# The Effect of a Precoder on Serially Concatenated Coding Systems with ISI Channel

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*Abstract*

The performance of a serially concatenated system which includes a channel with memory preceded by a precoder as a rate one inner coder is presented. The effect of different precoders on the Maximum Likelihood bit error performance is analyzed. The precoder weight gain is identified through a union bound analysis. It is shown that the choice of precoders is critical for a given channel to achieve a good bit error rate performance. Several important design considerations for the choice of precoders are derived based on analysis and these are confirmed through simulations with iterative decoding algorithm.

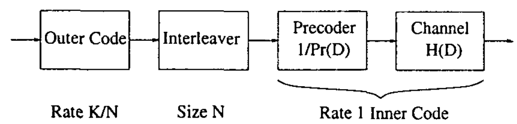


Fig. 1. Serially concatenated code system employing the channel with precoder

## I. Introduction

Since turbo codes [1] were first introduced in 1993, concatenated coding systems in conjunction with iterative decoding have attracted great interest in the communications area. The impressive bit error performance of parallel concatenated coding (PCC) systems employing a random interleaver has inspired people to consider several variations on its structure [2], [3], [4].

Benedetto *et al.* [4] proposed a serially concatenated coding (SCC) system, where two component encoders are connected serially through a random interleaver, and showed that the performance of the SCC is comparable to that of the PCC. In some situations, it was shown that SCC's do not exhibit an error floor which is normally observed in PCC's. In most of studies related to PCC's and SCC's, it is assumed that an encoded bit sequence is transmitted through a memoryless channel.

Recently, several researchers have proposed replacing the inner code of SCC's by other recursive structures [5], [6], [7]. In particular, Souvignier *et al.* [5] and Öberg *et al.* [7] investigated the application of SCC's which view a channel with memory as a rate 1 inner code. In order for this system to provide the required recursive structure for the inner code, a precoder is placed in front of the channel. The studies in [5] and [7] are limited to a particular partial response channel and a fixed precoder is assumed. Also, a concatenated code system in [8] is analyzed only for storage channel applications with high coding rate, so it does not fully exhibit the effect of precoders, which normally become notable in other situations such as low code rates. In this paper, we investigate the effect of different precoders on general intersymbol interference (ISI) channels with various coding rates. Throughout this work, the serially concatenated system with the ISI channel will be referred to as $\text{SCC}_{\text{ch}}$ to distinguish it from SCC's for memoryless channels. SCC's can also be applied to the ISI channel after converting the channel into a memoryless one using techniques such as equalization. However, this increases the decoder complexity considerably.

In general, the outer encoder in $\text{SCC}_{\text{ch}}$'s could represent any encoding scheme. "Turbo equalization" [9], [10] which uses the channel demodulation in a decoding iteration can be viewed as a $\text{SCC}_{\text{ch}}$ which takes a turbo code as an outer code. Because of the increased complexity in an outer decoder, a $\text{SCC}_{\text{ch}}$ with a turbo code as an outer code is not considered in this paper.

The precoders in $\text{SCC}_{\text{ch}}$'s takes different implication which is not identified in SCC's. We introduce the *precoder weight gain* to characterize the performance gain due to the precoder. The objective of this paper is to provide insights in the choice of precoders so as to derive design guidelines which are useful in the actual system design. Based on the Euclidean distance analysis, precoders are divided into two groups where each group exhibits a distinctive bit error curve. Two different bit error rate curve behaviors in each group are identified using computer simulations in various ISI channels.

This paper is organized as follows. The following section analyzes the bit error rate performance of $\text{SCC}_{\text{ch}}$'s which adopt the ISI channel with a precoder as a rate 1 inner code. An ensemble average Maximum Likelihood bit error rate upper-bound is derived using a union bound approach, and the asymptotic performance of systems both with and without precoders is derived based on a randomly generated interleaver. Section III investigates the effect of precoders on the $\text{SCC}_{\text{ch}}$ performance and addresses the issues of the precoder choice which can improve the bit error performance for a given ISI channel. Through analysis and computer simulations using iterative decoding techniques, we explain different asymptotic bit error rate behaviors de-


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pending on precoders and present some design considerations for the choice of precoders which improve the overall performance. Finally, section IV contains concluding remarks.

