Weighted Sum Rate Maximization for Multiuser Multirelay MIMO Systems

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***Abstract*—In this paper, we study a filter design that maximizes the weighted sum rate (WSR) in multiuser multirelay systems equipped with multiple antennas at each node. Since this problem is generally nonconvex, it is quite complicated to analytically find a solution. Hence, we transform the WSR maximization problem to an equivalent weighted sum mean-square-error (WSMSE) minimization problem, which is more amenable. Then, we identify the filters at the base station and the relays for minimizing the WSMSE with a proper weight and propose an alternating computation algorithm that guarantees a local optimum solution. Through simulations, we confirm the effectiveness of our proposed scheme.**

***Index Terms*—Amplify-and-forward (AF) relaying, minimum mean-squared-error (MMSE), multi-input multi-output (MIMO), multi-relay, multi-user, relay, weighted sum rate (WSR).**

I. Introduction

Over the last decade, point-to-point multi-input–multi-output (MIMO) techniques [1]–[3] and multiuser MIMO systems [4]–[7] have attracted a lot of attention as an effective means to increase reliability and capacity. Relaying [8]–[14] has become a promising technique for expanding the cell coverage and enhancing the system capacity in wireless communication systems. For this reason, multiuser MIMO relay systems are considered to be an important area in next-generation wireless networks.

The capacity of MIMO relay channels was studied in [8], but an exact capacity bound is still unknown. In [9] and [10], it was revealed that under the amplify-and-forward (AF) relaying strategy, the optimal filter at the relay station (RS) can be obtained using a singular value decomposition approach. Extending these to the case of multiple single-antenna users, many works have investigated MIMO relay channels in [11] and [13] and references therein. To maximize the sum rate, the optimal filter based on zero-forcing dirty-paper coding at the base station (BS) and a linear filter at the RS were designed in [11]. Xu *et al.* [13] jointly optimized the source and relay linear filters to support multiple mobile users (MUs) with a single RS. The sum rate maximization problem in multirelay MIMO systems with a single source and a single destination was addressed in [14], where a gradient descent (GD) algorithm is utilized with the assumption of full power transmission at all nodes. However, none of these works has considered multiuser multirelay MIMO systems.

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In this paper, we propose an algorithm to identify the BS and RS filters for maximizing the weighted sum rate (WSR) in multiuser multirelay MIMO systems that have multiple antennas at each node. It is worthwhile to note that our system configuration includes conventional single-user single-relay, multiuser single-relay, and single-user multirelay MIMO systems as special cases. To reduce implementation complexity, we only consider the linear filter at the BS and the AF filters at the RSs.

Since our problem for maximizing the WSR is generally nonconvex and NP-hard [15], it is quite complicated to directly find an analytical solution. Hence, we transform the WSR maximization problem to an equivalent weighted sum mean-square-error (WSMSE) minimization problem, which is more amenable. First, we propose an iterative algorithm for minimizing the WSMSE, which determines the receive filters and weight matrices and then identifies filters at the BS and the RSs. Through Monte Carlo simulations, it is shown that our proposed scheme performs very close to the schemes with MU or RS cooperation, where all RSs or all MUs operate as a single node, which confirms the effectiveness of our proposed scheme.

Throughout this paper, boldface uppercase and lowercase letters indicate matrices and vectors, respectively, and $\mathbb{C}^{M\times N}$ represents the set of all $M \times N$ matrices with complex entries. In addition, $\mathbb{E}[\mathbf{X}]$, $\mathbf{Tr}(\mathbf{X})$, and $|\mathbf{X}|$ stand for the expectation, trace, and determinant operations for a matrix $\mathbf{X}$, respectively.

II. System Model

Here, we provide a system model in Fig. 1, where N RSs help in the communication between a single BS and K MUs. Let us denote the number of antennas at BS, RS, and MU as M_b, M_r, and M_u, respectively. Throughout this paper, we will refer to this configuration as $(M_b, M_r \times N, M_u \times K)$ systems. For each user, M_u data streams are simultaneously transmitted, and $KM_u \leq \min\{M_b, NM_r\}$. In addition, we assume that the direct links between the BS and the MUs are neglected due to the large path loss between the BS and MUs.

