Zero-Forcing Beamforming in Multiuser MISO Downlink Systems Under Per-Antenna Power Constraint and Equal-Rate Metric

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***Abstract*—In this paper, we analyze the average sum rate of downlink multi-antenna systems with zero-forcing beamforming (ZFBF). In practical implementations, each antenna is equipped with its own power amplifier and is limited individually by linearity of the amplifier. Thus, this paper adopts a more realistic per-antenna power constraint instead of conventional sum-power constraint on transmit antennas. To this end, we first show that a distribution of the received signal-to-noise ratio (SNR) of the ZFBF scheme with per-antenna power constraint and equal-rate metric can be approximated as a minimum of chi-square random variables. Based on this result, we present an accurate formula of the average sum rate in a closed form. Furthermore, employing extreme value theory, an expression of the asymptotic average sum rate with large numbers of transmit antennas and users is derived from the limiting distribution of the received SNR. Simulation results verify the validity of our analysis even with not so large numbers of transmit antennas and users.**

***Index Terms*—Broadcast channel, zero-forcing precoding, per-antenna power constraints.**

I. INTRODUCTION

MULTIUSER multiple input multiple output (MU-MIMO) technologies are expected to play a key role in wireless communication systems. For the past few years, significant research efforts have been devoted to the investigation of the capacity region in MU-MIMO Gaussian broadcast channels (BC) [1]–[5], which serves as the information theoretic foundation for various MU-MIMO downlink schemes. It has been proved that dirty-paper coding (DPC) is a capacity achieving precoding strategy for the Gaussian MIMO BC [5]. However, due to nonlinearity, the DPC percoder is difficult to implement in practical systems. Thus, a lot of research has recently been focused on simpler linear precoding techniques, as suboptimal linear precoders can achieve a large portion of the rate region with moderate complexity. Among several practical precoding methods, a zero-forcing beamforming (ZFBF) scheme is one of appealing precoding techniques because of its simplicity [6] [7].

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Typically, sum-power constraint on transmit antennas has been assumed in most papers addressing transmission techniques for the MIMO BC. However, in realistic implementations, each antenna normally has its own amplifier and is thus limited individually as considered in [8]. In other words, the transmitter may not be able to allocate power arbitrarily among the transmit antennas. Another scenario which requires the per-antenna constraint is a distributed antenna system [9] [10] where geologically separated transmitters are connected via fiber links and are capable of cooperative transmission. In this case, the transmitters do not share power with each other. Thus, consideration of individual power constraint is important for understanding the performance in such systems [11].

Under per-antenna power constraint (PAPC), a few papers studied precoding designs or power allocation to optimize a certain objective function [8] [12] [13]. More specifically, the problem of finding the capacity region of a downlink system with PAPC was solved by using generalized uplink-downlink duality in [8]. This paper also addressed joint linear precoding and power allocation to minimize the total transmit power while satisfying a set of signal-to-interference-plus-noise ratio requirements.

Also, the ZFBF under PAPC was examined in [12] and [13]. It was shown in [12] that the optimal precoding matrix for fairness and throughput criteria can be found by utilizing the concept of generalized inverse instead of pseudo-inverse and standard convex optimization methods. In [13], for the special case of two users, an optimal power allocation problem for maximizing the weighted sum rate was solved by characterizing the intersections of the hyperplane constraints. As a simple non-iterative method under PAPC, we can consider a ZFBF addressed in [14]. Herein, to satisfy per-antenna power constraint, the ZFBF vectors are scaled by the maximum of all antennas' power. However, even in this case, analysis is nontrivial and has not been reported in the literature, which motivates us to explore an analytic expression of the system with the ZFBF.

In this paper, we consider ZFBF under PAPC, which uses the pseudo-inverse for the precoding matrix with a scale factor as in [14]. We refer to this precoder as ZFBF with scale down (ZFBF-SD). Our objective is to analyze the average sum rate of multiuser downlink systems adopting the ZFBF-SD over Rayleigh fadings. To this end, we first investigate a distribution of the received SNR for users, which depends on the minimum diagonal element of the pseudo-inverse Wishart matrix defined in [15]. However, the problem of determining the joint distribution of diagonal elements of this matrix is very hard and challenging. Thus, by employing approximation, we characterize an approximate distribution of the received SNR as the minimum distribution of chi-square random variables, the number of which equals that of transmit antennas. Based on this result, the average sum rate formula is derived in a closed form. From numerical simulations, we verify the accuracy of our analysis.

The next contribution in this paper is the derivation of an asymptotic sum rate expression for large-system analysis in wireless communication systems [16]–[19]. One attractive feature of large system approaches is that the resulting asymptotic expressions often turn out to be good estimates for finite-dimensional system performance. Moreover, such approaches may be more efficient for investigating the system performance, since we can obtain simple analytical expressions and determine key parameters for the performance. Among several tools for asymptotic analysis, extreme order statistics is one of important methods. For instance, the asymptotic analysis based on the extreme order statistics has been used for obtaining the log scaling law of MU diversity [20] [21] and throughput [22]. We also provide the asymptotic sum rate analysis for systems with large numbers of transmit antennas and users based on extreme value theory [23]. It will be shown that our asymptotic derivation has a simpler form compared to the finite dimensional case and is also quite accurate even for small numbers of transmit antennas and users.

This paper is organized as follows: In Section II, we describe a system model for the multiuser BC and formulate the received SNR subject to PAPC. Section III derives the average sum rate of MU-MISO systems with ZFBF-SD. The asymptotic sum rate analysis based on extreme value theory is presented in Section IV. We provide simulation results in Section V. Finally, Section VI concludes this paper.

Throughout the paper, we employ uppercase boldface letters for matrices and lowercase boldface for vectors. The superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ and $\|\cdot\|$ stand for transpose, conjugate transpose, element-wise conjugate and 2-norm, respectively. Also, $\text{tr}(\mathbf{A})$ denotes trace of a matrix $\mathbf{A}$, and $\mathbf{I}_N$ indicates an $N \times N$ identity matrix. The expectation of a random variable is given by $\mathbb{E}(\cdot)$.

