Achievable Degrees of Freedom on MIMO Two-way Relay Interference Channels

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***Abstract*—In this paper, we study new network information flow called multiple-input multiple-output (MIMO) two-way relay interference channels where two links of relay systems are interfering with each other. In this system, we characterize the achievable total degrees of freedom (DOF) when all user nodes and relays have M and N antennas, respectively. We provide three different methods, namely, time-division multiple access, signal space alignment for network coding (SSA-NC), and a new interference neutralization (IN) scheme. In the SSA-NC scheme, one relay is selected to fully exploit the dimension of the chosen relay for network coding. For the IN, we propose a new relay transmission scheme where two relays cooperatively design the beamforming vectors so that the interference signals are neutralized at each receiver. By adopting three different relaying strategies, we show that the DOF of $\max\left\{\min(4N, 2M), \min(2N, 2\lfloor\frac{4}{3}M\rfloor), \min(2N-1, 4M)\right\}$ is achieved for MIMO two-way relay interference channels.**

***Index Terms*—Degrees of freedom, two-way relay, interference channel.**

I. Introduction

WIRELESS relay networks have been studied with a lot of interest because relaying transmission is a promising technique which can be applied to extend the coverage or increase the system capacity [1] [2]. Conventional one-way relay systems suffer from a substantial performance loss in terms of spectral efficiency due to the pre-log factor 1/2 induced by the fact that two channel uses are required for one transmission. Two way relay systems have been suggested to overcome a loss of spectral efficiency in such one-way relay methods [3] [4] [5]. The two-way relay channel consists of two phases: the multiple access (MAC) phase and the broadcast (BC) phase. During the MAC phase, two users simultaneously send their messages to an intermediate relay. In the BC phase, the relay retransmits the received information to two users based on various relay operations such as amplify-and-forward (AF) and decode-and-forward (DF) [4]. By exploiting the knowledge of their own transmitted information, each user is able to cancel self-interference and decode the intended message.

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Two-way relay channels can be generalized to support a multi-pair setting where a single relay helps communication between multiple pairs of users [6]–[8]. In [6], the authors proposed a jointly demodulate-and-XOR forward relaying scheme for code division multiple access systems under interference limited environments. Also, the work in [7] and [8] examined the capacity of multi-pair two-way relay channels in the deterministic channel and the Gaussian channel, respectively.

General scenarios other than multi-pair two-way relay channels were investigated in [9]–[11]. Multiple interfering clusters of users were considered in [9], which communicate simultaneously with the help of a relay where the users within the same cluster exchange messages among themselves. They studied an achievable rate region of this multi-way relay channel according to different relaying schemes. By taking into account multiple transmit messages per user, multiple-input multiple-output (MIMO) Y channels with K-users and a single relay were introduced in [10] and [11], where each user wants to unicast independent messages for other $K-1$ users via the relay. They proposed an efficient scheme to deal with multiple interference signals in multi-user bi-directional communication systems, and characterized the achievable degrees of freedom (DOF) of K-user Y channels in [11].

Another direction of extension for relay systems is to consider multiple links of two-way relay systems. Motivation of this channel comes from a practical scenario in wireless networks, where two different user pairs want to simultaneously exchange their information in the same frequency band via two distinct relays. Hence, if all users concurrently transmit their own signals in the multi-link two-way channel, the relays suffer from interference signals, and an issue on interference management becomes critical. Also, this is an interesting problem from an information theoretical perspective, since this multi-link two-way relay channel can be considered as a combination of two-way relay channels and interference channels [12] [13]. We call this new model as *two-way relay interference channels*. In such a case, our fundamental question is how each user pair and relays should cooperate to maximize the transmission rate of the network. In general, the capacity of two-way relay interference channels is not known. Many researchers tried to investigate the DOF for such an interference limited channel, as the DOF reflects the capacity scaling behavior at high signal-to-noise ratio (SNR) [12] [14]. Thus, study of the achievable sum DOF is an initial step for addressing this open problem.

The main contribution of this paper is to provide a lower bound of the sum DOF for the two-way relay interference channel depending on antenna configurations when all user nodes have M antennas and relay nodes have N antennas. Specifically, we show that the DOF of $\min(4N, 2M)$ is achieved by time-division multiple access (TDMA) when $N \leq M$, and the DOF of $\min(2N, 2\lfloor\frac{4}{3}M\rfloor)$ is obtained by signal space alignment for network coding (SSA-NC) in [10] [11] when $M < N \leq \frac{4}{3}M$. Also, for the case of $N > \frac{4}{3}M$, we show that the DOF of $\min(2N-1, 4M)$ is achieved by performing the proposed interference neutralization scheme, which exploits side-information knowledge inherently coming from two-way communication so as to obtain both interference neutralization gain and network coding gain. As a result, the achievable sum DOF of $\max\{\min(4N, 2M), \min(2N, 2\lfloor\frac{4}{3}M\rfloor), \min(2N-1, 4M)\}$ will be attained for two-way relay interference channels. Through numerical simulations, we will confirm the derived achievable sum DOF results.

This paper is organized as follows: Section II describes the system model of MIMO two-way relay interference channels. In Section III, we propose a new relay transmission scheme where two relays cooperatively design the beamforming vectors so that the interference signals are neutralized at each receiver. In Section IV, we investigate the achievable sum DOF according to antenna configurations with three beamforming strategies using the AF relaying protocol. Section V provides discussions about transmission strategies and verifies the derived results through numerical simulations. Finally, the paper is terminated with conclusions in Section VI.

Throughout this paper, transpose, conjugate transpose, inverse and trace of a matrix $\mathbf{A}$ are represented by $\mathbf{A}^T$, $\mathbf{A}^H$, $(\mathbf{A})^{-1}$ and $\text{Tr}\{\mathbf{A}\}$, respectively. Also, $\text{span}(\mathbf{A})$ denotes the column space spanned by the columns of $\mathbf{A}$ and $\mathcal{E}(\cdot)$ indicates the expectation operator.

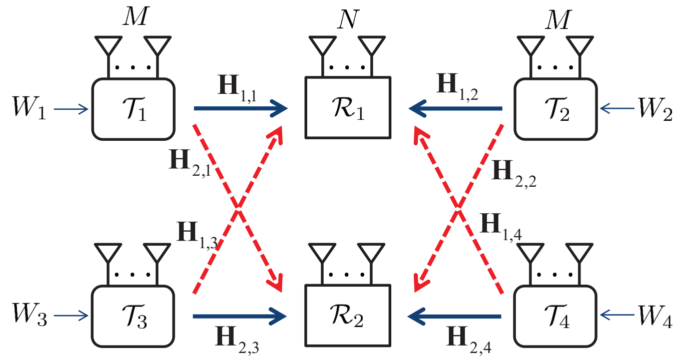
II. System Model

We describe a system model for MIMO two-way relay interference channels. We consider two links of two-way relay channels which are interfering with each other as shown in Fig. 1. In this channel, two users of the first link $\mathcal{T}_1$ and $\mathcal{T}_2$ exchange information W_1 and W_2 with each other via the relay $\mathcal{R}_1$, while users $\mathcal{T}_3$ and $\mathcal{T}_4$ of the second link exchange data messages W_3 and W_4 through the relay $\mathcal{R}_2$. We assume that all user nodes and relays have M and N antennas, respectively. In the MAC phase, $\mathcal{T}_i$ ($i \in \mathbb{U} \triangleq \{1,2,3,4\}$) sends the data symbol vector $\mathbf{s}_i = [s_{i,1}\ s_{i,2}\ \cdots s_{i,d_i}]^T$ by applying the beamforming matrix $\mathbf{P}_i = [\mathbf{p}_{i,1}\ \mathbf{p}_{i,2}\ \cdots \mathbf{p}_{i,d_i}]$ as $\mathbf{x}_i = \mathbf{P}_i\mathbf{s}_i$ where d_i is the number of data streams at $\mathcal{T}_i$. It is assumed that $\mathbf{P}_i$ satisfies transmit power constraint for $\mathcal{T}_i$ as $\mathcal{E}\left(\text{Tr}(\mathbf{P}_i\mathbf{P}_i^H)\right) \leq P$ and $\mathcal{E}(\mathbf{s}_i\mathbf{s}_i^H) = \mathbf{I}_{d_i}$ for $\forall i$.

