SINR Balancing Techniques in Coordinated Multi-Cell Downlink Systems

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***Abstract*—In this paper, we study coordinated multi-cell downlink systems where multiple base stations jointly design a transmission strategy by sharing channel state information. Particularly, we tackle the signal-to-interference-plus-noise ratio (SINR) balancing problem to maximize the worst-user rate. We consider single-input single-output (SISO) interference channels (IFC) where all nodes are equipped with a single antenna, and there is one active user in each cell. First, achievable rate regions with symmetric complex (SC) and asymmetric complex (AC) signaling techniques are investigated. Then, we present the optimal and near-optimal SINR balancing algorithms with the SC signaling for two and three user SISO IFC. Due to residual interference, the worst-user rate of the SC signaling is saturated at high signal-to-noise-ratio region. To alleviate this problem, we also propose efficient balancing schemes based on the AC signaling for both two and three-user cases. Simulation results confirm effectiveness of the proposed SINR balancing algorithms and show that a substantial gain of the AC signaling is achieved over the SC signaling in terms of the maximum worst-user rate.**

***Index Terms*—Interference channel, SINR balancing, multi-cell systems.**

I. INTRODUCTION

IN cellular networks, interference mitigation is an important issue since inter-cell interference seriously limits the overall system performance. Coordinated multipoint (CoMP) technologies have been recognized as good candidates for solving this issue, since coordination among neighboring cells can effectively reduce the inter-cell interference [1]–[7]. Depending on the base station (BS) cooperation level, the CoMP can be classified into two categories [8]. One is joint processing (JP) where BSs share both users' messages and channel state information (CSI), and the other is coordinated beamforming (CB) where BSs design their transmission strategy by sharing only users' CSI. Since the backhaul link among the BSs has the limited capacity, this paper focuses on the CB system which can reduce the system complexity compared to the JP.

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Currently, there have been lots of studies to improve the system performance in CB systems, for example [9]–[14]. The efforts to improve the network throughput have been made in [9]–[11]. In [12], authors have considered the CB for the minimization of the total weighted transmit power across the BSs subject to individual signal-to-interference-plus-noise-ratio (SINR) constraints at remote users. Other performance measure includes signal-to-interference-plus-noise ratio (SINR) balancing which maximizes the worst-user rate [13] [14].

The SINR-balancing technique has an important meaning since its solution introduces fairness to the system so that each user has the same rate, and characterizes different achievable rate regions [15]. In the context of the SINR balancing, the optimal power control algorithm with a fixed beamformer was firstly studied in [16] for single-cell broadcast channels (BC). A joint optimization of beamforming and power control using uplink-downlink duality was developed in [17], and authors in [18] introduced a distributed SINR balancing algorithm. For CB systems, schemes which maximize the worst-user rate for multiple input single output (MISO) interference channel (IFC) and MISO interference BC are proposed in [13] and [14], respectively. Algorithms in [13] and [14] solve the formulated problem with an aid of standard convex optimization tools [19].

In this paper, we tackle the problem of optimizing the worst-user rate for CB systems where one active user in each cell is served by its corresponding BS in a single antenna environment. The overall system can be modeled as single input single output (SISO) IFC systems with constant channel coefficients and no time-domain expansion as in [20]. First, we investigate achievable rate regions with symmetric complex (SC) and asymmetric complex (AC) signaling techniques. In the SC signaling, the inputs are chosen to be complex Gaussian symbols whose real and imaginary part have equal power and are independent Gaussian variates [21]. On the other hands, in the AC signaling, the inputs are set to be complex but not circularly symmetric . The advantage of the AC signaling has been well studied in terms of degrees of freedom (DOF) for interference channels with constant channel coefficients where the DOF accounts for the capacity behavior at the high signal-to-noise ratio (SNR) range [22]–[24]. Compared to an approach in [25], new compact characterization of Pareto boundary is presented for an arbitrary number of cells for the case of the SC signaling. For the AC signaling, we also provide new characterization of the Pareto boundary for the 2-user case. Note that other approaches which characterize the achievable rate region for 2-user SISO IFC were introduced in [26] and [27]. The authors in [26] proposed a rank-1 transmission scheme, and a rate-profile technique was employed in [27] for the rate region characterization in the AC signaling.

Second, we develop SINR balancing algorithms based on the SC signaling for 2 and 3-user SISO IFC. For the 2-user environment, the optimal power allocation scheme in terms of maximizing the worst-user rate is presented. Since the problem of the SINR balancing is somewhat complicated for 3-user systems, we employ an alternating optimization which iteratively finds a solution, and this provides the worst-user rate performance almost identical to the optimal exhaustive search method with the SC signaling. It is worth noting that our algorithms do not rely on the convex optimization tools unlike other existing methods in [13] and [14], and thus require less computational complexity.

With the SC signaling for the SISO IFC, the maximum worst-user rate is saturated as SNR grows due to the residual interference. Lastly, to improve this issue, we also propose a transmission strategy with the AC signaling. To the best of authors' knowledge, there exists no SINR balancing technique in the AC signaling for the SISO IFC. Under the AC signaling, the overall system can be considered as the K-user 2-by-2 MIMO IFC with real channel coefficients. Therefore, we can apply beamforming techniques which cannot be possible in the SC signaling. In our proposed AC signaling, a zero-forcing (ZF) transceiver design is employed to effectively eliminate interference terms, and this provides a significant improvement in terms of the worst-user rate. However, perfect interference cancelation using the ZF transceiver design is only allowed for the 2-user case due to lack of the receiver signal space for general K-user environments [20] [28]. In this context, for the 3-user case, we assume that only two users can perfectly cancel the interference via interference alignment (IA) techniques introduced in [28]. As a result, the worst-user rate performance is dominated by the user with residual interference. To maximize the worst-user rate, a power control method is introduced after IA beamforming. Simulation results confirm that the proposed SINR balancing algorithms are effective, and the AC signaling achieves a substantial gain over the SC signaling in terms of the maximum worst-user rate.

The rest of the paper is organized as follows: In Section II, we describe a general system model for multi-cell downlink systems. The achievable rate regions with both SC and AC signaling are presented in Section III. In Section IV, the SINR balancing algorithm with the SC signaling is introduced, and then Section V proposes transmission algorithms based on the AC signaling. Section VI illustrates the simulation results and the paper is closed with conclusions in Section VII.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices, lowercase boldface for vectors and normal letters for scalar quantities. Transpose and conjugate transpose of a matrix or a vector are represented by $(\cdot)^T$ and $(\cdot)^H$, respectively. The set of all real matrices of size M-by-N is denoted by $\mathbb{R}^{M\times N}$ and $\mathbb{Z}$ is the set of all integers. In addition, $\Re\{x\}$ and $\Im\{x\}$ indicate the real and imaginary part of a complex value x, respectively, $\det(\cdot)$ accounts for the determinant of a matrix.

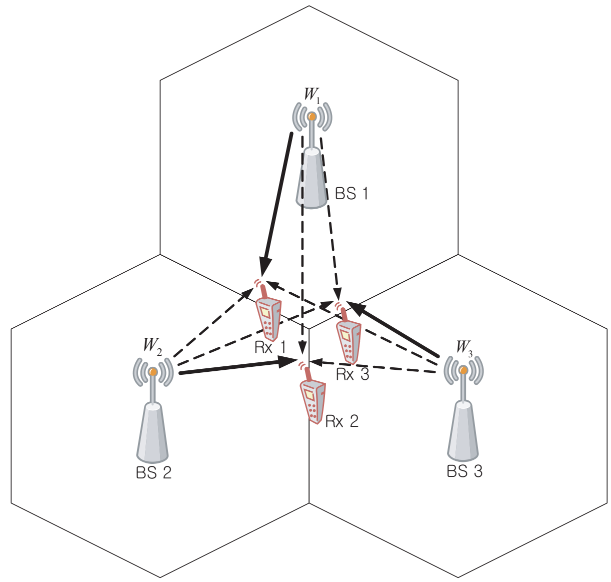


Fig. 1. Schematic diagram of 3-cell downlink systems.

II. System Model

In this section, we present a general description of multi-cell downlink systems where only CSI is exchanged among BSs in a single antenna environment. It is assumed that there is one active user in each cell. Then, the overall system can be modeled as K-user SISO IFC where BS i $(i = 1, \cdots, K)$ transmits the message W_i intended for user i. The 3-user SISO IFC system is illustrated in Figure 1 as an example. In the figure, the solid line indicates the desired signal and the dashed line represents the interference signal. For practical implementation issues, we consider K-user SISO IFC with constant channel coefficient [20].

