable even when the number of the transmit antennas is larger than that of the receive antennas. Simulation results show that the performance of our proposed scheme is very close to the conventional iterative scheme with significantly reduced complexity. Also, the proposed method achieves about a 3 dB performance gain over the conventional non-iterative coordination scheme with various configurations.

The following notations are used for the description throughout this paper. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. Also, $(\cdot)^{-1}$, $(\cdot)^T$ and $(\cdot)^\dagger$ stand for the inverse operation, transpose and conjugate transpose, respectively. Besides, $\mathbf{I}$ represents an identity matrix with proper dimension. We define diag$\{\cdots\}$ as a diagonal matrix with the elements inside the bracket. The notations $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote $m \times n$ real and complex matrix spaces, respectively.

II. System Description

We consider multiuser MIMO downlink systems where a base station is transmitting to K independent users simultaneously. In this system, the base station is equipped with N_t transmit antennas to support N_s data streams per user, and

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each user has N_r receive antennas. We assume that a proper user scheduling algorithm is adopted to select K users which satisfies $K \leq N_t/N_s$ [16]. Note that we consider general antenna configurations including $N_t \leq KN_r$ unlike other methods such as the CI and the BD. Also, perfect channel state information (CSI) of all users is assumed to be available at the base station via channel feedback or channel reciprocity.

In the discrete-time complex baseband MIMO case, the flat-fading channel from the base station to the k-th user $(k = 1, \cdots, K)$ is modeled by $\mathbf{H}_k = \left[\mathbf{h}_{k,1}^T \cdots \mathbf{h}_{k,N_r}^T\right]^T \in \mathbb{C}^{N_r \times N_t}$ where $\mathbf{h}_{k,i} \in \mathbb{C}^{1\times N_t}$ represents a row vector for $i = 1, \cdots, N_r$. We consider the entries of each user's channel matrix as independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0,1)$. We define the received signal vector $\mathbf{y}_k \in \mathbb{C}^{N_r\times 1}$ for user k as

$$\mathbf{y}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{T}_i \mathbf{x}_i + \mathbf{n}_k$$

where $\mathbf{x}_k \in \mathbb{C}^{N_s\times 1}$ denotes the transmit symbol vector for the k-th user, and $\mathbf{n}_k \in \mathbb{C}^{N_r \times 1}$ represents the additive white Gaussian noise vector observed at the receiver. $\mathbf{T}_k = [\mathbf{t}_{k,1} \cdots \mathbf{t}_{k,N_s}] \in \mathbb{C}^{N_t\times N_s}$ indicates the transmit precoder with unit-norm columns. Here the symbols in $\mathbf{x}_k$ are assumed to be independently generated with unit variance and the entries of $\mathbf{n}_k$ are i.i.d. with zero mean and variance N_0. Now the total received signal is given by

$$\mathbf{y} = [\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T]^T = \mathbf{H}\mathbf{T}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{H} = [\mathbf{H}_1^T \cdots \mathbf{H}_K^T]^T$, $\mathbf{x} = \left[\mathbf{x}_1^T, \cdots, \mathbf{x}_K^T\right]^T$, $\mathbf{n} = \left[\mathbf{n}_1^T, \cdots, \mathbf{n}_K^T\right]^T$ and $\mathbf{T} = [\mathbf{T}_1, \cdots, \mathbf{T}_K]$.

At the receiver, the received signal vector $\mathbf{w}_k \in \mathbb{C}^{N_s\times 1}$ for the k-th user after the receiver combining is expressed as

$$\mathbf{w}_k = \mathbf{R}_k^\dagger \mathbf{y}_k = \mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_k^\dagger \mathbf{H}_k \sum_{i=1, i\neq k}^{K} \mathbf{T}_i \mathbf{x}_i + \mathbf{R}_k^\dagger \mathbf{n}_k. \tag{2}$$

where $\mathbf{R}_k = [\mathbf{r}_{k,1} \cdots \mathbf{r}_{k,N_s}] \in \mathbb{C}^{N_r\times N_s}$ denotes the receive combining matrix for the k-th user.

When the coordination techniques in [4]–[8] are utilized, $\mathbf{T}_k$ causes no interference to user i $(i \neq k)$ by completely removing the interference term in (2). At the same time, the matrices $\mathbf{T}_k$ and $\mathbf{R}_k$ are jointly optimized in terms of the multiuser MIMO performance. Since it is complicated to obtain such optimized solutions which satisfy zero multiuser interference [4], we propose a low-complexity coordination method in the following section.

