Modified Maximum Likelihood Sequence Estimation in a Simple Partial Erasure Model*

Inkyu Lee† Takashi Yamauchi‡ John M. Cioffi†

†Information Systems Laboratory
Stanford University
Stanford, CA 94305, U.S.A.

‡Thin Film Laboratory
Mitsubishi Kasei Co.
Yokohama 227, JAPAN

Abstract

A simple partial erasure model caused by the effective reduction of a transition width in a magnetic recording system is presented. This model predicts the nonlinear distortion accurately at high recording densities. This paper proposes the modified maximum likelihood sequence detector (MLSD) incorporating this nonlinear model. Simulation shows that the modified maximum likelihood sequence detector outperforms the normal Viterbi detection in a nonlinearity-dominant channel.

1 Introduction

As recording densities grow in magnetic storage, nonlinear distortion becomes dominant, especially with thin-film disk. The nonlinear behavior is predominantly of two types: a transition shift, and an anomalous amplitude reduction or partial erasure [1]. Yamauchi has shown that partial erasure can be construed as the effective reduction of transition width across a track and has proposed a transition-width-reduction model based on both transition shift and transition-width reduction [2].

A magnetic recording channel model that can predict nonlinear distortion accurately enables one to design an improved receiver. However, if the channel model is too complicated, the complexity of receivers for that model may be prohibitive. A simpler model is appropriate for signal processing, while a more complex model is better for study of magnetic disk material.

In this paper, we present a simple partial-erasure model, which is simplified from the transition-width-reduction model in [2]. This model is shown to preserve the accuracy, while it is much simpler compared to the transition-width reduction model. The modified detection scheme to incorporate the nonlinear distortion in a high density magnetic recording channel is proposed in section 3. Then, performance of the modified scheme is compared to that of the linear sequence estimation. The simulation result shows that the modified scheme outperforms the normal sequence detection scheme in a nonlinearity-dominant channel.

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2 Channel Model

In this study, we will assume that transition shifts are compensated by appropriate time shifting of the write current reversals. Then, the output of a magnetic recording channel can be expressed as follows, using the transition-width-reduction model:

$$V(t) = \sum_k r_k \cdot b_k \cdot s(t - kT), \qquad (1)$$

where r_k is the effective transition-width ratio, b_k represents the NRZI modulated input data which is related to the binary data a_k by $b_k = a_k - a_{k-1}$, $s(t)$ is the step response, and T is the symbol period. The input data, b_k, takes on values {+2,0,-2}; a non-zero b_k indicates a transition. One future and several previous values of b_k are required to determine r_k [2].

For simplicity of the analysis, we will make a further assumption that the effective transition-width ratio, r_k is determined only by the number of neighboring transitions and is determined as follows:

$$r_k = \begin{cases} 1, & \text{if no transition in adjacent neighbors} \\ \gamma, & \text{if one transition in adjacent neighbors} \\ \gamma^2, & \text{if two transitions in adjacent neighbors} \end{cases}$$

where γ is a parameter which specifies the amount of partial erasure effect. For example, if $b_{k-1} = -2$ and $b_{k+1} = 0$, then $r_k = \gamma$.

The nonlinearity can be described by one parameter, γ. We will refer to this model as the simple partial erasure model and refer to the parameter, γ, as a reduction parameter. Note that when γ equals to 1, we are left with a linear superposition model. Also as density increases, γ decreases.

Table 1 compares the performance of the simple partial-erasure model and the linear superposition model as a function of normalized density, which is defined as $\frac{pw_{50}}{T}$ where pw_{50} is the width of the step response at 50% of its peak value. The performance measure is the signal-to-distortion ratio (SDR), which is defined as

$$SDR = 10 \log_{10} \left[\frac{E[y_k^2]}{E[(y_k - \tilde{y}_k)^2]} \right],$$

normalized density	γ	simple partial-erasure model	linear superposition model
1.0	0.90	41.55	15.18
1.25	0.83	32.13	10.53
1.5	0.775	26.25	7.86
1.75	0.73	22.06	6.11
2.0	0.70	18.90	4.80

Table 1: Comparison of SDR (dB)

where y_k is the output of the transition-width-reduction model and $\tilde{y}_k$ is the output of the simple partial model or the linear superposition model. The reduction parameter, γ, and the outputs of both models were calculated with the experimental data in [2]. This result indicates that the difference between the output of the transition-width-reduction model and that of the simple partial erasure model is small if transition shifts are compensated. Our new model yields much more accurate estimation of nonlinear distortion than the linear superposition model does.