Under the frequency-flat fading model, the received signal at user i is written as

$$y_i = h_{i,i}x_i + \sum_{j\neq i} h_{i,j}x_j + n_i, \quad \text{for } i = 1, \cdots, K \tag{1}$$

where x_i is the transmitted signal at BS i, $h_{i,j}$ indicates the channel coefficient from BS j to user i, and n_i represents the additive Gaussian noise at user i as $n_i \sim CN(0, \sigma_n^2)$. It is assumed that the CSI is globally available. Although the desired channel coefficient $h_{i,i}$ generally has power greater than that of the interference channel coefficient $h_{i,j}$ $(i \neq j)$ due to path loss, we consider the most challenging case where users are located in cell boundaries so that both $h_{i,i}$ and $h_{i,j}$ have the same power. Because implementation of multi-user detection may be difficult, we assume that the interference terms are treated as noise [29]. In addition, we consider per-BS power constraint as $E[|x_i|^2] \leq P_{\text{max}}$ for $i = 1, \cdots, K$ where P_{max} is the maximum transmit power, since each BS has its own power amplifier.

Using the real-valued representation, the input-output equation of (1) can be equivalently given as

$$\mathbf{y}_i = \mathbf{H}_{i,i}\mathbf{x}_i + \sum_{j\neq i} \mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i, \tag{2}$$

where $\mathbf{y}_i = [\Re\{y_i\}\ \Im\{y_i\}]^T$, $\mathbf{x}_i = [\Re\{x_i\}\ \Im\{x_i\}]^T$, $\mathbf{n}_i = [\Re\{n_i\}\ \Im\{n_i\}]^T$ and

$$\mathbf{H}_{i,j} = \begin{bmatrix} \Re\{h_{i,j}\} & -\Im\{h_{i,j}\} \\ \Im\{h_{i,j}\} & \Re\{h_{i,j}\} \end{bmatrix}. \tag{3}$$

Assuming that the input signal is chosen from a Gaussian codebook at each BS, the achievable rate of each link is determined by the covariance matrix of $\mathbf{x}_i$ denoted by $\mathbf{Q}_i \triangleq E[\mathbf{x}_i\mathbf{x}_i^H]$. Since $\mathbf{Q}_i$ is positive semi-definite, it can be expressed using eigenvalue decomposition [30] as

$$\mathbf{Q}_i = \mathbf{J}(\phi_i)\mathrm{diag}(\lambda_i, P_i - \lambda_i)\mathbf{J}(-\phi_i) \tag{4}$$

where $0 \leq P_i \leq P_{\max}$, $0 \leq \lambda_i \leq P_i$ and $\mathbf{J}(x)$ is a unitary rotation matrix defined as

$$\mathbf{J}(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}. \tag{5}$$

Here, P_i is the transmit power level at BS i and λ_i adjusts the degree of asymmetry. The distribution of $\mathbf{x}_i$ is circularly symmetric when $\lambda_i = \frac{P_i}{2}$ and the degree of asymmetry grows as $\left|\lambda_i - \frac{P_i}{2}\right|$ increases. Therefore, the SC signaling is a special case of the AC signaling with $\lambda_i = \frac{P_i}{2}$. We notice that $\mathbf{H}_{i,j}$ can also be represented using the rotation matrix as $\mathbf{H}_{i,j} = A_{i,j}\mathbf{J}(\theta_{i,j})$ where $A_{i,j} = |h_{i,j}|$ and $\theta_{i,j} = \angle h_{i,j}$.

III. Achievable Rate Region with SC and AC Signaling

In this section, we study achievable rate regions of both the SC and the AC signaling, and observe the efficiency of the AC signaling in terms of maximizing the worst user rate.

A. SC signaling

In the SC signaling, we can perform only the power control in $x_i = \sqrt{P_i}s_i$ where $s_i \sim CN(0,1)$ denotes the data symbol intended for user i. Then, an achievable rate of link k ($k = 1, \cdots, K$) is a function of the power levels $\{P_i\}$ as $R_k^{\mathrm{SC}}(\{P_i\}) = \log_2\left(1 + \gamma_k^{\mathrm{SC}}(\{P_i\})\right)$ where $\gamma_k^{\mathrm{SC}}(\{P_i\})$ is the individual SINR for link k defined as $\gamma_k^{\mathrm{SC}}(\{P_i\}) = \dfrac{g_{k,k}P_k}{\sigma_n^2 + \sum_{l\neq k} g_{k,l}P_l}$ and $g_{k,l} = \left|h_{k,l}\right|^2$ indicates the instantaneous channel gain. Now, we are ready to define an achievable rate region with the SC signaling as

$$\mathcal{R}^{\mathrm{SC}} = \bigcup_{0\leq P_i\leq P_{\max},\forall i} \left\{ \left(R_1^{\mathrm{SC}}(\{P_i\}), \cdots, R_K^{\mathrm{SC}}(\{P_i\})\right) \right\}. \tag{6}$$

A rate-tuple $(r_1, \cdots, r_K) \in \mathcal{R}^{\mathrm{SC}}$ is Pareto optimal if there is no other rate-tuple $(q_1, \cdots, q_K)$ in $\mathcal{R}^{\mathrm{SC}}$ with $(q_1, \cdots, q_K) \succeq (r_1, \cdots, r_K)$ where $\succeq$ represents the component-wise inequality [10]. In the following theorem, we obtain the exact Pareto-boundary set for $\mathcal{R}^{\mathrm{SC}}$. This result was first established in [25] using mathematical induction, but the proof there was somewhat lengthy. In this paper, we provide a new compact proof by using a partitioning approach.

Theorem 1: The Pareto boundary set of $\mathcal{R}^{\mathrm{SC}}$ is exactly given as

$$\mathcal{S}^{\mathrm{SC}} = \bigcup_{i=1,\cdots,K} \bigcup_{\substack{P_i=P_{\max},\\ 0\leq P_k\leq P_{\max}, k\neq i}} \left\{ \left(R_1^{\mathrm{SC}}(\{P_i\}), \cdots, R_K^{\mathrm{SC}}(\{P_i\})\right) \right\}.$$

Proof: To complete the proof, the following two statements should be shown

$$\text{The set of Pareto-optimal points} \subset \mathcal{S}^{\mathrm{SC}}, \tag{7}$$
$$\text{The set of Pareto-optimal points} \supset \mathcal{S}^{\mathrm{SC}}. \tag{8}$$

Since the proof for (7) is quite straightforward, we illustrate how to show (8). Without loss of generality, it is sufficient to show that if $P'_K = P_{\max}$, then $(P'_1, \cdots, P'_K)$ is Pareto-optimal for any $P'_1, \cdots, P'_{K-1}$. We do this by dividing all possible cases for $P'_1, \cdots, P'_{K-1}$ into the following two cases.

Case 1) $P'_1 = \cdots = P'_{K-1} = 0$

If we want to make some of $R_1, \cdots, R_{K-1}$ nonzero, then some of $P'_1, \cdots, P'_{K-1}$ become nonzero and the rate of link K decreases. Thus, $(P'_1, \cdots, P'_K)$ is Pareto-optimal for this case.

Case 2) L elements of $P'_1, \cdots, P'_{K-1}$ are nonzero

Without loss of generality, suppose that $P'_1, \cdots, P'_L$ are nonzero where $1 \leq L \leq K-1$. If we consider any other power tuple $(P''_1, \cdots, P''_K)$ such that $P''_1 \neq P'_1 \cdots$ or $P''_{K-1} \neq P'_{K-1}$ or $P''_K \neq P'_K = P_{\max}$, then the proof is completed by showing that there exists user i such that $R_i^{\mathrm{SC}}(\{P''_k\}) \leq R_i^{\mathrm{SC}}(\{P'_k\})$. To this end, all possible cases are classified into the following two subcases.

Case 2-1) $(P''_1, \cdots, P''_K) \succeq (P'_1, \cdots, P'_K)$

Since $P'_K = P''_K = P_{\max}$, we have $R_K^{\mathrm{SC}}(\{P''_i\}) \leq R_K^{\mathrm{SC}}(\{P'_i\})$.