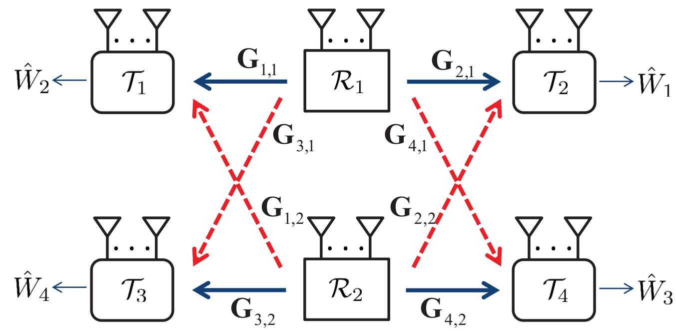
Then, the received signal vector $\mathbf{r}_l$ at the l-th link relay is given by

$$\mathbf{r}_l = \sum_{i\in\mathbb{U}} \mathbf{H}_{l,i}\mathbf{x}_i + \mathbf{z}_{\mathcal{R}_l} \quad \text{for} \quad l \in \mathbb{L} \triangleq \{1,2\}, \tag{1}$$

where $\mathbf{H}_{l,i}$ represents the $N\times M$ channel matrix from $\mathcal{T}_i$ to the relay $\mathcal{R}_l$ and $\mathbf{z}_{\mathcal{R}_l}$ denotes the additive white Gaussian noise (AWGN) at $\mathcal{R}_l$ with zero mean and $\mathcal{E}(\mathbf{z}_{\mathcal{R}_l}\mathbf{z}_{\mathcal{R}_l}^H) = \sigma^2\mathbf{I}_N$. It is assumed that channel elements are independent and identically distributed (i.i.d.) complex Gaussian random variables with



(a) MAC phase



(b) BC phase

Fig. 1. System model of two-way relay interference channels.

zero mean and unit variance. We also assume that channel state information (CSI) is perfectly known at all nodes.

After the MAC phase, each relay $\mathcal{R}_l$ generates the signal $\mathbf{x}_{\mathcal{R}_l} = \gamma_l\bar{\mathbf{W}}_l\mathbf{r}_l$ by multiplying the $N\times N$ relay filter matrix $\bar{\mathbf{W}}_l$ with power normalizing factor γ_l and broadcasts them to its users during the BC phase. Here, we denote $\mathbf{W}_l = \gamma_l\bar{\mathbf{W}}_l$ and the power normalizing factor γ_l is determined as $\gamma_l \leq \sqrt{P/\mathcal{E}\left(\text{Tr}\left(\bar{\mathbf{W}}_l\mathbf{r}_l\mathbf{r}_l^H\bar{\mathbf{W}}_l^H\right)\right)}$ to satisfy the relay power constraint. Then, the received signal vector of $\mathcal{T}_i$ is described as

$$\mathbf{y}_i = \sum_{l\in\mathbb{L}} \mathbf{G}_{i,l}\mathbf{x}_{\mathcal{R}_l} + \mathbf{z}_i \quad \text{for} \quad i \in \mathbb{U}, \tag{2}$$

where $\mathbf{G}_{i,l}$ stands for the $M\times N$ channel matrix from the l-th relay $\mathcal{R}_l$ to user $\mathcal{T}_i$ and $\mathbf{z}_i$ indicates the AWGN with variance σ^2 at $\mathcal{T}_i$. After removing self-interferences, user $\mathcal{T}_i$ detects its desired signal $\hat{\mathbf{s}}_{\bar{i}}$ by applying the receive combining matrix $\mathbf{D}_i$. Here, we define $\bar{i}$ as the index of user i's partner, e.g., $\bar{1} = 2$, $\bar{2} = 1$, $\bar{3} = 4$ and $\bar{4} = 3$.

Then, the $d_{\bar{i}}\times 1$ receive filter output vector for the message $\mathbf{s}_{\bar{i}}$ can be represented as

$$\begin{aligned}\hat{\mathbf{y}}_i &= \mathbf{D}_i^H\left(\mathbf{y}_i - \sum_{l\in\mathbb{L}} \mathbf{G}_{i,l}\mathbf{W}_l\mathbf{H}_{l,i}\mathbf{x}_i\right)\\ &= \mathbf{D}_i^H(\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,\bar{i}} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,\bar{i}})\mathbf{P}_{\bar{i}}\mathbf{s}_{\bar{i}} + \mathbf{i}_i + \mathbf{n}_i \end{aligned} \tag{3}$$

where $\mathbf{i}_i = \sum_{k\in\mathbb{U}\setminus\{i,\bar{i}\}} \mathbf{D}_i^H\left(\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,k} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,k}\right) \mathbf{P}_k\mathbf{s}_k$ indicates the interference signal vector from other link users to user $\mathcal{T}_i$ and $\mathbf{n}_i = \mathbf{D}_i^H\left(\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{z}_{\mathcal{R}_1} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{z}_{\mathcal{R}_2} + \mathbf{z}_i\right)$ denotes the noise vector at user $\mathcal{T}_i$. Because implementation of multi-user detection may be difficult, the interference terms $\mathbf{i}_i$ are treated as noise. Note that we should design $\mathbf{P}_i$, $\mathbf{W}_l$ and $\mathbf{D}_i$ for $\forall i, l$ so that the interference $\mathbf{i}_i$ is nullified while making the effective channel $\mathbf{D}_{\bar{i}}^H(\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,\bar{i}} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,\bar{i}})\mathbf{P}_{\bar{i}}$ have rank $d_{\bar{i}}$ for decoding the symbol $\mathbf{s}_{\bar{i}}$.

Throughout this paper, we assume that two relays operate in the amplify-and-forward (AF) protocol and the full-duplex mode[1] which implies that all nodes can transmit and receive simultaneously. Then, for a given set of the beamforming matrices $\mathbf{P}_i$, $\mathbf{W}_l$ and $\mathbf{D}_i$ for $\forall i, l$ satisfying the power constraint P, we can compute the achievable rate $R_i(\text{SNR})$ for the message W_i as a function of SNR as

$$\begin{aligned} R_i(\text{SNR}) \;&=\; \log_2 \det\left(\mathcal{E}\left\{\hat{\mathbf{y}}_{\bar{i}}\hat{\mathbf{y}}_{\bar{i}}^H\right\}\right) \\ &\quad - \log_2 \det\left(\mathcal{E}\left\{\mathbf{i}_{\bar{i}}\mathbf{i}_{\bar{i}}^H + \mathbf{n}_{\bar{i}}\mathbf{n}_{\bar{i}}^H\right\}\right) \end{aligned} \tag{4}$$

where SNR is defined as P/σ^2. Then, the achievable DOF of user i is expressed as $d_i = \lim_{\text{SNR}\to\infty} \frac{R_i(\text{SNR})}{\log_2(\text{SNR})}$. Finally, the sum of the achievable DOF η_{sum} for multi-link two-way relay interference channels is defined as

$$\eta_{sum} = \sum_{i\in\mathbb{U}} d_i = \lim_{\text{SNR}\to\infty} \sum_{i\in\mathbb{U}} \frac{R_i(\text{SNR})}{\log_2(\text{SNR})}.$$

III. Proposed Interference Neutralization Scheme

In this section, we first propose a new interference neutralization (IN) scheme for two-way relay interference channels. The IN scheme was considered in [15], which allows over-the-air interference removal before arriving at the undesired destination, and it was generalized by combining the ideas of interference alignment (IA) in the relay system. For the one-way relay interference channel, aligned interference neutralization was proposed in [16] and showed that the DOF of $2M-1$ is achievable when all nodes have M antennas. Further, interference-shaping and neutralization was presented in [17] in the context of multiuser two-hop MIMO interference channels where IA for neutralization is not required by the interference-shaping method.

In this paper, we extend the idea of interference-shaping and the neutralization method into the two-way relay interference channel. The key idea of the proposed method is to exploit side-information knowledge inherently given from two-way communications when the relay performs IN. Specifically, after using interference-shaping, the relays neutralize the inter-link interference signals over the air at the destination by utilizing the fact that each user has knowledge of the self-interference signal. Thus, unlike [17], the proposed method allows us to obtain not only an interference neutralization gain and but also a network coding gain.

