Regularized Transceiver Designs for Multi-User MIMO Interference Channels

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***Abstract*—For multi-user interference channels (IC), an altruistic approach based on the zero-forcing (ZF) criterion shows the near-optimal performance at high signal-to-noise ratio (SNR), whereas its performance at low SNR becomes poor compared to a simple egoistic algorithm (selfish beamforming). Thus, balancing between the egoism and the altruism has been an important issue to achieve good sum-rate performance at overall SNR regime. In this paper, we propose a new approach for enhancing the performance by regularizing the ZF based transceivers. To this end, we start with investigating efficient ZF transceivers for 2-user and 3-user ICs. First, coordinated spatial multiplexing (CSM) is proposed for 2-user IC. For the 3-user case, it is shown that the enhanced interference alignment (E-IA) introduced in our previous work is the optimal ZF transceivers in terms of the sum-rate performance. Next, to improve the performance of the CSM and E-IA schemes at low SNR, we propose a non-iterative regularization method under the high SNR approximation. The distributed implementation of the proposed regularization method is also presented where each node is able to compute its own precoding or decoding matrix using local channel state information. From simulations, it is observed that the proposed regularized design outperforms the conventional schemes in overall SNR regime. Also, we confirm that our distributed approach provides a substantial performance gain over the conventional distributed scheme with reduced computational complexity.**

***Index Terms*—Beamforming, interference alignment, MIMO interference channels.**

I. Introduction

MULTIPLE-input multiple-output (MIMO) systems are popular approaches to meet the demands for high data rate wireless communications [1] [2]. In theory, MIMO techniques enable spatial multiplexing which achieves extremely high spectral efficiency by transmitting independent data streams simultaneously. Nevertheless, their effectiveness may become much limited in next generation cellular systems since well designed cellular systems are interference-limited [3]. Therefore, it is quite important to study cellular networks as MIMO interference channels (IC). However, the capacity characterization of the IC is still an open problem even for two user and single antenna cases [4].

As an alternative measure, the degree-of-freedom (DOF) has emerged to assess the sum-rate performance at high signal-to-noise ratio (SNR) [5]–[10]. Finding the transceivers achieving the optimal DOF in general K-user MIMO IC with $K \geq 3$ is quite a difficult problem. The authors in [6] provide a closed-form solution for achieving the maximum achievable DOF in the 3-user MIMO IC where all nodes are equipped with the equal number of antennas. They introduced an idea of interference alignment (IA) to maximize the dimension of the desired signals. In [11], the enhanced interference alignment (E-IA) algorithm was proposed to improve the sum-rate performance of the conventional IA scheme. However, since both the IA and E-IA completely eliminate the inter-user interference while neglecting the noise effect, they suffer from a performance loss in comparison to a simple time-division multiple access (TDMA) scheme at low SNR.

To address this concern, the authors in [12] and [13] consider a scheme which balances between the egoism (selfish beamforming) and the altruism (interference cancellation). Especially in multiple-input single-output (MISO) IC, it was shown in [14] that the Pareto-optimal solutions can be obtained by well adjusting the balancing ratio. At low SNR, an altruistic approach based on the zero-forcing (ZF) criterion provides the performance inferior to a simple egoistic algorithm, since impairments mainly come from the additive noise. In contrast, at high SNR, the ZF algorithm shows the near-optimal sum-rate performance since co-channel interference becomes a dominant factor.

In this paper, we try to achieve good sum-rate performance at overall SNR regime by regularizing the ZF based transceivers instead of balancing. We start with investigating efficient ZF transceivers for 2-user and 3-user ICs. First, a non-iterative coordinated spatial multiplexing (CSM) scheme which achieves the maximum DOF for the 2-user MIMO IC is proposed. The proposed CSM can be considered as an extension of the ZF interference-aware coordinated beamforming (IA-CBF) in [15] which supports only single data stream for each user. For the 3-user case, we first prove that the E-IA scheme in [16] achieves the optimal sum-rate performance in terms of the ZF criteria. Next, to improve the performance of the proposed CSM and E-IA schemes at low SNR, we propose a method of regularizing the ZF based schemes. Since an iterative process is required for complete regularization, we employ a high SNR approximation to derive a non-iterative regularization algorithm.

Also, distributed implementation of the proposed regular-

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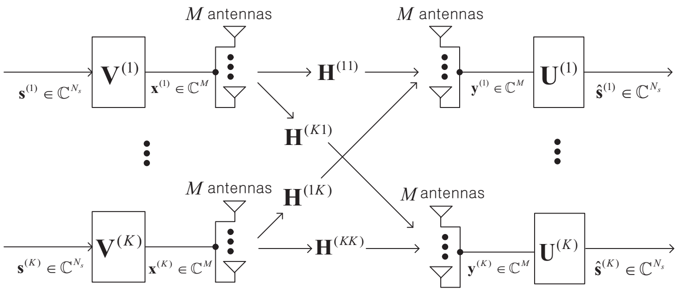

Fig. 1. System model for linear precoding and decoding in K-user MIMO interference channels.

ization method is presented where each node computes its own precoding or decoding matrix using local channel state information (CSI) [7] [17] [18]. In [17], the authors proposed to manage inter-user interference by adjusting input covariance based on an iterative semi-definite programming approximation algorithm. In addition, numerical IA feasibility checking methods have been studied in [7] and [18]. It is shown from simulations that our distributed approach provides a substantial performance gain over the conventional distributed scheme in [7] with reduced complexity.

Throughout the paper, the following notations are used for description. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The trace, conjugate, Hermitian transpose and the column space of a matrix or vector are represented by Tr$(\cdot)$, $(\cdot)^*$, $(\cdot)^H$ and $\mathcal{C}(\cdot)$, respectively. $||\cdot||$ indicates the Euclidean 2-norm of a vector. An identity and a zero matrix of size n are denoted by $\mathbf{I}_n$ and $\mathbf{0}_n$, respectively. $\mathbf{Q}(\mathbf{X})$ takes the orthonormal basis of $\mathcal{C}(\mathbf{X})$ as column vectors and we define $\bar{1} = 2$ and $\bar{2} = 1$.

The remainder of this paper is organized as follows: In Section II, we introduce the MIMO IC model. The linear transceivers based on the ZF criterion are discussed in Section III. In Section IV, we propose a method of regularizing the ZF transceivers. In Section V, the distributed implementation of the proposed regularization approach is presented. Numerical simulation results are illustrated in Section VI. The paper is closed with conclusions in Section VII.

II. System Model

In this paper, we consider a K-user MIMO IC where all transmit and receive nodes are equipped with M antennas as in [6]. As shown in Figure 1, the i-th transmitter attempts to send the information vector $\mathbf{s}^{(i)} \in \mathbb{C}^{N_s}$ to the i-th receiver $(i = 1, \cdots, K)$. Denoting $\mathbf{y}^{(k)}$ as the signal received by the k-th receiver, $\mathbf{y}^{(k)}$ can be written as

$$\mathbf{y}^{(k)} = \mathbf{H}^{(kk)}\mathbf{x}^{(k)} + \sum_{l \neq k} \mathbf{H}^{(kl)}\mathbf{x}^{(l)} + \mathbf{n}^{(k)} \tag{1}$$

where $\mathbf{x}^{(i)} \in \mathbb{C}^M$ stands for the signal vector transmitted from the i-th transmitter, $\mathbf{n}^{(i)}$ denotes the additive Gaussian noise vector at receiver i with covariance matrix $\sigma^2\mathbf{I}_M$ and $\mathbf{H}^{(kl)} \in \mathbb{C}^{M\times M}$ indicates the channel matrix from transmitter l to receiver k. It is assumed that the channel elements are sampled from independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance so that the probability of event that the channel is rank-deficient becomes zero. Also, all channel realizations are assumed to be perfectly known at all nodes except for Section V where each node requires only local CSI. In (1), $\mathbf{x}^{(i)}$ is related to $\mathbf{s}^{(i)}$ as $\mathbf{x}^{(i)} = \mathbf{V}^{(i)}\mathbf{s}^{(i)}$, where $\mathbf{V}^{(i)} \in \mathbb{C}^{M\times N_s}$ is a precoding matrix post-multiplied to the information vector $\mathbf{s}^{(i)}$. From the dimensions of $\mathbf{V}^{(i)}$ and $\mathbf{s}^{(i)}$, we can see that each user is served with N_s data streams. Throughout the paper, we consider the case of the maximum DOF of $N_s = \frac{M}{2}$ with even M [6] to maximize the network throughput. Especially, we focus on the cases of $K \leq 3$ where the achievable schemes for the DOF of $N_s = \frac{M}{2}$ was derived in [6]. Although the elements of $\mathbf{H}^{(kk)}$ are typically distributed with power larger than those of $\mathbf{H}^{(kl)} (k \neq l)$ due to a path loss in cellular networks, we consider the most challenging case where all of them have unit power. This situation arises for users located in cell boundaries.