Case 2-2) $P''_k < P'_k$ for some k

Denote two partition sets S_{down} and S_{up} for the whole index set $\{1, \cdots, K\}$ as

$$\begin{aligned} S_{\mathrm{down}} &= \{i | P''_i < P'_i\}, \\ S_{\mathrm{up}} &= \{1, \cdots, K\} \backslash S_{\mathrm{down}}. \end{aligned}$$

Then, we can show that $R_{i_{\min}}^{\mathrm{SC}}(\{P''_k\}) < R_{i_{\min}}^{\mathrm{SC}}(\{P'_k\})$ where $i_{\min} = \arg\min_{i\in S_{\mathrm{down}}}\{\frac{P''_i}{P'_i}\}$. First, $R_{i_{\min}}^{\mathrm{SC}}(\{P'_k\})$ is computed as

$$R_{i_{\min}}^{\mathrm{SC}}(\{P'_k\}) = \log_2\left(1 + \frac{g_{i_{\min},i_{\min}}P'_{i_{\min}}}{\sigma_n^2 + \sum_{j\in S_{\mathrm{up}}} g_{i_{\min},j}P'_j + \sum_{j\in S'_{\mathrm{down}}} g_{i_{\min},j}P'_j}\right)$$

where we denote S'_{down} as $S'_{\mathrm{down}} = S_{\mathrm{down}} \backslash i_{\min}$.

Defining $\alpha \triangleq \dfrac{P''_{i_{\min}}}{P'_{i_{\min}}} < 1$, an upperbound of $R_{i_{\min}}^{\mathrm{SC}}(\{P''_k\})$ is derived as

$$\begin{aligned} &R_{i_{\min}}^{\mathrm{SC}}(\{P''_k\}) \\ &= \log_2\left(1 + \frac{g_{i_{\min},i_{\min}}P''_{i_{\min}}}{\sigma_n^2 + \sum_{j\in S_{\mathrm{up}}} g_{i_{\min},j}P''_j + \sum_{j\in S'_{\mathrm{down}}} g_{i_{\min},j}P''_j}\right) \\ &\leq \log_2\left(1 + \frac{\alpha g_{i_{\min},i_{\min}}P'_{i_{\min}}}{\sigma_n^2 + \sum_{j\in S_{\mathrm{up}}} g_{i_{\min},j}P'_j + \alpha\sum_{j\in S'_{\mathrm{down}}} g_{i_{\min},j}P'_j}\right) \end{aligned}$$

where the inequality comes from $P''_j \geq P'_j$ if $j \in S_{\mathrm{up}}$ and $P''_j \geq \alpha P'_j$ if $j \in S_{\mathrm{down}}$. Consider a function $f(\eta)$ defined as

$$f(\eta) = \log_2\left(1 + \frac{\eta g_{i_{\min},i_{\min}}P'_{i_{\min}}}{\sigma_n^2 + \sum_{j\in S_{\mathrm{up}}} g_{i_{\min},j}P'_j + \eta\sum_{j\in S'_{\mathrm{down}}} g_{i_{\min},j}P'_j}\right)$$

$$R_k^{\text{AC}}(\{P_i,\lambda_i,\phi_i\}) = \frac{1}{2}\log_2 \frac{\det\left(\sum_j \mathbf{H}_{k,j}\mathbf{J}(\phi_j)\text{diag}(\lambda_j,(P_j-\lambda_j))\mathbf{J}(-\phi_j)\mathbf{H}_{k,j}^T + \frac{\sigma_n^2}{2}\mathbf{I}\right)}{\det\left(\mathbf{H}_{k,\bar{k}}\mathbf{J}(\phi_{\bar{k}})\text{diag}(\lambda_{\bar{k}},(P_{\bar{k}}-\lambda_{\bar{k}}))\mathbf{J}(-\phi_{\bar{k}})\mathbf{H}_{k,\bar{k}}^T + \frac{\sigma_n^2}{2}\mathbf{I}\right)} \tag{9}$$

$$R_k^{\text{AC}}(\{P_i,\lambda_i\},\phi_1) = \frac{1}{2}\log_2 \frac{\prod_{j=1}^{2}\left(\frac{\sigma_n^2}{2}+A_{k,j}^2\lambda_j+A_{k,\bar{j}}^2(P_{\bar{j}}-\lambda_{\bar{j}})\right)-\left(\prod_{j=1}^{2}A_{k,j}^2(P_j-2\lambda_j)\right)\cos^2\left(\phi_1+(-1)^{k-1}\beta_k\right)}{\left(\frac{\sigma_n^2}{2}+A_{k,\bar{k}}^2\lambda_{\bar{k}}\right)\left(\frac{\sigma_n^2}{2}+A_{k,\bar{k}}^2(P_{\bar{k}}-\lambda_{\bar{k}})\right)} \tag{10}$$

which is monotonically increasing with respect to (w.r.t.) η for positive η. Then, we have

$$R_{i_{\min}}^{\text{SC}}(\{P_k'\}) = f(\eta)|_{\eta=1} > f(\eta)|_{\eta=\alpha} \geq R_{i_{\min}}^{\text{SC}}(\{P_k''\}).$$

As a result, it is shown that $R_{i_{\min}}^{\text{SC}}(\{P_k'\}) > R_{i_{\min}}^{\text{SC}}(\{P_k''\})$, and this concludes the proof. ■

B. AC signaling

Compared to the SC signaling case, an achievable rate characterization with the AC signaling needs more parameters. Thus, we focus on the case of $K=2$ for simplicity. Then, an achievable rate of user k is given as (9) at the top of this page where we define $\bar{1}=2$ and $\bar{2}=1$.

An achievable rate region with the AC signaling is determined as

$$\mathcal{R}^{\text{AC}} = \bigcup_{\substack{0\leq\lambda_i\leq P_i,\ 0\leq\phi_i\leq\pi,\\ 0\leq P_i\leq P_{\max},\forall i}} \left\{\left(R_1^{\text{AC}}(\{P_i,\lambda_i,\phi_i\}), R_2^{\text{AC}}(\{P_i,\lambda_i,\phi_i\})\right)\right\}.$$

In order to plot the rate region $\mathcal{R}^{\text{AC}}$, the exhaustive search is required over 6 parameters. In the following theorem, we show that the reduced 4-dimensional search can be carried out without loss of optimality.

Theorem 2: A set $\mathcal{S}^{\text{AC}}$ includes all the Pareto-boundary points of $\mathcal{R}^{\text{AC}}$ where $\mathcal{S}^{\text{AC}}$ is defined as

$$\mathcal{S}^{\text{AC}} = \bigcup_{k=1,2}\ \bigcup_{\substack{P_k=P_{\max},\\ 0\leq P_{\bar{k}}\leq P_{\max},\\ 0\leq\lambda_i\leq P_i, i=1,2,\\ \phi_1^{\min}\leq\phi_1\leq\phi_1^{\max}}} \left\{\left(R_1^{\text{AC}}(\{P_i,\lambda_i,\phi_i\}), R_2^{\text{AC}}(\{P_i,\lambda_i,\phi_i\})\right)\right\}.$$

Here, $[\phi_1^{\min},\phi_1^{\max}]$ is the reduced range of ϕ_1 for given parameters $\{P_i,\lambda_i\}$ defined as

$$\begin{aligned}\phi_1^{\min} &= \min\{\phi_1^{R_1}(\{P_k,\lambda_k\}), \hat{n}\pi+\phi_1^{R_2}(\{P_k,\lambda_k\})\},\\ \phi_1^{\max} &= \max\{\phi_1^{R_1}(\{P_k,\lambda_k\}), \hat{n}\pi+\phi_1^{R_2}(\{P_k,\lambda_k\})\}\end{aligned}$$

where $\phi_1^{R_i}(\{P_k,\lambda_k\})$ and $\hat{n}$ are given as

$$\phi_1^{R_i}(\{P_k,\lambda_k\}) = \begin{cases}(-1)^i\beta_i, & \text{if } (P_{\bar{i}}-2\lambda_{\bar{i}})(2\lambda_i-P_i)\geq 0,\\ (-1)^i\beta_i+\frac{\pi}{2}, & \text{otherwise,}\end{cases}$$

$$\hat{n} = \arg\min_{n\in\mathbb{Z}}\left|\phi_1^{R_1}(\{P_k,\lambda_k\})-\phi_1^{R_2}(\{P_k,\lambda_k\})-n\pi\right|$$

and β_i is defined as $\beta_i = \theta_{i,i}-\theta_{i,\bar{i}}$.