In the first time slot, the transmitted signal at the BS is expressed as

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{V}_k \mathbf{s}_k$$

where $\mathbf{s}_k \in \mathbb{C}^{M_u\times 1}$ stands for the complex symbol with $\mathbb{E}[\mathbf{s}_k\mathbf{s}_k^H] = \mathbf{I}$ for the kth MU, and $\mathbf{V}_k \in \mathbb{C}^{M_b\times M_u}$ denotes the transmit filter applied to $\mathbf{s}_k$ at the BS. We consider the BS power constraint as

$$\mathrm{Tr}(\mathbf{V}\mathbf{V}^H) \leq P_b$$

where $\mathbf{V}$ is defined as $\mathbf{V} = [\mathbf{V}_1 \ldots \mathbf{V}_K]$, and P_b is the maximum transmit power at the BS.

At the nth RS side, the received signal is written as

$$\mathbf{r}_n = \mathbf{H}_n\mathbf{x} + \mathbf{n}_n.$$

Here, $\mathbf{H}_n \in \mathbb{C}^{M_r\times M_b}$ and $\mathbf{n}_n \in \mathbb{C}^{M_r\times 1}$ are denoted by the channel matrix from the BS to the nth RS, whose elements are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance and the additive white Gaussian noise (AWGN) vector with zero mean and $\mathbb{E}[\mathbf{n}_n\mathbf{n}_n^H] = \sigma_r^2\mathbf{I}$ at the nth RS, respectively.

In the second time slot, after the RSs apply the relay filter $\mathbf{F}_n \in \mathbb{C}^{M_r\times M_r}$ to the received signal in the first time slot, the nth RS

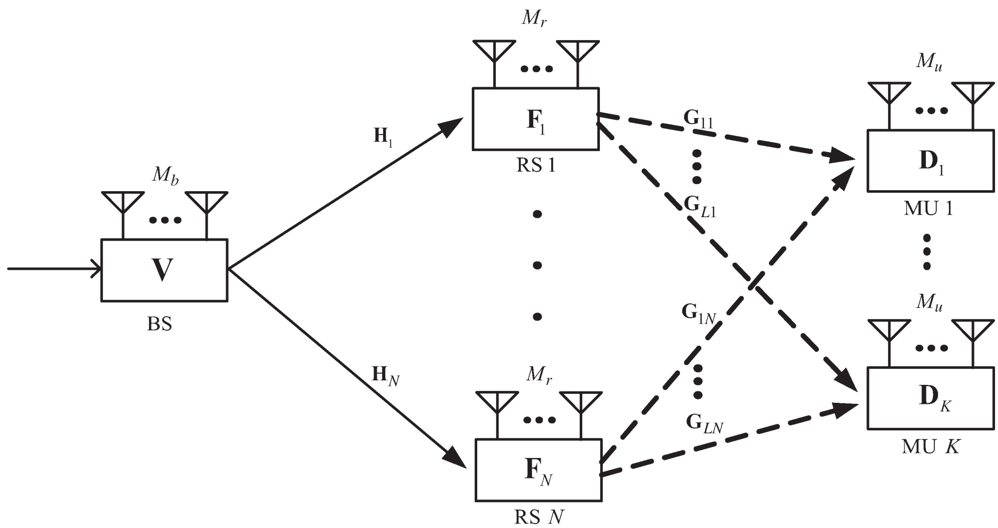

Fig. 1. $(M_b, M_r \times N, M_u \times K)$ systems.

transmits the signal $\mathbf{F}_n\mathbf{r}_n$. The RS power constraint is then given by

$$\mathrm{Tr}(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H) \leq P_{r_n} \quad \forall n$$

where $\mathbf{R}_{r_n}$ represents the receive covariance matrix at RS n as

$$\mathbf{R}_{r_n} = \mathbb{E}\left[\mathbf{r}_n\mathbf{r}_n^H\right] = \mathbf{H}_n\mathbf{V}\mathbf{V}^H\mathbf{H}_n^H + \sigma_r^2\mathbf{I}.$$

Let $\mathbf{A}_k$ and $\tilde{\mathbf{z}}_k$ be $\mathbf{A}_k = \sum_{n=1}^N \mathbf{G}_{kn}\mathbf{F}_n\mathbf{H}_n$ and $\tilde{\mathbf{z}}_k = \mathbf{A}_k\sum_{i\neq k}^K \mathbf{V}_i\mathbf{s}_i + \sum_{n=1}^N \mathbf{G}_{kn}\mathbf{F}_n\mathbf{n}_n + \mathbf{z}_k$, respectively, where $\mathbf{G}_{kn} \in \mathbb{C}^{M_u\times M_r}$ indicates the channel matrix from the nth RS to the kth MU, whose elements are i.i.d. complex Gaussian random variables with zero mean and unit variance, and $\mathbf{z}_k \in \mathbb{C}^{M_u\times 1}$ is equal to the AWGN vector with zero mean and $\mathbb{E}[\mathbf{z}_k\mathbf{z}_k^H] = \sigma_m^2\mathbf{I}$ at MU k. Then, the kth MU receives the aggregated signal from N RSs as

$$\mathbf{y}_k = \mathbf{A}_k\mathbf{V}_k\mathbf{s}_k + \tilde{\mathbf{z}}_k.$$

In the following section, we first formulate the WSR maximization problem under the assumption that all nodes have knowledge of global channel state information (CSI) including channel matrices $\mathbf{H}_n$ and $\mathbf{G}_{kn}$ and then identify the relationship between the WSR maximization problem and the WSMSE minimization problem.