II. System Model

We consider multiuser MISO downlink systems where a base station with M antennas communicates to K single antenna users as shown in Fig 1. Then, the received signal for user k is given by

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k$$

where $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ denotes the channel vector from the base station to the k-th user, $\mathbf{x} \in \mathbb{C}^{M\times 1}$ represents the transmitted signal vector from the base station, and n_k stands for the

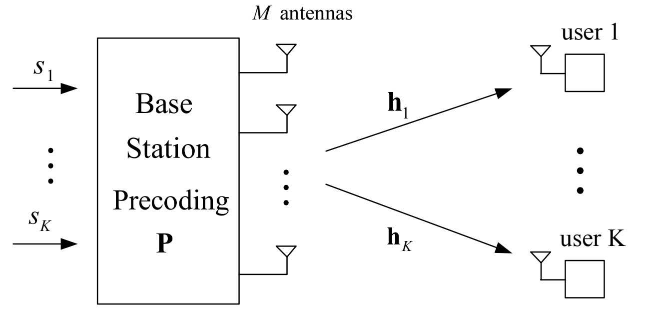

Fig. 1. Structure of a multiuser MISO downlink system.

additive white Gaussian noise (AWGN) with zero mean and unit variance. In the following matrix notation, we have

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

where $\mathbf{y} = [y_1, \dots, y_K]^T$, $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K]^H$, and $\mathbf{n} = [n_1, \dots, n_K]^T$. Here, the elements of $\mathbf{H}$ are independent and identically distributed (i.i.d.) with $\mathcal{CN}(0, 1)$. We assume that $K \le M$ and $\mathbf{H}$ is full row rank and known perfectly at the base station.

The transmitted vector is a linear transformation of the information symbols

$$\mathbf{x} = \mathbf{P}\mathbf{s}$$

where the information vector $\mathbf{s}$ of length K satisfies $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$. To achieve zero interference among users, the beamforming matrix $\mathbf{P}$ is chosen such that all off-diagonal elements of $\mathbf{HP}$ are zero. Therefore, the ZFBF precoding matrix can be determined as pseudo-inverse of $\mathbf{H}$

$$\mathbf{P} = \sqrt{\gamma}\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} = \sqrt{\gamma}\bar{\mathbf{P}}$$

where $\sqrt{\gamma}$ is a scale factor for satisfying power constraint and $\bar{\mathbf{P}}$ is defined as $\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1}$.

Applying the ZFBF precoder decouples a MISO BC into K independent scalar channels as

$$y_k = \sqrt{\gamma}s_k + n_k \quad \text{for } k = 1, \dots, K. \tag{1}$$

Therefore, the average sum rate can be expressed as

$$\mathbb{E}[R] = K\mathbb{E}[\log_2(1+\gamma)].$$

In the case of sum-power constraint, precoders are designed with the constraint

$$\mathbb{E}\left[\|\mathbf{x}\|^2\right] = \gamma \text{tr}\left\{\bar{\mathbf{P}}\bar{\mathbf{P}}^H\right\} \le \rho$$

where ρ represents total power constraint. Then, the maximum achievable SNR can be written as

$$\gamma = \frac{\rho}{\text{tr}\left\{\bar{\mathbf{P}}\bar{\mathbf{P}}^H\right\}}.$$

In contrast, assuming that ρ/M is the available power per antenna, the input covariance matrix subject to PAPC should satisfy the following equation

$$\mathbb{E}\left\{\left[\mathbf{x}\mathbf{x}^H\right]_{ii}\right\} = \gamma\left[\bar{\mathbf{P}}\bar{\mathbf{P}}^H\right]_{ii} \le \frac{\rho}{M}, \quad \forall i$$

where $[\mathbf{X}]_{ij}$ denotes the (i,j)-th element of a matrix $\mathbf{X}$. Employing the ZFBF-SD where the ZFBF vectors are scaled by the maximum of all antennas' power, it then follows that

$$\gamma \max_i \left\{ \left[\bar{\mathbf{P}}\bar{\mathbf{P}}^H\right]_{ii} \right\} = \frac{\rho}{M}.$$

Hence, the received SNR γ is given as

$$\begin{aligned} \gamma &= \frac{\rho}{M \max_i \left\{ \left[\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathbf{H}\right]_{ii} \right\}} \\ &= \frac{\rho}{M \max_i \left\{ \left[\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-2} \mathbf{H}\right]_{ii} \right\}}. \end{aligned} \tag{2}$$

It can be seen from (2) that in order to obtain the distribution of γ, we need to find the joint distribution of diagonal elements of $\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-2} \mathbf{H}$. However, since this matrix has a complex pseudo inverse Wishart distribution [15], there is no closed form of the joint probability density function of its diagonal elements. Thus, in the following section, we derive an approximate distribution of γ instead of an exact distribution for the sum rate analysis.

III. Performance Analysis with Finite M and K

In this section, we provide a formula of the average sum rate of multiuser downlink systems which employ ZFBF-SD under PAPC. From (1), it is clear that all users have the same SNR value for fixed channel instants. Then, the received SNR γ for all users is represented as

$$\gamma = \min_i \xi_i \quad \text{for} \quad i = 1, \ldots, M \tag{3}$$

where ξ_i is defined by

$$\xi_i = \frac{\rho}{M \left[\mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-2} \mathbf{H}\right]_{ii}}. \tag{4}$$

Notice that the minimum operation in (3) is related to the number of transmit antennas M, not the number of users K. It can be easily seen from (3) and (4) that obtaining a distribution of γ requires the joint distribution of ξ_i's which is correlated with each other. However, as mentioned earlier, it is very difficult to compute its distribution due to the lack of the joint distribution of diagonal elements of a pseudo inverse Wishart matrix. For this reason, we establish an approximate distribution instead of an exact one in the following.

To overcome the difficulty of finding the joint distribution of ξ_i's, we assume that ξ_i's are independent. Under this assumption, the marginal distribution of ξ_i is only required in order to determine the distribution of γ. Although correlation in ξ_i's is neglected, it will be shown in the simulation section that the derived results based on this assumption are quite accurate. The following lemma characterizes an approximate distribution of ξ_i.

Lemma 1: The distribution of ξ_i can be approximately expressed as a chi-square distribution with the scale parameter

$$\xi_i \sim \frac{\rho}{\alpha M} \chi^2(2(M-K+1))$$

where α is defined as $\mathbb{E}\left[\|\bar{\mathbf{v}}_i\|^2\right]$. Here, $\bar{\mathbf{v}}_i$ denotes the subvector containing the first K components of the i-th row vector of $\mathbf{V}$ where $\mathbf{V} \in \mathbb{C}^{M\times M}$ is composed of right singular vectors of $\mathbf{H}$.

Proof: See Appendix A. ■

From now on, we evaluate $\mathbb{E}\left[\|\bar{\mathbf{v}}_i\|^2\right]$. Note that each row of $\mathbf{V}$ is an isotropically distributed (i.d.) complex unit vector. In the subsequent lemma, we derive the mean of the sum of the magnitude square of any L elements of an i.d. complex unit vector.