Proof: Following the same steps as in the proof of Theorem 1, we can show that at least one of P_1 and P_2 should be $P_{\max}$ for any Pareto boundary points. Also, it is easy to see that for given other parameters, the above rate depends only on $\phi_1-\phi_2$. Thus, we can set $\phi_2=0$ without loss of generality. Using some trigonometric formula, we can obtain a simplified rate expression as (10) at the top of this page where the pre-log factor $\frac{1}{2}$ is caused by employing real-valued data symbols. For given other parameters, both $R_1^{\text{AC}}(\{P_i,\lambda_i\},\phi_1)$ and $R_2^{\text{AC}}(\{P_i,\lambda_i\},\phi_1)$ are periodic functions w.r.t. ϕ_1 with period π. Also, $R_k^{\text{AC}}(\{P_i,\lambda_i\},\phi_1)$ hits its maximum at $\phi_1=\phi_1^{R_k}(\{P_i,\lambda_i\})$. Thus, for given $\{P_i,\lambda_i\}$, it is sufficient to search within the range between $\phi_1^{R_1}(\{P_i,\lambda_i\})$ and $n\pi+\phi_1^{R_2}(\{P_i,\lambda_i\})$ for any $n\in\mathbb{Z}$. The integer $\hat{n}$ is chosen to minimize the search range. ■

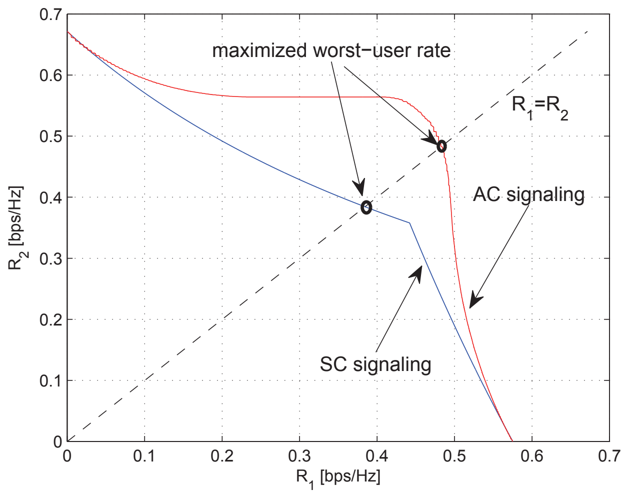
It is interesting to see that in the rate expression (10), the achievable rate does not depend on ϕ_1 if at least one of two BSs uses the SC signaling (i.e., $\lambda_i=\frac{P_i}{2}$). This tells us that if either the desired signal or the interference-plus-noise term is circularly symmetric, then the dominant direction of the distribution of the asymmetric complex term has no influence on the achievable rate. In Figure 2, we present the achievable rate region for a sample channel realization at low and high SNR. The optimal point in terms of the worst-user rate is the intersection between the line $R_1=R_2$ and the Pareto boundary of the rate region. It is shown in the plot that the AC signaling can achieve the higher worst-user rate than the SC signaling and a performance gain increases as SNR grows. Note that if time sharing is employed where the achievable rate region is obtained from the convex hull operation over all the rate tuples, we may achieve larger rate regions in certain channel realizations [25] [27]. However, for simple implementation issue, we focus on the system with pure SC and AC signaling methods in this paper, i.e., we do not consider the time sharing.

IV. COORDINATED SINR BALANCING WITH SC SIGNALING

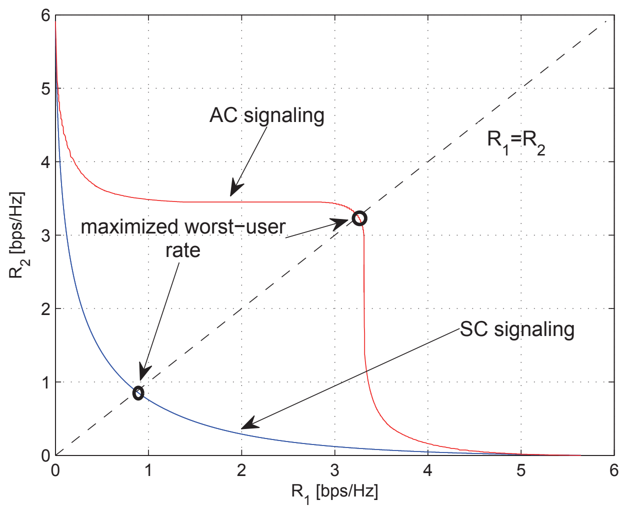
In this section, we establish an SINR balancing scheme with the SC signaling. Since an individual rate $R_k^{\text{SC}}(\{P_i\})$ is monotonically increasing w.r.t. $\gamma_k^{\text{SC}}(\{P_i\})$, the optimization problem for the SINR balancing can be formulated as

$$\max_{0\leq P_i\leq P_{\max},\forall i}\ \min_{k=1,\cdots,K}\ \gamma_k^{\text{SC}}(\{P_i\}). \tag{11}$$

For illustrative purposes, we begin by presenting the proposed SC signaling for the 2-user case and will extend to 3-user SISO IFC systems.



(a) Achievable rate region for SNR = 0 dB



(b) Achievable rate region for SNR = 20 dB

Fig. 2. Achievable rate region for a sample channel realization.

A. 2-user SISO IFC

According to Theorem 1, the optimal solution for (11) is one of the solutions of the following two problems

$$\max_{0\leq P_1\leq P_{\max}, P_2=P_{\max}} \min_k \gamma_k^{\text{SC}}(\{P_i\}), \tag{12}$$

$$\max_{0\leq P_2\leq P_{\max}, P_1=P_{\max}} \min_k \gamma_k^{\text{SC}}(\{P_i\}). \tag{13}$$

Let $\hat{P}_1$ and $\hat{P}_2$ denote the solutions of the problems (12) and (13), respectively. To illustrate the key idea of the proposed algorithm, we describe how to find $\hat{P}_1$, i.e., the optimization of P_1 for fixed $P_2 = P_{\max}$. To this end, we define two functions $\gamma_1^{P_2}(P_1) \triangleq \gamma_1(\{P_i\})\big|_{P_2=P_{\max}}$ and $\gamma_2^{P_2}(P_1) \triangleq \gamma_2(\{P_i\})\big|_{P_2=P_{\max}}$.
Then, $\hat{P}_1$ can be obtained as

$$\hat{P}_1 = \arg\max_{0\leq P_1\leq P_{\max}} \min\{\gamma_1^{P_2}(P_1), \gamma_2^{P_2}(P_1)\}. \tag{14}$$

To solve the above problem efficiently, we adopt the following useful properties. First, $\gamma_2^{P_2}(0) > \gamma_1^{P_2}(0) = 0$. Second, $\gamma_1^{P_2}(P_1)$ and $\gamma_2^{P_2}(P_1)$ are monotonically increasing and decreasing w.r.t. P_1, respectively. From these properties, we see that if $\gamma_1^{P_2}(P_{\max}) > \gamma_2^{P_2}(P_{\max})$, there exists a cross-over point within $0 \leq P_1 \leq P_{\max}$, and the minimum of $\gamma_1^{P_2}(P_1)$ and $\gamma_2^{P_2}(P_1)$ is maximized when $\gamma_1^{P_2}(P_1) = \gamma_2^{P_2}(P_1)$. Otherwise, the solution for (12) is just $P_1 = P_{\max}$. As a result, $\hat{P}_i$ $(i = 1, 2)$ is written as (15) at the top of the next page.

In summary, the optimal solution $(P_1^{\text{opt}}, P_2^{\text{opt}})$ for the problem (11) is given by

$$(P_1^{\text{opt}}, P_2^{\text{opt}}) = \arg\max_{(P_1,P_2)\in\Upsilon} \min_k R_k^{\text{SC}}(\{P_i\})$$

where $\Upsilon = \{(\hat{P}_1, P_{\max}), (P_{\max}, \hat{P}_2)\}$. Note that our proposed solution provides the optimal worst-user rate performance in SC signaling by searching over only two candidates.

B. 3-user SISO IFC

Similar to the 2-user case, we employ Theorem 1. Then, (11) can be equivalently expressed as

$$\left(P_1^{\text{opt}}, P_2^{\text{opt}}, P_3^{\text{opt}}\right) = \arg\max_{(P_1,P_2,P_3)\in S} \min_k \gamma_k^{SC}(\{P_i\}) \tag{16}$$

where $S = \{(P_{\max}, P_2^{(1)}, P_3^{(1)}), (P_1^{(2)}, P_{\max}, P_3^{(2)}), (P_1^{(3)}, P_2^{(3)}, P_{\max})\}$ and $P_j^{(i)}$ represents the optimized power level at transmitter j for given $P_i = P_{\max}$. For the ease of explanation, we describe how to obtain $P_1^{(3)}$ and $P_2^{(3)}$, i.e., optimization of P_1 and P_2 with fixed $P_3 = P_{\max}$.

Since the individual SINR is a function of both P_1 and P_2, a solution can be computed via joint optimization. However, it is somewhat complicated to derive a closed form solution due to non-convexity of the formulated problem (16). Instead, we apply an alternating optimization method which iteratively finds a local optimal solution. In this algorithm, we first identify the optimal P_1 with fixed P_2 and P_3. Then, P_2 is optimized with fixed P_1 and P_3. This procedure is repeated until convergence occurs.