III. Proposed CSM Scheme

In this section, we present a new CSM algorithm which yields the transmit precoding and receive combining matrices without an iterative process. In the first phase, the proposed scheme determines the channels combined with the receive combining filters $\mathbf{R}_k$ through a successive process. Next, in the second phase, the receive combining matrices are computed by utilizing the effective channels obtained in the first phase, and then the corresponding transmit precoding matrices are determined.

A. Determination of the effective channel

At each step in the first phase, one user is identified in order to determine its effective channel which corresponds to each data stream. Here the effective channel $\widetilde{\mathbf{H}}$ at the base station after the receive combining filters are applied at the receivers can be expressed as

$$\widetilde{\mathbf{H}} = \left[\widetilde{\mathbf{H}}_1^T \cdots \widetilde{\mathbf{H}}_K^T\right]^T = \left[\left(\mathbf{R}_1^\dagger \mathbf{H}_1\right)^T \cdots \left(\mathbf{R}_K^\dagger \mathbf{H}_K\right)^T\right]^T$$

where $\widetilde{\mathbf{H}}_k = \left[\widetilde{\mathbf{h}}_{k,1}^T \cdots \widetilde{\mathbf{h}}_{k,N_s}^T\right]^T = \mathbf{R}_k^\dagger \mathbf{H}_k \in \mathbb{C}^{N_s\times N_t}$ indicates the effective channel for the k-th user. First, our method successively assigns one data stream for each user in order to determine the first row $\widetilde{\mathbf{h}}_{k,1} \in \mathbb{C}^{1\times N_t}$ $(k = 1, \cdots, K)$ of the effective channels $\widetilde{\mathbf{H}}_k$ one after another. After that, the same process is repeated for the next data stream to compute the subsequent channel rows $\widetilde{\mathbf{h}}_{k,i} \in \mathbb{C}^{1\times N_t}$ $(i = 2, \cdots, N_s)$ in similar ways.

Now we address how to identify a user in order to determine its effective channel row $\widetilde{\mathbf{h}}_{k,i}$ for assigning a data stream at each step. As mentioned before, our algorithm starts by computing the first effective channel row $(i = 1)$ of user k. Let us denote a set $\mathcal{A}_1 = \{1, \cdots, K\}$ which specifies available users to be considered for the first step. We define an intermediate matrix $\mathbf{G}_k \in \mathbb{C}^{N_r\times N_t}$ and initialize as $\mathbf{G}_k = \mathbf{H}_k$ for all k. For the first step $(j = 1)$ which determines the first user p_1 and the channel row $\widetilde{\mathbf{h}}_{p_1,1}$ corresponding to its first data stream $i = 1$, we apply singular value decomposition (SVD) to $\mathbf{G}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^\dagger$ for all users $k \in \mathcal{A}_1$, where $\mathbf{U}_k = [\mathbf{u}_{k,1} \cdots \mathbf{u}_{k,N_r}]$ and $\mathbf{V}_k = [\mathbf{v}_{k,1} \cdots \mathbf{v}_{k,N_t}]$ indicate the unitary matrices composed of the left and right singular vectors of $\mathbf{G}_k$, respectively, and $\boldsymbol{\Sigma}_k = \mathrm{diag}\{\sigma_{k,1} \ \cdots \ \sigma_{k,\min(N_t,N_r)}\}$ denotes a diagonal matrix contains the corresponding singular values.

By assuming the first column of $\mathbf{U}_k$ as the receive combining vector for user $k \in \mathcal{A}_1$, we can obtain the effective channel row $\mathbf{u}_{k,1}^\dagger \mathbf{H}_k$ with the channel gain $\|\mathbf{u}_{k,1}^\dagger \mathbf{H}_k\| = \sigma_{k,1}$ which is equivalent to the maximum singular value of $\mathbf{G}_k$. To maximize the effective channel gain, we want to choose the first user as one with the maximum gain as $p_1 = \arg\max_{k\in\mathcal{A}_1} \sigma_{k,1}$. Then the effective channel $\widetilde{\mathbf{h}}_{p_1,1}$ for user p_1 is computed as $\widetilde{\mathbf{h}}_{p_1,1} = \mathbf{u}_{p_1,1}^\dagger \mathbf{H}_{p_1} = \sigma_{p_1,1} \mathbf{v}_{p_1,1}^\dagger$ whose channel gain is the same as the maximum singular value, i.e., $\|\widetilde{\mathbf{h}}_{p_1,1}\| = \sigma_{p_1,1}$.