Lemma 2: Defining $\mathbf{w}$ and $\mathbf{w}^{(L)}$ as an i.d. M-dimensional complex unit vector and any L elements of $\mathbf{w}$ ($L \le M$), respectively, the mean of $\|\mathbf{w}^{(L)}\|^2$ is computed as

$$\mathbb{E}\left[\|\mathbf{w}^{(L)}\|^2\right] = \frac{L}{M}.$$

Proof: See Appendix B. ■

From Lemma (2), $\mathbb{E}\left[\|\bar{\mathbf{v}}_i\|^2\right]$ is obtained as $\frac{K}{M}$. Finally, combining this result and Lemma 1, ξ_i follows a chi-square distribution with scale factor

$$\xi_i \sim \frac{\rho}{K} \chi^2(2(M-K+1)) \tag{5}$$

whose probability density function (PDF) and cumulative density function (CDF) of ξ_i are given respectively by

$$f_{\xi_i}(x) = \frac{\bar{\alpha}^N x^{N-1}}{\Gamma(N)} \exp(-\bar{\alpha}x) \quad \text{and} \quad F_{\xi_i}(x) = \frac{\gamma(N, \bar{\alpha}x)}{\Gamma(N)} \tag{6}$$

where $\bar{\alpha} = \frac{K}{\rho}$, $N = M - K + 1$ and $\gamma(s,x)$ indicates the incomplete gamma function $\int_0^x t^{s-1} e^{-t} \mathrm{d}t$.

In what follows, based on (5), we try to compute $\gamma = \min_i \xi_i$ using the result of the order statistics [23]. Let $X_1, X_2, \cdots, X_L$ be i.i.d. random samples from a continuous random variable with the PDF $f_X(x)$ and the CDF $F_X(x)$. Let us denote $X_{l:L}$ as the l-th largest value among $X_1, \cdots, X_L$ ($X_{1:L} \le X_{2:L} \le \cdots \le X_{L:L}$). Then $X_{l:L}$ for $1 \le l \le L$ is called the l-th order statistic and its PDF can be expressed by

$$f_{X_{l:L}}(x_l) = \frac{L! [F_X(x_l)]^{l-1} [1 - F_X(x_l)]^{L-l} f_X(x_l)}{(l-1)!(L-l)!}. \tag{7}$$

For an integer N, the CDF of ξ_i in (6) can be rewritten in terms of a finite series expansion as [24]

$$F_{\xi_i}(x) = \frac{\gamma(N, \bar{\alpha}x)}{\Gamma(N)} = 1 - \exp(-\bar{\alpha}x) \sum_{k=0}^{N-1} \frac{(\bar{\alpha}x)^k}{k!}.$$

Inserting the above expression for the CDF and the PDF in (6) into (7), a distribution of γ can be represented as

$$\begin{aligned} f_\gamma(x) &= \frac{M\bar{\alpha}^N x^{N-1}}{\Gamma(N)} \left[\exp(-\bar{\alpha}x) \sum_{k=0}^{N} \frac{(\bar{\alpha}x)^k}{k!} \right]^{M-1} \exp(-\bar{\alpha}x) \\ &= \frac{M\bar{\alpha}^N x^{N-1}}{\Gamma(N)} \exp(-\bar{\alpha}Mx) \left[\sum_{k=0}^{N-1} \frac{(\bar{\alpha}x)^k}{k!} \right]^{M-1}. \end{aligned}$$

Then, using the identity [25]

$$\left[\sum_{k=0}^{N-1} \frac{(\bar{\alpha}x)^k}{k!} \right]^{M-1} = \sum_{k=0}^{(N-1)(M-1)} b_k (\bar{\alpha}x)^k$$

where

$$b_0 = 1, \quad b_1 = M-1, \quad b_{(N-1)(M-1)} = \frac{1}{((N-1)!)^{M-1}},$$

$$b_k = \frac{1}{k} \sum_{j=1}^{J_0} \frac{jM-k}{j!} b_{k-j} \text{ with } J_0 = \min(k, N-1)$$

for $2 \le k \le (M-1)(N-1) - 1$,

we can finally express $f_\gamma(x)$ as

$$f_\gamma(x) = \frac{M}{\Gamma(N)} \exp(-\bar{\alpha} M x) \sum_{k=0}^{(N-1)(M-1)} b_k \bar{\alpha}^{k+N} x^{k+N-1}. \quad (8)$$

Next, based on the above results (8), we derive the average sum rate in a closed form. The average sum rate $\mathbb{E}[R]$ is computed as

$$\mathbb{E}[R] = \mathbb{E}\left[K \log_2(1+\gamma)\right] = K \int_0^\infty \log_2(1+x) f_\gamma(x) dx.$$

Applying the integral identity [26]

$$\int_0^\infty \log(1+t)\exp(-\mu t) t^{n-1} dt = (n-1)! \exp(\mu) \sum_{i=1}^{n} \frac{\Gamma(i-n,\mu)}{\mu^i}, \quad \text{for } n = 1, 2, \cdots,$$

we can obtain $\mathbb{E}[R]$ as

$$\begin{aligned} \mathbb{E}[R] &= \frac{KM}{\Gamma(N)} \sum_{k=0}^{(N-1)(M-1)} b_k \bar{\alpha}^{N+k} (N+k-1)! \\ &\times \exp(M\bar{\alpha}) \sum_{j=1}^{N+k} \frac{\Gamma(j-N-k, M\bar{\alpha})}{(M\bar{\alpha})^j} \end{aligned} \quad (9)$$

where $\Gamma(s,x)$ stands for the complementary incomplete gamma function defined as $\Gamma(s,x) = \int_x^\infty t^{s-1}\, e^{-t}\, \mathrm{d}t$.

So far, we have characterized the received SNR as a minimum of M chi-square random variables with parameter N, and derived a closed-form expression of the average sum rate. The accuracy of our derived analysis will be confirmed in the simulation section. Still, the derived sum rate expression (9) is complicated. In the following section, we will provide a simplified result through asymptotic analysis.

IV. Asymptotic Sum Rate Analysis

In this section, we now consider the large system regime where the number of antennas at the base station grows large ($M \to \infty$) while $M - K$ is kept constant. Based on extreme value theory [23], we will provide an asymptotic sum rate of MU-MISO systems with ZFBF-SD. Before starting, we will briefly introduce some major results of the extreme value theory which deals with asymptotic distributions of extreme values such as maxima or minima.

A. Extreme Value Theory

Let Z_n be the maximum of n i.i.d. random variables X_i's with the distribution function $F(x)$. If there exist constants $b_n \in \mathbb{R},\ a_n > 0$, and some nondegenerate distribution functions G such that a distribution of the standardized maxima $(Z_n - b_n)/a_n$ converges to G, then G belongs to one of the following three standard extreme value distributions (10) on the next page. Similarly, the asymptotic distribution of the standardized minimum of X_i's must be one of just three types which are given as

$$G_i^*(x) = 1 - G_i(-x) \qquad \text{for } i = 1, 2, 3. \quad (11)$$

In order to identify the asymptotic distribution and the mean of the received SNR γ, we provide the following two lemmas.