As a first step, we explain the optimization of P_1 with fixed P_2 and P_3. For simple explanation, we denote $\bar{\gamma}_k^{l,m}(P_i)$ as the SINR of link k for fixed P_l and P_m where $l, m \in \{1, 2, 3\}\backslash\{i\}$ and $l \neq m$. Then, finding the optimal P_1^* can be formulated as

$$P_1^* = \arg\max_{0\leq P_1\leq P_{\max}} \min\left\{\bar{\gamma}_1^{2,3}(P_1), \bar{\gamma}_2^{2,3}(P_1), \bar{\gamma}_3^{2,3}(P_1)\right\}. \tag{17}$$

Note that $\bar{\gamma}_1^{2,3}(0)$ is smaller than $\min\{\bar{\gamma}_2^{2,3}(0), \bar{\gamma}_3^{2,3}(0)\}$. Also, $\bar{\gamma}_1^{2,3}(P_1)$ is monotonically increasing, and $\bar{\gamma}_2^{2,3}(P_1)$ and $\bar{\gamma}_3^{2,3}(P_1)$ are monotonically decreasing with respect to P_1. From these properties, if $\bar{\gamma}_1^{2,3}(P_{\max}) \geq \min\{\bar{\gamma}_2^{2,3}(P_{\max}), \bar{\gamma}_3^{2,3}(P_{\max})\}$, we can figure out that there always exists at least one cross-over point within $0 \leq P_1 \leq P_{\max}$ as shown in an example in Figure 3. Thus, a solution for (17) is given by $P_1^* = \min\{P_1^{1,2}, P_1^{1,3}\}$ where $P_1^{1,2}$ and $P_1^{1,3}$ represent the power level P_1 which satisfies $\bar{\gamma}_1^{2,3}(P_1) = \bar{\gamma}_2^{2,3}(P_1)$ and $\bar{\gamma}_1^{2,3}(P_1) = \bar{\gamma}_3^{2,3}(P_1)$, respectively. Otherwise, $P_1^* = P_{\max}$ is the solution for (17) obviously. The optimal P_2^* with fixed P_1 and P_3 can be calculated in a similar fashion.

As a result, P_i^* $(i = 1, 2)$ is obtained as (18) at the top of the next page where $\delta_i = g_{i,i}\left(\sigma_n^2 + g_{i,\bar{i}}P_{\bar{i}} + g_{i,3}P_{\max}\right)$, $\beta_{i,\bar{i}} = g_{i,i}\left(\sigma_n^2 + g_{\bar{i},3}P_{\max}\right)$ and $\beta_{i,3} = g_{i,i}\left(\sigma_n^2 + g_{3,\bar{i}}P_{\bar{i}}\right)$. This

$$\hat{P}_i = \begin{cases} \dfrac{-\sigma_n^2 g_{i,i} + \sqrt{(\sigma_n^2 g_{i,i})^2 + 4 g_{i,i} g_{\bar{i},i} g_{\bar{i},\bar{i}} P_{\max}(\sigma_n^2 + g_{i,\bar{i}} P_{\max})}}{2 g_{i,i} g_{\bar{i},i}}, & \text{if } \dfrac{g_{\bar{i},\bar{i}}}{g_{i,i}} < \dfrac{(\sigma_n^2 + g_{\bar{i},i} P_{\max})}{(\sigma_n^2 + g_{i,\bar{i}} P_{\max})}, \\ P_{\max}, & \text{otherwise.} \end{cases} \tag{15}$$

$$P_i^* = \begin{cases} \min\limits_{j \in \{\bar{i},3\}} \dfrac{-\beta_{i,j} + \sqrt{\beta_{i,j}^2 + 4\delta_i g_{j,i} g_{j,j} P_j}}{2 g_{i,i} g_{j,i}}, & \text{if } \bar{\gamma}_i^{\bar{i},3}(P_{\max}) \geq \min\{\bar{\gamma}_{\bar{i}}^{\bar{i},3}(P_{\max}), \bar{\gamma}_3^{\bar{i},3}(P_{\max})\}, \\ P_{\max}, & \text{otherwise.} \end{cases} \tag{18}$$

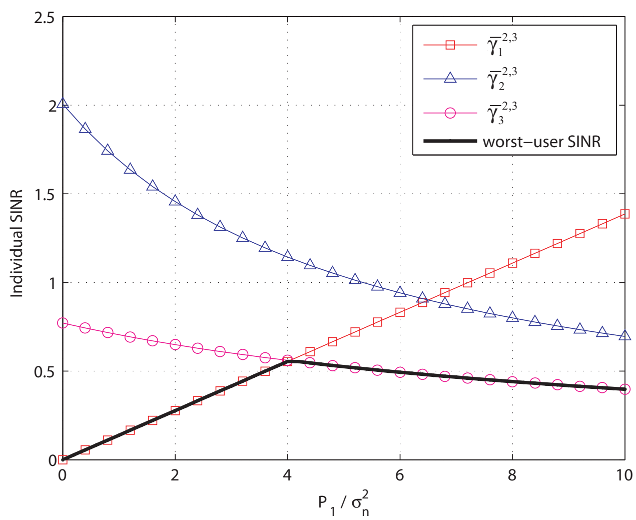


Fig. 3. Instantaneous SINR for fixed $P_2 = P_3 = P_{\max}$ at $\frac{P_{\max}}{\sigma_n^2} = 10$ dB for a sample channel realization.

alternating optimization procedure is repeated until convergence.

Normally, the power level of BS i $(i = 1, 2)$ at n-th iteration, $P_i(n)$, can be updated simply by P_i^*. On the other hand, we update $P_i(n)$ by $\frac{P_i^* + P_i(n-1)}{2}$ in our algorithm. As a result, our algorithm identifies a solution by comparing multiple points which include a solution of the simple updating method as $P_i(n) = P_i^*$. Thus, we can further improve the worst-user SINR. The proposed algorithm of finding $P_1^{(3)}$ and $P_2^{(3)}$ for fixed $P_3 = P_{\max}$ is summarized below.

Initialization

1) Set $P_1 \leftarrow P_{\max}$ and $P_2 \leftarrow P_{\max}$.

Main Loop

2) Compute P_1^* using (18), and update $P_1 \leftarrow \dfrac{P_1^* + P_1}{2}$.

3) Compute P_2^* using (18), and update $P_2 \leftarrow \dfrac{P_2^* + P_2}{2}$.

4) Go back to step 2 until convergence.

The convergence proof of our proposed scheme is presented in the following theorem.

Theorem 3: The proposed SINR balancing algorithm with SC signaling always converges.

Proof: Let us define $\hat{\gamma}_k$ as the instantaneous SINR of user k for given $P_i(n-1)$. Then, we have 6 possible states of the individual SINRs: $\hat{\gamma}_3 \leq \hat{\gamma}_2 \leq \hat{\gamma}_1$, $\hat{\gamma}_3 \leq \hat{\gamma}_1 \leq \hat{\gamma}_2$, $\hat{\gamma}_1 \leq \hat{\gamma}_2 \leq \hat{\gamma}_3$, $\hat{\gamma}_1 \leq \hat{\gamma}_3 \leq \hat{\gamma}_2$, $\hat{\gamma}_2 \leq \hat{\gamma}_1 \leq \hat{\gamma}_3$ and $\hat{\gamma}_2 \leq \hat{\gamma}_3 \leq \hat{\gamma}_1$. In our scheme, if $\hat{\gamma}_3 \leq \hat{\gamma}_2 \leq \hat{\gamma}_1$ or $\hat{\gamma}_3 \leq \hat{\gamma}_1 \leq \hat{\gamma}_2$, $P_1(n)$ and $P_2(n)$ decrease which leads to an improvement of the minimum SINR $\hat{\gamma}_3$. Also, $P_1(n)$ increases and $P_2(n)$ decreases with an increased $\hat{\gamma}_1$ when $\hat{\gamma}_1 \leq \hat{\gamma}_2 \leq \hat{\gamma}_3$ or $\hat{\gamma}_1 \leq \hat{\gamma}_3 \leq \hat{\gamma}_2$. Similarly, if $\hat{\gamma}_2 \leq \hat{\gamma}_1 \leq \hat{\gamma}_3$ or $\hat{\gamma}_2 \leq \hat{\gamma}_3 \leq \hat{\gamma}_1$, $\hat{P}_1(n)$ is decreased and $P_2(n)$ is increased. As a result, the worst-user SINR improves at the expense of a decrease of other user's SINR, and the state of the individual SINRs is changed to one of 6 cases. This means that the gap between the first and second worst-user SINRs approaches zero as iterations go. As a result, we reach at a stop criterion which is given as $\hat{\gamma}_1 = \hat{\gamma}_2 \leq \hat{\gamma}_3$. In this case, $P_1(n)$ and $P_2(n)$ cannot be increased (or decreased) due to a decrease of $\hat{\gamma}_2$ (or $\hat{\gamma}_1$). Thus, the algorithm converges, and this concludes the proof. ∎

TABLE I
THE NUMBER OF AVERAGE ITERATIONS OF THE PROPOSED SC SIGNALING FOR 3-USER SISO IFC

SNR (dB)	-10	-5	0	5	10	15	20	25
# of average iterations	2.6	4.5	6.4	7.5	7.9	8.0	8.1	8.1

In summary, we can obtain $P_1^{(3)}$ and $P_2^{(3)}$ using the above algorithm. The other power levels $P_1^{(2)}$, $P_3^{(2)}$, $P_2^{(1)}$ and $P_3^{(1)}$ are computed in a similar fashion. Finally, we have a solution of (16) by choosing the best one among three candidates in S. Table I provides the number of average iterations of the proposed SC signaling. It is worth noting that our proposed SC signaling does not employ the convex optimization tools unlike the algorithms in [13] and [14], and provides the worst-user rate almost identical to the optimal SC signaling performance as will be shown in the simulation section. However, the worst-user rate is saturated as the SNR grows due to residual interference.