For the next step, to find the other users' effective channel rows, the set $\mathcal{A}_2$ which contains the available users in the second step is established as

$$\mathcal{A}_2 = \left\{ \ k \ \middle| \ k \in \mathcal{A}_1, \ k \neq p_1 \right\}. \tag{3}$$

At step j $(j = 2, \cdots, K)$ where a user p_j and its effective channel $\widetilde{\mathbf{h}}_{p_j,1}$ are determined for assigning a data stream $(i = 1)$, we should consider the previously obtained effective channels $\widetilde{\mathbf{h}}_{p_1,1}, \cdots, \widetilde{\mathbf{h}}_{p_{j-1},1}$ to minimize multiuser interference while maximizing the channel gain. According to (2), this can be achieved by designing $\widetilde{\mathbf{h}}_{p_j,1}$ near-orthogonal to $\widetilde{\mathbf{h}}_{p_1,1}, \cdots, \widetilde{\mathbf{h}}_{p_{j-1},1}$.

To this end, we first update the matrix $\mathbf{G}_k$ by projecting the rows of $\mathbf{H}_k$ into the nullspace of the already determined channels $\left[\widetilde{\mathbf{h}}_{p_1,1}^T \ \cdots \ \widetilde{\mathbf{h}}_{p_{j-1},1}^T\right]$ for $k \in \mathcal{A}_j$. When identifying $\widetilde{\mathbf{h}}_{p_j,1}$, we can ignore the vector space already occupied by the previous users $p_1, \cdots, p_{j-1}$ by treating $\mathbf{G}_k$ as the channel matrix. In what follows, we describe how to compute $\mathbf{G}_k$ in detail.

Let us denote $\{\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j-1,1}\}$ as an orthonormal basis vector set of the vector space spanned by $\left[\widetilde{\mathbf{h}}_{p_1,1}^T \cdots \widetilde{\mathbf{h}}_{p_{j-1},1}^T\right]$. Here the basis row vector $\mathbf{d}_{m,n} \in \mathbb{C}^{1\times N_t}$ corresponds to $\widetilde{\mathbf{h}}_{p_m,n}$. Specifically, at the second step ($j=2$), we obtain $\mathbf{d}_{1,1}$ as $\mathbf{d}_{1,1} = \widetilde{\mathbf{h}}_{p_1,1}/\|\widetilde{\mathbf{h}}_{p_1,1}\|$ by definition. Using the basis vector set, $\mathbf{G}_k$ at the j-th step is given as

$$\begin{aligned}\mathbf{G}_k &= \mathbf{H}_k - \left[\left(\mathbf{h}_{k,1}\sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\right)^T \cdots \left(\mathbf{h}_{k,N_r}\sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\right)^T\right]^T \quad (5)\\ &= \mathbf{H}_k - \left[\mathbf{h}_{k,1}^T \cdots \mathbf{h}_{k,N_r}^T\right]^T \sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\\ &= \mathbf{H}_k\left(\mathbf{I} - \sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\right). \quad (6)\end{aligned}$$

The second term of the right hand side (RHS) in (5) stands for the components of $\mathbf{H}_k$ spanning the subspace determined by $\{\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j-1,1}\}$. Applying the SVD to $\mathbf{G}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^\dagger$ for all $k \in \mathcal{A}_j$, we choose the j-th user as $p_j = \arg\max_{k\in\mathcal{A}_j} \sigma_{k,1}$ which shows the maximum channel gain $\sigma_{p_j,1} = \|\mathbf{u}_{p_j,1}^\dagger\mathbf{G}_{p_j}\|$ without considering the other users' channel subspace. After that, we utilize $\mathbf{u}_{p_j,1}$ to obtain its corresponding effective channel $\widetilde{\mathbf{h}}_{p_j,1} = \mathbf{u}_{p_j,1}^\dagger\mathbf{H}_{p_j}$.