Lemma 3 (Sufficient condition for Weibull distribution [23]): Define ζ_p as $F^{-1}(p)$. If $\zeta_1 < \infty$ and

$$\lim_{x \to \zeta_1} \frac{(\zeta_1 - x) f(x)}{1 - F(x)} = \beta > 0,$$

then there exist constants $a_n > 0$ and b_n such that $(Z_n - b_n)/a_n$ uniformly converges in distribution to a normalized Weibull random variable as $n \to \infty$. The normalizing constant a_n and b_n can be selected by

$$a_n = \zeta_1 - F^{-1}\left(1 - \frac{1}{n}\right) \qquad \text{and} \qquad b_n = 0.$$

Lemma 4 ([22]): If $(Z_n - b_n)/a_n$ converges in distribution to a random variable Z that has a nondegenerate distribution function, and if $\mathbb{E}\{[(Z_n)^-]^p\} < \infty$ for any positive real number p where $(x)^- = \max(-x, 0)$, then

$$\lim_{n\to\infty} \mathbb{E}\left(\frac{Z_n - b_n}{a_n}\right)^p = \mathbb{E}\{Z^p\}.$$

Lemma 3 indicates a sufficient condition for $F(x)$ to belong to attraction to the maximal Weibull distribution $G_2(x;\beta)$, and explains how the normalizing constant a_n can be determined. From Lemma 4, the relation between convergence in distribution and moment convergence is established. They will be used to identify the limiting distribution of ξ_i and the mean of the received SNR.

B. Limiting Distribution of SNR and Asymptotic Sum Rate

In this subsection, we will prove that the limiting distribution of γ belongs to the domain of attraction of the minimal Weibull distribution $G_2^*(x;\beta)$. In the following lemma, by showing that a distribution of $-\xi_i$ satisfies the sufficient condition shown in Lemma 3, we derive the limiting distribution of γ.

Lemma 5: As $M \to \infty$, γ/a_M converges in distribution to a normalized Weibull random variable whose distribution function is $G_2^*(x;\beta)$ where $\beta = N$. The normalization parameter a_M can be selected as

$$a_M = F_{\xi_i}^{-1}\left(\frac{1}{M}\right).$$

Proof: See Appendix C. ■

Note that the shape parameter β equals the degree of freedom N of a chi-square distribution.

From now on, we will derive the asymptotic sum rate by utilizing the limiting distribution of γ. Using Jensen's inequality, $\mathbb{E}[R]$ can be evaluated by

$$\mathbb{E}[R] \quad \approx \quad K \log_2\left(1 + \mathbb{E}[\gamma]\right).$$

By setting $p = 1$ in Lemma 4 and applying the result from Lemma 5, $\mathbb{E}[\gamma]$ is given as

$$\mathbb{E}[\gamma] \approx a_M \mathbb{E}[\Upsilon] = F_{\xi_i}^{-1}\left(\frac{1}{M}\right)\mathbb{E}[\Upsilon] \quad (12)$$

where Υ represents a random variable with the normalized Weibull distribution $G_2^*(x;N)$.

Using the result [27]

$$\mathbb{E}[\Upsilon] = \Gamma\left(1 + \frac{1}{N}\right),$$

$$
\begin{aligned}
&\text{(Fréchet)} \;\; G_1(x;\beta) = \begin{cases} 0 & \text{for } x \le 0, \quad \beta > 0 \\ \exp(-x^{-\beta}) & \text{for } x > 0 \end{cases} \\
&\text{(Weibull)} \;\; G_2(x;\beta) = \begin{cases} \exp(-(-x)^{\beta}) & \text{for } x \le 0, \quad \beta > 0 \\ 1 & \text{for } x > 0 \end{cases} \\
&\text{(Gumbel)} \;\; G_3(x) \quad = \exp(-e^{-x}) \qquad \text{for } -\infty < x < \infty.
\end{aligned} \tag{10}
$$

$\mathbb{E}[\gamma]$ is computed as

$$\mathbb{E}[\gamma] \approx F_{\xi_i}^{-1}\left(\frac{1}{M}\right)\Gamma\left(1+\frac{1}{N}\right). \tag{13}$$

Here, we can easily check from (6) that

$$F_{\xi_i}^{-1}\left(\frac{1}{M}\right) = \frac{\rho}{K}\Gamma(N)\gamma^{-1}\left(N, \frac{1}{M}\right) \tag{14}$$

where $\gamma^{-1}(\cdot,\cdot)$ denotes the inverse of the incomplete gamma function $\gamma(s,x)$.

After inserting (14) into (13), $\mathbb{E}[R]$ can be finally written as

$$\mathbb{E}[R] \approx K\log_2\left(1+\frac{\rho}{K}\Gamma(N)\gamma^{-1}\left(N,\frac{1}{M}\right)\Gamma\left(1+\frac{1}{N}\right)\right). \tag{15}$$

Although there is no closed form for the inverse incomplete gamma function, it is usually provided in common softwares such as Matlab and mathematica.

It should be emphasized that (15) has a simpler form compared with (9) and is expressed in the form of $C(\rho) = K\log_2(1+c\cdot\rho)$ where c is a constant. Denoting the multiplexing gain S_∞ and the high SNR power offset L_∞ as [28]

$$S_\infty = \lim_{\rho\to\infty}\frac{C(\rho)}{\log_2(\rho)} \quad \text{and} \quad L_\infty = \lim_{\rho\to\infty}\left(\log_2(\rho) - \frac{C(\rho)}{S_\infty}\right),$$

we can easily observe from (15) that S_∞ equals K and L_∞ is

$$L_\infty = \log_2\frac{K}{\Gamma(N)\gamma^{-1}\left(N,\frac{1}{M}\right)\Gamma\left(1+\frac{1}{N}\right)}.$$

These quantitative performance measures are useful for characterization of the system performance and comparison between two different systems. For illustration purposes, we compare our derived results with the average sum rate of ZFBF under sum power constraint (ZFBF-SPC). In the sum power case, the asymptotic sum rate is given as [29]

$$C^{SP}(\rho) = K\log_2\left(1+\frac{M-K}{K}\rho\right). \tag{16}$$

This implies that S_∞^{SP} equals K and L_∞^{SP} is computed as $\log_2\frac{K}{M-K}$. Thus, given M and K, the system with ZFBF-SD needs an additional transmit power ΔP by

$$\Delta P = 10\log_{10}\frac{M-K}{\Gamma(N)\gamma^{-1}\left(N,\frac{1}{M}\right)\Gamma\left(1+\frac{1}{N}\right)} \quad \text{dB} \tag{17}$$

compared to that of ZFBF-SPC for the same spectral efficiency. This will be verified in the following section.