III. Problem Formulation

A. Objective Function

Our objective is to jointly determine the linear filters at both the BS and the RSs for maximizing the WSR. This problem can be formulated as

$$\begin{aligned}
&\min_{\mathbf{V}_k,\mathbf{F}_n} -\sum_{k=1}^K w_k R_k \\
&\text{s.t. } \mathrm{Tr}\left(\mathbf{V}\mathbf{V}^H\right) \leq P_b \\
&\qquad \mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \leq P_{r_n} \quad \forall n
\end{aligned} \tag{1}$$

where w_k and R_k are the weight and the individual rate of the kth MU, respectively. The weight w_k is a predetermined value depending on the required quality of service for applications. Assuming Gaussian signaling, the individual rate of the kth MU R_k is given by

$$R_k = \frac{1}{2}\log_2\left|\mathbf{I} + \mathbf{V}_k^H\mathbf{A}_k^H\mathbf{R}_{\tilde{z}_k}^{-1}\mathbf{A}_k\mathbf{V}_k\right| \tag{2}$$

where the effective noise covariance matrix $\mathbf{R}_{\tilde{z}_k}$ is defined as

$$\mathbf{R}_{\tilde{z}_k} = \mathbf{A}_k\sum_{i\neq k}^K \mathbf{V}_i\mathbf{V}_i^H\mathbf{A}_k^H + \sigma_r^2\sum_{n=1}^N \mathbf{G}_{kn}\mathbf{F}_n\mathbf{F}_n^H\mathbf{G}_{kn}^H + \sigma_m^2\mathbf{I}.$$

In (2), the scaling factor $1/2$ results from the fact that messages are transmitted over two time slots.

Now, we find another expression for the individual rate. Let us assume that minimum-mean-square-error (MMSE) receive filters are applied to each MU. Then, at the kth MU, the MMSE receive filter $\hat{\mathbf{D}}_k$ is written as

$$\begin{aligned}
\hat{\mathbf{D}}_k &= \arg\min_{\mathbf{D}_k}\mathbb{E}\left[\|\mathbf{D}_k\mathbf{y}_k - \mathbf{s}_k\|^2\right] \\
&= \mathbf{V}_k^H\mathbf{A}_k^H\left(\mathbf{A}_k\mathbf{V}_k\mathbf{V}_k^H\mathbf{A}_k^H + \mathbf{R}_{\tilde{z}_k}\right)^{-1}.
\end{aligned} \tag{3}$$

Applying this result, the MMSE matrix for MU k as in [16] is given by

$$\begin{aligned}
\mathbf{E}_k &= \mathbb{E}\left[(\hat{\mathbf{D}}_k\mathbf{y}_k - \mathbf{s}_k)(\hat{\mathbf{D}}_k\mathbf{y}_k - \mathbf{s}_k)^H\right] \\
&= \left(\mathbf{I} + \mathbf{V}_k^H\mathbf{A}_k^H\mathbf{R}_{\tilde{z}_k}^{-1}\mathbf{A}_k\mathbf{V}_k\right)^{-1}.
\end{aligned} \tag{4}$$

Then, the individual rate of MU k in (2) can be alternatively expressed as

$$R_k = \frac{1}{2}\log_2\left|\mathbf{E}_k^{-1}\right|.$$

The objective function of problem (1) is nonconvex with respect to both $\mathbf{V}_k$ and $\mathbf{F}_n$. Hence, it is quite difficult to identify a solution. In the following, by utilizing the relationship between the WSR maximization problem and the WSMSE minimization problem as in [17], we will show that the WSR maximization problem (1) can be transformed into an equivalent WSMSE minimization problem.