In order to compute $\mathbf{G}_k$ at the next step, we need to derive $\mathbf{d}_{j,1}$ which constitutes the orthonormal basis vector set $\{\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j,1}\}$ for the vector space $\left[\widetilde{\mathbf{h}}_{p_1,1}^T \cdots \widetilde{\mathbf{h}}_{p_j,1}^T\right]$ containing the newly identified channel row $\widetilde{\mathbf{h}}_{p_j,1}$. From (5), $\widetilde{\mathbf{h}}_{p_j,1}$ is expressed as (4) at the bottom of this page. In (4), the first term has the component orthogonal to $\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j-1,1}$ by the definition of $\mathbf{G}_k$, whereas the second term shows the components which lie in the vector space of the basis $\{\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j-1,1}\}$.

Since $\mathbf{d}_{j,1}$ needs to be the component of $\widetilde{\mathbf{h}}_{p_j,1}$ which is orthonormal to $\{\mathbf{d}_{1,1}, \cdots, \mathbf{d}_{j-1,1}\}$, we compute $\mathbf{d}_{j,1}$ as

$$\mathbf{d}_{j,1} = \frac{\mathbf{u}_{p_j,1}^\dagger\mathbf{G}_{p_j}}{\|\mathbf{u}_{p_j,1}^\dagger\mathbf{G}_{p_j}\|} = \mathbf{v}_{p_j,1}^\dagger$$

which is the normalized value of the first term in (4). In addition, the set $\mathcal{A}_{j+1}$ which contains the valid users for the next step is updated similar to (3) as $\mathcal{A}_{j+1} = \left\{ k \mid k \in \mathcal{A}_j,\ k \neq p_j \right\}$. Then we proceed to the next step. This process is repeated until all users determine their respective channel rows $\widetilde{\mathbf{h}}_{p_1,1}, \cdots, \widetilde{\mathbf{h}}_{p_K,1}$ and a data stream is assigned for each user ($j=K$).

For computing the subsequent effective channel rows ($i = 2, \cdots, N_s$) for each user, we repeat our process with $j=1$ and the set $\mathcal{A}_1 = \{1, \cdots, K\}$. Since we want to consider all of the determined channels at the previous steps when computing $\mathbf{G}_k$, the equation in (6) is generalized as

$$\mathbf{G}_k = \mathbf{H}_k\left(\mathbf{I} - \left(\sum_{n=1}^{i-1}\sum_{m=1}^{K}\mathbf{d}_{m,n}^\dagger\mathbf{d}_{m,n} + \sum_{m=1}^{j-1}\mathbf{d}_{m,i}^\dagger\mathbf{d}_{m,i}\right)\right). \quad (7)$$

This process is ended when all $\widetilde{\mathbf{H}}_1, \cdots, \widetilde{\mathbf{H}}_K$ are obtained for assigning N_s data streams per user ($i = N_s$ and $j = K$).

B. Matrix computation for joint processing

By utilizing the effective channel $\widetilde{\mathbf{H}}$ obtained in the first phase, we now derive the receive combining matrix $\mathbf{R}_k$ for each user k. To this end, we consider a transmit precoding matrix $\widetilde{\mathbf{T}} = \left[\widetilde{\mathbf{T}}_1 \cdots \widetilde{\mathbf{T}}_K\right] = \widetilde{\mathbf{H}}^\dagger(\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^\dagger)^{-1} \in \mathbb{C}^{N_t\times KN_s}$ which satisfies zero multiuser interference with respect to $\widetilde{\mathbf{H}}$, i.e., $\widetilde{\mathbf{H}}_k\widetilde{\mathbf{T}}_i = 0$ for all $k \neq i$. Now let us assume that the base station adopts $\widetilde{\mathbf{T}}$ as the transmit precoding matrix $\mathbf{T}$ in (1) with unit-norm columns to satisfy the power constraint. Note that this assumption is made only for deriving $\mathbf{R}_k$.