V. IMPROVED SINR BALANCING WITH AC SIGNALING

In this section, to improve the worst-user rate of the SC signaling, we propose an efficient SINR balancing scheme based on the AC signaling. Using a rotation matrix, (2) can be rewritten as

$$\mathbf{y}_i = A_{i,i}\mathbf{J}(\theta_{i,i})\mathbf{x}_i + \sum_{j \neq i} A_{i,j}\mathbf{J}(\theta_{i,j})\mathbf{x}_j + \mathbf{n}_i. \tag{19}$$

We assume a rank-1 transmission for each BS to achieve the optimal DOF for K-user SISO IFC which is equal to $\frac{K}{2}$

[22]. This choice corresponds to an extremely asymmetric distribution of $\mathbf{x}_i$, i.e., $\lambda_i = P_i$. Then, the transmit signal vector $\mathbf{x}_i$ is related to u_i as $\mathbf{x}_i = \mathbf{v}_i u_i$ where $\mathbf{v}_i \in \mathbb{R}^{2\times 1}$ denotes the transmit beamformer and $u_i \sim N(0,1)$ indicates the data symbol intended for receiver i.

Thus, the individual rate for link i is computed as

$$R_i^{AC} = \frac{1}{2}\log_2 \frac{\det\left(\frac{\sigma_n^2}{2}\mathbf{I} + \sum_{k=1}^{K} A_{i,k}^2 \mathbf{J}(\theta_{i,k})\mathbf{v}_k\mathbf{v}_k^T\mathbf{J}(\theta_{i,k})^T\right)}{\det\left(\frac{\sigma_n^2}{2}\mathbf{I} + \sum_{k\neq i} A_{i,k}^2 \mathbf{J}(\theta_{i,k})\mathbf{v}_k\mathbf{v}_k^T\mathbf{J}(\theta_{i,k})^T\right)}. \quad (20)$$

Then, the SINR balancing problem with the AC signaling is formulated as

$$\begin{aligned} &\max_{\mathbf{v}_i,\forall i} \ \min_k \{R_k^{AC}\}, \quad (21)\\ &\text{subject to } ||\mathbf{v}_i||^2 \le P_{\max}, \forall i. \end{aligned}$$

Analogous to the SC signaling, we first provide the SINR balancing technique with the AC signaling for the 2-user case. After that, the AC signaling for 3-cell downlink systems is proposed.

A. 2-user SISO IFC

Since identifying the optimal solution for (21) is somewhat complicated even in the simplest 2-user case, we propose an one-shot suboptimal algorithm by assuming zero-forcing (ZF) receivers, i.e., the receive combiner at user i, $\mathbf{w}_i \in \mathbb{R}^{2\times 1}$ $(i = 1,2)$, is designed such that $A_{1,2}\mathbf{w}_1^T\mathbf{J}(\theta_{1,2})\mathbf{v}_2 = A_{2,1}\mathbf{w}_2^T\mathbf{J}(\theta_{2,1})\mathbf{v}_1 = 0$. Then, the individual SINR for user i, γ_i^{AC}, is computed as $\gamma_i^{AC} = \frac{A_{i,i}^2|\mathbf{w}_i^T\mathbf{J}(\theta_{i,i})\mathbf{v}_i|^2}{\frac{\sigma_n^2}{2}||\mathbf{w}_i||^2}$, since the filtered noise is given as $\mathbf{w}_i\mathbf{n}_i \sim N(0, \frac{\sigma_n^2}{2}||\mathbf{w}_i||^2)$. Our problem is then simplified as

$$\begin{aligned} &\max_{\mathbf{v}_i,\mathbf{w}_i,\forall i} \ \min_{i=1,2} \frac{A_{i,i}^2|\mathbf{w}_i^T\mathbf{J}(\theta_{i,i})\mathbf{v}_i|^2}{||\mathbf{w}_i||^2}\\ &\text{subject to } ||\mathbf{v}_i||^2 \le P_{\max}\\ &\qquad \mathbf{w}_i \perp A_{i,\bar{i}}\mathbf{J}(\theta_{i,\bar{i}})\mathbf{v}_{\bar{i}},\ \forall i \end{aligned}$$

where the interference terms are eliminated due to the second constraint.

In this formulation, we should optimize the power levels and directions of the beamforming vectors $\{\mathbf{v}_i\}$. Since we have no inter-cell interference, full power transmission is optimal, i.e., $||\mathbf{v}_i||^2 = P_{\max}$. Then, the transmit beamforming vectors have a form of $\mathbf{v}_i = \sqrt{P_{\max}}\mathbf{j}(\phi_i)$ for $i = 1,2$ where $\mathbf{j}(x)$ is defined as $\mathbf{j}(x) = [\cos(x)\ \sin(x)]^T$ and ϕ_i determines the direction of $\mathbf{v}_i$. Note that we have $\mathbf{J}(x)\mathbf{j}(y) = \mathbf{j}(x+y)$ and $\mathbf{j}(x)^T\mathbf{j}(y) = \cos(-x+y)$. Using these properties, (19) can be rewritten as

$$\mathbf{y}_i = \sqrt{P_{\max}}A_{i,i}\mathbf{j}(\theta_{i,i}+\phi_i)u_i + \sqrt{P_{\max}}A_{i,\bar{i}}\mathbf{j}(\theta_{i,\bar{i}}+\phi_{\bar{i}})u_{\bar{i}} + \mathbf{n}_i.$$

For given $\{\phi_i\}$, the ZF receiver is computed as $\mathbf{w}_i = \mathbf{j}\left(\theta_{i,\bar{i}} + \phi_{\bar{i}} - \frac{\pi}{2}\right)$. Thus, the ZF filter output is given as

$$\hat{u}_i = \sqrt{P_{\max}}A_{i,i}\cos\left(\frac{\pi}{2} + \theta_{i,i} - \theta_{i,\bar{i}} + \phi_i - \phi_{\bar{i}}\right)u_i + \tilde{n}_i,$$

where $\tilde{n}_k \sim N(0, \frac{\sigma_n^2}{2})$ since $||\mathbf{w}_k||^2 = 1$. Now, the remaining problem is to optimize ϕ_1 and ϕ_2 as

$$\max_{\phi_1,\phi_2} \ \min_{i=1,2} A_{i,i}^2\cos^2(\phi_1 - \phi_2 + \alpha_i) \quad (22)$$

where $\alpha_k = (-1)^{k-1}\left(\frac{\pi}{2} + \theta_{k,k} - \theta_{k,\bar{k}}\right)$. Since the objective function depends only on $\phi_1 - \phi_2$, we set $\phi_2 = 0$ without loss of generality and obtain the modified problem as $\phi_1^{\text{opt}} = \arg\max_{\phi_1} f(\phi_1)$ where $f(\phi_1) = \min_{i=1,2}|A_{i,i}\cos(\phi_1+\alpha_i)|$.

It is straightforward to see that the maximum value of $f(\phi_1)$ occurs only when $|A_{i_{\min},i_{\min}}\cos(\phi_1+\alpha_{i_{\min}})|$ is maximized or $|A_{1,1}\cos(\phi_1+\alpha_1)| = |A_{2,2}\cos(\phi_1+\alpha_2)|$ where we define $i_{\min} = \arg\min_i A_{i,i}$. In other words, the objective function is maximized only when two envelopes have the same absolute value or the absolute value of the envelope with the small amplitude is maximized. According to the ratio $\frac{A_{1,1}}{A_{2,2}}$ and the phase offset $\alpha_1 - \alpha_2$, the point satisfying the above conditions is optimal for maximizing the objective function.