At receiver k, the receive filter $\mathbf{U}^{(k)} \in \mathbb{C}^{M\times\frac{M}{2}}$ is post-multiplied to $\mathbf{y}^{(k)}$ to yield

$$\hat{\mathbf{s}}^{(k)} = \mathbf{U}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}^{(k)}\mathbf{s}^{(k)} + \mathbf{U}^{(k)H}\sum_{l\neq k}\mathbf{H}^{(kl)}\mathbf{V}^{(l)}\mathbf{s}^{(l)} + \check{\mathbf{n}}^{(k)} \tag{2}$$

where $\hat{\mathbf{s}}^{(k)}$ is the estimate vector for $\mathbf{s}^{(k)}$ and $\check{\mathbf{n}}^{(k)} = \mathbf{U}^{(k)H}\mathbf{n}^{(k)}$ represents the filtered noise with covariance $\sigma^2\mathbf{U}^{(k)H}\mathbf{U}^{(k)}$. Under the assumption of $E\left[\mathbf{s}^{(i)}\mathbf{s}^{(i)H}\right] = \mathbf{I}_{\frac{M}{2}}$ and single-user detection, the achievable sum-rate for given $\{\mathbf{V}^{(i)}\}$ and $\{\mathbf{U}^{(i)}\}$ is computed as

$$R_\Sigma\left(\{\mathbf{V}^{(i)}\}, \{\mathbf{U}^{(i)}\}\right) = \sum_{k=1}^{K}\sum_{i=1}^{\frac{M}{2}}\log_2\left(1 + \frac{|\mathbf{u}_i^{(k)}\mathbf{H}^{(kk)}\mathbf{v}_i^{(k)}|^2}{\sum\limits_{(l,j)\neq(k,i)}|\mathbf{u}_i^{(k)}\mathbf{H}^{(kl)}\mathbf{v}_j^{(l)}|^2 + \sigma^2||\mathbf{u}_i^{(k)}||^2}\right)$$

where $\mathbf{u}_i^{(k)}$ and $\mathbf{v}_i^{(k)}$ denote the i-th column of $\mathbf{U}^{(k)}$ and $\mathbf{V}^{(k)}$, respectively.

Since each transmitter has its own power amplifier, we consider the per-user power constraint as

$$E\left[||\mathbf{x}^{(k)}||^2\right] = \text{Tr}\left(\mathbf{V}^{(k)}\mathbf{V}^{(k)H}\right) \leq P, \quad \text{for } k = 1, \cdots, K.$$

In this MIMO IC scenario, the simplest method of managing inter-user interference is an orthgonal access technique such as TDMA where at each time slot only one link operates with the other links being turned off. Especially, in Section VI, we will consider the TDMA scheme with round-robin scheduling where each active MIMO link operates with singular value decomposition (SVD) based beamforming combined with water-filling (WF) power allocation [19].

III. Coordinated Transceiver Designs Based on ZF Criterion

In this section, we propose a non-iterative CSM scheme which achieves full DOF of M [5] in the 2-user MIMO IC case. Also, we show that the E-IA in [16] is the optimal ZF scheme for the 3-user MIMO IC in terms of sum-rate performance. Our proposed precoders and decoders consist of the two-stage process as

$$\mathbf{V}^{(i)} = \mathbf{V}_{\text{MU}}^{(i)}\mathbf{V}_{\text{SU}}^{(i)} \quad \text{and} \quad \mathbf{U}^{(i)} = \mathbf{U}_{\text{MU}}^{(i)}\mathbf{U}_{\text{SU}}^{(i)},$$

where inter-user interference is mitigated by the multi-user precoders $\mathbf{V}_{\text{MU}}^{(i)} \in \mathbb{C}^{M\times\frac{M}{2}}$ and decoders $\mathbf{U}_{\text{MU}}^{(i)} \in \mathbb{C}^{M\times\frac{M}{2}}$. With an aid of $\mathbf{V}_{\text{MU}}^{(i)}$ and $\mathbf{U}_{\text{MU}}^{(i)}$, the MIMO IC is decoupled into K parallel single-user MIMO systems $\mathbf{U}_{\text{MU}}^{(i)H}\mathbf{H}^{(ii)}\mathbf{V}_{\text{MU}}^{(i)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$ and we can maximize the single-user capacity by adjusting the single-user precoders $\mathbf{V}_{\text{SU}}^{(i)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$ and decoders $\mathbf{U}_{\text{SU}}^{(i)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$.

Similar two-stage algorithms have been proposed in [8], [11], [20] and [21]. The transmit filters in [8], [11], [21] are restricted to the ZF criterion and the effect of minimum mean-squared error (MMSE) receiver is studied in [8] and [11]. In [20], the authors proposed two-layer transceivers where the inner and outer filters correspond to the single-user and multi-user transceivers, respectively. The transceiver algorithm in [20] is analogous to our proposed scheme such that it starts with a judicious choice of initial transceivers and performs regularization to enhance the sum-rate performance in overall SNR regime. However, since the criterion of streamwise signal-to-noise-plus-interference ratio (SINR) is adopted to update the transmit and receiver filters, it suffers from slow convergence in high SNR scenarios as in [7].

For given $\{\mathbf{V}_{\text{MU}}^{(i)}\}$ and $\{\mathbf{U}_{\text{MU}}^{(i)}\}$, the optimal single-user precoders and decoders under the assumption of no inter-user interference are represented as

$$\mathbf{V}_{\text{SU}}^{(i)} = \bar{\mathbf{V}}^{(i)}\mathbf{\Phi}^{(i)} \text{ and } \mathbf{U}_{\text{SU}}^{(i)} = \bar{\mathbf{U}}^{(i)}, \quad \text{for } i = 1, \cdots, K \tag{3}$$

where $\bar{\mathbf{V}}^{(i)}$ and $\bar{\mathbf{U}}^{(i)}$ are the right and left singular matrices of $\mathbf{U}_{\text{MU}}^{(i)H}\mathbf{H}^{(ii)}\mathbf{V}_{\text{MU}}^{(i)}$, respectively, and the power allocation matrix $\mathbf{\Phi}^{(i)}$ is a real diagonal matrix whose diagonal elements can be optimized according to the water-filling algorithm [19] subject to the per-use power constraint $\text{Tr}\left(\mathbf{\Phi}^{(i)\,2}\right) = P$. We now discuss the optimization of multi-user transceivers for the case of $K = 2$ and 3.

A. CSM for Two-User IC ($K = 2$)

For the 2-user MIMO IC, the filter outputs $\hat{\mathbf{s}}^{(1)}$ and $\hat{\mathbf{s}}^{(2)}$ are given as

$$\begin{bmatrix} \hat{\mathbf{s}}^{(1)} \\ \hat{\mathbf{s}}^{(2)} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^{(1)H}\mathbf{y}^{(1)} \\ \mathbf{U}^{(2)H}\mathbf{y}^{(2)} \end{bmatrix} =$$
$$\begin{bmatrix} \mathbf{U}^{(1)H}\mathbf{H}^{(11)}\mathbf{V}^{(1)} & \mathbf{U}^{(1)H}\mathbf{H}^{(12)}\mathbf{V}^{(2)} \\ \mathbf{U}^{(2)H}\mathbf{H}^{(21)}\mathbf{V}^{(1)} & \mathbf{U}^{(2)H}\mathbf{H}^{(22)}\mathbf{V}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \end{bmatrix} + \begin{bmatrix} \check{\mathbf{n}}^{(1)} \\ \check{\mathbf{n}}^{(2)} \end{bmatrix}.$$

The ZF constraint can be satisfied only when the off-diagonal terms are zero (i.e., $\mathbf{U}_{\text{MU}}^{(1)H}\mathbf{H}^{(12)}\mathbf{V}_{\text{MU}}^{(2)} = \mathbf{U}_{\text{MU}}^{(2)H}\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)} = \mathbf{0}_{\frac{M}{2}}$). Also, to maximize the system performance, we design the decoders $\mathbf{U}_{\text{MU}}^{(i)}$ to be matched to the subspace of the desired signals $\mathbf{H}^{(ii)}\mathbf{V}_{\text{MU}}^{(i)}$, which is a reasonable choice when no inter-user interference exists.