The received signal $\mathbf{y}_k$ at user k is given as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{T}\mathbf{x} + \mathbf{n}_k = \mathbf{H}_k\widetilde{\mathbf{T}}_k\mathbf{x}_k + \mathbf{H}_k\sum_{i=1,i\neq k}^{K}\widetilde{\mathbf{T}}_i\mathbf{x}_i + \mathbf{n}_k.$$

Since the effective channel columns of $\mathbf{H}\widetilde{\mathbf{T}}$ yielded from $\widetilde{\mathbf{H}}$ are near-orthogonal, we can achieve the maximum channel gains with negligible multiuser interferences by utilizing maximum ratio combining (MRC). Applying MRC to $\mathbf{y}_k$, the receive combining matrix $\mathbf{R}_k$ is designed as

$$\mathbf{R}_k = \mathbf{H}_k\widetilde{\mathbf{T}}_k. \quad (8)$$

Finally, we determine the transmit precoding matrix $\mathbf{T}$ by using the receive combining matrices obtained in (8). Let $\mathbf{Z}$ denote the effective channel matrix combined with the determined $\mathbf{R}_k$ in (8) as

$$\mathbf{Z} = \left[\left(\mathbf{R}_1^\dagger\mathbf{H}_1\right)^T \cdots \left(\mathbf{R}_K^\dagger\mathbf{H}_K\right)^T\right]^T.$$

To eliminate the multiuser interference, we design $\mathbf{T}$ as

$$\mathbf{T} = \mathbf{Z}^\dagger(\mathbf{Z}\mathbf{Z}^\dagger)^{-1}. \quad (9)$$

Then, in order to satisfy the power constraint, each column of $\mathbf{T}$ is normalized to have a unit energy. Although the receive combining matrix $\mathbf{R}_k$ can be updated again as $\mathbf{R}_k = \mathbf{H}_k\mathbf{T}_k$ by using $\mathbf{T}$ in (9) subsequently, simulation shows that only a marginal gain is achieved. Thus we do not consider further updates to reduce the complexity. We summarize our scheme in **Algorithm 1** below.

IV. Complexity Analysis

In this section, we present the complexity analysis of the proposed algorithm compared with the conventional iterative method in [4]. We first clarify the complexity of the SVD regarding the number of floating point multiplications. The

$$\widetilde{\mathbf{h}}_{p_j,1} = \mathbf{u}_{p_j,1}^\dagger\mathbf{H}_{p_j} = \mathbf{u}_{p_j,1}^\dagger\mathbf{G}_{p_j} + \mathbf{u}_{p_j,1}^\dagger\left[\left(\mathbf{h}_{p_j,1}\sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\right)^T \cdots \left(\mathbf{h}_{p_j,N_r}\sum_{m=1}^{j-1}\mathbf{d}_{m,1}^\dagger\mathbf{d}_{m,1}\right)^T\right]^T \quad (4)$$

TABLE I
COMPLEXITY OF THE SVD

Configurations ($N_s = 1$)			$N_r = 2$	$N_r = 3$	$N_r = 4$
$N_t = 4$ and $K = 2$	Proposed	Operations	3		
		Multiplications	5388	7602	9816
	Conventional	Operations	12.4	16.2	19
		Multiplications	16864	31104	47120
$N_t = 6$ and $K = 4$	Proposed	Operations	10		
		Multiplications	26120	36780	47440
	Conventional	Operations	51.6	72.4	86.4
		Multiplications	70176	139008	214272

Algorithm 1 Proposed CSM scheme

for $i = 1 : N_s$

Set $\mathcal{A}_1 = \{1, \cdots, K\}$

for $j = 1 : K$

Obtain $\mathbf{G}_k$ for $k \in \mathcal{A}_j$ using (7)

Apply SVD to $\mathbf{G}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^\dagger$ for $k \in \mathcal{A}_j$

$p_j = \arg\max_{k \in \mathcal{A}_j} \sigma_{k,1}$

$\mathbf{d}_{j,i} = \mathbf{v}_{p_j,1}^\dagger$

$\widetilde{\mathbf{h}}_{p_j,i} = \mathbf{u}_{p_j,1}^\dagger \mathbf{H}_{p_j}$

$\mathcal{A}_{j+1} = \{\, k \mid k \in \mathcal{A}_j,\ k \neq p_j \}$

end

end

Compute $\widetilde{\mathbf{T}} = \widetilde{\mathbf{H}}^\dagger (\widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^\dagger)^{-1}$ and normalize $\widetilde{\mathbf{T}}$

Obtain $\mathbf{R}_k = \mathbf{H}_k \widetilde{\mathbf{T}}_k$ for all k

Set $\mathbf{Z} = \left[\left(\mathbf{R}_1^\dagger \mathbf{H}_1\right)^T \cdots \left(\mathbf{R}_K^\dagger \mathbf{H}_K\right)^T \right]^T$