3 Modified Detection Scheme

Here we assume that the channel is equalized to the class IV partial response (PR4), characterized by discrete time impulse response $1 - D^2$. Since the NRZI modulation already includes $1 - D$ factor, the channel in the equation (1) is equalized to $1 + D$ as a result.

In order to estimate the current input b_k, we need to know b_{k+1} and b_{k-1}. When MLSD is used, this estimation scheme can be implemented using a delay. The incoming current sampled channel output is considered as a future output, which is used for estimation of the previous input bit. Thus, a $D + D^2$ channel is used to incorporate this scheme instead of $1 + D$. The corresponding trellis description considering the nonlinear distortion can be obtained as shown in figure 1. In the trellis, X denotes "don't care" input, and '*' indicates that out-

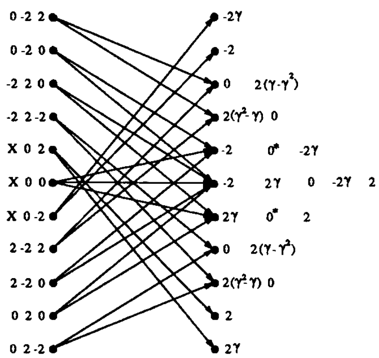

Figure 1: trellis diagram of a simple partial erasure model

put is possible only for the valid input sequences. 3 digits in the left hand side of the trellis represents b_{k-2}, b_{k-1}, b_k, respectively, and the values in the right hand side of the trellis indicate the trellis output. Some input states are ruled out, since the non-zero input bits +2 and -2 should alternate according to the NRZI code constraint. Note that γ illustrates the partial-erasure effect in the trellis. When γ equals to 1, which is a linear channel case, this trellis is equivalent to the normal PR4 channel description. Thus, the modified MLSD using this trellis is proposed to compute the error metric.

Performance of the modified MLSD is determined by the minimum distance, d_{min}, of any possible output sequence pairs from the trellis description. It can be shown that the minimum distance is

$$d_{min}^2 = 8\gamma^2.$$

The output error sequences corresponding to this d_{min}^2 are shown in figure 2 with trellis.

In the following section, performance degradation of the normal MLSD in a nonlinear channel is examined and the SNR's are compared.

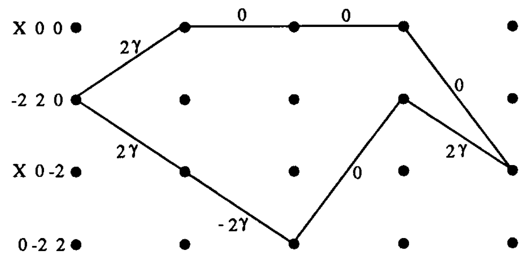

Figure 2: error sequences corresponding to dmin

4 Performance comparison

Now we will derive the SNR of the normal MLSD when used in a nonlinearity-dominant channel. We are refering to the normal MLSD as Partial Response Maximum Likelihood (PRML) which uses Viterbi detector tuned to $1 - D^2$ channel. When the normal MLSD is used ignoring the nonlinearity, this situation becomes a mismatched channel problem, since the target channel of Viterbi detector is mismatched to $1 - D^2$ channel in a nonlinear channel.

Define the discrete sampled channel output $\mathbf{z}$ expressed in the vector form as

$$\mathbf{z} = H\mathbf{x} + \mathbf{n}$$

where H is the channel response including the nonlinearity, $\mathbf{x}$ is the input sequence and $\mathbf{n}$ is an additive Gaussian noise with covariance matrix R_n.

Define the noiseless channel output $\mathbf{y} = H\mathbf{x}$, the mismatched channel output $\tilde{\mathbf{y}} = \bar{H}\mathbf{x}$ and the erroneous mismatched channel output $\hat{\mathbf{y}} = \bar{H}\hat{\mathbf{x}}$ which corresponds to the erroneous input $\hat{\mathbf{x}}$. Here we denote $\bar{H}$ as the mismatched channel equal to $1 - D^2$. The probability of error is the probability that in Viterbi detection, $\hat{\mathbf{y}}$ has greater likelihood than $\tilde{\mathbf{y}}$, or