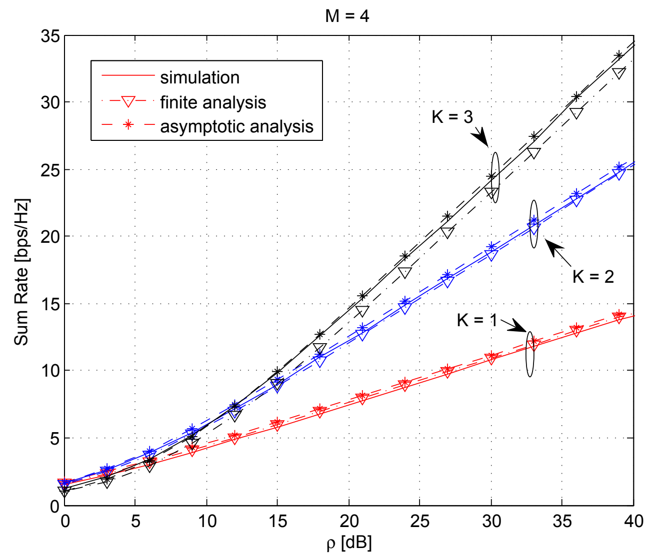

Fig. 2. Average sum rate with respect to SNR with $M = 4$ for various K.

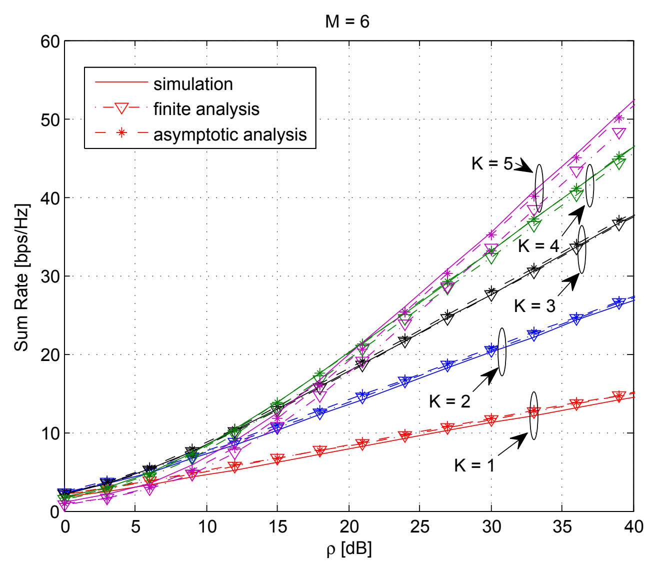

Fig. 3. Average sum rate with respect to SNR with $M = 6$ for various K.

V. Numerical Results

In this section, we compare our sum rate analysis for MISO downlink systems with the ZFBF-SD with numerical simulations to confirm the validity of our analysis. For simulations, we employ spatially uncorrelated Rayleigh fading channels which are generated randomly and independently for each transmission. Figures 2 and 3 exhibit the average sum rate of ZFBF-SD with different numbers of users for $M = 4$ and 6

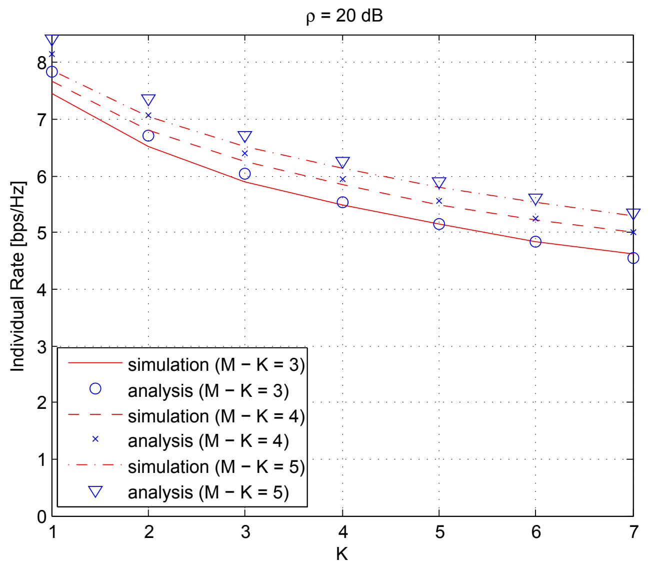

Fig. 4. Asymptotic individual rate with respect to K for $\rho = 20$ dB and fixed $M - K$.

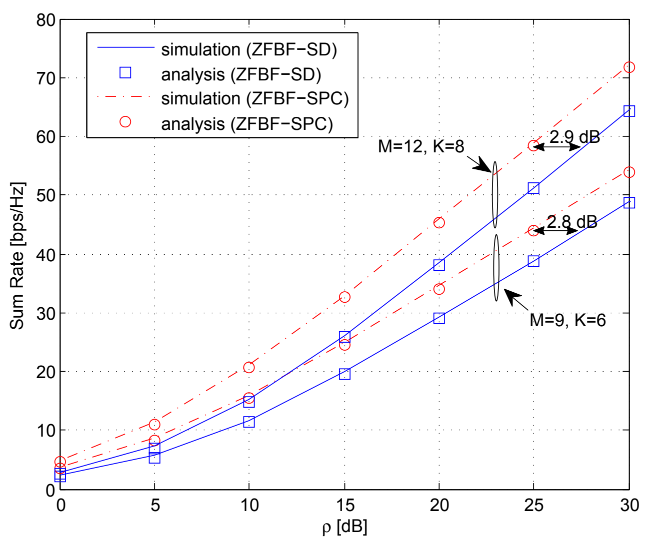

Fig. 5. Performance comparison between ZFBF-SD and ZFBF-SPC.

with respect to SNR in dB, respectively. The results of finite and asymptotic analysis in these plots are evaluated by (9) and (15), respectively. From these figures, we emphasize that our analysis in (9) matches well with the empirical curves for various K. It can also be seen that our asymptotic result (15) matches quite well with actual sum rate performance even with a small number of M and K. On the other hand, for the cases of $K = 3$ in Figure 2 and $K = 4$ and 5 in Figure 3, our finite analysis is slightly less accurate than the asymptotic analysis. This implies that compared to the finite analysis, the asymptotic analysis is less sensitive to the loss which results from the assumption that ξ_i's in (4) are independent.