Now, we illustrate how to find the optimal ϕ_1. First, two envelopes have the same absolute value when $A_{1,1}\cos(\phi_1 + \alpha_1) = \pm A_{2,2}\cos(\phi_1 + \alpha_2)$. After some manipulations, we can see that this is satisfied when $\phi_1 = \tan^{-1}\left(\frac{A_{1,1}\cos\alpha_1 \pm A_{2,2}\cos\alpha_2}{A_{1,1}\sin\alpha_1 \pm A_{2,2}\sin\alpha_2}\right)$. Second, it is easy to check that the envelope with the small amplitude has its maximum absolute value at $\phi_1 = -\alpha_{i_{\min}}$. As a result, the transmit beamformers $\mathbf{v}_k$ and the receiver combiners $\mathbf{w}_k$ in our proposed scheme for $k = 1,2$ are computed as

$$\mathbf{v}_k = \sqrt{P_{\max}}\mathbf{j}(\phi_k^{\text{opt}}) \text{ and } \mathbf{w}_k = \mathbf{j}\left(\theta_{k,\bar{k}} + \phi_{\bar{k}}^{\text{opt}} - \frac{\pi}{2}\right),$$

where $\phi_1^{\text{opt}} = \arg\max_{\phi_1\in\chi} f(\phi_1)$. Here, the search candidate set χ of size 3 is given as

$$\chi = \left\{\tan^{-1}\left(\frac{A_{1,1}\cos\alpha_1 \pm A_{2,2}\cos\alpha_2}{A_{1,1}\sin\alpha_1 \pm A_{2,2}\sin\alpha_2}\right), -\alpha_{i_{\min}}\right\}.$$

B. 3-user SISO IFC

In this subsection, we extend our algorithm to the 3-user case. Unlike the 2-user SISO IFC, a closed form ZF design does not exist for the 3-user SISO IFC with constant channel coefficients and no symbol extension [20] [22]. This is due to lack of the receiver signal space. For this reason, in our strategy, we only allow two users to have the ZF transceiver using IA in [28]. As a result, the worst-user rate performance is dominated by the user with residual interference especially at the high SNR range. To solve this issue, the power control method is also proposed after the IA beamforming.

Similar to the 2-user case, the beamforming vector for user i is designed to have a form of $\mathbf{v}_i = \sqrt{P_i}\mathbf{j}(\phi_i)$ where P_i and $\mathbf{j}(\phi_i)$ determine the power level and the direction of $\mathbf{v}_i$, respectively. Then, the received signal for user i in (19) can be expressed as

$$\mathbf{y}_i = \sqrt{P_i}A_{i,i}\mathbf{j}(\theta_{i,i}+\phi_i)u_i + \sum_{j\neq i}\sqrt{P_j}A_{i,j}\mathbf{j}(\theta_{i,j}+\phi_j)u_j + \mathbf{n}_i. \quad (23)$$

First, we apply the IA scheme. To decode the desired information symbol in (23), two interference terms should be aligned. Without loss of generality, we assume that user 1 and 2 have the perfectly aligned interference. To this end, we adjust ϕ_i's to satisfy the following IA constraints for user 1 and 2 as

$$\text{span}(\mathbf{j}(\theta_{1,2}+\phi_2)) = \text{span}(\mathbf{j}(\theta_{1,3}+\phi_3)) \quad (24)$$
$$\text{span}(\mathbf{j}(\theta_{2,1}+\phi_1)) = \text{span}(\mathbf{j}(\theta_{2,3}+\phi_3)) \quad (25)$$

where span$(\mathbf{x})$ indicates the signal space spanned by a column vector $\mathbf{x}$.

Then, both (24) and (25) are fulfilled if $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are expressed as

$$\begin{aligned} \mathbf{v}_1 &= \sqrt{P_1}\mathbf{j}(\theta_{2,3} - \theta_{2,1} + \phi_3) \\ \mathbf{v}_2 &= \sqrt{P_2}\mathbf{j}(\theta_{1,3} - \theta_{1,2} + \phi_3) \\ \mathbf{v}_3 &= \sqrt{P_3}\mathbf{j}(\phi_3). \end{aligned} \quad (26)$$

Since the choice of ϕ_3 does not change the individual rate, we can set $\phi_3 = 0$ without loss of optimality [28].

With $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ in (26), link i for $i = 1, 2$ has the perfectly aligned interference as

$$\begin{aligned} \mathbf{y}_i = &\sqrt{P_i}A_{i,i}\mathbf{j}(\theta_{i,i} + \theta_{\bar{i},3} - \theta_{\bar{i},i})u_i \\ &+ (\sqrt{P_{\bar{i}}}A_{i,\bar{i}}u_{\bar{i}} + \sqrt{P_3}A_{i,3}u_3)\mathbf{j}(\theta_{i,3}) + \mathbf{n}_i. \end{aligned} \quad (27)$$

Hence, we can easily remove the interference terms by adopting a ZF receiver $\mathbf{w}_i = \mathbf{j}(\theta_{i,3} - \frac{\pi}{2})$. Then, the ZF filter output of user i $(i = 1, 2)$ is given as

$$\hat{u}_i = \mathbf{w}_i^T \mathbf{y}_i = \sqrt{P_i}A_{i,i}\cos(\alpha_{i,i}) + \tilde{n}_i \quad (28)$$

where we have $\alpha_{i,i} = \theta_{i,i} + \theta_{\bar{i},3} - \theta_{\bar{i},i} - \theta_{i,3} + \frac{\pi}{2}$ and $\tilde{n}_i = \mathbf{w}_i^T \mathbf{n}_i \sim N(0, \frac{\sigma_n^2}{2})$ is the filtered noise.

On the other hand, the received signal at link 3 contains interference signals which are not fully aligned and is expressed as

$$\mathbf{y}_3 = \sqrt{P_3}A_{3,3}\mathbf{j}(\theta_{3,3})u_3 + \sum_{i=1}^{2}\sqrt{P_i}A_{i,i}\mathbf{j}(\beta_{3,i})u_i + \mathbf{n}_3 \quad (29)$$

where we define $\beta_{3,i} = \theta_{3,i} + \theta_{\bar{i},3} - \theta_{\bar{i},i}$ for $i = 1, 2$. Since the condition of IA for user 3, span$(\mathbf{j}(\beta_{3,1}))$ = span$(\mathbf{j}(\beta_{3,2}))$, is measured as a zero event due to channel randomness, we cannot completely eliminate the interference terms in (29).

For this reason, we employ a receive filter which eliminates only one out of two interference terms in (29) for user 3 in our algorithm. We assume that the interference from BS 1 is eliminated using the receive filter $\mathbf{w}_3 = \mathbf{j}(\beta_{3,1} - \frac{\pi}{2})$ for the ease of explanation. The other case of interference nulling can be similarly explained. Then, the output of the filter $\mathbf{w}_3$ is written as

$$\begin{aligned} \hat{u}_3 = \mathbf{w}_3^T \mathbf{y}_3 = &\sqrt{P_3}A_{3,3}\cos(\theta_{3,3} - \beta_{3,1} + \frac{\pi}{2})u_3 \\ &+ \sqrt{P_2}A_{3,2}\cos(\beta_{3,2} - \beta_{3,1} + \frac{\pi}{2})u_2 + \tilde{n}_3, \end{aligned} \quad (30)$$

which contains the interference term from user 2. From (28) and (30), we have the following SINR expressions as

$$\gamma_1^{AC} = \frac{G_{1,1}P_1}{\frac{\sigma_n^2}{2}}, \quad \gamma_2^{AC} = \frac{G_{2,2}P_2}{\frac{\sigma_n^2}{2}}, \quad \gamma_3^{AC} = \frac{G_{3,3}P_3}{\frac{\sigma_n^2}{2} + G_{3,2}P_2} \quad (31)$$

where $G_{i,j} = (A_{i,j})^2\cos^2(\alpha_{i,j}), \alpha_{3,3} = \theta_{3,3} - \beta_{3,1} + \frac{\pi}{2}$ and $\alpha_{3,2} = \beta_{3,2} - \beta_{3,1} + \frac{\pi}{2}$.