## II. Performance of Serially Concatenated Codes with a Channel with Memory

Consider a serially concatenated code system which takes the intersymbol interference (ISI) channel $H(D)$ with a precoder as a rate 1 inner code, see Figure 1. Here the outer encoder with the free distance $d^o_{\text{free}}$ has rate $R = \frac{K}{N}$ where $K$ and $N$ represent the length of input words and codewords, respectively. Thus, the size of the interleaver is equal to $N$. Also, note that $H(D)$ represents the transfer polynomial in the field of real numbers, whereas $Pr(D)$ assumes the polynomial in the finite field.

As for the precoder structure, many different forms of precoders could be used such as $Pr_1(D)/Pr_2(D)$ or Tomlinson-Harashima precoders [11], [12]. However, these precoder structures require increased complexity in the inner decoder. Therefore, in this work, we limit our focus to $1/Pr(D)$ precoders whose memory is equal to or smaller than the channel memory. With this condition, for a given channel response $H(D) = \sum_i h_i D^i$, the number of states in the inner decoder remains the same. Maximum a Posteriori (MAP) sequence detector accounts for an inner decoder based on this channel description.

For simplicity of the presentation, convolutional outer codes are viewed as its equivalent block code by terminating sequences of convolutional codes. In this system, codewords of $\text{SCC}_{\text{ch}}$'s are defined as the precoder output words. Let $x$ and $\epsilon$ be a correct codeword and an error codeword of length $N$ respectively. Then, the erroneous codeword $x' = x \oplus \epsilon$ at the precoder output generates the input error event $e = x - x'$ at the inner decoder. The probability of bit error caused by choosing $x \oplus \epsilon$ over $x$ in the ISI channel corrupted by Gaussian noise with two-sided noise power spectral density $\sigma^2$ in Maximum Likelihood (ML) detector is $\frac{w}{K}\Pr\{\text{ML chooses } x \oplus \epsilon \text{ over } x\}$ where $w$ denotes the input weight.

For a given error codeword $\epsilon$ with weight $h$, $2^h$ different error events are possible depending on the transmitted codeword $x$. For example, assuming binary phase shift keying (BPSK) with the input alphabet $\{0,1\}$, an error codeword $\epsilon = \cdots 0110 \cdots$ can generate four error events $(\cdots 0,1,1,0\cdots)$, $(\cdots 0,1,-1,0\cdots)$, $(\cdots 0,-1,1,0\cdots)$, $(\cdots 0,-1,-1,0\cdots)$ depending on $x$. Because of a random interleaver, we can assume that each $2^h$ error event is equally probable for low weight $h$'s which are of importance in our analysis. Now we can compute the probability of error in the ISI channel for every possible codewords $x$ caused by the $k$th error codeword $\epsilon_k$ as

$$\Pr\{\text{ML chooses } x \oplus \epsilon_k \text{ over any } x\} = \frac{1}{2^{h_k}} \sum_{n=1}^{2^{h_k}} Q\left(\frac{d(e_{k,n})}{2\sigma}\right)$$

where $h_k$ represents the weight of the $k$th error codeword $\epsilon_k$, $e_{k,n}$ specifies one of $2^{h_k}$ error events generated by the $k$th codeword, and $d(e_{k,n})$ denotes its Euclidean distance for the ISI channel $H(D)$.

Finally, using the union bound approach, the probability of bit error for $\text{SCC}_{\text{ch}}$'s under Maximum Likelihood (ML) decoding for BPSK can be shown to have an upper-bound as

$$P_b \le \sum_{k=1}^{2^K-1} \frac{w_k}{K} \frac{1}{2^{h_k}} \sum_{n=1}^{2^{h_k}} Q\left(\frac{d(e_{k,n})}{2\sigma}\right). \qquad (1)$$