Compute $\mathbf{T} = \mathbf{Z}^\dagger (\mathbf{Z}\mathbf{Z}^\dagger)^{-1}$ and normalize $\mathbf{T}$

required number of multiplications for computing the maximum singular value and its singular vector from a matrix of size $m \times n$ is given by [17] [18]

$$C_{\max}(m,n) = (8mn + 4m + 4n) M_{\text{itr}} + (4mn + 2\min(m,n)) \quad (10)$$

where M_{itr} indicates the number of iterations in the power method algorithm for the SVD. We confirm in computer simulations that $M_{\text{itr}} = 20$ on average is enough for the power method. In contrast, the required number of multiplications for obtaining N_s dominant singular vectors from a matrix of size $m \times n$ is given by [17] [18]

$$C_{\text{dom}}(m,n,N_s) = (8mn + 4m + 4n) M_{\text{itr}} N_s + 2\,(4mn + 2\min(m,n))\,(N_s - 1). \quad (11)$$

Our proposed scheme applies the SVD to the $N_r \times N_t$ matrix $\mathbf{G}_k$ in (7) for calculating both the maximum singular value and its corresponding singular vector. In contrast, the conventional algorithm derives the singular vector from an $N_r \times (N_t - (K-1)N_s)$ matrix which spans the nullspace of the other $K-1$ users' effective channels. Therefore, the complexity of the SVD for the proposed and the conventional scheme becomes $C_{\max}(N_r, N_t)$ and $C_{\text{dom}}(N_r, N_t - (K-1)N_s, N_s)$, respectively.

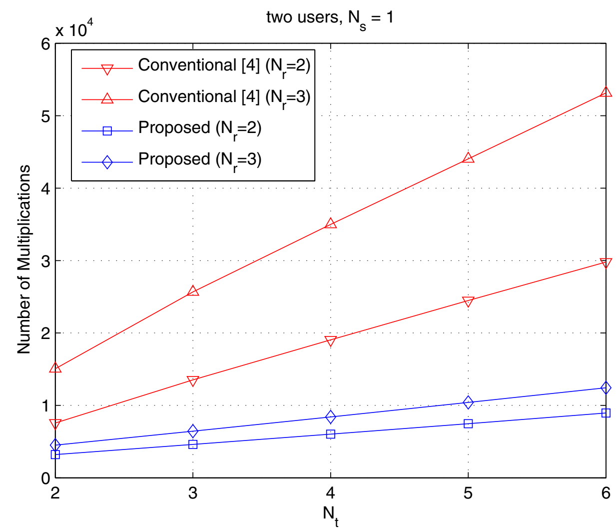

Fig. 1. multiplication numbers comparison with various N_t.

Table I lists the average number of the SVD operations and its corresponding number of floating point multiplications computed by utilizing (10) and (11) with various system configurations. Note that computer simulations are carried out to assess the number of the SVD operations for the conventional scheme due to its dependence on channel realizations. In contrast, the proposed scheme is implemented with $\frac{K(K+1)}{2} N_s$ SVD operations which is irrelevant to N_r. This makes our scheme more suitable in practical systems. For $N_t = 4$ and $K = 2$, the proposed scheme exhibits a significant complexity reduction of 68%, 76% and 79% in comparison to the conventional scheme with $N_r = 2$, 3 and 4, respectively. A similar trend is observed with $N_t = 6$ and four users ($K = 4$).

Figure 1 depicts the overall complexity by considering the SVD operations as well as the matrix multiplications with different numbers of transmit antennas and $K = 2$. This figure shows that our proposed scheme exhibits much reduced complexity compared to the conventional one. Similarly, a similar trend can be observed in Figure 2 with various numbers of users and $N_t = 8$. In the following section, we will verify that our low-complexity proposed scheme achieves the performance very close to the conventional scheme.