To provide an intuition of the proposed scheme, we first consider a simple case with $M=2$ and $N=4$. Through this example, we will show that the DOF of $2N-1$ is achieved. To this end, we assume that three users $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3$ send two message symbols and one user $\mathcal{T}_4$ transmits one message symbol simultaneously to the corresponding partner user so that the DOF of 7 is achieved. Here, each user does not use the beamforming matrix, i.e., $\mathbf{P}_i = \mathbf{I}$ with $\mathcal{E}\left(\text{Tr}(\mathbf{s}_i\mathbf{s}_i^H)\right) = P$ for $i \in \{1,2,3\}$ and user $\mathcal{T}_4$ selects one antenna with $\mathcal{E}\left(|s_4|^2\right) = P$.

1) MAC Phase: During the MAC phase, user $\mathcal{T}_i$ transmits $\mathbf{s}_i$ and then the received signal at the l-th link relay becomes

$$\mathbf{r}_l = \mathbf{H}_{l,1}\mathbf{s}_1 + \mathbf{H}_{l,2}\mathbf{s}_2 + \mathbf{H}_{l,3}\mathbf{s}_3 + \mathbf{h}_{l,4}s_4 + \mathbf{z}_{\mathcal{R}_l} \quad \text{for} \quad l \in \mathbb{L}.$$

Each relay multiplies the receive combining vectors $\mathbf{V}_l = \left[\mathbf{v}_{l,(1)}, \mathbf{v}_{l,(2)}\right]$ for $l \in \mathbb{L}$ and obtains the linear combination of the transmitted data symbols $\mathbf{s}_i$ with their channel gains $\mathbf{V}_l^H\mathbf{H}_{l,i}$, $\forall i, l$.

We will design these receive combining vectors by exploiting the interference-shaping scheme proposed in [17]. The main idea of interference-shaping is to select the relay combining vectors so that the ratios of the effective channel gains observed by two relays $\mathcal{R}_1$ and $\mathcal{R}_2$ are the same for the first link symbols ($\mathbf{s}_1$, $\mathbf{s}_2$) and the second link symbols ($\mathbf{s}_3$, s_4), i.e., $\frac{\mathbf{v}_{1,(n)}^H\mathbf{H}_{1,1}\mathbf{s}_1}{\mathbf{v}_{2,(n)}^H\mathbf{H}_{2,1}\mathbf{s}_1} = \frac{\mathbf{v}_{1,(n)}^H\mathbf{H}_{1,2}\mathbf{s}_2}{\mathbf{v}_{2,(n)}^H\mathbf{H}_{2,2}\mathbf{s}_2} \triangleq \alpha_{(n)}$ and $\frac{\mathbf{v}_{1,(n)}^H\mathbf{H}_{1,3}\mathbf{s}_3}{\mathbf{v}_{2,(n)}^H\mathbf{H}_{2,3}\mathbf{s}_3} = \frac{\mathbf{v}_{1,(n)}^H\mathbf{h}_{1,4}s_4}{\mathbf{v}_{2,(n)}^H\mathbf{h}_{2,4}s_4} \triangleq \beta_{(n)}$ for $n=1$ and 2 where $\alpha_{(n)}$ and $\beta_{(n)}$ represent arbitrary non-zero real values. Here, we assume that the channel gain ratio values $\alpha_{(n)}$ and $\beta_{(n)}$ can be chosen artificially in order to satisfy the conditions of $\alpha_{(n)} \neq \beta_{(n)}$ and $\frac{\alpha_{(1)}}{\beta_{(1)}} \neq \frac{\alpha_{(2)}}{\beta_{(2)}}$.

For given $\alpha_{(n)}$ and $\beta_{(n)}$, the receive combining vectors are determined such that the following conditions are satisfied

$$\mathbf{v}_{1,(n)}^H\left[\mathbf{H}_{1,1}\ \mathbf{H}_{1,2}\right] = \alpha_{(n)}\mathbf{v}_{2,(n)}^H\left[\mathbf{H}_{2,1}\ \mathbf{H}_{2,2}\right], \tag{5}$$

and

$$\mathbf{v}_{1,(n)}^H\left[\mathbf{H}_{1,3}\ \mathbf{h}_{1,4}\right] = \beta_{(n)}\mathbf{v}_{2,(n)}^H\left[\mathbf{H}_{2,3}\ \mathbf{h}_{2,4}\right]. \tag{6}$$

Let us denote the channel matrix $\left[\mathbf{H}_{l,i}\ \mathbf{H}_{l,\bar{i}}\right]$ as $\mathbf{H}_{li,l\bar{i}}$ and by using the fact that all elements of channel vectors are drawn from a continuous distribution (equivalently generic channel matrices) which implies that $\mathbf{H}_{11,12}$ and $\mathbf{H}_{21,22}$ have full rank with probability one, it is rewritten as

$$\begin{aligned} \frac{1}{\alpha_{(n)}}\mathbf{v}_{1,(n)}^H\mathbf{H}_{11,12}\mathbf{H}_{21,22}^{-1} &= \mathbf{v}_{2,(n)}^H, \\ \mathbf{v}_{1,(n)}^H\mathbf{H}_{13,14} &= \beta_{(n)}\mathbf{v}_{2,(n)}^H\mathbf{H}_{23,24}. \end{aligned} \tag{7}$$

Then, it follows

$$\mathbf{v}_{1,(n)}^H\left(\mathbf{H}_{13,14} - \frac{\beta_{(n)}}{\alpha_{(n)}}\mathbf{H}_{11,12}\mathbf{H}_{21,22}^{-1}\mathbf{H}_{23,24}\right) = \mathbf{0}. \tag{8}$$

From (8), for given $\alpha_{(n)}$ and $\beta_{(n)}$, the receive combining vector for $\mathbf{v}_{1,(n)}$ is determined by the left nullspace of the matrix $\mathbf{H}_{13,14} - \frac{\beta_{(n)}}{\alpha_{(n)}}\mathbf{H}_{11,12}\mathbf{H}_{21,22}^{-1}\mathbf{H}_{23,24}$ whose size equals 4×3. Due to the assumption of genetic channel matrices, the dimension of the left nullspace is almost surely one and $\mathbf{v}_{1,(n)}$ always exists. Also, since we assume $\frac{\beta_{(1)}}{\alpha_{(1)}} \neq \frac{\beta_{(2)}}{\alpha_{(2)}}$, $\mathbf{v}_{1,(1)}$ and $\mathbf{v}_{1,(2)}$ must be linearly independent. After designing $\mathbf{v}_{1,(n)}$, the receive combining vector $\mathbf{v}_{2,(n)}$ of the second link relay is obtained from (7).

From (5) and (6), the filter output at the relay becomes

$$\begin{aligned} \mathbf{v}_{1,(n)}^H\mathbf{r}_1 \;&=\; L^{(n)}(\mathbf{s}_1,\mathbf{s}_2) + L^{(n)}(\mathbf{s}_3,s_4) + \tilde{z}_{\mathcal{R}_{1,(n)}} \\ \mathbf{v}_{2,(n)}^H\mathbf{r}_2 \;&=\; \frac{1}{\alpha_1}L^{(n)}(\mathbf{s}_1,\mathbf{s}_2) + \frac{1}{\beta_1}L^{(n)}(\mathbf{s}_3,s_4) + \tilde{z}_{\mathcal{R}_{2,(n)}} \end{aligned}$$

[1] It can be easily shown that the DOF is reduced by the factor of two if the half-duplex mode is employed.

where $L^{(n)}(\mathbf{s}_1,\mathbf{s}_2) = \mathbf{v}_{1,(n)}^H\mathbf{H}_{1,1}\mathbf{s}_1 + \mathbf{v}_{1,(n)}^H\mathbf{H}_{1,2}\mathbf{s}_2$ and $L^{(n)}(\mathbf{s}_3,s_4) = \mathbf{v}_{1,(n)}^H\mathbf{H}_{1,3}\mathbf{s}_3 + \mathbf{v}_{1,(n)}^H\mathbf{h}_{1,4}s_4$ denote the linear combination of the link 1 symbols and link 2 symbols, respectively, and $\tilde{z}_{\mathcal{R}_l,(n)} = \mathbf{v}_{l,(n)}^H\mathbf{z}_{\mathcal{R}_l}$. Note that two relays contain the same linear combinations $L^{(n)}(\mathbf{s}_1,\mathbf{s}_2)$ and $L^{(n)}(\mathbf{s}_3,s_4)$ with different scaling factors $\alpha_{(n)}$ and $\beta_{(n)}$.