Now we want to rearrange the above expression in terms of all error events. Let us first define $\mathcal{E}$ be a set of all possible input error events for the ISI channel $H(D)$ where each error event takes on values $\{+1, 0, -1\}$. For notational convenience, $+$ and $-$ will be used instead of $+1$ and $-1$, for error event descriptions. An error event $e$ can be uniquely decomposed into a concatenation of disjoint error sub-events $e_i, i = 1, 2, \cdots m$. We will denote this error event as $e = (e_1, e_2, \cdots e_m)$. So, the Euclidean distance of $e$ can be obtained by summing all the Euclidean distance of each sub-event ($d^2(e) = \sum_{i=1}^{m} d^2(e_i)$). Note that the appearing order of error sub-events $e_i$'s in $e$ does not affect $d(e)$. For a given error event $e \in \mathcal{E}$, let us denote $E(e)$ as a set of codewords $\epsilon$ which produce $e$. Every error event is assumed to start with $+$. If a codeword $\epsilon$ belongs to $E(e)$, then any shifted codewords are also members of the same group. For example, codewords generating an error event $+-$ are $E(+-) = \{110\cdots 0, 0110\cdots 0, \cdots, 0\cdots 011\}$.

Reorganizing the above expression with respect to error events $e \in \mathcal{E}$ yields

$$P_b \le \sum_{e\in\mathcal{E}} Q\left(\frac{d(e)}{2\sigma}\right) \sum_{\epsilon \in E(e)} \frac{1}{2^{h(\epsilon)}} \frac{w(\epsilon)}{K}$$

where $d(e)$ is the Euclidean distance of error event $e$, $w(\epsilon)$ represents weight of input words which generate a codeword $\epsilon$, and $h(\epsilon)$ denotes weight of codeword $\epsilon$. We will use the notation of $h(e)$ instead of $h(\epsilon)$, since weight remains the same for every codewords $\epsilon \in E(e)$.

From the above expression, we refer to the coefficient of $Q$ function, $\sum_{\epsilon\in E(e)} w(\epsilon)$, as $W(e)$ and this represents the total weights of all input words which generate every codewords $\epsilon \in E(e)$. Combining notations defined above, we now obtain a compact expression for an upper-bound of probability of bit error as

$$P_b \le \sum_{e\in\mathcal{E}} \frac{W(e)}{K \cdot 2^{h(e)}} Q\left(\frac{d(e)}{2\sigma}\right). \qquad (2)$$

One important coefficient which affects the above expression is $1/2^{h(e)}$, and we will refer to the performance gain due to this factor as the *precoder weight gain*. This is unique to $\text{SCC}_{\text{ch}}$'s which employ the ISI channel as an inner structure. This precoder weight gain is achieved when a precoder is employed before the channel and this will be clearly explained later.

### A. Error rate performance with no precoder

We first present the asymptotic BER performance analysis for no precoder case ($Pr(D) = 1$). Consider a nonsystematic convolutional code $[G_1(D)\ G_2(D)]$ with generating polynomials $G_1(D) = 1 \oplus D^2$ and $G_2(D) = 1 \oplus D \oplus D^2$ as an outer code. A puncturing pattern listed in [13] is used to achieve a rate 2/3 and the channel response $H(D) = 1 + 2D - 2D^3 - D^4$ is assumed with a white Gaussian noise. Error events for this channel response $H(D)$ are tabulated in [14] and its minimum Euclidean distance is found to be $d^2(+ - +) = 6$. With this outer encoding scheme, the outer code sequences corresponding to the free distance $d^o_{\text{free}} = 3$ are $D^{3j}(D^2 \oplus D^4 \oplus D^5), j \geq 0$. Applying the search algorithm described in [15] to this system with a particular interleaver of length $N = 1023$, it is found that outer codewords with weight $d^o_{\text{free}} = 3$ generate four precoder output words corresponding to the error event $e = (+, +00+)$ and 336 precoder output words corresponding to the error event $e = (+, +, +)$. Therefore, their corresponding Euclidean distances are equal to $d^2(+, +00+) = d^2(+) + d^2(+00+) = 10 + 12 = 22$ and $d^2(+, +, +) = 3d^2(+) = 30$, respectively. Then, we obtain the asymptotic bit error probability as

$$P_e \approx \frac{4}{682 \cdot 2} Q(\frac{\sqrt{22}}{2\sigma}) + \frac{336}{682} Q(\frac{\sqrt{30}}{2\sigma})$$

where 682 is the input word length $K$.