B. Transformation to the WSMSE Minimization Problem

The WSMSE minimization problem can be mathematically formulated as

$$\begin{aligned}
&\min_{\mathbf{V}_k,\mathbf{F}_n,\mathbf{W}_k}\sum_{i=1}^K \mathrm{Tr}(\mathbf{W}_i\mathbf{E}_i) \\
&\text{s.t. } \mathrm{Tr}\left(\mathbf{V}\mathbf{V}^H\right) \leq P_b \\
&\qquad \mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \leq P_{r_n}, \quad \forall n
\end{aligned} \tag{5}$$

where $\mathbf{W}_k \in \mathbb{C}^{M_u \times M_u}$ is a weight matrix for MU k. Note that this problem is jointly nonconvex with respect to both the BS and the RS filters. In contrast to the WSR maximization problem (1), however, the WSMSE minimization problem (5) is convex with respect to a single filter while all the other filters are fixed, which makes the problem more amenable.

The following proposition illustrates a relation between the WSR maximization problem and the WSMSE minimization problem in multiuser multirelay MIMO systems.

Proposition 1: For a given set of filters and corresponding MMSE matrices, the Karush–Kuhn–Tucker (KKT) conditions of the WSMSE minimization problem are identical to those of the WSR maximization problem with respect to $\mathbf{V}_k$ and $\mathbf{F}_n \forall n$, if the weight matrices are settled as $\mathbf{W}_k = (w_k/2\ln 2)\mathbf{E}_k^{-1} \forall k$.

Proof: The proof simply follows the result in [17] and, thus, is omitted here. ■

This proposition implies that the result for conventional multiuser MIMO systems in [17] can be generalized to multiuser multirelay MIMO systems. Now, we are ready to propose an algorithm to find a local optimal solution for the WSR maximization problem utilizing the WSMSE minimization problem.

IV. Proposed Algorithm

Here, we propose an iterative algorithm for minimizing the WSMSE, which determines the receive MMSE filters and the weight matrices and identifies the filters at the BS and the RSs. The new WSMSE minimization problem, which is the same as the original WSR maximization problem (1), is represented as

$$\begin{aligned}\min_{\substack{\mathbf{V}_k,\mathbf{F}_n,\\ \mathbf{D}_k,\mathbf{W}_k}} & \sum_{i=1}^{K}\left\{\mathrm{Tr}(\mathbf{W}_i\tilde{\mathbf{E}}_i) - \frac{w_i}{2}\log_2\left|\frac{2\ln 2}{w_i}\mathbf{W}_i\right| - \frac{w_i M_u}{2\ln 2}\right\}\\ \text{s.t. } & \mathrm{Tr}\left(\mathbf{V}\mathbf{V}^H\right) \le P_b\\ & \mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \le P_{r_n} \quad \forall n \end{aligned} \tag{6}$$

where the mean-square-error (MSE) matrix $\tilde{\mathbf{E}}_i$ is denoted as $\tilde{\mathbf{E}}_i = \mathbb{E}[(\mathbf{D}_i\mathbf{y}_i - \mathbf{s}_i)(\mathbf{D}_i\mathbf{y}_i - \mathbf{s}_i)^H]$.

First, if the BS filter and the RS filters are fixed, the receive MMSE filters $\hat{\mathbf{D}}_k$ at MUs are simply given as (3), which make $\tilde{\mathbf{E}}_k$ equal to $\mathbf{E}_k$, and the weight matrices $\mathbf{W}_k$ are determined by Proposition 1. Second, the WSMSE minimization problem with respect to the relay filters is reformulated as

$$\begin{aligned}\min_{\mathbf{F}_n} & \sum_{i=1}^{K}\mathrm{Tr}(\mathbf{W}_i\mathbf{E}_i)\\ \text{s.t. } & \mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \le P_{r_n}, \quad \forall n. \end{aligned} \tag{7}$$

For a given set of $\{\mathbf{V}_i, \mathbf{F}_j, \mathbf{D}_l, \mathbf{W}_k \forall i, j \ne n, l, k\}$, the KKT conditions of the problem in (7) with respect to the nth relay filter $\mathbf{F}_n$ are represented as

$$\nabla_{\mathbf{F}_n}\mathcal{L}_{\mathrm{WSMSE}}^{\mathrm{RS}_n} = 2\frac{\partial\mathcal{L}_{\mathrm{WSMSE}}^{\mathrm{RS}_n}}{\partial\mathbf{F}_n^*} = \mathbf{0} \tag{8}$$

$$\lambda_n\left(\mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) - P_{r_n}\right) = 0 \tag{9}$$

where $\mathcal{L}_{\mathrm{WSMSE}}^{\mathrm{RS}_n}$ is the Lagrangian function given by

$$\mathcal{L}_{\mathrm{WSMSE}}^{\mathrm{RS}_n} = \sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{W}_k\mathbf{E}_k\right) + \lambda_n\left(\mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) - P_{r_n}\right)$$

and $\lambda_n \ge 0$ is the Lagrangian multiplier with respect to the power constraint of RS n.

