Synchronization of Symbol Timing in the OFDM Single-Frequency Networks

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***Abstract*— In this paper, we propose the method to estimate OFDM symbol timing in Single-Frequency Networks (SFN). For the efficient data transmission, the method employs redundant cyclic prefix (CP) instead of the train sequence and requires only a small proportion of CP not corrupted by inter-symbol interference (ISI). The proposed algorithm can estimate two points which indicate the beginning and end of the CP region in the absence of ISI with low variance. As a result, we can achieve the SFN gain as well as minimizing ISI.**

I. Introduction

OFDM system is very sensitive to synchronization errors such as symbol timing offset (STO) and frequency offset (FO). In this paper, we focus on the synchronization of symbol timing (ST) in single-frequency network (SFN) in the presence of frequency offset (FO). SFN is a broadcast network where several transmitters simultaneously send the same signal over the same frequency. Thus, the receiver receives several echoes of the same signal, and the constructive or destructive interference among these echoes results in fadings. Actually, we can consider the SFN transmission as a severe form of multipath propagation. For brevity, we refer to this multipath propagation as the SFN channel. Since the SFN channel is so lengthy that most of cyclic prefix (CP) are corrupted by inter-symbol interference (ISI), it is very challenging to estimate ST by using the CP. The ST estimator in [1] is based on the fact that the slope of correlation function in [1] varies when ever each echo arrives at the receiver. Furthermore, the variance of symbol timing offset (STO) can be reduced by the heuristic rules. However, the algorithm also estimates delayed ST which yields inter-carrier interference (ICI) and ISI when the power of post-echoes is much higher than that of pre-echoes. In this paper, we propose the ST estimator based on the correlation between CP and its duplicated part of the OFDM symbol. First, the whole CP is characterized by the correlation function whose window size is so short to include small proportion of CP not corrupted by ISI. Then, the optimal ST is simply obtained by using the property of the correlation function. Although our proposed algorithm can not ensure unbiased estimates, but obtain one point in the CP region without ISI with low variance at the moderate and high signal to noise ratio (SNR). Hence, the received OFDM signal does not undergo the ISI which results in significant degradation of the system performance. If two more received antenna are employed, our estimator can obtain the optimal ST with moderate variance at the low SNR.

II. Proposed Symbol Timing Estimator

Let us define the channel impulse response of the pth echo as $\mathbf{h}_p = [h(0)_p \cdots h(L-1_p)]^T$ where $h(k)_p$ is identical independent distributed (i.i.d.) Rayleigh distributed and L stands for the channel length, N as the number of subcarriers, N_g as the length of CP and $N_u = N + N_g$. The summation of received echoes can be represented by for $-N_g \le n$

$$r(n) = \sum_{p=0}^{G-1} \sum_{l=0}^{l=L-1} h_p(l) x(n-l-d(p)) e^{j2\pi\varepsilon n/N} + w(n)$$

where $x(n)$ indicates the transmitted signal, $w(n)$ represents the white gaussian noise, ε denotes FO and $\mathbf{d} = [d(0) \cdots d(G-1)]$ stands for the propagation delay with G being the number of echoes. Let us assume that $d(G-1)+L-1$ is less than N_g and the difference between both is larger than window size N_w. In this case, the whole CP can be classified by the following metric

$$\Gamma(\delta) = \sum_{n=\delta}^{\delta+N_w-1} \Big(|r(n)|^2 + |r(n+N)|^2\Big)/2 - \left| \sum_{n=\delta}^{\delta+N_w-1} r^*(n) r(n+N) \right|. \quad (1)$$

Note that the proposed metric is independent of the FO unlike [2] which also employs shortened window. For $d(G-1) + L - N_g \le \delta \le -N_w$, the value of $\Gamma(\delta)$ is close to 0, which indicates that the CP region is not affected by ISI. Otherwise, $\Gamma(\delta)$ always has positive value. By using the property of the metric, we can estimate the optimal ST as follows

$$\hat{\delta}_1 = \arg\max_{\delta} \frac{\Gamma(\delta)}{\Gamma(\delta - N_w)} = \arg\max_{\delta} \Phi(\delta). \quad (2)$$

For $\delta = 0$, $\Gamma(0)$ which does not include any samples in CP would have larger value over $\Gamma(\delta)$ for $\delta \le -1$, while $\Gamma(-N_w)$ is close to zero. Therefore, $\Phi(0)$ has the largest value among all other $\Phi(\delta)$ for $\delta \neq 0$. Conversely, $\Phi(\delta)$ has the smallest value at $\delta = d(G-1)+L-N_g$. As a result, there would be no ISI in the received OFDM symbol if ST is chosen from one among $d(G-1)+L-N_g \le \delta \le 0$. Rather, the phase distortion due to pre-receive can be compensated by the equalizer.