This asymptotic performance is dominated by the second term which is determined by three single error events $+$, because of the large multiplicity. This indicates that without precoder, error event $e$ which consists of error event $+$ dominates the asymptotic performance with $d^2(e) = d^o_{\text{free}} d^2(+)$. Therefore, when the error event $+$ produces a small Euclidean distance for a given channel $H(D)$, the slope of the minimum distance asymptote becomes lower in $\text{SCC}_{\text{ch}}$'s with no precoder.

We plot the simulation results obtained by applying iterative decoding techniques described in [4], [16] with 10 iterations in Figure 2. To incorporate the energy in the ISI channel $H(D)$, Signal-to-Noise Ratio (SNR) defined above is used in the $x$ axis. It should be noted that since weight of channel input words $h(e)$ with no precoder is the same as that of outer codewords, $\text{SCC}_{\text{ch}}$'s without a precoder are unable to generate the precoder weight gain $1/2^{h(e)}$.

### *B. Error rate performance with a precoder*

Let us consider a $\text{SCC}_{\text{ch}}$ with a precoder $1/(1 \oplus D)$. Since input words with odd weights to $1/(1 \oplus D)$ precoder generate very large weights, only even weights input words are of our interest. Considering the same generating polynomials and puncturing pattern as in no precoder case, we can easily find that there are four outer codewords of weight 4 and they are $D^{3j}(1 \oplus D \oplus D^3 \oplus D^5), D^{3j}(D^2 \oplus D^3 \oplus D^6 \oplus D^7), D^{3j}(D^2 \oplus D^4 \oplus D^7 \oplus D^8), D^{3j}(D^2 \oplus D^3 \oplus D^9 \oplus D^{11}), j \geq 0$. These weight 4 sequences are permuted by the random interleaver, resulting in precoder input words with a form of $D^{i_1} \oplus D^{i_2} \oplus D^{i_3} \oplus D^{i_4}$ for $0 \leq i_1 < i_2 < i_3 < i_4$. Then, most of precoder output sequences are in a form of $D^{i_1} \oplus D^{i_1+1} \cdots \oplus D^{i_2-1} \oplus D^{i_3} \oplus D^{i_3+1} \cdots \oplus D^{i_4-1}$ and these sequences can support error events consisting of two error sub-events with lengths $i_2 - i_1$ and $i_4 - i_3$. Among all error events $e$ of length $l$, one with alternating signs

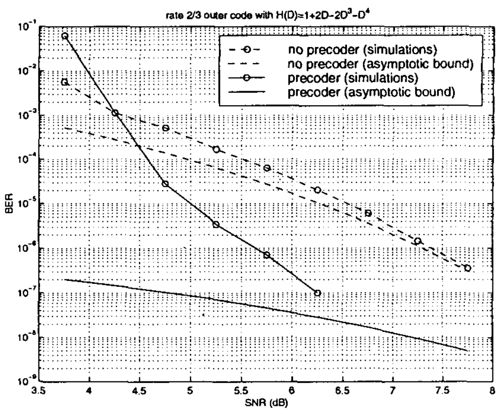

Fig. 2. Performance comparison between no precoder and $1/1 \oplus D$ precoder

(i.e. $e = + - + - \cdots +$) produces the smallest Euclidean distance for most channels $H(D)$. The Euclidean distance for $+ - + - \cdots$ is found to be 8 as long as the length of the error event is greater than 3 [14]. Therefore, the overall Euclidean distance for the precoder $1/(1 \oplus D)$ is equal to $d^2(e) = 2 \cdot 8 = 16$ as long as error sub-events are longer than three symbols. With similar approaches applied to the no precoder case, it is found that this particular interleaver generates one dominant error event $e = (+-+, +-+-+-+)$ and the corresponding Euclidean distance is $d^2(e) = d^2(+-+) + d^2(+-+-+-+) = 6+8 = 14$. Note again that the probability that the precoder output codeword $\cdots 011 \cdots 10 \cdots$ of weight $h$ supports the error event $+ - + \cdots +$ is $1/2^h$. The same codeword can support other error events such as $- - \cdots -$, but the corresponding Euclidean distance would be much higher, thus its contribution to the bit error rate becomes negligible compared to the dominant error event. So the asymptotic performance approaches

$$P_e \approx \frac{2 \cdot 4}{682 \cdot 2^{10}} Q(\frac{\sqrt{14}}{2\sigma}). \tag{3}$$

The simulation results are also plotted in Figure 2. Compared to no precoder case, the system with precoders exhibits a larger separation between the asymptotic bound and simulation results. This can be attributed to a fact that only one error event case is included when computing the asymptotic performance in (3). It is also interesting to see that at low SNR's, $\text{SCC}_{\text{ch}}$ without precoder achieves the better performance than one with a precoder, and this becomes pronounced with more powerful outer codes with the higher outer free distance $d^o_{\text{free}}$, as observed in [5], [7].