In Figure 4, we compare the asymptotic individual rate with the empirical results for $\rho = 20$ dB with respect to the number of users. This plot shows the behavior of the individual rate with different K and the fixed gap between M and K. We can see that the individual rate performance is degraded as K becomes larger with fixed $M - K$ or as $M - K$ becomes smaller for a given K. Also, it can easily be checked that our asymptotic result converges to the actual rate when M and K grow with fixed $M - K$.

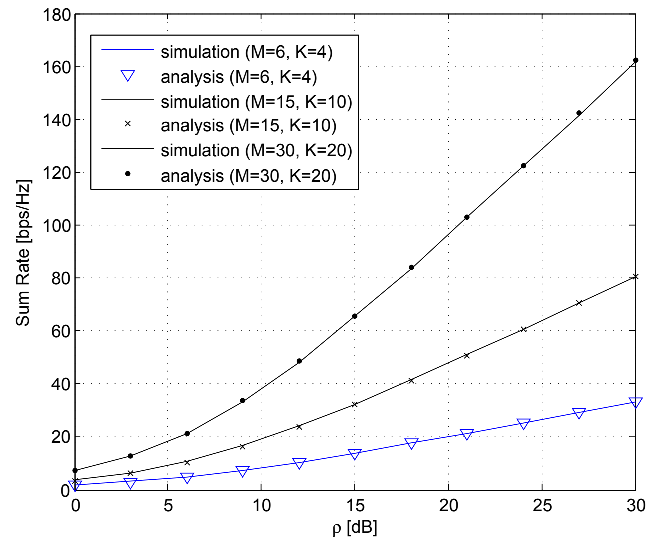

Fig. 6. Asymptotic average sum rate for various M and K.

Figure 5 shows the performance gap between ZFBF-SD and ZFBF-SPC. The analysis plots for the ZFBF-SPC are based on (16). Notice that there are 2.9 dB and 2.8 dB gaps between the actual ZFBF-SD and ZFBF-SPC curves for $M = 12, K = 8$ and $M = 9, K = 6$, respectively. Correspondingly, our theoretical results computed from (17) are 2.78 dB and 2.61 dB, which are quite close to the actual value. Therefore, we can confirm that the actual gap is well estimated by our derived result. Also, this gap can be interpreted as a performance loss due to the simple structure of the ZFBF-SD and PAPC which is more restrictive than SPC. This loss may be reduced by using more complicated precoder under PAPC.

Finally, we present the comparison of the asymptotic sum rate and numerical results. Figure 6 illustrates the average sum rate for various M and K. Once again we can verify from this plot that our asymptotic analysis is very tight and accurately approximates the performance of the ZFBF-SD even for small K.

VI. Conclusions

In this paper, we have investigated the average sum rate of the MISO-BC with ZFBF-SD under PAPC. We have first derived an approximate distribution of the received SNR. Based on this result, the formula of the average sum rate has been derived in a closed form. Furthermore, we have presented the asymptotic average sum rate analysis for asymptotically large number of transmit antennas and users, which not only has concise expressions but also provides accurate results. From numerical simulations, we have confirmed that our analysis is valid for even small M and K.

APPENDIX

A. Proof of Lemma 1

By singular value decomposition, $\mathbf{H} \in \mathbb{C}^{K\times M}$ can be decomposed as

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{18}$$

where a unitary matrix $\mathbf{U} \in \mathbb{C}^{K\times K}$ contains left singular vectors, a matrix $\mathbf{\Sigma} \in \mathbb{C}^{K\times M}$ consists of ordered singular values of $\mathbf{H}$ and a unitary matrix $\mathbf{V} \in \mathbb{C}^{M\times M}$ is composed of right singular vectors. Applying (18) into $\left[\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-2}\mathbf{H}\right]_{ii}$ in (4), we obtain

$$\left[\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-2}\mathbf{H}\right]_{ii} = \left[\mathbf{V}\mathbf{S}\mathbf{V}^H\right]_{ii}$$

where $\mathbf{S} = \mathbf{\Sigma}^H(\mathbf{\Sigma}\mathbf{\Sigma}^H)^{-2}\mathbf{\Sigma} \in \mathbb{C}^{M\times M}$. Note that $\mathbf{S}$ is a diagonal matrix whose diagonal elements consist of K eigenvalues of $(\mathbf{H}\mathbf{H}^H)^{-1}$ and $M-K$ zeros.

Denoting $\mathbf{\Lambda} \in \mathbb{C}^{K\times K}$ as a diagonal matrix whose diagonal elements are the eigenvalues of $(\mathbf{H}\mathbf{H}^H)^{-1}$, ξ_i can be expressed as

$$\xi_i = \frac{\rho}{M\mathbf{v}_i\mathbf{S}\mathbf{v}_i^H} = \frac{\rho}{M\bar{\mathbf{v}}_i\mathbf{\Lambda}\bar{\mathbf{v}}_i^H} \triangleq \frac{\rho}{M\phi_i}$$

where $\mathbf{v}_i$ stands for the i-th row of $\mathbf{V}$, $\bar{\mathbf{v}}_i$ denotes the subvector containing the first K components of $\mathbf{v}_i$ and ϕ_i is defined as $\bar{\mathbf{v}}_i\mathbf{\Lambda}\bar{\mathbf{v}}_i^H$.

To compute a distribution of ϕ_i, we now observe a distribution of $\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]_{ii}$. Defining ψ_i as $\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]_{ii}$, it follows

$$\psi_i = \mathbf{u}_i\mathbf{\Lambda}\mathbf{u}_i^H$$

where $\mathbf{u}_i$ represents the i-th row of $\mathbf{U}$. The major difference between ϕ_i and ψ_i is that $\|\bar{\mathbf{v}}_i\|^2$ is less than or equal to 1, while $\|\mathbf{u}_i\|^2$ equals 1. According to Lemma 1 in [30], $\mathbf{U}$ is uniformly distributed on the group of unitary matrices and is independent of $\mathbf{\Lambda}$, i.e., the joint distribution of $(\mathbf{H}\mathbf{H}^H)^{-1}$ depends only on $\mathbf{\Lambda}$. Consequently, the distribution of the i-th diagonal element of $(\mathbf{H}\mathbf{H}^H)^{-1}$, ψ_i, does not change even if $\mathbf{u}_i$ is replaced by any isotropically distributed complex unit vector.

Since $\bar{\mathbf{v}}_i/|\bar{\mathbf{v}}_i\|$ is also an isotropically distributed complex unit vector, we can conclude that $\phi_i/\|\bar{\mathbf{v}}_i\|^2$ and ψ_i have the same distribution. By the fact that ψ_i^{-1} follows $\chi^2(2(M-K+1))$ [31] and taking the expectation value of $\|\bar{\mathbf{v}}_i\|^2$, the distribution of ξ_i can finally be approximated as

$$\xi_i \sim \frac{\rho}{\alpha M}\chi^2(2(M-K+1))$$

where α defines $\mathbb{E}\left[\|\bar{\mathbf{v}}_i\|^2\right]$.