III. Improved Proposed Estimator

A. Selecting the Optimal Symbol Timing

In this section, we analyze the metric in Eq. (1) according to the SFN channels to find drawback and propose the improved method to overcome that. In fig. 3, there are two echoes in the SFN channels, the power of pre-echo is less than that of post-echo by $10dB$ and the delay difference of both is $d(1)-d(0) = 110$ samples with $N = 256$ and $N_g = 128$. The optimal ST can be estimated by selecting the largest magnitude of $\Phi(0)$, but it would yield large symbol timing errors at the low SNR. This can be explained that $\delta = d(1)$ may be estimated instead of $\delta = 0$ at the low SNR since $\Phi(d(1))$ which actually indicates the starting point of the second echo

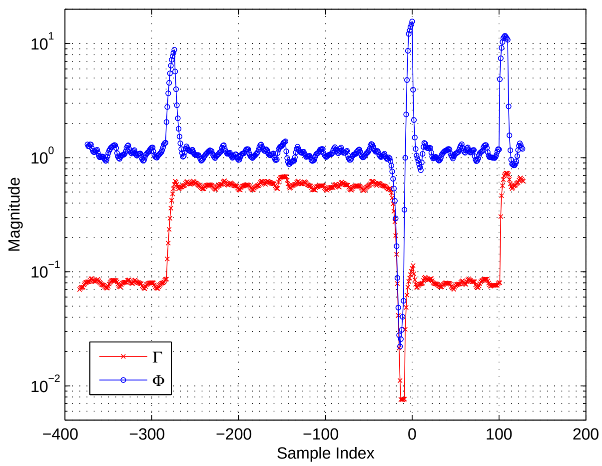

Fig. 1. The Proposed Metrics

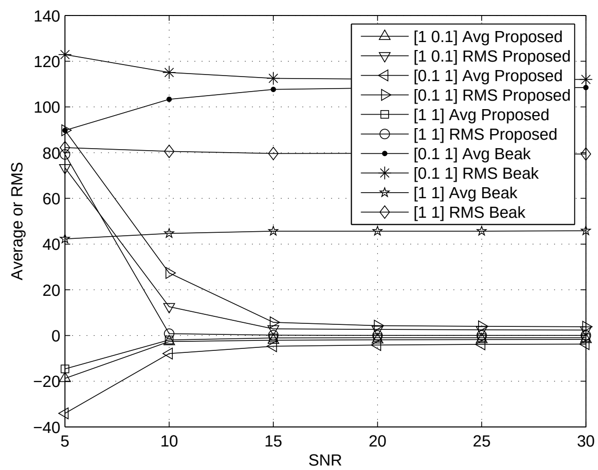

Fig. 2. Average and Root Mean Square of STO

also has fairly large magnitude. To the best of my knowledge, most of ST estimators with CP suffer from the same problem and would estimate the delayed ST in this SFN channel. On the contrary, the magnitude of $\Phi(d(1)+L-N_g)$ which stands for the starting point of cyclic prefix of the second echo still remains and there would be no candidates which has relatively low magnitude. Therefore, it would be better to select the smallest value of $\Phi(\delta)$ instead of the largest value of $\Phi(\delta)$ if the power of pre-echoes is much less than that of post-echoes. Conversely, we can deduce that the largest value of $\Phi(\delta)$ should be chosen if the power of pre-echoes is equal or larger than that of post-echoes.

B. Estimation with Diversity

Since our proposed estimator uses only small samples of CP, the correlation function in Eq. (1) is vulnerable to noise. Under low SNR conditions, this will degrade the performance of the proposed ST estimator, which can be confirmed from large variance in fig .3. To remedy the vulnerability, the proposed estimator employs multiple received antennas. Actually, two received antennas are sufficient for accurate estimation of ST.

IV. Simulation Results

In this section, we simulate the performance of the proposed ST estimator with following system parameters. The number

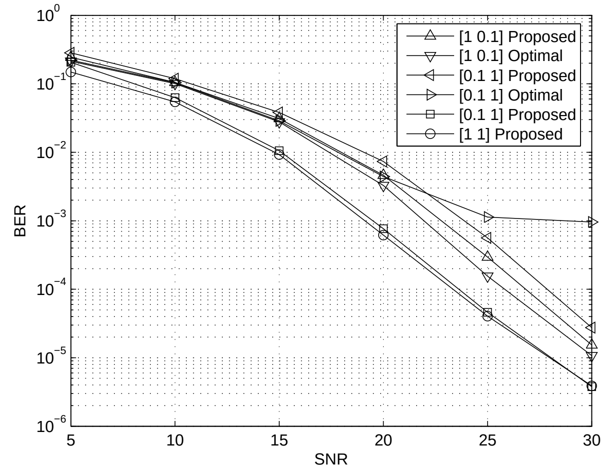

Fig. 3. BER

of subcarriers, cyclic prefix samples, the number of received antennas, the normalized FO, the modulation level and the number of simulation runs are set to $N = 256, N_g = 128, N_r = 2, \epsilon = 0.23$, 16QAM and $N_R = 50000$, respectively. The power ratios of the power of pre-echo to that of post-echo are given by $[1\ 0.1], [0.1\ 1]$ and $[1\ 1]$. The delay difference of both is set to 110 samples. Fig. 3 shows the average and variance of STO for the proposed ST estimator and conventional estimator in [3]. It should be noted for the proposed estimator that the average of STO is less than 0 and the variance is low although most of CP are corrupted by ISI. Hence, the bit error rate (BER) is at most 1dB worse than that of the optimal receiver at the high SNR. If the residual FO is negligible, the performance loss would be further reduced. For the case of $[0.1\ 1]$, the BER curve of the optimal receiver reaches saturation at the high SNR. This can be due to that the coarse FO estimator by using CP can not estimate sufficiently the FO since the first echo has low power.

V. Conclusion

A CP-based estimation algorithm is proposed in this paper for estimating the ST in SFN. The proposed algorithm classifies the CP by using the proposed metric and finds the CP region free from ISI with simple divide operation. Then, the optimal ST can be chosen according to given the SFN channel. Simulation results show that the proposed estimator performs well, even under low SNR with two received antennas. Furthermore, the estimator is computationally efficient in that it does not requires expensive operations such as inverse and employs shortened sliding window which enables most of multiplications to be shared.

References

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