2) BC Phase: After obtaining the desired superposition form of the transmitted data symbols, two relays cooperatively transmit $\mathbf{v}_{1,(n)}^H\mathbf{r}_1$ and $\mathbf{v}_{2,(n)}^H\mathbf{r}_2$ to all users along with the transmit beamforming vectors $\mathbf{u}_{1,(n)}$ and $\mathbf{u}_{2,(n)}$, respectively. The objective of the beamforming vector design is to neutralize inter-link interference signals. Since each user has knowledge of self-interference, each user can decode the desired signal if there is no inter-link interference. For example, user 1 wants to decode $\mathbf{s}_2$ and has side-information $\mathbf{s}_1$. Thus, user 1 can decode $\mathbf{s}_2$ if it receives the signal $L^{(n)}(\mathbf{s}_1,\mathbf{s}_2)$ from two relays by removing the self-interference. Similarly, user 2 is able to decode $\mathbf{s}_1$ if the signal $L^{(n)}(\mathbf{s}_1,\mathbf{s}_2)$ is obtained during the BC phase. Therefore, we only need to neutralize inter-link interference signals comprised of $L^{(n)}(\mathbf{s}_3,s_4)$ by selecting the transmit beamforming vectors appropriately during the BC phase.

To accomplish this, the relay transmit beamforming vectors are designed such that the following condition is satisfied as

$$-\beta_{(n)}\begin{bmatrix}\mathbf{G}_{1,1}\\ \mathbf{G}_{2,1}\end{bmatrix}\mathbf{u}_{1,(n)} = \begin{bmatrix}\mathbf{G}_{1,2}\\ \mathbf{G}_{2,2}\end{bmatrix}\mathbf{u}_{2,(n)}.$$

By symmetry, $L^{(n)}(\mathbf{s}_1,\mathbf{s}_2)$ should be neutralized at the receiver of the link 2 users, i.e., user 3 and user 4 by satisfying the condition

$$-\alpha_{(n)}\begin{bmatrix}\mathbf{g}_{3,1}\\ \mathbf{G}_{4,1}\end{bmatrix}\mathbf{u}_{1,(n)} = \begin{bmatrix}\mathbf{g}_{3,2}\\ \mathbf{G}_{4,2}\end{bmatrix}\mathbf{u}_{2,(n)}.$$

Here, we denote user $\mathcal{T}_3$'s channel as a row vector $\mathbf{g}_{3,l}$ for $l = 1,2$ by operating only one antenna by antenna selection. This is due to the fact that user $\mathcal{T}_3$ will receive only one symbol s_4 from user $\mathcal{T}_4$ and this enables to design the relay beamforming vectors which satisfy the above conditions.

Then denoting $\mathbf{G}_{il,\bar{i}l} = \left[\mathbf{G}_{i,l}^T\ \mathbf{G}_{\bar{i},l}^T\right]^T$, the above two conditions can be described as

$$-\beta_{(n)}\mathbf{G}_{12,22}^{-1}\mathbf{G}_{11,21}\mathbf{u}_{1,(n)} = \mathbf{u}_{2,(n)}, \tag{9}$$
$$\mathbf{G}_{31,41}\mathbf{u}_1 = -\frac{1}{\alpha_{(n)}}\mathbf{G}_{32,42}\mathbf{u}_2.$$

This becomes

$$\left(\mathbf{G}_{31,41} - \frac{\beta_{(n)}}{\alpha_{(n)}}\mathbf{G}_{32,42}\mathbf{G}_{12,22}^{-1}\mathbf{G}_{11,21}\right)\mathbf{u}_{1,(n)} = \mathbf{0}. \tag{10}$$

The relay beamforming vector $\mathbf{u}_{1,(n)}$ can be calculated as the nulling vector of the matrix $\mathbf{G}_{31,41} - \frac{\beta_{(n)}}{\alpha_{(n)}}\mathbf{G}_{32,42}\mathbf{G}_{12,22}^{-1}\mathbf{G}_{11,21}$ whose size is 3×4. In a similar manner as in the MAC phase, $\mathbf{u}_{1,(1)}$ and $\mathbf{u}_{1,(2)}$ always exist and are linearly independent. We can subsequently decide the transmit beamforming vectors $\mathbf{u}_{2,(1)}$ and $\mathbf{u}_{2,(2)}$ for $\mathcal{R}_2$ from (9).

Finally, the relays amplify and forward its received signal $\mathbf{r}_1$ and $\mathbf{r}_2$ by multiplying the relay beamforming matrices $\bar{\mathbf{W}}_1 = \sum_{n=1}^2 \mathbf{u}_{1,(n)}\mathbf{v}_{1,(n)}^H = \mathbf{U}_1\mathbf{V}_1^H$ and $\bar{\mathbf{W}}_2 = \sum_{n=1}^2 \mathbf{u}_{2,(n)}\mathbf{v}_{2,(n)}^H = \mathbf{U}_2\mathbf{V}_2^H$, respectively, where $\mathbf{U}_l = \left[\mathbf{u}_{l,(1)}\ \mathbf{u}_{l,(2)}\right]$. In order to satisfy power constraints and the neutralization conditions, the relay beamforming matrices are obtained as $\mathbf{W}_l = \gamma_{min}\bar{\mathbf{W}}_l$ where γ_{min} represents the power normalizing factor as $\gamma_{min} = \min(\gamma_1,\gamma_2)$ where $\gamma_l = \sqrt{P/\mathcal{E}\left(\mathrm{Tr}\left(\bar{\mathbf{W}}_l\mathbf{r}_l\mathbf{r}_l^H\bar{\mathbf{W}}_l^H\right)\right)}$ for $l = 1,2$.

Let us consider the received signal of user $\mathcal{T}_1$ during the BC phase as

$$\begin{aligned}
\mathbf{y}_1 &= \mathbf{G}_{1,1}\mathbf{W}_1\mathbf{r}_1 + \mathbf{G}_{1,2}\mathbf{W}_2\mathbf{r}_2 + \mathbf{n}_1\\
&= \gamma_{min}\Bigg(\mathbf{G}_{1,1}\Big(\sum_{n=1}^2\mathbf{u}_{1,(n)}\mathbf{v}_{1,(n)}^H\Big)\mathbf{r}_1\\
&\qquad + \mathbf{G}_{1,2}\Big(\sum_{n=1}^2\mathbf{u}_{2,(n)}\mathbf{v}_{2,(n)}^H\Big)\mathbf{r}_2\Bigg) + \mathbf{n}_1\\
&= \gamma_{min}\Bigg(\Big(\sum_{n=1}^2\mathbf{G}_{1,1}\mathbf{u}_{1,(n)}\mathbf{v}_{1,(n)}^H\Big)\mathbf{r}_1\\
&\qquad + \Big(\sum_{n=1}^2 -\beta_{(n)}\mathbf{G}_{1,1}\mathbf{u}_{1,(n)}\mathbf{v}_{2,(n)}^H\Big)\mathbf{r}_2\Bigg) + \mathbf{n}_1\\
&= \gamma_{min}\Bigg(\sum_{n=1}^2\mathbf{G}_{1,1}\mathbf{u}_{1,(n)}\Big(\mathbf{v}_{1,(n)}^H\mathbf{r}_1 - \beta_{(n)}\mathbf{v}_{2,(n)}^H\mathbf{r}_2\Big)\Bigg) + \mathbf{n}_1\\
&= \gamma_{min}\Bigg(\sum_{n=1}^2\mathbf{G}_{1,1}\mathbf{u}_{1,(n)}\Big(L^{(n)}(\mathbf{s}_1,\mathbf{s}_2) - \frac{\beta_{(n)}}{\alpha_{(n)}}L^{(n)}(\mathbf{s}_1,\mathbf{s}_2)\Big)\Bigg)\\
&\quad + \tilde{\mathbf{n}}_1
\end{aligned}$$

where $\tilde{\mathbf{n}}_i = \mathbf{G}_{i,1}\mathbf{W}_1\mathbf{z}_{\mathcal{R}_1} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{z}_{\mathcal{R}_2} + \mathbf{n}_i$. The last equality follows from the fact that the interference signals of $L^{(n)}(s_3,s_4)$ are canceled (neutralized) over the air at the destination.