When compared to the no precoder case, it is clear that $\text{SCC}_{\text{ch}}$'s with a precoder perform better even with a smaller Euclidean distance. This is mainly due to the coefficient of $Q$ function $1/2^{h(e)}$ in (2), which is determined by weight of the precoder output codewords. Normally a precoder can generate codewords with large weights even when weight of precoder input words is small. So $\text{SCC}_{\text{ch}}$'s with a precoder can achieve the high precoder weight gain. This effect of weight $h$ on the bit error rate performance has not been addressed before. Also, this precoder weight gain has not been observed in other serially concatenated structures. For example, the performance of SCC's in [4] is determined by the codewords weight and not by the weight distribution in a codeword. In other words, two different codewords with the same weight contribute the same amount to the bit error rate of SCC's in [4], but could have different contributions to the BER in $SCC_{ch}$'s since they support different error events.

As shown in this example with a precoder, for a given interleaver, the precoder $1/Pr(D)$ determines the multiplicity while the ISI channel $H(D)$ determines Euclidean distance property. In other words, the asymptotic slope of the BER curve is determined by the channel $H(D)$, and multiplicities of error events are lowered by the precoder weight gain. When precoders generate similar precoder weight gains, one naturally chooses the precoder which maximizes the BER slope. In the above example, we can further improve the asymptotic BER performance by having precoders whose precoder output words yield a high Euclidean distance for the channel $H(D)$. Also, the same goal can be achieved by enhancing the interleaver to avoid certain output streams which might generate a small Euclidean distance [16], [17].

## III. Effect of different precoders

We will now analyze the effect of the choice of different precoders. For a given channel $H(D)$, as long as memory of precoder $Pr(D)$ does not exceed that of $H(D)$, the inner decoder complexity is determined by the channel memory $n_h$ of $H(D)$ and is independent of the precoder memory. Therefore, we focus on precoders whose memory is smaller than or equal to the channel memory. We now present $SCC_{ch}$'s with convolutional codes. In this section, all simulations are carried out by iterative decoding algorithm with 10 iterations.

Consider a channel $H(D) = (1 + D)(1 - 0.5D)(1 + 0.25D) = 1 + 0.75D - 0.375D^2 - 0.125D^3$ and assume the same convolutional outer code polynomials as in II-A. Also a rate 4/5 puncturing pattern in [13] is assumed to yield $d^o_{\text{free}} = 2$. Since the dominant error events come from precoder output words with low weight, we summarize precoder output words with input weight two and its Euclidean distance in the channel $H(D)$ in Table III. For each precoder output, Euclidean distances for all possible error events are considered and among them the smallest ones are listed. For example, a precoder output 11 could support both error events $+-$ and $++$ which generate Euclidean distance 2.4062 and 4.4687, respectively for $H(D)$. A similar analysis can be made using a technique by viewing a ISI channel with precoder as a trellis code [8]. However, by treating a precoder and $H(D)$ separately, we can gain clearer insight on how error events associated with a precoder play a role in $SCC_{ch}$.

The actual bit error performance of this $SCC_{ch}$ is dependent upon the choice of a random interleaver, since the actual precoder output codewords are determined by the interleaver. A random interleaver can be designed to eliminate certain precoder input words which could result in small Euclidean distances. For example, a $S$-random interleaver proposed in [18] could be used to avoid precoder output codewords such as 1 and 11 which yields small Euclidean distances as shown in table III.