Then, our goal is to find the precoders and decoders satisfying the following conditions:

$$\mathcal{C}\left(\mathbf{U}_{\text{MU}}^{(i)}\right) = \mathcal{C}\left(\mathbf{H}^{(ii)}\mathbf{V}_{\text{MU}}^{(i)}\right), \text{ for } i = 1, 2. \tag{4}$$

$$\mathcal{C}\left(\mathbf{U}_{\text{MU}}^{(i)}\right) \perp \mathcal{C}\left(\mathbf{H}^{(i\bar{i})}\mathbf{V}_{\text{MU}}^{\bar{i}}\right), \text{ for } i = 1, 2. \tag{5}$$

Here, the single-user transceivers are not included since $\mathcal{C}\left(\mathbf{U}^{(i)}\right)$ and $\mathcal{C}\left(\mathbf{H}^{(ij)}\mathbf{V}^{(j)}\right)$ depend only on the multi-user transceivers. The following theorem is useful for solving the above problem.

Theorem 1: For $\{\mathbf{V}_{\text{MU}}^{(i)}\}$ and $\{\mathbf{U}_{\text{MU}}^{(i)}\}$ to satisfy (4) and (5), $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right)$ should be defined as

$$\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{d}_{i_1}\,\mathbf{d}_{i_2}\,\cdots\,\mathbf{d}_{i_{\frac{M}{2}}}\right]\right), \tag{6}$$

for $1 \le i_1 < i_2 < \cdots < i_{\frac{M}{2}} \le M$, where $\mathbf{d}_1, \cdots, \mathbf{d}_M$ are the eigenvectors of $\mathbf{D} = \left(\mathbf{H}^{(12)H}\mathbf{H}^{(11)}\right)^{-1}\mathbf{H}^{(22)H}\mathbf{H}^{(21)}$.

Proof: First, we show that if $\mathbf{V}_{\text{MU}}^{(1)}$ and $\mathbf{V}_{\text{MU}}^{(2)}$ satisfy (4) and (5), then it follows $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{d}_{i_1}\,\mathbf{d}_{i_2}\,\cdots\,\mathbf{d}_{i_{\frac{M}{2}}}\right]\right)$ with $1 \le i_1 < i_2 < \cdots < i_{\frac{M}{2}} \le M$. For $\mathbf{V}_{\text{MU}}^{(1)}$ and $\mathbf{V}_{\text{MU}}^{(2)}$ which satisfy (4) and (5), substituting (4) into (5) leads to

$$\mathbf{V}_{\text{MU}}^{(1)H}\mathbf{H}^{(11)H}\mathbf{H}^{(12)}\mathbf{V}_{\text{MU}}^{(2)} = \mathbf{V}_{\text{MU}}^{(2)H}\mathbf{H}^{(22)H}\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)} = \mathbf{0}_{\frac{M}{2}}.$$

Then, we have

$$\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(2)}\right) \perp \mathcal{C}\left(\mathbf{H}^{(12)H}\mathbf{H}^{(11)}\mathbf{V}_{\text{MU}}^{(1)}\right),$$
$$\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(2)}\right) \perp \mathcal{C}\left(\mathbf{H}^{(22)H}\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)}\right).$$

Note that in $\mathbb{C}^M$, the $\frac{M}{2}$-dimensional subspace orthogonal to $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(2)}\right)$ of dimension $\frac{M}{2}$ is unique. Thus, we obtain

$$\mathcal{C}\left(\mathbf{H}^{(12)H}\mathbf{H}^{(11)}\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\mathbf{H}^{(22)H}\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)}\right).$$

Since $\mathbf{H}^{(12)H}\mathbf{H}^{(11)}$ is invertible almost surely, we get

$$\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\left(\mathbf{H}^{(12)H}\mathbf{H}^{(11)}\right)^{-1}\mathbf{H}^{(22)H}\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)}\right).$$

As $\mathbf{d}_1, \cdots, \mathbf{d}_M$ span the whole $\mathbb{C}^M$, we can express $\mathbf{V}_{\text{MU}}^{(1)}$ as

$$\mathbf{V}_{\text{MU}}^{(1)} = [\mathbf{d}_1 \cdots \mathbf{d}_M]\,\mathbf{C}$$

where $\mathbf{C} \in \mathbb{C}^{M\times\frac{M}{2}}$ is a coefficient matrix. Then, it follows from $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\mathbf{D}\mathbf{V}_{\text{MU}}^{(1)}\right)$ that

$$\mathcal{C}\left([\mathbf{d}_1 \cdots \mathbf{d}_M]\,\mathbf{C}\right) = \mathcal{C}\left(\mathbf{D}\,[\mathbf{d}_1 \cdots \mathbf{d}_M]\,\mathbf{C}\right). \tag{7}$$

Let $\lambda_1, \cdots, \lambda_M$ denote the eigenvalues of $\mathbf{D}$. Then, $\mathbf{D}\,[\mathbf{d}_1 \cdots \mathbf{d}_M]$ is computed as

$$\mathbf{D}\,[\mathbf{d}_1 \cdots \mathbf{d}_M] = [\mathbf{d}_1 \cdots \mathbf{d}_M]\,\text{diag}(\lambda_1, \cdots, \lambda_M). \tag{8}$$

By substituting (8) into (7), we have

$$\mathcal{C}\left(\mathbf{C}\right) = \mathcal{C}\left(\text{diag}\left(\lambda_1, \cdots, \lambda_M\right)\mathbf{C}\right), \tag{9}$$

since $[\mathbf{d}_1 \cdots \mathbf{d}_M]$ is invertible with probability one.

We have shown that if $\mathbf{V}_{\text{MU}}^{(1)}$ and $\mathbf{V}_{\text{MU}}^{(2)}$ satisfy (4) and (5), then the coefficient matrix $\mathbf{C}$ for $\mathbf{V}_{\text{MU}}^{(1)}$ should satisfy (9). From now on, it will be shown that (9) is true only when there are at most $\frac{M}{2}$ nonzero vectors in $\mathbf{c}_1, \cdots, \mathbf{c}_M$ where $\mathbf{c}_i$ denotes the i-th row vector of $\mathbf{C}$. For simplicity, we provide a rigorous proof for the cases of $M = 2$ and $M = 4$ since extension to the general case is straightforward.

First, suppose $M = 2$. Denoting $\mathbf{C} = [a_1, a_2]^T$, (4) can be written as

$$\mathcal{C}\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \mathcal{C}\left(\begin{bmatrix} \lambda_1 a_1 \\ \lambda_2 a_2 \end{bmatrix}\right).$$

This can be satisfied only if either a_1 or a_2 is zero as λ_i's are distinct nonzero values with probability one.

Now, we move to the case of $M = 4$ where (4) becomes

$$\mathcal{C}\left(\begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \\ \mathbf{c}_4 \end{bmatrix}\right) = \mathcal{C}\left(\begin{bmatrix} \lambda_1\mathbf{c}_1 \\ \lambda_2\mathbf{c}_2 \\ \lambda_3\mathbf{c}_3 \\ \lambda_4\mathbf{c}_4 \end{bmatrix}\right). \tag{10}$$

Here, $\mathbf{c}_i$'s are two-dimensional row vectors and the column space has dimensionality of 2 to meet rank $\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = 2$. Thus, $\mathbf{c}_1$ and $\mathbf{c}_2$ are linearly independent without loss of generality.