After removing self-interference, user $\mathcal{T}_1$ can detect its desired symbol $\mathbf{s}_2$ without any interference signal as

$$\mathbf{y}_1^{'} = \gamma_{min}\mathbf{G}_{1,1}\mathbf{U}_1\Sigma_1\mathbf{V}_1^H\mathbf{H}_{1,2}\mathbf{s}_2 + \tilde{\mathbf{n}}_1, \tag{11}$$

where $\Sigma_1 = \mathrm{diag}\left(\frac{\alpha_{(1)}-\beta_{(1)}}{\alpha_{(1)}},\ \frac{\alpha_{(2)}-\beta_{(2)}}{\alpha_{(2)}}\right)$. The relay beamforming vectors $\mathbf{U}_1$ and $\mathbf{V}_1$ have been designed to be linearly independent so that the effective channel is guaranteed to be full rank. Thus, user $\mathcal{T}_1$ is able to decode the desired symbol $\mathbf{s}_2$ by multiplying the zero-forcing matrix $\mathbf{D}_i$ of the effective channel.

In a similar way, for users $\mathcal{T}_2$, $\mathcal{T}_3$ and $\mathcal{T}_4$, we obtain

$$\begin{aligned}
\mathbf{y}_2^{'} &= \gamma_{min}\mathbf{G}_{2,1}\mathbf{U}_1\Sigma_1\mathbf{V}_1^H\mathbf{H}_{1,1}\mathbf{s}_1 + \tilde{\mathbf{n}}_2,\\
y_3^{'} &= \gamma_{min}\mathbf{g}_{3,1}\mathbf{U}_1\Sigma_2\mathbf{V}_1^H\mathbf{h}_{1,4}s_4 + \tilde{n}_3,\\
\mathbf{y}_4^{'} &= \gamma_{min}\mathbf{G}_{4,1}\mathbf{U}_1\Sigma_2\mathbf{V}_1^H\mathbf{H}_{1,3}s_3 + \tilde{\mathbf{n}}_4,
\end{aligned}$$

where $\Sigma_2 = \mathrm{diag}\left(\frac{\beta_{(1)}-\alpha_{(1)}}{\beta_{(1)}},\ \frac{\beta_{(2)}-\alpha_{(2)}}{\beta_{(2)}}\right)$. All users can also decode their desired symbol by multiplying zero-forcing matrices. As a result, user $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ achieve the DOF of 2 and $\mathcal{T}_4$ obtains the DOF of 1. Therefore $\eta_{sum} = 7$ is achieved when $M = 2$ and $N = 4$. So far, we have proposed the new IN relay scheme for two-way relay interference channels. In the following section, we will consider more general cases according to antenna configurations and derive the achievable sum DOF of two-way relay interference channels.

IV. Achievable Degrees of Freedom of Two-way Relay Interference Channels

In this section, we study the achievable sum DOF of two-way relay interference channels. We characterize the achievable sum DOF in terms of the number of antennas N and M by applying three different relay transmission schemes. We present the main result of this section in the following theorem.

Theorem 1: In a complex Gaussian MIMO two-way relay interference channel where all user nodes and relays have M and N antennas, the achievable sum DOF becomes

$$\eta_{sum} = \max\Bigg\{ \min(4N, 2M), \min\left(2N, 2\lfloor\frac{4}{3}M\rfloor\right), \min(2N-1, 4M)\Bigg\}.$$

Theorem 1 will be proved by presenting TDMA, SSA-NC, and the proposed IN schemes in the following subsections.

A. Case of $N \le M$

We first consider the case when relays do not have more antennas than users ($N \le M$). In this case, we will show that the DOF of $\min(2M, 4N)$ is achieved by the TDMA method. We assume that two relays cooperatively help each link transmission with time-sharing such as two-way MIMO multiple relay systems [5]. During the first time slot, the first link users $\mathcal{T}_1$ and $\mathcal{T}_2$ exchange $d_i = \min(M, 2N)$ message symbols via two relays, while the other link users $\mathcal{T}_3$ and $\mathcal{T}_4$ keep silence. Then the filter output of $\mathcal{T}_i$ for $i = 1, 2$ in (3) becomes

$$\hat{\mathbf{y}}_i = \mathbf{D}_i^H \left(\sum_{l\in\mathbb{L}} \mathbf{G}_{i,l}\mathbf{W}_l\mathbf{H}_{l,\bar{i}} \right) \mathbf{P}_{\bar{i}}\mathbf{s}_{\bar{i}} + \mathbf{n}_i.$$

Note that there is no interference term $\mathbf{i}_i$.

To determine the number of data streams, let us check the rank of the effective channels $\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,\bar{i}} + \mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,\bar{i}}$ which is the sum of two $M \times M$ matrices. Since all elements of the channels $\mathbf{H}_{l,i}$ and $\mathbf{G}_{i,l}$ for $\forall i, l$ are i.i.d. Gaussian random variables, the rank of $\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,\bar{i}}$ and $\mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,\bar{i}}$ depends on the $N\times N$ relay beamforming matrix $\mathbf{W}_1$ and $\mathbf{W}_2$, respectively. By choosing the relay beamforming matrix as an arbitrary rank N matrix, e.g., $\mathbf{W}_l = \mathbf{I}_N$, $\forall l$, we can achieve the maximum rank $\min(M, 2N)$ for the effective channels, because N basis vectors of the first matrix $\mathbf{G}_{i,1}\mathbf{W}_1\mathbf{H}_{1,\bar{i}}$ and $\min(N, M-N)$ basis vectors of the second matrix $\mathbf{G}_{i,2}\mathbf{W}_2\mathbf{H}_{2,\bar{i}}$ are linearly independent with probability one by the assumption of generic channels.

Then, the number of transmit data streams of each user is obtained as $d_i = M$ if $\frac{M}{2} \le N \le M$, and $d_i = 2N$ if $N < \frac{M}{2}$ for $i = 1, 2$ and users are able to decode the transmitted data streams. Thus the total of $\min(2M, 4N)$ data streams are exchanged for the first link users during the first time slot. Similarly, the second link users can communicate with each other during the second time slot with the same DOF. As a result, the TDMA scheme allows us to achieve the DOF of $\min(2M, 4N)$.

B. Case of $M < N \le \frac{4}{3}M$

Now, we explore the case where the number of relay antennas is greater than that of user's ($N > M$). We adopt the SSA-NC scheme to achieve the DOF of $2N$ and investigate the necessary number of relay antennas for this scheme. Before proving the achievability of $\eta_{sum} = \min\{2N, 2\lfloor\frac{4}{3}M\rfloor\}$, we review the SSA-NC scheme introduced in [10] and [11]. In this scheme, the beamforming vectors are designed such that two desired signal vectors coming from two users are aligned to the same vector space at the relay. For example, $\mathcal{T}_i$ transmits its symbol $s_{i,1}$ to $\mathcal{R}_1$ utilizing the beamforming vector $\mathbf{p}_{i,1}$ for $i = 1, 2$. Each user applies the beamforming vector to receive two desired symbols at the same spatial dimension of the relay, i.e., $\text{span}(\mathbf{H}_{1,1}\mathbf{p}_{1,1}) = \text{span}(\mathbf{H}_{1,2}\mathbf{p}_{2,1})$. From these aligned received signals, the relay is able to detect and encode these two symbols using analog network coding (ANC) [18] or physical layer network coding (PNC) [19]. Both methods allow simultaneous transmission of two users by performing joint detection. The difference is that the ANC performs symbol-wise encoding at the BC phase, whereas bit-wise encoding is employed for the PNC.

Let us start with applying a relay selection method which chooses one of relays to help both link users communicate with each other. Then this channel can be interpreted as multi-pair two-way relay channels in [7]. In the following, we will perform SSA-NC for both the MAC and BC phase at the selected relay.

1) MAC phase: We first determine the user's beamforming matrix $\mathbf{P}_i$ to align the signal space for network coding at the selected relay. We assume that users at each link transmit the same number of data streams, i.e., $d_1 = d_2$ and $d_3 = d_4$ and that the first link relay $\mathcal{R}_1$ is selected without loss of generality. In the MAC phase, $\mathbf{P}_1$ and $\mathbf{P}_2$ for the first link users are designed which satisfy the following conditions

$$\text{span}\left(\mathbf{H}_{1,1}\mathbf{p}_{1,m}\right) = \text{span}\left(\mathbf{H}_{1,2}\mathbf{p}_{2,m}\right) = \text{span}\left(\mathbf{a}_{1,m}\right), \quad (12)$$

for $m = 1, \cdots, d_1$, where $\mathbf{a}_{l,m}$ is a unitary vector which spans the aligned space at $\mathcal{R}_l$ for network coding of the m-th data stream of the l-th link users.