$$Pr\{||\mathbf{z} - \tilde{\mathbf{y}}||^2 > ||\mathbf{z} - \hat{\mathbf{y}}||^2\}$$
$$= Pr\{||\mathbf{y} - \tilde{\mathbf{y}} + \mathbf{n}||^2 > ||\mathbf{y} - \hat{\mathbf{y}} + \mathbf{n}||^2\}$$
$$= Pr\{||\tilde{\mathbf{y}}||^2 - 2\tilde{\mathbf{y}}\cdot\mathbf{y} - 2\tilde{\mathbf{y}}\cdot\mathbf{n} > ||\hat{\mathbf{y}}||^2 - 2\hat{\mathbf{y}}\cdot\mathbf{y} - 2\hat{\mathbf{y}}\cdot\mathbf{n}\}$$

where $||\cdot||$ denotes vector 2-norm and $\cdot$ represents vector inner product.

Define $\mathbf{e} = \hat{\mathbf{y}} - \tilde{\mathbf{y}}$ and substitute into the above equation. Then,

$$Pr\{2\mathbf{e}\cdot\mathbf{n} > ||\hat{\mathbf{y}}||^2 - ||\tilde{\mathbf{y}}||^2 - 2\mathbf{e}\cdot\mathbf{y}\}$$
$$= Pr\{\mathbf{e}\cdot\mathbf{n} > \frac{1}{2}\mathbf{e}\cdot(\hat{\mathbf{y}} + \tilde{\mathbf{y}} - 2\mathbf{y})\}.$$

Since $\mathbf{n}$ is the only random variable, the variance of the LHS of the above inequality is $\mathbf{e}^T R_n \mathbf{e}$. If we neglect the number of the nearest neighbors, the probability of error is

$$P_e \approx Q\left(\min_{\mathbf{x},\hat{\mathbf{x}}} \frac{<\mathbf{e}, \hat{\mathbf{y}} + \tilde{\mathbf{y}} - 2\mathbf{y}>}{2\sqrt{\mathbf{e}^T R_n \mathbf{e}}}\right)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-x^2/2}dx$$

and $<\cdot,\cdot>$ represents vector inner product.

Thus, the effective SNR is the squared value of the arguement of Q function. Simulation has been done over all possible input sequence pair $\mathbf{x}, \hat{\mathbf{x}}$ to find the effective SNR of the normal MLSD in a nonlinear channel.

Performance comparison between the modified MLSD and the normal MLSD is shown in figure 3. In this simulation, a Lorentzian channel is assumed so that a step response is

$$s(t) = \frac{1}{1 + (2t/pw_{50})^2}.$$

The x-axis represents the reduction parameter γ described in section 2. The effective SNR's of the both systems are computed including the performance degradation due to a colored error sequence. 5 taps of feedforward filter is assumed and Matched Filter Bound (MFB) is set to 30 dB at $\gamma = 1$. From the plot, we can notice that the modified MLSD achieves about 3 - 4 dB gain over the normal MLSD at high densities with almost the same complexity. The performance degradation of the normal MLSD becomes severe as the nonlinearty increases. This is consistent with the well-known fact that the MLSD is very sensitive to the variation of parameters and the nonlinear distortion, which makes the mismatched channel environment [3].

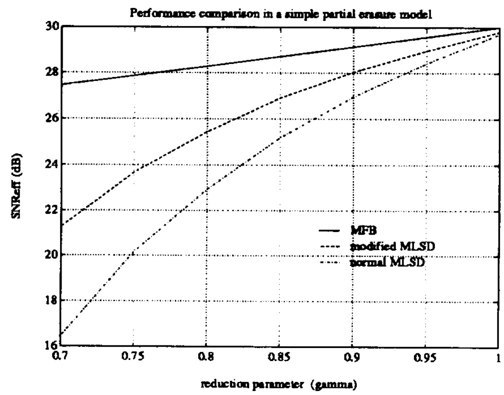

Figure 3: Comparison of SNR

5 Conclusion

We have shown that the simple partial erasure model based on the effective reduction of a transition width predicts nonlinear distortion quite accurately. Its simpler structure leads to an efficient architecture for an improved receiver with negligible extra cost in implementation. Our new model is represented by a single parameter γ, and requires knowledge of one future and one past input bit. Based on this channel model, the modified MLSD has been proposed. Performance analysis for other receivers in this simple partial erasure model has been made in [4]. Simulation indicates that the modified MLSD can achieve a significant improvement over the normal MLSD. It has been also shown that the normal MLSD performs poorly in the presence of nonlinear distortion.

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