From (8), the nth relay filter $\mathbf{F}_n(\lambda_n)$, which is a function of λ_n, is calculated as

$$\begin{aligned}\mathbf{F}_n = & \left(\sum_{l=1}^{K}\mathbf{G}_{ln}^H\mathbf{D}_l^H\mathbf{W}_l\mathbf{D}_l\mathbf{G}_{ln} + \lambda_n\mathbf{I}\right)^{-1}\\ & \times\sum_{l=1}^{K}\mathbf{G}_{ln}^H\mathbf{D}_l^H\mathbf{W}_l\left(\mathbf{V}_l^H - \mathbf{D}_l\sum_{j\ne n}\mathbf{G}_{lj}\mathbf{F}_j\mathbf{H}_j\mathbf{V}\mathbf{V}^H\right)\mathbf{H}_n^H\\ & \times\left(\mathbf{H}_n\mathbf{V}\mathbf{V}^H\mathbf{H}_n^H + \sigma_r^2\mathbf{I}\right)^{-1}. \end{aligned} \tag{10}$$

Here, λ_n should be determined from the second KKT condition (9). We search λ_n for two cases, i.e., $\lambda_n = 0$ and $\lambda_n > 0$ as in [18] and [19]. If the inverse of $\sum_{l=1}^{K}\mathbf{G}_{ln}^H\mathbf{D}_l^H\mathbf{W}_l\mathbf{D}_l\mathbf{G}_{ln}$ exists and $\mathbf{F}_n(0)$ satisfies the power constraint at RS n, then the optimum filter $\hat{\mathbf{F}}_n$ at RS n is obtained as $\hat{\mathbf{F}}_n = \mathbf{F}_n(0)$. Otherwise, the full power constraint, i.e., $\mathrm{Tr}(\mathbf{F}_n(\lambda_n)\mathbf{R}_{r_n}\mathbf{F}_n(\lambda_n)^H) = P_{r_n}$, should be met. In this case, since $\mathrm{Tr}(\mathbf{F}_n(\lambda_n)\mathbf{R}_{r_n}\mathbf{F}_n(\lambda_n)^H)$ is a monotonically decreasing function with respect to λ_n, we can find $\lambda_n > 0$ efficiently by employing a bisection method.

Now, we consider problem (5) to identify the BS filter as

$$\begin{aligned}\min_{\mathbf{V}_k} & \sum_{i=1}^{K}\mathrm{Tr}(\mathbf{W}_i\mathbf{E}_i)\\ \text{s.t. } & \mathrm{Tr}\left(\mathbf{V}\mathbf{V}^H\right) \le P_b\\ & \mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \le P_{r_n} \quad \forall n. \end{aligned} \tag{11}$$

Note that the BS filter should fulfill the BS power constraint and multiple RS power constraints. Applying Proposition 4 and 5 in [20] to problem (11), we realize that the optimal value of the WSMSE minimization problem (11) is equal to that of the problem given by

$$\begin{aligned}\max_{\mu,\lambda_n}\min_{\mathbf{V}_k} & \sum_{i=1}^{K}\mathrm{Tr}(\mathbf{W}_i\mathbf{E}_i)\\ \text{s.t. } & \mu\left(\mathrm{Tr}\left(\mathbf{V}\mathbf{V}^H\right) - P_b\right)\\ & + \sum_{n=1}^{N}\lambda_n\left(\mathrm{Tr}(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H) - P_{r_n}\right) \le 0 \end{aligned} \tag{12}$$

where $\mu \ge 0$ and $\lambda_n \ge 0$ are auxiliary variables. However, it is still difficult to directly search an optimal BS filter in (12). Hence, we first fix μ and λ_n as nonnegative constants and then utilize the subgradient algorithm [21] to optimize μ and λ_n for maximizing the WSMSE.

For a fixed set of $\{\mu \ge 0, \lambda_1 \ge 0, \ldots, \lambda_N \ge 0\}$, problem (12) is reduced to the WSMSE minimization problem with a single sum power constraint as

$$\begin{aligned} g(\mu,\lambda_n) : \min_{\mathbf{V}_k} & \sum_{i=1}^{K}\mathrm{Tr}(\mathbf{W}_i\mathbf{E}_i)\\ \text{s.t. } & \mu\mathrm{Tr}(\mathbf{V}\mathbf{V}^H) + \sum_{n=1}^{N}\lambda_n\mathrm{Tr}\left(\mathbf{F}_n\mathbf{R}_{r_n}\mathbf{F}_n^H\right) \le P_t \end{aligned} \tag{13}$$

where the constant P_t is defined as $P_t = \mu P_b + \sum_{n=1}^{N}\lambda_n P_{r_n}$.