Denoting the i-th user's beamforming vector as $\mathbf{p}_{i,m} = \gamma_{i,m}\bar{\mathbf{p}}_{i,m}$ with the power allocation factor $\gamma_{i,m}$ and the direction vector $\bar{\mathbf{p}}_{i,m}$, the beamforming vectors $\bar{\mathbf{p}}_{1,m}$ and $\bar{\mathbf{p}}_{2,m}$ can be obtained by solving the following linear equation

$$\underbrace{\begin{bmatrix} \mathbf{I}_N & -\mathbf{H}_{1,1} & \mathbf{0} \\ \mathbf{I}_N & \mathbf{0} & -\mathbf{H}_{1,2} \end{bmatrix}}_{2N\times(N+2M)} \begin{bmatrix} \mathbf{a}_{1,m} \\ \bar{\mathbf{p}}_{1,m} \\ \bar{\mathbf{p}}_{2,m} \end{bmatrix} = \mathbf{0}, \quad (13)$$

for $m = 1, \cdots, d_1$, and $\gamma_{1,m}$ and $\gamma_{2,m}$ are determined by the power constraint. Note that the vectors $\mathbf{a}_{1,m}$ for $m = 1, \ldots, d_1$ should be linearly independent with each other to have d_1 network coded messages.

To show existence of a solution of equation (13), we should check the dimension of the nullspace of the matrix of size $2N \times (N + 2M)$ in (13). We can see that the rank of the null space of the above matrix is equal to $2M - N$ with probability one, since all channel matrices are assumed to be generic. Thus the first link users are able to transmit $d_1 = d_2 \le 2M - N$ data streams using the SSA-NC.

In a similar way, the second link users can also transmit $d_3 = d_4 \leq 2M - N$ data streams and determine the beamforming vectors by solving the equation

$$\begin{bmatrix} \mathbf{I}_N & -\mathbf{H}_{1,3} & \mathbf{0} \\ \mathbf{I}_N & \mathbf{0} & -\mathbf{H}_{1,4} \end{bmatrix} \begin{bmatrix} \mathbf{b}_{1,m} \\ \bar{\mathbf{p}}_{3,m} \\ \bar{\mathbf{p}}_{4,m} \end{bmatrix} = \mathbf{0},$$

for $m = 1, \cdots, d_3$, where $\mathbf{b}_{1,m}$ denotes a unitary vector which spans the aligned space of the m-th interfering signals from the other link users to the relay $\mathcal{R}_1$. However, the vectors $\mathbf{a}_{1,m}$ for $m = 1, \ldots, d_1$ and $\mathbf{b}_{1,n}$ for $n = 1, \ldots, d_3$ should be linearly independent to decode the messages at the aligned signal spaces. Since the signal space dimension of $\mathcal{R}_1$ is N, the sum of two links' data streams cannot exceed N. If $\mathcal{T}_1$ and $\mathcal{T}_2$ send $d_1 = 2M - N$ data streams, the remaining dimension at $\mathcal{R}_1$ equals $N - d_1 = 2N - 2M$. As a result, the number of transmit data symbols for the second link users becomes $d_3 \leq \min(2M - N, 2N - 2M)$. Note that $\min(2N - 2M, 2M - N)$ is maximized when $3N = 4M$. Thus, if $M < N \leq \frac{4}{3}M$, users $\mathcal{T}_3$ and $\mathcal{T}_4$ can send $d_3 = 2N - 2M$ data streams and finally the total number of data streams is given by $\sum_{i \in \mathbb{U}} d_i = 2(2M - N) + 2(2N - 2M) = 2N$ when the SSA-NC scheme is adopted at $\mathcal{R}_1$.

2) BC phase: During the BC phase, we compute the i-th user decoding matrix $\mathbf{D}_i = \left[\mathbf{d}_{i,1} \cdots \mathbf{d}_{i,d_{\bar{i}}}\right]$ and the relay beamforming matrix $\mathbf{W}_1$ for decoding these network coded messages at the corresponding users. Note that the relay has N network coded messages and all users have M antennas which is smaller than the number of messages N. Thus, we should carefully design the beamforming matrices so that the number of the transmitted messages from the relay does not exceed each user's dimension for decoding the corresponding messages.

In a similar way as in the MAC phase, we can also align each user pair's received signal space as

$$\operatorname{span}\left(\mathbf{d}_{i,m}^H \mathbf{G}_{i,1}\right) = \operatorname{span}\left(\mathbf{d}_{\bar{i},m}^H \mathbf{G}_{\bar{i},1}\right), \quad \forall i, m.$$

Let us denote $\dot{\mathbf{a}}_{1,m}$ and $\dot{\mathbf{b}}_{1,m}$ as the unitary aligned vectors at which the network coded messages from $\mathbf{a}_{1,m}$ for link 1 and $\mathbf{b}_{1,m}$ for link 2 arrive, respectively. Then, we calculate $\mathbf{D}_i$ so that the BC channels are aligned as $\dot{\mathbf{A}} = [\dot{\mathbf{a}}_{1,1} \cdots \dot{\mathbf{a}}_{1,d_1}]$ and $\dot{\mathbf{B}} = \left[\dot{\mathbf{b}}_{1,1} \cdots \dot{\mathbf{b}}_{1,d_3}\right]$. For example, the first link users' receive combining matrices $\mathbf{D}_1$, $\mathbf{D}_2$ and $\dot{\mathbf{A}}_1 = [\dot{\mathbf{a}}_{1,1} \cdots \dot{\mathbf{a}}_{1,d_1}]$ are obtained by solving the equation

$$\begin{bmatrix} \dot{\mathbf{a}}_{1,m}^H & \mathbf{d}_{1,m}^H & \mathbf{d}_{2,m}^H \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{I}_N & \mathbf{I}_N \\ -\mathbf{G}_{1,1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}_{2,1} \end{bmatrix}}_{(N+2M)\times 2N} = \mathbf{0}, \tag{14}$$

for $m = 1, \cdots, d_1$.

Then, $\dot{\mathbf{a}}_{1,m}$, $\mathbf{d}_{1,m}$ and $\mathbf{d}_{2,m}$ are computed from the left nullspace of the matrix of size $(N + 2M) \times 2N$ in (14) for $m = 1, \cdots, d_1$. Since the rank of the left nullspace is also $2M - N$ with probability one, we can get $\mathbf{D}_1$ and $\mathbf{D}_2$ for d_1 network coded messages. In a similar way, we can also design $\mathbf{D}_3$, $\mathbf{D}_4$ and $\dot{\mathbf{B}}_1 = \left[\dot{\mathbf{b}}_{1,1} \cdots \dot{\mathbf{b}}_{1,d_3}\right]$ for d_3 network coded messages from the equations

$$\begin{bmatrix} \dot{\mathbf{b}}_{1,m}^H & \mathbf{d}_{3,m}^H & \mathbf{d}_{4,m}^H \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{I}_N & \mathbf{I}_N \\ -\mathbf{G}_{3,1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}_{4,1} \end{bmatrix}}_{(N+2M)\times 2N} = \mathbf{0}, \tag{15}$$

for $m = 1, \cdots, d_3$. By performing this BC channel alignment, from the relay perspective, this system is equivalent to two-user MIMO broadcast channels where the effective channels for the first link users and the second link users equal is $\dot{\mathbf{A}}_1^H$ and $\dot{\mathbf{B}}_1^H$, respectively.

Given $\dot{\mathbf{A}}_1$, $\dot{\mathbf{B}}_1$, $\mathbf{A}_1$ and $\mathbf{B}_1$, we calculate the relay beamforming matrix $\bar{\mathbf{W}}_1 = \mathbf{U}_1 \mathbf{V}_1^H$ where $\mathbf{U}_1$ and $\mathbf{V}_1$ are determined by $\mathbf{U}_1^H = \left[\dot{\mathbf{A}}_1 \ \ \dot{\mathbf{B}}_1\right]^{-1}$ and $\mathbf{V}_1^H = [\mathbf{A}_1 \ \ \mathbf{B}_1]^{-1}$, respectively. As a result, the interference vector $\mathbf{i}_i$ in equation (3) is perfectly canceled as

$$\hat{\mathbf{y}}_i = \mathbf{D}_i^H \mathbf{G}_{i,1} \mathbf{W}_1 \mathbf{H}_{1,\bar{i}} \mathbf{P}_{\bar{i}} \mathbf{s}_{\bar{i}} + \mathbf{n}_i, \quad \text{for } i \in \mathbb{U}.$$

Then all users can communicate with their partners without the other link users' interference. Therefore, we can achieve the DOF of $2N$ when $M < N \leq \frac{4}{3}M$ by selecting one relay to operate.