From these conditions, it can be shown that assuming $\mathbf{c}_3 \neq \mathbf{0}$ or $\mathbf{c}_4 \neq \mathbf{0}$ results in contradiction. Since (10) implies that

$$\text{rank}\left(\begin{bmatrix} \mathbf{c}_1 & \lambda_1\mathbf{c}_1 \\ \mathbf{c}_2 & \lambda_2\mathbf{c}_2 \\ \mathbf{c}_3 & \lambda_3\mathbf{c}_3 \end{bmatrix}\right) = 2,$$

and the last row of the above matrix is nonzero, there exists unique pair $(l_1, l_2) \neq (0,0)$ such that $l_1\mathbf{c}_1 + l_2\mathbf{c}_2 = \mathbf{c}_3$ and $l_1\lambda_1\mathbf{c}_1 + l_2\lambda_2\mathbf{c}_2 = \lambda_3\mathbf{c}_3$. By combining two equalities, we get

$$\lambda_3\mathbf{c}_3 = l_1\lambda_1\mathbf{c}_1 + l_2\lambda_2\mathbf{c}_2 = l_1\lambda_3\mathbf{c}_1 + l_2\lambda_3\mathbf{c}_2.$$

Due to the linear independence of $\mathbf{c}_1$ and $\mathbf{c}_2$, we have $l_1\lambda_1 = l_1\lambda_3$ and $l_2\lambda_2 = l_2\lambda_3$ which cannot be achieved. Thus, we can conclude that both $\mathbf{c}_3$ and $\mathbf{c}_4$ should be zero vectors.

It is proved that if (9) is true, the coefficient matrix $\mathbf{C}$ contains only $\frac{M}{2}$ nonzero row vectors. Denoting the $\frac{M}{2}$ nonzero vectors by $\mathbf{c}_{i_1}, \cdots, \mathbf{c}_{i_{\frac{M}{2}}}$, $\mathbf{V}_{\text{MU}}^{(1)}$ can be expressed as

$$\mathbf{V}_{\text{MU}}^{(1)} = [\mathbf{d}_1 \cdots \mathbf{d}_M]\begin{bmatrix} \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_M \end{bmatrix} = \left[\mathbf{d}_{i_1} \cdots \mathbf{d}_{i_{\frac{M}{2}}}\right]\begin{bmatrix} \mathbf{c}_{i_1} \\ \vdots \\ \mathbf{c}_{i_{\frac{M}{2}}} \end{bmatrix}.$$

This tells us that $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{d}_{i_1} \cdots \mathbf{d}_{i_{\frac{M}{2}}}\right]\right)$ if (4) and (5) are true. ■

Once $\mathbf{V}_{\text{MU}}^{(1)}$ is computed with some index combinations $i_1, \cdots, i_{\frac{M}{2}}$ in (6), $\mathbf{V}_{\text{MU}}^{(2)}$ is obtained using $\mathbf{V}_{\text{MU}}^{(1)H}\mathbf{H}^{(11)H}\mathbf{H}^{(12)}\mathbf{V}_{\text{MU}}^{(2)} = \mathbf{0}_{\frac{M}{2}}$, which is achieved by constructing $\mathbf{V}_{\text{MU}}^{(2)}$ as the $\frac{M}{2}$ right singular vectors of $\mathbf{V}_{\text{MU}}^{(1)H}\mathbf{H}^{(11)H}\mathbf{H}^{(12)} \in \mathbb{C}^{\frac{M}{2}\times M}$ corresponding to zero singular values. Then, we choose the multi-user decoder $\mathbf{U}_{\text{MU}}^{(k)}$ to be matched to the desired signals $\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}$. Thus, for given $\mathbf{V}_{\text{MU}}^{(1)}$, the remaining multi-user transceivers are computed as

$$\mathbf{V}_{\text{MU}}^{(2)} = \mathbf{V}_0^{\text{sing}}\left(\mathbf{V}_{\text{MU}}^{(1)H}\mathbf{H}^{(11)H}\mathbf{H}^{(12)}\right) \tag{11}$$

$$\mathbf{U}_{\text{MU}}^{(k)} = \mathbf{U}_1^{\text{sing}}\left(\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}\right) \text{ for } k = 1, 2, \tag{12}$$

where the columns of $\mathbf{V}_0^{\text{sing}}(\mathbf{X})$ are right singular vectors of $\mathbf{X}$ corresponding to zero singular values. Similarly, the left singular vectors corresponding to nonzero singular values consistute the columns of $\mathbf{U}_1^{\text{sing}}(\mathbf{X})$.

Since $\mathbf{V}_{\text{MU}}^{(1)}$ determined from any index combination $1 \leq i_1 < \cdots < i_{\frac{M}{2}} \leq M$ satisfies (4) and (5), it is straightforward to select the best index corresponding to the maximum sum-rate performance with the search size $\binom{M}{\frac{M}{2}}$ as

$$(i_1^{\text{opt}}, \cdots, i_{\frac{M}{2}}^{\text{opt}}) = \arg\max_{(i_1,\cdots,i_{\frac{M}{2}})\in\pi} R_\Sigma\left(\{\mathbf{V}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})}\}, \{\mathbf{U}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})}\}\right) \tag{13}$$

where π is the set of all index combinations as

$$\pi = \left\{(i_1, \cdots, i_{\frac{M}{2}}) \middle| \ 1 \leq i_1 < \cdots < i_{\frac{M}{2}} \leq M\right\},$$

and $\mathbf{V}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})}$ and $\mathbf{U}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})}$ are defined as

$$\mathbf{V}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})} = \mathbf{V}^{(k)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})}\mathbf{V}^{(k)}_{\text{SU},(i_1,\cdots,i_{\frac{M}{2}})}, \quad \text{for } k = 1, 2,$$

$$\mathbf{U}^{(k)}_{(i_1,\cdots,i_{\frac{M}{2}})} = \mathbf{U}^{(k)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})}\mathbf{U}^{(k)}_{\text{SU},(i_1,\cdots,i_{\frac{M}{2}})}, \quad \text{for } k = 1, 2.$$

Here, the multi-user transceivers associated with the index combination $i_1, \cdots, i_{\frac{M}{2}}$ are given as

$$\mathbf{V}^{(1)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})} = \mathbf{Q}\left([\mathbf{d}_{i_1} \cdots \mathbf{d}_{i_{\frac{M}{2}}}]\right),$$

$$\mathbf{V}^{(2)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})} = \mathbf{V}_0^{\text{sing}}\left(\mathbf{V}^{(1)H}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})}\mathbf{H}^{(11)H}\mathbf{H}^{(12)}\right),$$

$$\mathbf{U}^{(k)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})} = \mathbf{U}_1^{\text{sing}}\left(\mathbf{H}^{(kk)}\mathbf{V}^{(k)}_{\text{MU},(i_1,\cdots,i_{\frac{M}{2}})}\right), \quad \text{for } k = 1, 2,$$

and the corresponding single-user transceivers can be computed using (3). The algorithm of the proposed CSM scheme can be summarized as follows:

Step 1. Compute $\mathbf{d}_1, \cdots \mathbf{d}_M$ which are the eigenvectors of $\left(\mathbf{H}^{(12)H}\mathbf{H}^{(11)}\right)^{-1}\mathbf{H}^{(22)H}\mathbf{H}^{(21)}$.

Step 2. Choose the best index combination $(i_1^{\text{opt}}, \cdots, i_{\frac{M}{2}}^{\text{opt}})$ using (13).

Step 3. Set $\mathbf{V}_{\text{MU}}^{(1)} = \mathbf{Q}\left([\mathbf{d}_{i_1^{\text{opt}}} \cdots \mathbf{d}_{i_{\frac{M}{2}}^{\text{opt}}}]\right)$ and compute the remaining multi-user and single-user transceivers using (3), (11), (12).

The above CSM algorithm where each user is served with the maximum available DOF of $N_s = \frac{M}{2}$ derived in [5] can be considered as a generalization of the ZF IA-CBF in [15] which is restricted to the case of single data stream per user, i.e., $N_s = 1$. It is shown in [15] that the ZF IA-CBF is the sum-rate optimal ZF scheme in the 2-user IC with $M = 2$. Although at this time a proof is unavailable, we conjecture that the proposed CSM for general M is the optimal ZF scheme as long as the full DOF is supported (i.e., $N_s = \frac{M}{2}$).