The Lagrangian function of problem (13) is written as

$$\mathcal{L}_{\mathrm{RWSMSE}}^{\mathrm{BS}} = \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{W}_i \mathbf{E}_i) + \zeta \left(\mu \mathrm{Tr}(\mathbf{V}\mathbf{V}^H) + \sum_{n=1}^{N} \lambda_n \mathrm{Tr}\left(\mathbf{F}_n \mathbf{R}_{r_n} \mathbf{F}_n^H\right) - P_t \right)$$

where $\zeta \geq 0$ is a Lagrangian multiplier, and then, the KKT conditions are represented as

$$\nabla_{\mathbf{V}_k} \mathcal{L}_{\mathrm{RWSMSE}}^{\mathrm{BS}} = 2 \frac{\partial \mathcal{L}_{\mathrm{RWSMSE}}^{\mathrm{BS}}}{\partial \mathbf{V}_k^*} = \mathbf{0} \tag{14}$$

$$\zeta \left(\mu \mathrm{Tr}(\mathbf{V}\mathbf{V}^H) + \sum_{n=1}^{N} \lambda_n \mathrm{Tr}\left(\mathbf{F}_n \mathbf{R}_{r_n} \mathbf{F}_n^H\right) - P_t \right) = 0. \tag{15}$$

To satisfy the first KKT condition (14), the transmit filters should have the form given by

$$\mathbf{V}_k = \left(\sum_{i=1}^{K} \mathbf{A}_i^H \mathbf{D}_i^H \mathbf{W}_i^H \mathbf{D}_i \mathbf{A}_i + \zeta \left(\mu \mathbf{I} + \sum_{n=1}^{N} \lambda_n \mathbf{H}_n^H \mathbf{F}_n^H \mathbf{F}_n \mathbf{H}_n \right) \right)^{-1} \times \mathbf{A}_k^H \mathbf{D}_k^H \mathbf{W}_k^H \tag{16}$$

where the Lagrangian multiplier ζ should be determined to satisfy the second KKT condition (15) similar to searching λ_n in determining $\mathbf{F}_n$.

Now, the remaining work is to maximize $g(\mu, \lambda_n)$ with respect to nonnegative μ and λ_n. Note that the function $g(\mu, \lambda_n)$ is not necessarily differentiable. Therefore, the subgradient method [21] can be applied to find a solution. The subgradient direction of the function $g(\mu, \lambda_n)$ is determined as $\mathrm{Tr}(\hat{\mathbf{V}}\hat{\mathbf{V}}^H) - P_b, \mathrm{Tr}(\mathbf{F}_n \mathbf{R}_{r_n} \mathbf{F}_n^H) - P_{r_n}$, where $\hat{\mathbf{V}}$ is the optimal solution for problem (13) with the fixed set of $\{\mu, \lambda_1, \ldots, \lambda_N\}$. Then, the solution for problem (12) can be searched in an iterative fashion summarized in Algorithm 1.

Algorithm 1 Iterative Subgradient Algorithm for $\mathbf{V}$
Initialize $\mu^{(0)}$ and $\lambda_n^{(0)}$ for $\forall n$.
Iterate
1. Solve problem (13).
2. Update $\mu^{(j+1)}$ and $\lambda_n^{(j+1)}$ as
   $\mu^{(j+1)} = \mu^{(j)} + t(\mathrm{Tr}(\mathbf{V}\mathbf{V}^H) - P_b)$,
   $\lambda_n^{(j+1)} = \lambda_n^{(j)} + t(\mathrm{Tr}(\mathbf{F}_n \mathbf{R}_{r_n} \mathbf{F}_n^H) - P_{r_n})$ for $\forall n$.

Until $|\mu(\mathrm{Tr}(\mathbf{V}\mathbf{V}^H) - P_b)| \leq \epsilon$ and
$|\lambda_n(\mathrm{Tr}(\mathbf{F}_n \mathbf{R}_{r_n} \mathbf{F}_n^H) - P_{r_n})| \leq \epsilon$ for $\forall n$.

In Algorithm 1, t denotes the step size of the subgradient algorithm, and ϵ indicates the tolerance for the complementary slackness of KKT conditions with respect to the BS and RS power constraints. Finally, the overall algorithm for the WSR maximization problem is summarized in Algorithm 2, which utilizes the WSMSE minimization at each iteration and where δ denotes the tolerance of convergence. In Algorithm 1 and 2, the superscript stands for the number of iteration. The proposed scheme in Algorithm 2 is the gradient search method in the sense of finding a local optimal point, where the gradient with respect to $\mathbf{V}$, $\mathbf{F}_n$, and $\mathbf{D}_k$ is equal to zero, and the weight matrices are the gradients of the objective in terms of the individual MSE terms.