Note that if $N > \frac{4}{3}M$, the second link users can transmit $d_3 = \min(2N - 2M, 2M - N) = 2M - N$ data streams. Thus, the total of $2(2M - N)$ dimensions are utilized for the SSA-NC at the relay $\mathcal{R}_1$ and $N - 2(2M - N) = 3N - 4M$ dimensions remain. We cannot use these remaining dimensions for network coding and only one symbol per each dimension can be transmitted. For example, consider the case of $M = 3$ and $N = 5$. Then both link users can align only one signal space at the selected relay. Even though 3 remaining dimensions are occupied by any user at the relay without network coding, the achievable DOF is only 7. Instead, we can achieve the DOF of 8 by selecting 4 antennas among 5 relay antennas and adopting network coding for full dimensions. Therefore, choosing $\lfloor \frac{4}{3}M \rfloor$ antennas among $N > \frac{4}{3}M$ antennas is better than applying all antennas for the SSA-NC and this achieves the DOF of $2\lfloor \frac{4}{3}M \rfloor$ by obtaining a network coding gain. Consequently, we can show that the DOF of $\min(2N, 2\lfloor \frac{4}{3}M \rfloor)$ is achieved by adopting the SSA-NC.

C. Case of $N > \frac{4}{3}M$

For $N > \frac{4}{3}M$, we show that the DOF of $\min(2N - 1, 4M)$ is achievable by the proposed IN scheme described in Section III. Let us consider the feasibility condition for the IN scheme. We assume that user $\mathcal{T}_i$ selects d_i antennas and the size of the channel matrix $\mathbf{H}_{l,i}$ equals $N \times d_i$. In order to design the receive combining vector $\mathbf{v}_{1,(n)}$ which satisfies the equation (8), the size of the channel matrices $\mathbf{H}_{11,12}$ and $\mathbf{H}_{21,22}$ should be $N \times N$ for inverse operations and the size of $\mathbf{H}_{13,14}$ and $\mathbf{H}_{22,24}$ should be $N \times (N - 1)$ in order to have left nullspace. In other words, the feasibility condition for (8) is satisfied when the total number of message symbols for link 1 users is N ($d_1 + d_2 = N$) and that of message symbols for link 2 users is $N - 1$ ($d_3 + d_4 = N - 1$), or vice versa, with $d_i \leq M$, $\forall i$. In the same way, user $\mathcal{T}_i$ selects $d_{\bar{i}}$ antennas during the BC phase so that the size of the channel matrix $\mathbf{G}_{i,l}$ becomes $d_{\bar{i}} \times N$.

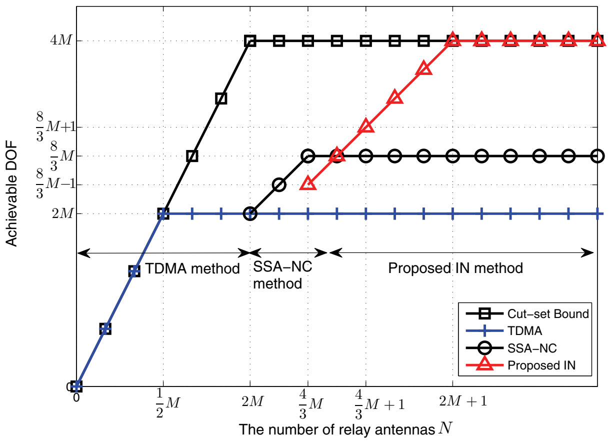


Fig. 2. Achievable DOF of two-way relay interference channels with respect to relay antennas N.

Then, a solution of equation (10) always exists for the relay transmit beamforming vector $\mathbf{u}_{1,(n)}$ for all and n.

Next, we should make $L = \max\{d_1, d_2, d_3, d_4\}$ independent vectors for $\mathbf{U}_l = \left[\mathbf{u}_{l,(1)} \cdots \mathbf{u}_{l,(L)}\right]$ and $\mathbf{V}_l = \left[\mathbf{v}_{l,(1)} \cdots \mathbf{v}_{l,(L)}\right]$ for $l = 1, 2$. As described in the previous section, it is possible to determine arbitrary real values $\alpha_{(n)}$ and $\beta_{(n)}$ which satisfy the conditions $\alpha_{(n)} \neq \beta_{(n)}$ for $n \in \{1, \cdots, L\}$ and $\frac{\alpha_{(i)}}{\beta_{(i)}} \neq \frac{\alpha_{(j)}}{\beta_{(j)}}$, $\forall i \neq j$. Then, we can guarantee the effective channel $\mathbf{G}_{i,1}\mathbf{U}_1\Sigma_l\mathbf{V}_1^H\mathbf{H}_{1,\bar{i}}$ in (11) has the rank of $d_{\bar{i}}$, i.e., rank $\left(\mathbf{G}_{i,1}\mathbf{U}_1\Sigma_l\mathbf{V}_1^H\mathbf{H}_{1,\bar{i}}\right) = d_{\bar{i}}$, for $i \in \mathbb{U}$. This means that each user $\mathcal{T}_i$ is able to decode all desired $d_{\bar{i}}$ data streams sent by the partner $\mathcal{T}_{\bar{i}}$. Since we chose $d_1 + d_2 = N$ and $d_3 + d_4 = N - 1$, the sum DOF of $\eta_{sum} = 2N - 1$ is finally achieved.

Note that if $N \geq 2M + 1$, all nodes are able to transmit maximum $d_i = M$ data streams by relay antenna selection as $\mathcal{R}_1$ and $\mathcal{R}_2$ use $2M$ and $2M + 1$, respectively, among N relay antennas. Then the sum DOF of $\eta_{sum} = 4M$ is obtained, which achieves the cut-set bound of two-way relay interference channels. As a result, the total achievable DOF becomes $\eta_{sum} = \sum_{i\in\mathbb{U}} d_i = \min(2N - 1, 4M)$ by adopting the proposed IN scheme.

D. Cut-set outer bound

For the comparison of the achievable sum DOF, we provide a cut-set outer bound for MIMO two-way relay interference channels. For conventional one-link two-way relay channels where two user nodes have M antennas and the relay is equipped with N antennas, the optimal DOF is given by $\eta^*_{link1} = d^*_1 + d^*_2 = \min\{2M, 2N\}$. Similarly, if only the second link operates, the optimal DOF also becomes $\eta^*_{link2} = d^*_3 + d^*_4 = \min\{2M, 2N\}$. Adding up these optimal DOF, a cut-set outer bound of the two-way relay interference channel is obtained as $\eta_{cut-set} = d^*_1 + d^*_2 + d^*_3 + d^*_4 \leq \min\{4M, 4N\}$.

V. Discussion and Numerical Results

So far, we have investigated the achievable sum DOF of two-way relay interference channels with various antenna configurations. Depending on the number of relay antennas for given user antennas, different transmission strategies are employed to efficiently deal with interference signals. We summarize the derived achievable DOF in Figure 2. For the case of $N \leq M$, we can avoid the other link users' interference signals by using TDMA and achieve the DOF of $\min(4N, 2M)$. Note that the cut-set upper bound is achieved by TDMA when $N \leq \frac{1}{2}M$. For the case of $M < N \leq \frac{4}{3}M$, we adopt the SSA-NC method which can deal with more data streams than the TDMA. By fully exploiting the relay dimension for network coding, we are able to achieve a network coding gain of 2 and thus the achievable DOF in this region becomes $2N$. However if $N > \frac{4}{3}M$, since all users cannot fully utilize the relay dimensions for the network coding, both relays cooperatively mitigate the interferences, and jointly design the beamforming matrices to neutralize one link users' signals at the other link users. Using the proposed IN method, we can obtain higher achievable DOF than SSA-NC when $N > \frac{4}{3}M$ and achieve the cut-set upper bound of $4M$ when $N \geq 2M + 1$. In summary, the DOF of $\max\left\{\min(4N, 2M), \min(2N, 2\lfloor\frac{4}{3}M\rfloor), \min(2N - 1, 4M)\right\}$ is achieved for two-way relay interference channels. Note that the curves of SSA-NC and IN have a cross-over point when $\frac{4}{3}M < N < \frac{4}{3}M + 1$. However, for any integer N greater than $\frac{4}{3}M$, the DOF of IN is always higher than that of SSA-NC.