B. Enhanced Interference Alignment for Three-User IC ($K = 3$)

In this subsection, we briefly review the E-IA scheme in [16] and show that the E-IA is sum-rate optimal in the ZF criteria. We start with presenting the feasibility conditions for IA [6] as

$$\begin{aligned} \mathcal{C}\left(\mathbf{H}^{(12)}\mathbf{V}_{\text{MU}}^{(2)}\right) &= \mathcal{C}\left(\mathbf{H}^{(13)}\mathbf{V}_{\text{MU}}^{(3)}\right), \\ \mathcal{C}\left(\mathbf{H}^{(21)}\mathbf{V}_{\text{MU}}^{(1)}\right) &= \mathcal{C}\left(\mathbf{H}^{(23)}\mathbf{V}_{\text{MU}}^{(3)}\right), \\ \mathcal{C}\left(\mathbf{H}^{(31)}\mathbf{V}_{\text{MU}}^{(1)}\right) &= \mathcal{C}\left(\mathbf{H}^{(32)}\mathbf{V}_{\text{MU}}^{(2)}\right). \end{aligned} \tag{14}$$

We notice that as long as the full DOF is supported (i.e., $N_s = \frac{M}{2}$), the precoders of all ZF schemes (e.g., conventional IA [6] and E-IA [16]) should satisfy the above conditions for the inter-user interference to be completely eliminated at the receiver side.

Now, we describe how to determine $\mathbf{V}_{\text{MU}}^{(i)}$ and $\mathbf{U}_{\text{MU}}^{(i)}$ in the E-IA scheme. The feasibility conditions (14) can be satisfied by setting the multi-user precoders as $\mathbf{V}_{\text{MU}}^{(i)} = \mathbf{Q}\left(\mathbf{B}^{(i)}\right)$ where

$$\begin{aligned} \mathbf{B}^{(1)} &= \left[\mathbf{e}_{i_1}\, \mathbf{e}_{i_2}\, \cdots\, \mathbf{e}_{i_{\frac{M}{2}}}\right], \\ \mathbf{B}^{(2)} &= \mathbf{F}\mathbf{B}^{(1)} \text{ and } \mathbf{B}^{(3)} = \mathbf{G}\mathbf{B}^{(1)}, \end{aligned} \tag{15}$$

for $1 \le i_1 < i_2 < \cdots < i_{\frac{M}{2}} \le M$. Here, $\mathbf{e}_1, \cdots, \mathbf{e}_M$ are the eigenvectors of $\mathbf{E}$ [6] where $\mathbf{E}, \mathbf{F}$ and $\mathbf{G}$ are defined as

$$\begin{aligned} \mathbf{E} &= (\mathbf{H}^{(31)})^{-1}\mathbf{H}^{(32)}(\mathbf{H}^{(12)})^{-1}\mathbf{H}^{(13)}(\mathbf{H}^{(23)})^{-1}\mathbf{H}^{(21)}, \\ \mathbf{F} &= (\mathbf{H}^{(32)})^{-1}\mathbf{H}^{(31)} \text{ and } \mathbf{G} = (\mathbf{H}^{(23)})^{-1}\mathbf{H}^{(21)}. \end{aligned}$$

As any choice of the indexes $1 \le i_1 < i_2 < \cdots < i_{\frac{M}{2}} \le M$ satisfies the feasibility conditions, the E-IA in [16] chooses the best $\frac{M}{2}$ indexes out of $\{1, \cdots, M\}$ which guarantee the maximum sum-rate performance. Once $\mathbf{V}_{\text{MU}}^{(i)}$ is determined, the inter-user interference can be nulled at the receiver side by constituting the columns of $\mathbf{U}_{\text{MU}}^{(i)}$ as $\frac{M}{2}$ left singular vectors of $\mathbf{H}^{(ij)}\mathbf{V}_{\text{MU}}^{(j)}$ $(j \ne i)$ corresponding to zero singular values.

Now, we show that from the ZF criteria, the precoders and decoders of the E-IA scheme achieve the optimal sum-rate performance. To this end, we need to confirm the joint optimality of the multi-user and single-user transceivers of the E-IA scheme. Since it is obvious that the single-user transceiver design described in the beginning of Section III is optimal for given multi-user transceivers, it is sufficient to check the optimality of the multi-user precoders and decoders of the E-IA scheme. Recognizing that the E-IA selects the best $\frac{M}{2}$ indexes $i_1, \cdots, i_{\frac{M}{2}}$ out of $\{1, \cdots, M\}$ in (15) corresponding to the maximum sum-rate, the proof is completed by showing that if $(\mathbf{V}_{\text{MU}}^{(1)}, \mathbf{V}_{\text{MU}}^{(2)}, \mathbf{V}_{\text{MU}}^{(3)}) \in \mathcal{S}$ where $\mathcal{S}$ is a set of all precoding matrices satisfying the feasibility conditions (14), then it should be $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(k)}\right) = \mathcal{C}\left(\mathbf{B}^{(k)}\right)$ with $\mathbf{B}^{(k)}$ in (15) for $k = 1, \cdots, 3$. As the proof for $k = 2, 3$ can be directly derived, we concentrate on the proof of $k = 1$.

Since all of $\{\mathbf{H}^{(ij)}\}$ are invertible almost surely, it is straightforward to see that if $(\mathbf{V}_{\text{MU}}^{(1)}, \mathbf{V}_{\text{MU}}^{(2)}, \mathbf{V}_{\text{MU}}^{(3)}) \in \mathcal{S}$, then $\mathcal{C}\left(\mathbf{E}\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right)$. Then, following the same steps in the proof of Theorem 1, we arrive at

$$\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\left[\mathbf{e}_{i_1} \cdots \mathbf{e}_{i_{\frac{M}{2}}}\right]\right),$$

for $1 \le i_1 < i_2 < \cdots < i_{\frac{M}{2}} \le M$. Since $\mathcal{C}\left(\mathbf{V}_{\text{MU}}^{(1)}\right) = \mathcal{C}\left(\mathbf{B}^{(1)}\right)$, the proof for $k = 1$ is completed. As a result, since the E-IA scheme in [16] selects the best one corresponding to the maximum sum-rate among all transceivers satisfying the feasiblity conditions (14), the E-IA is the sum-rate optimal ZF algorithm.

IV. Regularizing the ZF Transceivers

The main limitation of the ZF based algorithms is their poor performance at low SNR due to noise enhancement [22]. In this section, we present improved transceivers which regularize the precoders and decoders of the ZF schemes. As we are interested in the regularized nulling of inter-user interference, we focus on the design of the multi-user transceivers $\{\mathbf{V}_{\text{MU}}^{(k)}\}$ and $\{\mathbf{U}_{\text{MU}}^{(k)}\}$. It is conventional to employ the MMSE criterion to achieve improved sum-rate performance by regularizing the ZF transceivers [16] [23]. Although the MMSE design is straightforward for the receiver filters $\{\mathbf{U}_{\text{MU}}^{(k)}\}$, this is not the case for the transmitters $\{\mathbf{V}_{\text{MU}}^{(k)}\}$. Thus, we design the transmit processing $\{\mathbf{V}_{\text{MU}}^{(k)}\}$ as the MMSE receivers for the corresponding reciprocal channels [7].

The proposed regularization consists of the following two parts:

Part 1. For given $\{\mathbf{V}^{(k)}\}$, regularized design of $\{\mathbf{U}_{\text{MU}}^{(k)}\}$.

Part 2. For given $\{\mathbf{U}^{(k)}\}$, regularized design of $\{\mathbf{V}_{\text{MU}}^{(k)}\}$.

It is complicated to achieve the goal of two parts simultaneously since the $2K$-matrices $\{\mathbf{V}_{\text{MU}}^{(k)}\}, \{\mathbf{U}_{\text{MU}}^{(k)}\}$ are coupled with each other. To obtain a non-iterative algorithm, we apply high SNR approximation.

First we illustrate the optimization of the decoder $\mathbf{U}_{\text{MU}}^{(k)}$ when the precoders $\{\mathbf{V}^{(k)}\}$ are given. The optimal decoder in terms of the signal-to-interference-plus-noise ratio (SINR) is the MMSE receiver computed as [24]

$$\begin{aligned} \mathbf{U}_{\text{MU}}^{(k)} &= \left(\sum_l \mathbf{H}^{(kl)}\mathbf{V}^{(l)}\mathbf{V}^{(l)H}\mathbf{H}^{(kl)H} + \sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}^{(kk)}\mathbf{V}^{(k)} \\ &\triangleq \mathbf{U}_{\text{MMSE}}^{(k)}. \end{aligned}$$

Since the intra-user interference is controled by the single-user transceivers, we choose the multi-user decoders as

$$\mathbf{U}_{\text{MU}}^{(k)} = \mathbf{Q}\left(\mathbf{U}_{\text{MMSE}}^{(k)}\right) \tag{16}$$

for $k = 1, \cdots, K$.