Algorithm 2 Proposed WSR Maximization Algorithm
Initialize $\mathbf{V}^{(0)}$ and $\mathbf{F}_n^{(0)}$ for $\forall n$.
Iterate
1. Compute $\mathbf{D}_k^{(j+1)}$ and $\mathbf{W}_k^{(j+1)}$ for $\forall k$ for fixed $\mathbf{V}^{(j)}$ and $\mathbf{F}_n^{(j)}$.
2. Update $\mathbf{F}_n^{(j+1)}$ for $\forall n$ for fixed $\mathbf{D}_k^{(j+1)}$, $\mathbf{W}_k^{(j+1)}$, and $\mathbf{V}^{(j)}$.
3. Update $\mathbf{V}^{(j+1)}$ for fixed $\mathbf{D}_k^{(j+1)}$, $\mathbf{W}_k^{(j+1)}$, and $\mathbf{F}_n^{(j+1)}$ using Algorithm 1.

Until $\mathrm{Tr}((\mathbf{V}^{(j+1)} - \mathbf{V}^{(j)})(\mathbf{V}^{(j+1)} - \mathbf{V}^{(j)})^H) \leq \delta$ and
$\mathrm{Tr}((\mathbf{F}_n^{(j+1)} - \mathbf{F}_n^{(j)})(\mathbf{F}_n^{(j+1)} - \mathbf{F}_n^{(j)})^H) \leq \delta$.

Now, we consider the complexity of the proposed algorithm. In general, it is difficult to measure the complexity of iterative algorithms directly. Instead, we provide a brief complexity analysis. The majority part of the complexity at each step includes inverse operations whose complexity is $\mathcal{O}(KM_u^3)$ for $\mathbf{D}_k$ or $\mathbf{W}_k$, $\mathcal{O}(2NM_r^3)$ for $\mathbf{F}_n$, and $\mathcal{O}(KM_b^3)$ for $\mathbf{V}$. Moreover, the bisection search for $(\mu, \lambda_1, \ldots, \lambda_N)$ guarantees the error tolerance of τ after $\mathcal{O}(\log 1/\tau)$ iterations [22]. Denoting the number of iterations as J_{itr}, the total complexity is aggregated to $J_{\mathrm{itr}}(2\mathcal{O}(KM_u^3) + \mathcal{O}(2NM_r^3 \log 1/\tau) + \mathcal{O}(K(N+1)M_b^3 \log 1/\tau))$. The convergence of our proposed scheme can be guaranteed as in [17], but its proof is omitted due to the page limitation.

V. Simulation Results

Here, we provide simulation results to illustrate the effectiveness of the proposed algorithm. For all simulations, we assume that noise variances at both the RSs and the MUs are σ_n^2. All performance curves are plotted as a function of the signal-to-noise ratio, which is defined as P/σ_n^2, where the BS transmit power and the nth RS power are set to $P_b = P$ and $P_{r_n} = P/N$, respectively. In addition, ten random points are employed as an initial value for our iterative scheme, and the best output is selected to increase the probability of finding a better local optimal value. In our simulations, ϵ and δ, which influence the convergence speed and the solution accuracy, are settled as 10^{-4}. We compare the WSR performance of the following schemes:

- GD: the GD algorithm with all RS or MU cooperation;
- Proposed-joint: our scheme jointly optimized at the BS and the RSs;
- Proposed-BS: our scheme optimized only at the BS with fixed RS filters $\mathbf{F}_n = \sqrt{(P_{r_n}/\mathrm{Tr}(\mathbf{R}_{r_n}))}\mathbf{I}$;
- Proposed-RS: our scheme optimized only at the RSs with a fixed BS filter $\mathbf{V} = \sqrt{(P_b/M_b)}\mathbf{I}$.

In Fig. 2, the sum rate performance in $(4, 2\times 2, 2\times 2)$ systems is plotted with weights $[w_1\ w_2] = [11]$. In this configuration, we can recognize that the proposed-joint scheme performs very close to both the GD algorithms in $(4, 2\times 2, 4\times 1)$ and $(4, 4\times 1, 2\times 2)$ systems. Since in the GD algorithms for $(4, 2\times 2, 4\times 1)$ and $(4, 4\times 1, 2\times 2)$ systems, all MUs and RSs are operated as a single node with the twice number of antennas at each MU and RS, the GD algorithm is served as a performance upper bound.