TABLE I
Required CSI for Achieving the DOF

Transmission scheme	Required CSI at nodes	
TDMA	Source $\mathcal{T}_i$	-
	Relay $\mathcal{R}_l$	-
SSA-NC	Source $\mathcal{T}_i$	$\{\mathbf{H}_{l,i}, \mathbf{H}_{l,\bar{i}}\}$
	Relay $\mathcal{R}_l$	$\{\mathbf{H}_{l,i}, \mathbf{G}_{i,l}, \ \forall i\}$
IN	Source $\mathcal{T}_i$	-
	Relay $\mathcal{R}_l$	$\{\mathbf{H}_{l,i}, \mathbf{H}_{\bar{l},i}, \mathbf{G}_{i,l}, \mathbf{G}_{i,\bar{l}}, \ \forall i\}$

Now, we discuss the required CSI at each node for performing the above transmission strategies. From a perspective of the achievable DOF, both the TDMA and IN methods do not require CSI at the source during the MAC phase and can be applied without incurring a DOF loss. On the other hand, when the SSA-NC method is employed, each user should know both its channel $\mathbf{H}_{l,i}$ and its partner's channel $\mathbf{H}_{l,\bar{i}}$ to the selected relay $\mathcal{R}_l$. Thus, the required CSI level at the source node is higher for the SSA-NC scheme in the MAC phase.

During the BC phase, however, the relay needs the highest CSI level for performing the IN. The relay $\mathcal{R}_l$ should acquire not only its uplink and downlink channels $\mathbf{H}_{l,i}$ and $\mathbf{G}_{i,l}$, but also the other relay's uplink and downlink channels $\mathbf{H}_{\bar{l},i}$ and $\mathbf{G}_{i,\bar{l}}$. While the SSA-NC method requires the knowledge on $\mathbf{G}_{i,l}$ for the aligned signal space between link users, the channel information is not necessary for the TDMA method and just amplify-and-forward at the relay achieves the derived DOF. These requirements at each node for achieving the derived DOF are illustrated in Table I.

Next, we confirm the accuracy of our achievable DOF analysis for two-way relay interference channels through simulations. We evaluate the sum rate performance of the precoding and relaying scheme proposed in the previous section and

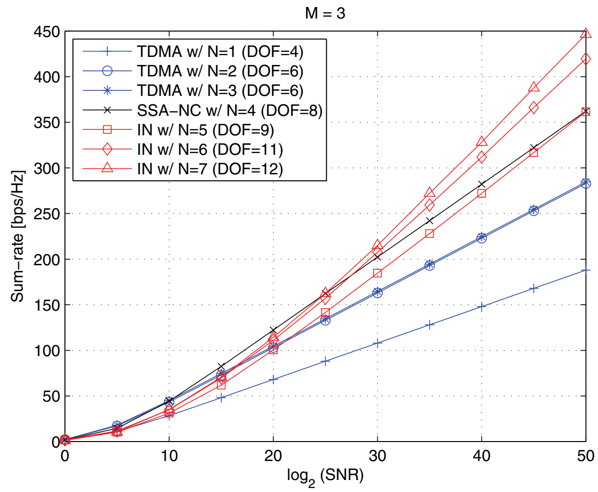


Fig. 3. Sum rate performance of various transmission strategies for $M = 3$.

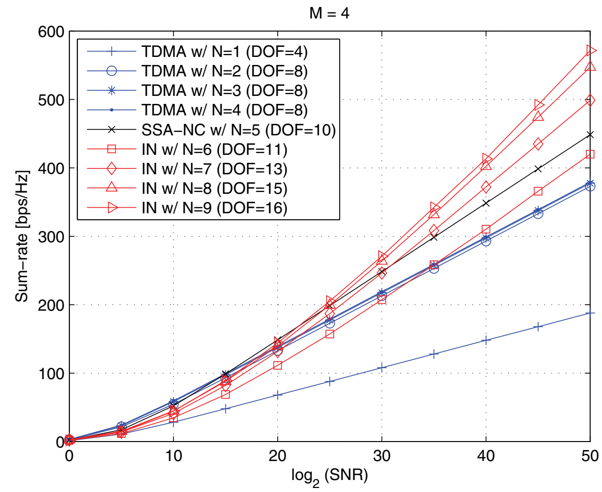


Fig. 4. Sum rate performance of various transmission strategies for $M = 4$.

demonstrate that the proposed scheme achieves the DOF illustrated in Figure 2. Throughout simulations, we assume that each node has the same transmission power constraint P and the same noise variance σ^2. If the number of data streams is determined, equal power is allocated to the precoding vector for each data symbol as $\|\mathbf{p}_{i,m}\|^2 = P/d_i$ for $\forall i, m$. For the TDMA method, we simply design the precoding matrices $\mathbf{P}_i$ as an arbitrary $M \times d_i$ unitary matrix and the relay beamforming matrix $\bar{\mathbf{W}}_l$ as an identity matrix $\bar{\mathbf{W}}_l = \mathbf{I}_{N\times N}$ for $\forall i, l$. For the SSA-NC scheme, the first link relay $\mathcal{R}_1$ is selected to operate for both link communications and all precoding and decoding vectors are designed to satisfy the alignment conditions and power constraints. Also, $\alpha_{(n)}$ and $\beta_{(n)}$ for the proposed IN scheme are randomly chosen by real values in $(0\ 1]$. Finally, the sum rate is calculated by (4) and is plotted as a function of $\log_2\left(\frac{P}{\sigma^2}\right)$.

In Figure 3, we exhibit the average sum rate performance of various transmission schemes according to the number of relay antennas for the case of $M = 3$. The curves for the TDMA with $N = 1, 2, 3$ show the slope of 4, 6 and 6, respectively, at the high SNR regime. As we predicted, the DOF of $\min(4N, 2M)$ is achieved by the help of the other link relay. When $N = 4$, the SSA-NC method obtains the rate with the slope of 8 which equals the derived DOF of $\min(2N, 2\lfloor\frac{4}{3}M\rfloor)$. As the number of relay antennas increases, we can attain higher DOF of $\min(2N-1, 4M)$ by using the IN method, which reaches the cut-set bound of $4M = 12$ when relays have $N = 7$ antennas.

Figure 4 presents the average sum rate when each user has $M = 4$ antennas. We can observe that when $2 \le N \le 4$, the TDMA achieves the same DOF of 8 and just obtains small SNR gains as the number of relay antennas increases. Note that when $N = 5$, the sum-rate of SSA-NC is worse than that of TDMA in the low SNR region. However, the SSA-NC obtains a higher rate with the slope of 10 in the high SNR regime. The slope for the case of $6 \le N \le 9$ becomes $\min(2N-1, 4M)$ by performing the IN method. As expected, the plot shows that for all schemes, the sum rate slopes $\sum R_i(\mathrm{SNR}) / \log_2(\mathrm{SNR})$ at high SNR match well with our analysis results. As a result, we verify that we can achieve the DOF of $\max\left\{\min(4N, 2M), \min(2N, 2\lfloor\frac{4}{3}M\rfloor), \min(2N-1, 4M)\right\}$ for two-way relay interference channels through numerical results.

VI. Conclusion

In this paper, the achievable sum DOF of MIMO two-way relay interference channels has been studied. We have shown that the sum DOF of $\max\left\{\min(4N, 2M), \min(2N, 2\lfloor\frac{4}{3}M\rfloor), \min(2N-1, 4M)\right\}$ is achievable by applying three different relaying schemes: TDMA, SSA-NC, and the proposed IN. One consequence of this result is that a cut-set outer bound for the DOF performance is achieved when the number of relay antennas $N \ge \frac{4M-1}{2}$ and $N \le \frac{1}{2}M$. Through numerical simulations, the derived achievable DOF results have been verified. These results can serve as a lower bound of the capacity for MIMO two-way relay interference channels. Optimal beamforming vector designs of TDMA, SSA-NC and IN will remain as future works.

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