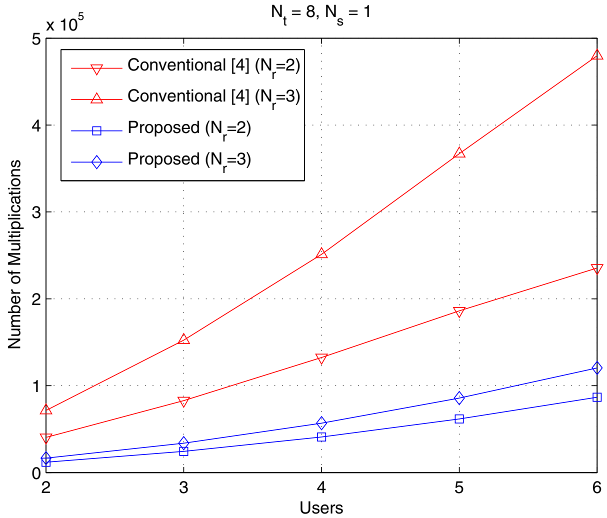

Fig. 2. multiplication numbers comparison with various numbers of users.

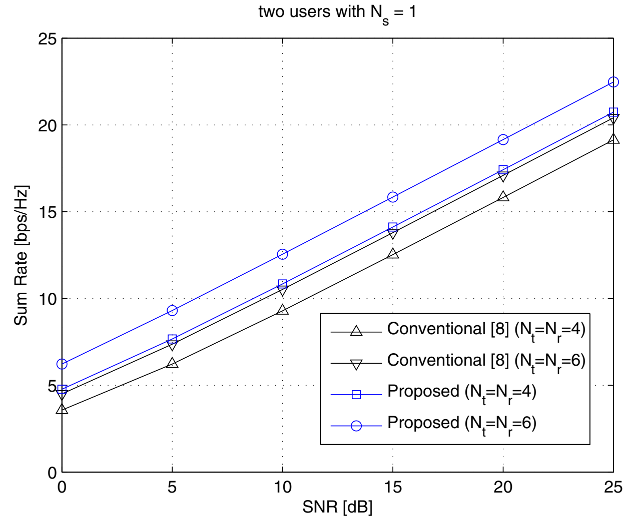

Fig. 4. Sum rate comparison.

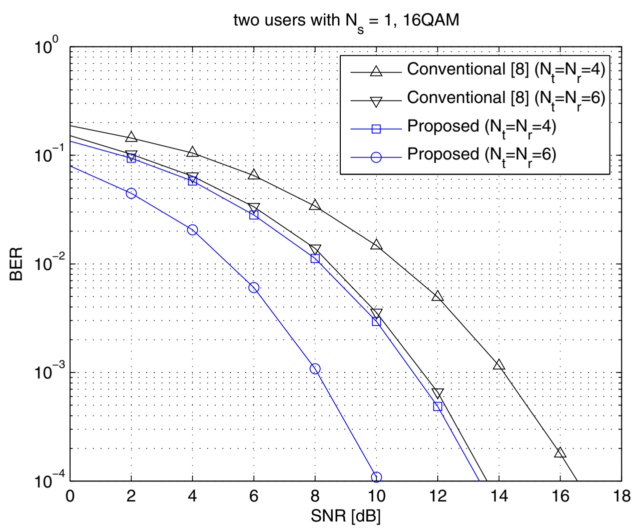

Fig. 3. BER comparison in 16-QAM.

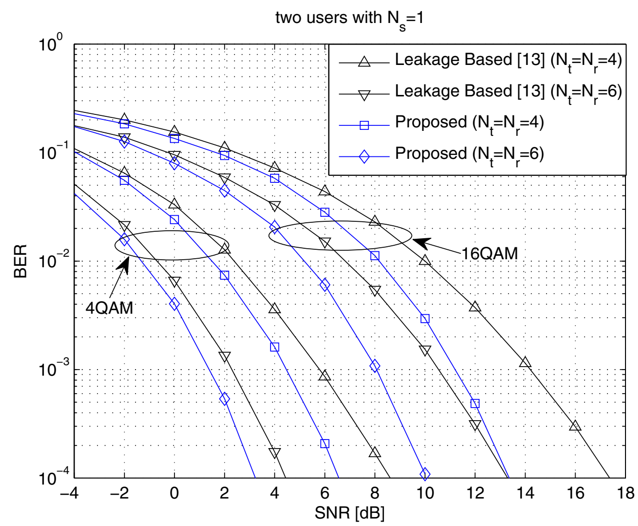

Fig. 5. BER comparison with the leakage based scheme.