B. Proof of Lemma 2

From [32], the joint probability density function of any L elements of $\mathbf{w}$ is given by

$$f(\mathbf{w}^{(L)}) = \frac{\Gamma(M)}{\pi^L\Gamma(M-L)}(1-\mathbf{w}^H\mathbf{w})^{M-L-1}, \quad \text{for} \quad L = 1,\ldots,M-1.$$

Let us define $w_j = r_je^{j\theta_j}$ as the j-th element of $\mathbf{w}$. After integrating over θ_j, the joint density function of $\{r_j\}_{j=1}^L$ is obtained as (19) at the top of the next page.

Now consider the change of variables from $\{r_j\}_{j=1}^L$ to $u, \phi_1, \ldots, \phi_{L-1}$ given by

$$\begin{aligned}
r_1 &= u\cos\phi_1\cos\phi_2\cdots\cos\phi_{L-3}\cos\phi_{L-2}\cos\phi_{L-1}\\
r_2 &= u\cos\phi_1\cos\phi_2\cdots\cos\phi_{L-3}\cos\phi_{L-2}\sin\phi_{L-1}\\
r_3 &= u\cos\phi_1\cos\phi_2\cdots\cos\phi_{L-3}\sin\phi_{L-2}\\
&\vdots\\
r_{L-1} &= u\cos\phi_1\sin\phi_2\\
r_L &= u\sin\phi_1.
\end{aligned} \tag{20}$$

Note that ϕ has a range $[0, \pi/2]$ since r_i is a non-negative value for each i. Also, u goes from 0 to 1.

The Jacobian of this transformation in (20) is calculated as

$$|J| = u^{L-1}\cos\phi_1^{L-2}\cos\phi_2^{L-3}\cdots\cos\phi_{L-2}. \tag{21}$$

Substituting (20) and (21) together into (19) and integrating over ϕ_i's, we obtain (22) at the top of the next page.

Then, the change of variable $x = u^2$ leads to the PDF of x as

$$f(x) = \frac{\Gamma(M)}{\Gamma(M-L)\Gamma(L)}(1-x)^{M-L-1}x^{L-1}.$$

It is obvious that x equals $\|\mathbf{w}^{(L)}\|^2$ from the previous steps of our derivation, i.e., $x = u^2 = \sum_{i=1}^L r_i^2$. It also turns out that this PDF is equivalent to a beta distribution with parameters M and L. Then, we easily evaluate $\mathbb{E}[\|\mathbf{w}^{(L)}\|^2]$ as

$$\mathbb{E}[\|\mathbf{w}^{(L)}\|^2] = \int_0^1 xf(x)dx = \frac{L}{M}.$$

C. Proof of Lemma 5

Denoting $Y_i = -\xi_i$, the CDF and PDF of Y_i from (6) are calculated as

$$\begin{aligned}
F_{Y_i}(x) &= 1 - \frac{\gamma(-\bar{\alpha}x, N)}{\Gamma(N)}, \quad x \le 0,\\
f_{Y_i}(x) &= \frac{\bar{\alpha}^N}{\Gamma(N)}(-x)^{m-1}\exp(\bar{\alpha}x), \quad x \le 0.
\end{aligned} \tag{23}$$

We can check from the above CDF that ζ_1 equals zero. From Lemma 3, it follows that

$$\begin{aligned}
\lim_{x\to 0}\frac{-xf_{Y_i}(x)}{1-F_{Y_i}(x)} &= \lim_{x\to 0}\frac{-f_{Y_i}(x) - xf'_{Y_i}(x)}{-f_{Y_i}(x)}\\
&= 1 + \lim_{x\to 0}\frac{xf'_{Y_i}(x)}{f_{Y_i}(x)}
\end{aligned} \tag{24}$$

where the first equality results from L'Hospital's rule.

Taking a derivative of (23), we obtain $f'_{Y_i}(x)$ as

$$\begin{aligned}
f'_{Y_i}(x) &= \frac{\bar{\alpha}^N}{\Gamma(N)}(-x)^{N-2}\exp(\bar{\alpha}x)(1-N-\bar{\alpha}x)\\
&= f_{Y_i}(x)\frac{1-N-\bar{\alpha}x}{-x}.
\end{aligned}$$

Based on the above result, the limit in (24) then becomes

$$1 + \lim_{x\to 0}\frac{xf'_{Y_i}(x)}{f_{Y_i}(x)} = 1 + \lim_{x\to 0}\bar{\alpha}x + N - 1 = N.$$

Finally, we can conclude that

$$\lim_{M\to\infty} Pr\{Y_{M:M} \le a_M \cdot x\} = G_2(x;N) = \exp(-(-x)^N)$$

$$\begin{aligned} f(\{r_j\}_{j=1}^{L}) &= \int_0^{2\pi}\cdots\int_0^{2\pi}\frac{\Gamma(M)}{\pi^L\Gamma(M-L)}\left(1-\sum_{i=1}^{L}r_j^2\right)^{M-L-1}\prod_{i=1}^{L}r_j d\theta_1 d\theta_2\cdots d\theta_L \\ &= \frac{2^L\Gamma(M)}{\Gamma(M-L)}\left(1-\sum_{i=1}^{L}r_j^2\right)^{M-L-1}\prod_{i=1}^{L}r_j. \end{aligned} \quad (19)$$

$$\begin{aligned} f(u) &= \frac{2^L\Gamma(M)}{\Gamma(M-L)}(1-u^2)^{M-L-1}u^{2L-1}\times \\ &\quad \int_0^{\pi/2}\cdots\int_0^{\pi/2}\cos\phi_1^{2L-3}\sin\phi_1\cos\phi_2^{2L-5}\sin\phi_2\cdots\cos\phi_{L-1}\sin\phi_{L-1}d\phi_1 d\phi_2\dots d\phi_{L-1} \\ &= \frac{2^L\Gamma(M)}{\Gamma(M-L)}(1-u^2)^{M-L-1}u^{2L-1}\frac{1}{2L-2}\frac{1}{2L-4}\cdots\frac{1}{4}\frac{1}{2} \\ &= \frac{2\Gamma(M)}{\Gamma(M-L)\Gamma(L)}(1-u^2)^{M-L-1}u^{2L-1}. \end{aligned} \quad (22)$$

where a_M is determined by $a_M = -F_{Y_i}^{-1}(1-\frac{1}{M})$. From $Y_i = -\xi_i$, the limiting distribution of the SNR $\gamma = \min_i \xi_i$ can be given as

$$\begin{aligned} \lim_{M\to\infty} Pr\{\xi_{1:M} \le a_M x\} &= \lim_{M\to\infty} Pr\{-Y_{M:M} \le a_M x\} \\ &= 1 - \lim_{M\to\infty} Pr\{Y_{M:M} \le -a_M x\} \\ &= 1-\exp(-x^N) = G_2^*(x;N). \end{aligned}$$

Also, a_M can be rewritten as

$$a_M = -F_{Y_i}^{-1}\left(1-\frac{1}{M}\right) = F_{\xi_i}^{-1}\left(\frac{1}{M}\right).$$

REFERENCES

[1] G. Caire and S. Shamai, “On the achievable throughput of a multi-antenna gaussian broadcast channel,” *IEEE Trans. Inf. Theory*, vol. 49, pp. 1691–1706, July 2003.