Next, we address the regularized design of $\mathbf{V}_{\text{MU}}^{(k)}$ when the decoders $\{\mathbf{U}^{(k)}\}$ are fixed. In [25], the authors proposed a linear precoding method for the multi-user MIMO broadcast channels where the multi-user precoders are obtained as generalized MMSE basis vectors combined with a combining matrix computed from the minimum interference-plus-noise power criterion. The MMSE basis vectors are given by an orthonormal matrix $\mathbf{Q}_k = \mathbf{Q}\left(\left(\sum_l \mathbf{H}_l\mathbf{H}_l^H + \alpha\mathbf{I}_M\right)\mathbf{H}_k\right)$ where $\mathbf{H}_j$ denotes the channel transfer matrix from the BS to user j in the multi-user downlink channels and α is inverse SNR.

In the MIMO IC, we incorporate this idea to construct $\mathbf{V}_{\text{MU}}^{(k)}$ by replacing $\mathbf{H}_l$ with $\mathbf{U}^{(l)H}\mathbf{H}^{(lk)}$ such that $\mathbf{V}_{\text{MU}}^{(k)}$ becomes a linear combination of columns of $\bar{\mathbf{Q}}^{(k)} \in \mathbb{C}^{M \times \frac{M}{2}}$ where $\bar{\mathbf{Q}}^{(k)}$ is the MMSE channel inversion (CI) matrix computed as

$$\begin{aligned} &\bar{\mathbf{Q}}^{(k)} = \\ &\mathbf{Q}\left(\left(\sum_l \mathbf{H}^{(lk)H}\mathbf{U}^{(l)}\mathbf{U}^{(l)H}\mathbf{H}^{(lk)} + \alpha\mathbf{I}_M\right)^{-1}\mathbf{H}^{(kk)H}\mathbf{U}^{(k)}\right) \end{aligned} \tag{17}$$

where the inverse SNR α is computed as the ratio of the total noise variance at the filter output to the total transmit power at each user, i.e., $\alpha = \frac{\frac{M}{2}\sigma^2}{P}$. From this setting, we can make $\mathbf{V}^{(k)}$ lie near the null space of $\mathbf{U}^{(l)H}\mathbf{H}^{(lk)}$ $(\forall l \ne k)$ while taking the

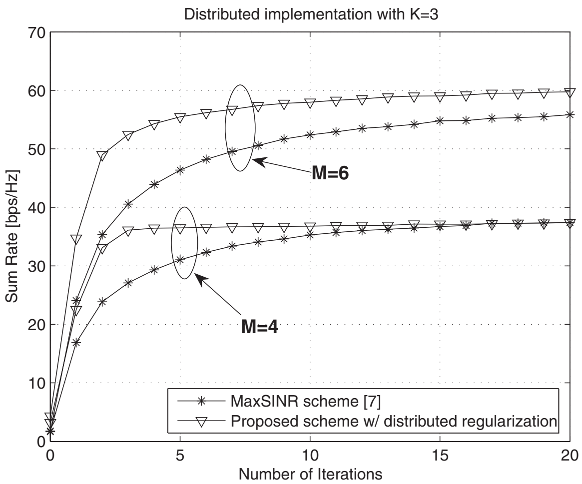

Fig. 2. Average sum-rate performance of the proposed distributed regularization scheme in 3-user MIMO IC at SNR of 30 dB.

noise into account [25]. We note that the transmit filter $\bar{\mathbf{Q}}^{(k)}$ can also be interpreted as an MMSE receiver for a reciprocal channel [7] and will be multiplied by further processing $\mathbf{T}^{(k)}$ for basis modification within subspace identified by $\bar{\mathbf{Q}}^{(k)}$.

Then, $\mathbf{V}_{\text{MU}}^{(k)}$ can be expressed as $\mathbf{V}_{\text{MU}}^{(k)} = \bar{\mathbf{Q}}^{(k)}\mathbf{T}^{(k)}$ where the combination matrix $\mathbf{T}^{(k)} \in \mathbb{C}^{\frac{M}{2}\times\frac{M}{2}}$ is determined according to the minimum interference-plus-noise power criterion under the per-user power constraint $\text{Tr}\left(\mathbf{T}^{(k)}\mathbf{T}^{(k)H}\right) = \frac{M}{2}$. From (2), the power at other users' interference induced by $\mathbf{V}^{(k)}$ plus the total noise power at receiver k is given by

$$E\left[\sum_{l\neq k}\left|\left|\mathbf{U}^{(l)H}\mathbf{H}^{(lk)}\bar{\mathbf{Q}}^{(k)}\mathbf{T}^{(k)}\mathbf{V}_{\text{SU}}^{(k)}\mathbf{s}^{(k)}\right|\right|^2\right] + E\left[\left|\left|\mathbf{U}^{(k)H}\mathbf{n}^{(k)}\right|\right|^2\right]. \quad (18)$$

From our setting, the decoder matrix $\mathbf{U}^{(k)}$ is column-orthonormal and it is assumed for simplicity that $\mathbf{V}_{\text{SU}}^{(k)H}\mathbf{V}_{\text{SU}}^{(k)} = \frac{P}{M/2}\mathbf{I}_{\frac{M}{2}}$ (i.e., $\mathbf{\Phi} = \sqrt{\frac{P}{M/2}}\mathbf{I}_{\frac{M}{2}}$) which is true at high SNR regime since uniform power allocation is near-optimal for high $\frac{P}{\sigma^2}$.

Then, (18) can be computed as

$$\frac{P}{M/2}\text{Tr}\left(\mathbf{T}^{(k)H}\mathbf{\Omega}^{(k)}\mathbf{T}^{(k)}\right) \quad (19)$$

where $\mathbf{\Omega}^{(k)}$ is defined as

$$\mathbf{\Omega}^{(k)}=\left[\bar{\mathbf{Q}}^{(k)H}\left(\sum_l\mathbf{H}^{(lk)H}\mathbf{U}^{(l)}\mathbf{U}^{(l)H}\mathbf{H}^{(lk)}\right)\bar{\mathbf{Q}}^{(k)}+\frac{\frac{M}{2}\sigma^2}{P}\mathbf{I}_{\frac{M}{2}}\right].$$

Since $\mathbf{\Omega}^{(k)}$ is Hermitian and positive semi-definite, we can compute the Cholesky factorization as $\mathbf{\Omega}^{(k)} = \mathbf{L}^{(k)H}\mathbf{L}^{(k)}$ [25]. The combining matrix $\mathbf{T}^{(k)}$ which minimizes the metric (19) can be obtained as $\mathbf{T}^{(k)} = \gamma^{(k)}\left(\mathbf{L}^{(k)}\right)^{-1}$ where $\gamma^{(k)}$ is determined such that $\text{Tr}(\mathbf{T}^{(k)}\mathbf{T}^{(k)H}) = \frac{M}{2}$.

We are now ready to explain the proposed one-shot algorithm. Actually, the optimal $\{\mathbf{V}_{\text{MU}}^{(k)}\}$ and $\{\mathbf{U}_{\text{MU}}^{(k)}\}$ depend mutually on each other, which means that **Part 1** and **Part 2** should be performed iteratively for complete regularization. However, since we are interested in a non-iterative algorithm with low-complexity, **Part 1** is performed only once using the high SNR approximation $\mathbf{V}^{(k)} \approx \mathbf{V}_{\text{ZF}}^{(k)}$ where $\mathbf{V}_{\text{ZF}}^{(k)}$ is the precoding matrix computed from the ZF design in Section III. The proposed regularization algorithm is summarized as follows:

Step 1. Compute $\{\mathbf{V}_{\text{ZF}}^{(k)}\}$ from the ZF algorithm in Section III.

Step 2. Set
$\mathbf{U}_{\text{MU}}^{(k)}\leftarrow\mathbf{Q}\left(\left(\sum_l\mathbf{H}^{(kl)}\mathbf{V}_{\text{ZF}}^{(l)}\mathbf{V}_{\text{ZF}}^{(l)H}\mathbf{H}^{(kl)H}+\sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}^{(kk)}\mathbf{V}_{\text{ZF}}^{(k)}\right)$
for $k = 1,\cdots,K$.