V. Simulation Results

In this section, we present the BER performance of the proposed scheme comparing with the conventional schemes in [4], [8], and [13] through Monte Carlo simulations where flat fading channels are assumed. Figure 3 shows a performance gain of our scheme over the conventional method in [8] for 16-QAM with $N_t = N_r = 4$ and 6 where the number of user is two $(K = 2)$. Note that the number of the data stream is set to $N_s = 1$ since the method in [8] can support only single data stream per user. The proposed algorithm exhibits a 3.5 dB power gain over the conventional scheme at a BER of 10^{-4}. Figure 4 illustrates the performance curves in terms of the sum rate. In this plot, we can observe a similar performance trend.

In Figure 5, we compare the proposed method with another non-iterative scheme in [13] which yields a solution by maximizing the SLNR. Again our method exhibits a performance gain over the scheme in [13]. For instance, the proposed scheme outperforms the leakage based scheme by 2 dB at a BER of 10^{-4} with $N_t = N_r = 4$ and 4-QAM. The performance gap is increased to 4 dB in 16-QAM.

Now, we illustrate the performance of our proposed scheme and the iterative scheme in [4] with various numbers of receive antennas N_r. First, we compare the BER of the systems in Figure 6 for 4-QAM and 16-QAM with $N_t = 4$ and $N_s = 1$ where two users exist $(K = 2)$. We observe in this figure that the proposed scheme shows the performance almost identical to the iterative scheme with much reduced complexity for all N_r. Note that the scheme in [8] is not applicable when $N_t > N_r$ due to rank deficiency. Figure 7 illustrates the BER plots when the number of users is increased to $K = 4$ with $N_t = 8$. Again the proposed scheme achieves almost identical BER compared to the iterative scheme with much reduced complexity.

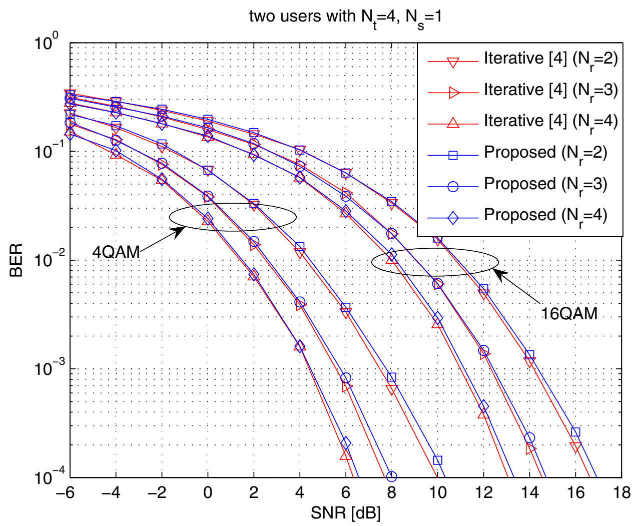

Fig. 6. BER comparison with $N_t = 4$.

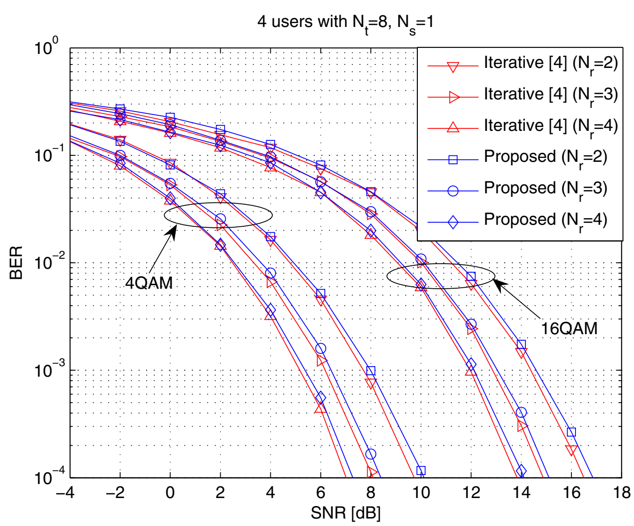

Fig. 7. BER comparison with $K = 4$.

VI. Conclusions

In this paper, we have presented a new low-complexity CSM algorithm for multiuser MIMO downlink systems. We maximize the performance of each data stream successively by making the effective channel rows orthogonal. Then the transmit precoding and the corresponding receive combining matrices are obtained from those orthogonalized effective channels without requiring an iterative process. Also, the proposed scheme is applicable in general systems with more than two transmit antennas and users. Our scheme achieves the performance almost identical to the conventional iterative scheme with much reduced complexity.

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