[2] P. Viswanath and D. Tse, “Sum capacity of the vector gaussian broadcast channel and uplink-downlink duality,” *IEEE Trans. Inf. Theory*, vol. 49, pp. 1912–1921, Aug. 2003.

[3] S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 49, pp. 2658–2668, Oct. 2003.

[4] W. Yu and J. Cioffi, “Sum capacity of Gaussian vector broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 50, pp. 1875–1892, Sep. 2004.

[5] H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the Gaussian multiple-input multiple-output broadcast channel,” *IEEE Trans. Inf. Theory*, vol. 52, pp. 3936–3964, Sep. 2006.

[6] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multiantenna multiuser communication–part I: channel inversion and regularization,” *IEEE Trans. Commun.*, vol. 53, pp. 195–202, Jan. 2005.

[7] H. Sung, S.-R. Lee, and I. Lee, “Generalized channel inversion methods for multiuser MIMO systems,” *IEEE Trans. Commun.*, vol. 57, pp. 3489–3499, Nov. 2009.

[8] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” *IEEE Trans. Signal Process.*, vol. 55, pp. 2646–2660, June 2007.

[9] W. Choi and J. G. Andrews, “Downlink performance and capacity of distributed antenna systems in a multicell environment,” *IEEE Trans. Wireless Commun.*, vol. 6, pp. 69–73, Jan. 2007.

[10] S.-R. Lee, S.-H. Moon, J.-S. Kim, and I. Lee, “Capacity analysis of distributed antenna systems in a composite fading channel,” *IEEE Trans. Wireless Commun.*, vol. 11, pp. 1076–1086, Mar. 2012.

[11] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, “Multi-cell MIMO cooperative networks: a new look at interference,” *IEEE J. Sel. Areas Commun.*, vol. 28, pp. 1380–1408, Dec. 2010.

[12] A. Wiesel, Y. Eldar, and S. Shamai, “Zero-forcing precoding and generalized inverses,” *IEEE Trans. Signal Process.*, vol. 56, pp. 4409–4418, Sep. 2008.

[13] F. Boccardi and H. Huang, “Zero-forcing precoding for the MIMO broadcast channel under per-antenna power constraints,” in *Proc. 2006 IEEE Workshop Signal Process. Adv. Wireless Commun.*, pp. 1–5.

[14] J. Andrews, A. Ghosh, and R. W. Heath, “Networked MIMO with clustered linear precoding,” *IEEE Trans. Wireless Commun.*, vol. 8, pp. 1910–1921, Apr. 2009.

[15] Z. Zhang, “Pseudo-inverse multivariate/matrix-variate distributions,” *J. Multivariate Analysis*, vol. 98, pp. 1684–1692, Sep. 2007.

[16] D. Tse and S. Hanly, “Linear multiuser receivers: effective interference, effective bandwidth and user capacity,” *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 641–657, Mar. 1999.

[17] A. M. Tulino, A. Lozano, and S. Verdú, “Impact of antenna correlation on the capacity of multiantenna channels,” *IEEE Trans. Inf. Theory*, vol. 7, pp. 2491–2509, July 2005.

[18] D. Aktas, M. N. Bacha, J. S. Evans, and S. V. Hanly, “Scaling results on the sum capacity of cellular networks with MIMO links,” *IEEE Trans. Inf. Theory*, vol. 52, pp. 3264–3274, July 2006.

[19] H. Huh, S.-H. Moon, Y.-T. Kim, I. Lee, and G. Caire, “Multi-cell MIMO downlink with cell cooperation and fair scheduling: a large-system limit analysis,” *IEEE Trans. Inf. Theory*, vol. 57, no. 12, pp. 7771–7786, Dec. 2011.

[20] S.-H. Moon, S.-R. Lee, and I. Lee, “Sum-rate capacity of random beamforming for multi-antenna broadcast channels with other cell interference,” *IEEE Trans. Wireless Commun.*, vol. 10, pp. 2440–2444, Aug. 2011.

[21] D. Gesbert and M. Kountouris, “Rate scaling laws in multicell networks under distributed power control and user scheduling,” *IEEE Trans. Inf. Theory*, vol. 57, pp. 234–244, Jan. 2011.

[22] G. Song and Y. Li, “Asymptotic throughput analysis for channel-aware scheduling,” *IEEE Trans. Commun.*, vol. 54, pp. 1827–1834, Oct. 2006.

[23] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd edition. Wiley, 2003.

[24] M.-S. Alouini and M. Simon, “Performance of coherent receivers with hybrid SC/MRC over Nakagami-m fading channels,” *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1155–1164, July 1999.

[25] G. Fedele, “N-branch diversity reception of M-ary DPSK signals in slow and nonselective Nakagami fading,” *European Trans. Telecommun.*, vol. 7, pp. 119–123, Mar./Apr. 1996.

[26] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th edition. Academic Press, 2000.

[27] E. Castillo, A. S. Hadi, N. Balakrishnan, and J. M. Sarabia, *Extreme Value and Related Models with Applications in Engineering and Science*. John Wiley and Sons, 2005.

[28] A. Lozano, A. M. Tulino, and S. Verdu, “High-SNR power offset in multiantenna communication,” *IEEE Trans. Inf. Theory*, vol. 51, pp. 4134–4151, Dec. 2005.

[29] R. Couillet, S. Wagner, and M. Debbah, “Asymptotic analysis of correlated multi-antenna broadcast channels,” in *Proc. 2009 IEEE WCNC*.

[30] D. J. Love and R. W. Heath, "Grassmannian beamforming on correlated MIMO channels," in *Proc. 2004 IEEE GLOBECOM*, pp. 106–110.

[31] D. A. Gore, R. W. Heath, and A. J. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Commun. Lett.*, vol. 11, pp. 491–493, Nov. 2002.

[32] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 45, pp. 139–157, Jan. 1999.

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