Fig. 3 shows the WSR performance in $(4, 2\times 2, 2\times 2)$ systems with weights $[w_1 w_2] = [3]$. In this system configuration, where the weights for each user are unequal, as the WSR for the system with MU cooperation cannot be defined, only the GD performance with RS cooperation is plotted for comparison. It is shown that the performance of our proposed-joint scheme achieves GD scheme within 0.64 b/s/Hz. In addition, compared with the results in Figs. 2 and 3, we can realize that the proposed-BS scheme outperforms the proposed-RS scheme. These results are originated from the fact that while the RS filter (10) considers CSI from only itself to MUs, the BS filter (16) utilizes CSI

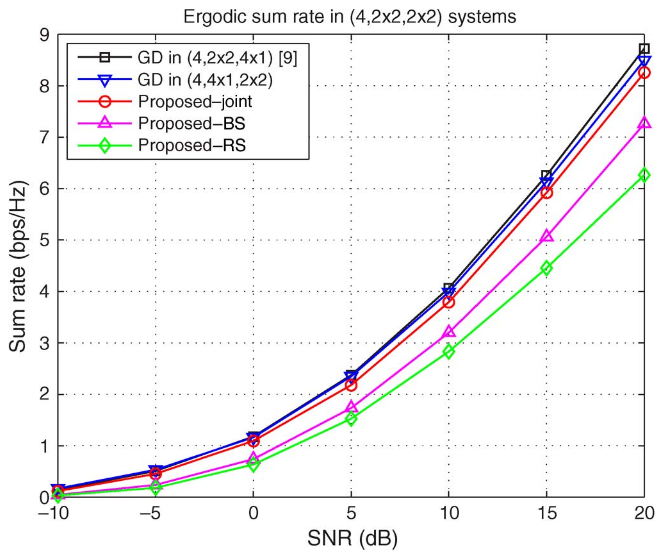

Fig. 2. Sum rate performance in $(4, 2 \times 2, 2 \times 2)$ systems.

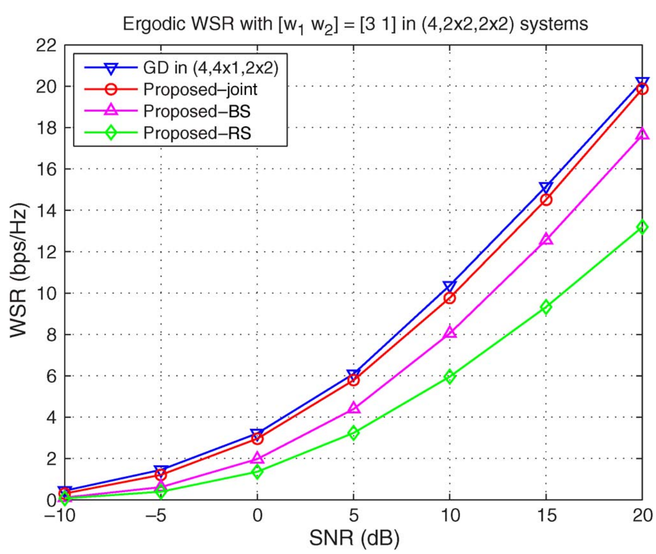

Fig. 3. WSR performance in $(4, 2 \times 2, 2 \times 2)$ systems with weights $[w_1 w_2] = [31]$.

from all the RSs to MUs. Hence, interuser interference is reduced more efficiently by the BS filter in multiuser multirelay MIMO systems.

From our simulation results, we learn that by applying the proposed scheme, the WSR performance of multiuser MIMO relay systems with multiple distributed relays employing a small number of antennas is very close to that of multiuser MIMO relay systems with a single cooperative relay equipped with a large number of antennas. Compared with systems with a single relay equipped with a large number of antennas, multiple distributed relays with a small number of antennas may be more beneficial to overcome large-scale fading and to increase cell coverage. Note that our systems under consideration impose increased signaling overhead and information sharing between nodes in practical implementations, and these issues remain as future work.

VI. Conclusion

In this paper, we have studied the WSR maximization problem in multiuser multirelay MIMO systems. The WSR maximization problem has been transformed to an equivalent WSMSE minimization problem, which is more amenable. We have proposed an algorithm for minimizing the WSMSE, which determines the weight matrix to identify the filters at the BS and the RSs iteratively. Simulation results confirm that our proposed scheme performs very close to the GD schemes with RS or MU cooperation, where all RSs or MUs operate as a single node.

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