Step 3. For $k = 1,\cdots,K,$ compute $\bar{\mathbf{Q}}^{(k)}$ and $\mathbf{L}^{(k)}$ as
$\bar{\mathbf{Q}}^{(k)}=\mathbf{Q}\left(\left(\sum_l\mathbf{H}^{(lk)H}\mathbf{U}_{\text{MU}}^{(l)}\mathbf{U}_{\text{MU}}^{(l)H}\mathbf{H}^{(lk)}+\frac{M\sigma^2}{2P}\mathbf{I}_M\right)^{-1}\mathbf{H}^{(kk)H}\mathbf{U}_{\text{MU}}^{(k)}\right),$
$\mathbf{L}^{(k)H}\mathbf{L}^{(k)}=\bar{\mathbf{Q}}^{(k)H}\left(\sum_l\mathbf{H}^{(lk)H}\mathbf{U}_{\text{MU}}^{(l)}\mathbf{U}_{\text{MU}}^{(l)H}\mathbf{H}^{(lk)}\right)\bar{\mathbf{Q}}^{(k)}+\frac{M\sigma^2}{2P}\mathbf{I}_{\frac{M}{2}}.$

Step 4. Compute $\mathbf{T}^{(k)} \leftarrow \gamma^{(k)}\left(\mathbf{L}^{(k)}\right)^{-1}$ for $k = 1,\cdots,K.$

Step 5. Construct $\mathbf{V}_{\text{MU}}^{(k)} \leftarrow \bar{\mathbf{Q}}^{(k)}\mathbf{T}^{(k)}$ for $k = 1,\cdots,K.$

Step 6. Compute the single-user transceivers $\{\mathbf{V}_{\text{SU}}^{(k)}\}, \{\mathbf{U}_{\text{SU}}^{(k)}\}$ with the same way as in Section III.

In the sequel, the proposed non-iterative regularized schemes from the ZF transceivers in Section III will be referred to as the regularized CSM (R-CSM) and regularized IA (R-IA) for $K = 2$ and $K = 3$, respectively. It should be noted that we can further improve the performance of the proposed R-CSM and R-IA by repeating from **Step 2** to **Step 6**. However, considering the tradeoff between complexity and performance gain, an iterative algorithm is not adopted in this section.

As will be shown in the next section, the proposed regularization approach can also be employed without initialization with the ZF and IA transceivers in a distributed manner. Nonetheless, we propose the ZF and IA initialization that provides us a benefit of fast convergence as well as near optimal performance especially at high SNR regime.

V. Distributed Implementation of the Proposed Regularization Approach

The proposed R-CSM and R-IA algorithms require the global CSI for every nodes, and this may be impractical in actual systems. In this section, we introduce a distributed regularization algorithm where the distributed approach means that what transmitter k (receiver k) needs when computing the precoder $\mathbf{V}^{(k)}$ (decoder $\mathbf{U}^{(k)}$) is $\mathbf{U}^{(l)H}\mathbf{H}^{(lk)}$ ($\mathbf{H}^{(kl)}\mathbf{V}^{(l)}$) for $l = 1,\cdots,K$ [7] [26]. In other words, each node can compute its own precoder and decoder using only local CSI connected to itself and the covariance matrix of its effective noise-plus-interference terms regardless of system configuration.

First, since the ZF based precoders cannot be found in a distributed manner, we start with a random choice of precoders and perform an iterative process. The proposed distributed

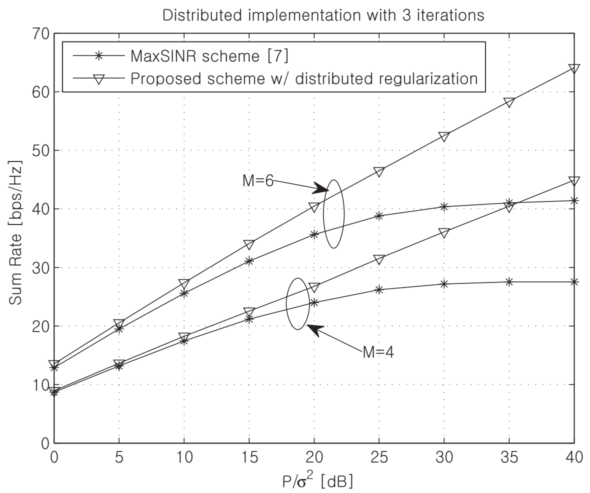

Fig. 3. Average sum-rate performance of the proposed distributed regularization scheme in 3-user MIMO IC with 3 iterations.

algorithm can be summarized as follows:

Initialization:

Set $\{\mathbf{V}_{\text{MU}}^{(k)}\}$ as arbitrary $M \times \frac{M}{2}$ matrices and set $n = 1$.

Main Iteration:

While $n \leq N_{iter}$

For $k = 1, \cdots, K,$ receiver k updates its decoder as

$$\mathbf{U}_{\text{MU}}^{(k)} \leftarrow \mathbf{Q}\left(\left(\sum_l \mathbf{H}^{(kl)}\mathbf{V}_{\text{MU}}^{(l)}\mathbf{V}_{\text{MU}}^{(l)H}\mathbf{H}^{(kl)H}+\sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}\right).$$

For $k = 1, \cdots, K,$ transmitter k computes

$$\mathbf{V}_{\text{MU}}^{(k)} \leftarrow \bar{\mathbf{Q}}^{(k)}\mathbf{T}^{(k)}.$$

Set $n \leftarrow n + 1$.

End

Termination:

For $k = 1, \cdots, K,$ receiver k sets

$$\mathbf{U}_{\text{SU}}^{(k)} \leftarrow \mathbf{U}_1^{\text{sing}}\left(\mathbf{U}_{\text{MU}}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}\right).$$

For $k = 1, \cdots, K,$ receiver k sets

$$\mathbf{V}_{\text{SU}}^{(k)} \leftarrow \mathbf{V}_1^{\text{sing}}\left(\mathbf{U}_{\text{MU}}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}\right).$$

In the above algorithm, the columns of $\mathbf{V}_1^{\text{sing}}(\mathbf{X})$ consist of the right singular vectors of $\mathbf{X}$ corresponding to nonzero singular values. N_{iter} can be considered as the number of training iterations in time-division duplexing systems [26]. Unlike the SINR maximization algorithm (MaxSINR) in [7] where the training iterations are performed with $\{\mathbf{U}^{(k)}\}$ and $\{\mathbf{V}^{(k)}\}$, the proposed scheme executes the iteration with the multi-user transceivers $\{\mathbf{U}_{\text{MU}}^{(k)}\}$ and $\{\mathbf{V}_{\text{MU}}^{(k)}\}$, and the single-user transceivers are determined at the last step based on $\mathbf{U}_{\text{MU}}^{(k)H}\mathbf{H}^{(kk)}\mathbf{V}_{\text{MU}}^{(k)}$.

We briefly compare the proposed distributed regularization algorithm with the MaxSINR scheme where the transmit and receive filters are alternately updated to improve the stream-wise SINR. As shown in Figure 2, the proposed scheme with the distributed regularization outperforms the conventional distributed MaxSINR scheme in terms of the convergence behaviour. This is because the MaxSINR algorithm tries to mitigate the inter-user plus intra-user interference at each iteration which results in slow convergence of the algorithm. Thus, Figure 3 illustrates that with finite number of iterations, the MaxSINR scheme shows the poor performance for high SNR. In contrast, the proposed regularization attempts to reduce only the inter-user interference at each step and the intra-user interference is perfectly cancelled at the last step. As a result, the proposed scheme exhibits the reasonable performance with only a few iterations.