Finally, for given $\{\mathbf{v}_i\}$, our original SINR balancing problem (21) is changed to a simple power allocation problem as

$$(P_1^*, P_2^*, P_3^*) = \arg\max_{0 \le P_i \le P_{\max},\ \forall i} \min\{\gamma_1^{AC}, \gamma_2^{AC}, \gamma_3^{AC}\}. \quad (32)$$

Since increasing P_1 and P_3 does not decrease the other user's SINR in (31), we set $P_1^* = P_3^* = P_{\max}$ without loss of generality. Then, γ_2^{AC} and γ_3^{AC} are a monotonically increasing and monotonically decreasing function with respect to P_2, respectively, and γ_1^{AC} becomes a constant value. Thus, we only adjust P_2 to maximize the worst-user SINR.

For simple explanation of deriving P_2^*, we define $\bar{\gamma}_i^{AC}(P_2) = \gamma_i^{AC}|_{P_1=P_3=P_{\max}}$ for $i = 1, 2, 3$. Then, if $\bar{\gamma}_2^{AC}(P_{\max}) > \bar{\gamma}_3^{AC}(P_{\max})$, the minimum of γ_i^{AC}'s is maximized when $\bar{\gamma}_2^{AC}(P_2) = \bar{\gamma}_3^{AC}(P_2)$, since there always exists a cross-over point within $0 \le P_2 \le P_{\max}$. Otherwise, we set $P_2^* = P_{\max}$ as a solution of (32). As a result, P_2^* is computed as (33) at the top of the next page.

In the above illustration, the beamforming vectors $\{\mathbf{v}_k\}$ are designed to have the perfectly aligned interference only for user 1 and 2. In contrast, user 3 has misaligned interference terms from BS 1 and 2, and we only eliminate the interference from BS 1 using the receive filters $\{\mathbf{w}_k\}$. In general, the beamforming vectors $\{\mathbf{v}_k\}$ can be determined so that misaligned interference remains at one of three users. Besides, between the interference terms from two aligned links, we choose one to which interference nulling is applied. Thus, there are 6 possible different designs in total, and the worst-user rate can be further improved if we choose the best one out of 6 possible cases.

For simple presentation, let us denote $\gamma_{\text{max-min}}^{AC}(i, j, k)$ as the maximized worst-user SINR corresponding to a system where beamformers perfectly align interference terms in link i and j $(i, j \in \{1, 2, 3\}, i \ne j)$ and the receivers null out the interference from BS k $(k \in \{i, j\})$ for the link with misaligned interference. Then, the optimal solution $\hat{\gamma}^{AC}$ is computed as

$$\hat{\gamma}^{AC} = \max_{\substack{i,j \in \{1,2,3\},\ i \ne j \\ k \in \{i,j\}}} \gamma_{\text{max-min}}^{AC}(i, j, k). \quad (34)$$

As a result, an additional selection diversity is expected by choosing the best one of 6 possible cases. In the following section, we show that our proposed schemes achieve much improved performance over the power control schemes with the SC signaling in terms of the worst-user rate.

VI. Simulation Results

In this section, we present simulation results to confirm the effectiveness of the AC signaling in terms of the worst-user rate. In our simulation, it is assumed that the channel coefficients are sampled from independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. We present the average worst-user rate of the following schemes as a function of $\frac{P_{\max}}{\sigma_n^2}$.

- non-cooperative: full power transmission at all BSs, i.e., $P_1 = \cdots = P_K = P_{\max}$, which is well known to approach Nash equilibrium [31].
- exhaustive search: the optimal solution obtained by searching all possible power levels and phase values for the SC and AC signaling, respectively.

Figure 4 plots the average worst-user rate performance for 2-user SISO IFC. Here, the AC with exhaustive search of ϕ_i indicates the optimal phase search under ZF constraint in (22). As expected, the proposed SC signaling scheme exhibits

$$P_2^* = \begin{cases} \dfrac{-2G_{2,2} + \sqrt{\sigma_n^4 G_{2,2}^2 + 8\sigma_n^2 G_{2,2} G_{3,2} G_{3,3} P_{\max}}}{4G_{2,2}G_{3,2}}, & \text{if } \bar{\gamma}_2^{AC}(P_{\max}) > \bar{\gamma}_3^{AC}(P_{\max}), \\ P_{\max}, & \text{otherwise.} \end{cases} \tag{33}$$

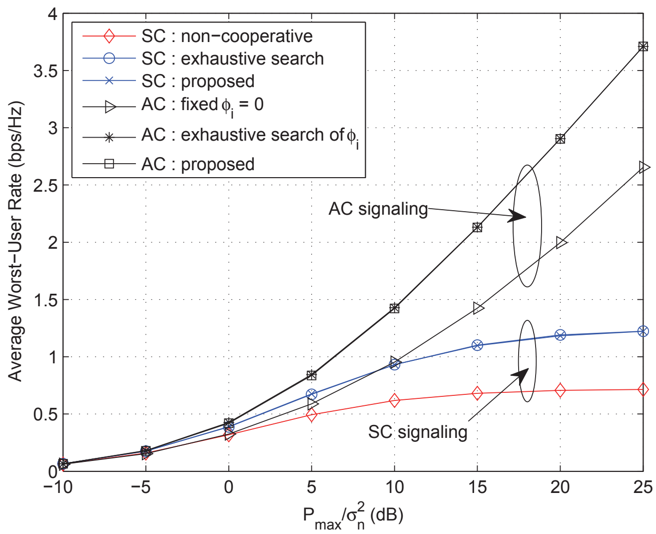


Fig. 4. Average worst-user rate for 2-user SISO IFC systems.

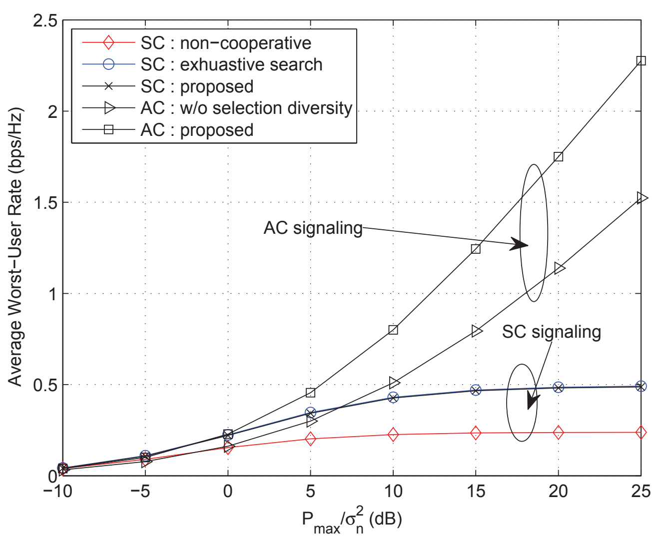


Fig. 5. Average worst-user rate for 3-user SISO IFC systems.

the same performance as the optimal exhaustive search, and outperforms the non-cooperative method. However, the SINR balancing schemes with the SC signaling cannot attain a linear increase w.r.t. SNR, since the number of data streams is 2 and this is greater than the optimal DOF which is given by 1 [32]. On the other hand, the transmission schemes based on the AC signaling provide the worst-user rate which increases linearly with the SNR. The proposed choice of ϕ_1 and ϕ_2 achieves the same performance as the exhaustive search algorithm and obtains a performance gain of about 7 dB compared to the choice of $\phi_1 = \phi_2 = 0$.

In Figure 5, we illustrate the average worst-user rate performance for 3-user SISO IFC. The AC signaling without selection diversity indicates the proposed AC signaling which considers only one possible transceiver design in (34), i.e., the selection diversity is not exploited. Similar to the 2-user case, the transmission schemes based on the SC signaling does not exhibit a linear increase w.r.t. $\frac{P_{\max}}{\sigma_n^2}$, whereas the worst-user rate of the proposed AC signaling grows linearly. Although the worst-user rate of the AC signaling is dominated by the misaligned link due to residual interference at the high SNR region, the optimal power control (33) compensates for a loss from the residual interference. Also, the proposed AC signaling exploits selection diversity which leads to an additional gain of 7.5 dB over the AC signaling without selection at 1.5 bps/Hz, and as a result, the proposed AC signaling outperforms the SC signaling algorithms for overall SNR region at the expense of increased computational complexity.

VII. Conclusion

In this paper, we have addressed the problem of SINR balancing for 2 and 3-user SISO IFC systems. We have first studied achievable rate regions of both SC and AC signalings. It is shown that the AC signaling has a potential to outperform the SC signaling as SNR increases, since the SC signaling schemes transmit streams more than the optimal DOF. Thus, we have proposed an efficient SINR balancing algorithms based on the AC signaling to realize a performance gain over the proposed SC signaling schemes for both two and three-user SISO IFCs. Simulation results confirm the effectiveness of the proposed SINR balancing algorithms and show that a substantial gain of the AC signaling is achieved over the SC signaling in terms of the maximum worst-user rate.

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