Second, we can obtain the complexity savings for the same number of iterations N_{itr}. The proposed scheme and the MaxSINR scheme require floating point operations (flops) of $M^3 \cdot \left((6K^2 + \frac{15}{2}K)N_{\text{itr}} + \frac{29}{8}K\right)$ and $M^4 \cdot N_{\text{itr}}\left(2K^2 + \frac{4}{3}K\right) + M^3 \cdot 4KN_{\text{itr}}$, respectively. The flop evaluation is performed according to [27] as

- Multiplication of $m \times n$ and $n \times p$ matrices: $2mnp$.
- Orthogonalization process $\mathbf{Q}(\mathbf{X})$ for an $m \times n$ matrix $\mathbf{X}$ ($m \geq n$): $2n^2(m - n/3)$.
- Cholesky factorization of an $m \times m$ matrix: $m^3/3$.
- Inversion of an $m \times m$ matrix with Gauss-Jordan elimination method: $4m^3/3$.
- SVD of an $m \times n$ matrix ($m \leq n$): $4n^2m + 13m^3$.

Notice that the required number of flops of the proposed scheme is proportional to M^3 while that of the MaxSINR is in proportion to M^4. This saving mainly comes from the fact that the proposed algorithm needs the inverse operation of an M-by-M matrix only once to update $\mathbf{V}^{(i)}$(or $\mathbf{U}^{(i)}$) at each step, whereas the MaxSINR algorithm requires $\frac{M}{2}$ times. In summary, the proposed algorithm with distributed regularization provides the performance superior to the MaxSINR with reduced complexity.

VI. Numerical Results

In this section, we provide numerical results evaluating the sum-rate performance of the proposed schemes. For comparison, we plot sum-rates of the TDMA schemes where only one link operates at each channel use with round-robin scheduling. When operating, each link adopts the SVD beamforming combined with the WF algorithm (SVD-WF) with each user's transmit power of KP and P. Although we should compare the proposed schemes with the TDMA with P for per-user power constraint, the performance of the TDMA with KP is also presented since the other schemes are consuming power of KP in total. Also, the performance of the sum-rate maximizing algorithm based on the gradient ascent (GA) method in [11] and [28] is presented to check the near-optimality of the proposed schemes. The GA-based algorithm finds the sum-rate maximizing input covariance in an iterative fashion without consideration on the receiver structure and can guarantee only a local optimal solution. In order to increase the probability of achieving the global optimal solution, we have repeated the GA algorithm for multiple randomly chosen initial precoders and selected one corresponding to the maximum sum-rate. It should be noted that the GA is far from practical implementation in spite of its superior performance since it requires flops of $M^3 \cdot N_{\text{itr}}\left(4K^3 + \frac{23}{3}K^2 - \frac{29}{6}K\right)$ which is much higher than the proposed regularization process.

In Figure 4, average sum-rate performance of various schemes are plotted for the 2-user IC with $M = 4$. The selfish beamforming differs from the proposed CSM in a sense

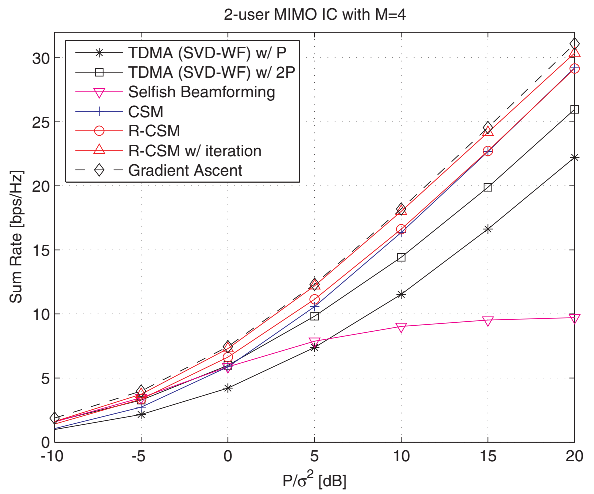

Fig. 4. Average sum-rate performance for 2-user MIMO IC with $M = 4$

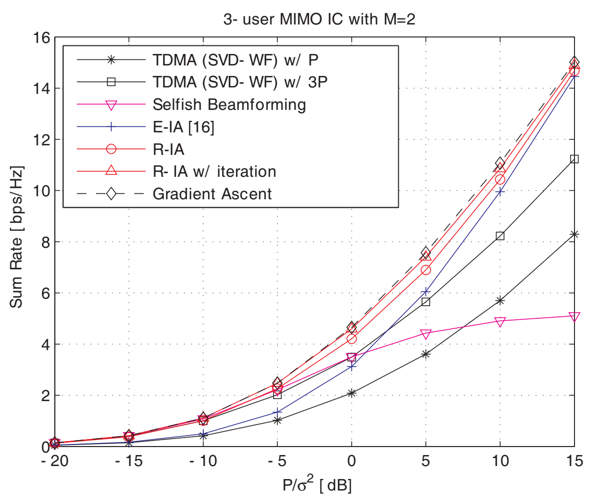

Fig. 5. Average sum-rate performance for 3-user MIMO IC with $M = 2$

that the multi-user transceivers $\mathbf{V}_{\text{MU}}^{(i)}$ and $\mathbf{U}_{\text{MU}}^{(i)}$ are set to the first $\frac{M}{2}$ right and left singular vectors of $\mathbf{H}^{(ii)}$, respectively. This means that the selfish beamforming performs precoding and decoding neglecting the effect of inter-user interference. The selfish beamforming is thus considered as non-cooperative eigenbeamforming in [15] and the egoistic approach in [13]. From Figure 4, the proposed CSM based on the ZF criterion shows sum-rate performance slightly inferior to the TDMA with $2P$ at low SNR. However, the proposed regularization provides an improvement over the CSM scheme and outperforms the TDMA for all SNR regime. Since the performance impairment mainly comes from the noise effect at low SNR, it is observed that the selfish beamforming shows reasonable performance. At high SNR, the CSM outperforms the TDMA with $2P$ in terms of the sum-rate performance. In contrast, the selfish beamforming which neglects the effect of inter-user interference shows quite poor performance as the performance is interference-limited in high SNR regime. Also, we can see that the proposed regularization method approaches the sum-rate of the near optimal GA method with enough iterations.

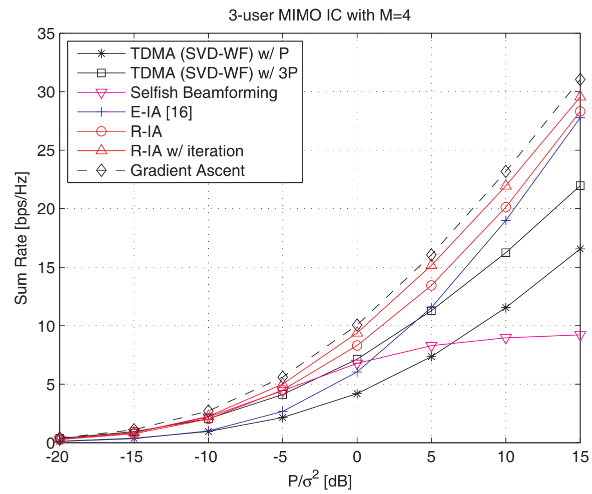

Fig. 6. Average sum-rate performance for 3-user MIMO IC with $M = 4$

In Figure 5, the average sum-rate performance is presented for several transmission techniques in the 3-user MIMO IC with $M = 2$. Although the curve of the E-IA shows a slope steeper than the TDMA scheme, the sum-rate performance of the E-IA at low SNR is poor compared to the TDMA with $3P$. On the other hand, the proposed R-IA scheme outperforms the TDMA scheme over all SNR regime. Figure 6 demonstrates the sum-rate results for the case of $M = 4$. We observe an increase in the performance gap between the R-IA and the E-IA with a larger number of antennas. This is because the effect of the regularization tends to be more significant when the number of data streams is increased. Also, it is shown that the proposed non-iterative R-IA provides performance within only 2.5 dB from the near-optimum GA-based algorithm with significantly reduced complexity. When iterations are performed in the R-IA, the performance difference to the optimal solution is further decreased at the expense of the increased complexity.

VII. Conclusion

In this paper, we have proposed efficient transceiver algorithms for 2-user and 3-user MIMO IC. For the 2-user case, a non-iterative CSM algorithm is proposed. Also, we have shown that the E-IA in [16] is an optimal ZF transceiver for the 3-user IC. To overcome the degraded performance of the ZF-based algorithms at low SNR, we have proposed a regularization method. Moreover, the distributed implementation of the regularization process is discussed. From the simulation results, it is observed that the proposed regularized transceivers outperform the conventional algorithms over all SNR regime. Also, our distributed approach provides a substantial performance gain over the conventional distributed scheme [7] with reduced complexity.

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