| $Pr(D)$ | Precoder Output/Euclidean distance |
|---|---|
| $1 \oplus D$ | $1/1.7188, 11/2.4062, 111/2.1562, \cdots$ |
| $1 \oplus D^2$ | $1/1.7188, 101/2.5, 10101/3.2813, \cdots$ |
| $1 \oplus D \oplus D^2$ | $11/2.4062, 11011/4.375, 11011011/6.3438 \cdots$ |
| $1 \oplus D^3$ | $1/1.7188, 1001/3.1875, 1001001/4.6562, \cdots$ |
| $1 \oplus D^2 \oplus D^3$ | $10111/3.1875, 101110010111/6.125 \cdots$ |
| $1 \oplus D \oplus D^3$ | $11101/3.1875, 111010011101/6.125, \cdots$ |
| $1 \oplus D \oplus D^2 \oplus D^3$ | $11/2.4062, 110011/4.5625, \cdots$ |

TABLE I

Codewords generated by weight 2 input and Euclidean distance in $H(D)$

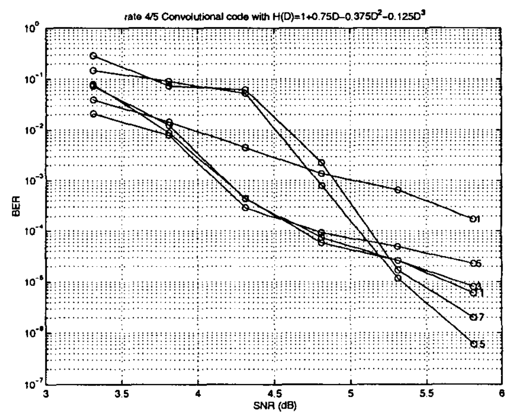


Fig. 3. rate 4/5 convolutional outer code with different precoders

Based on the Euclidean distance analysis shown in table III, we can predict that precoders $Pr(D) = 1 \oplus D$ and $1 \oplus D^2$ will have much lower slope in bit error curve than precoders $Pr(D) = 1 \oplus D^2 \oplus D^3$ and $1 \oplus D \oplus D^3$ for high SNR's. This is confirmed by simulation results shown in Figure 3. Let $pr$ denote the octal representation of $Pr(D)$ (For example, $pr = 13$ in octal indicates $Pr(D) = 1 \oplus D^2 \oplus D^3$). Numbers on each line in this figure indicate $pr$. The size of a random interleaver $N$ is set to 500.

Two interesting observations can be made on this plot. First, for low to moderate SNR's, precoders $pr = 3, 5, 11$ perform better than other choices of precoder, but as SNR increases, BER's of precoders such as $pr = 15, 17$ decrease rapidly and eventually outperform the others. This is exactly what we expect from the Euclidean distance analysis made in Table III which determines the asymptotic BER slope. Second, the slopes of precoders $pr = 3, 5, 11$ are similar to that of no precoder case ($pr = 1$). This indicates that the minimum Euclidean distances for these cases are all similar, while multiplicities of the minimum distance er-

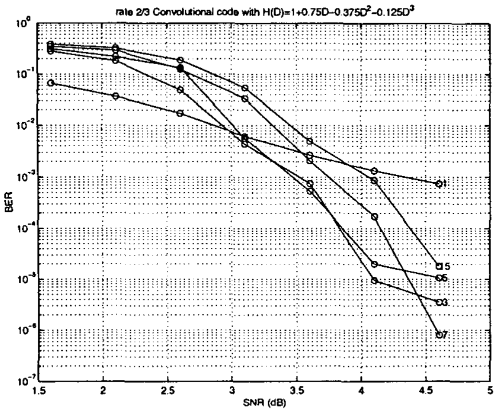


Fig. 4. rate 2/3 convolutional outer code with different precoders

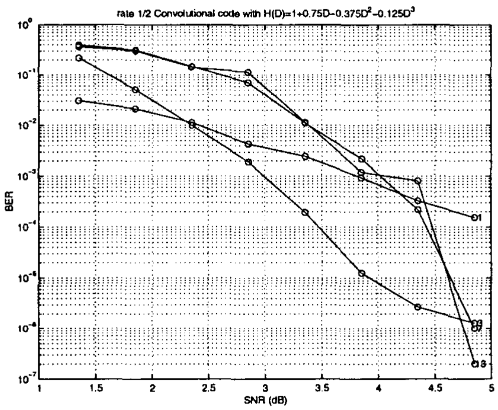


Fig. 5. rate 1/2 convolutional outer code with different precoders

ror event for $pr = 3, 5, 11$ are lowered by precoder weight gain. Therefore, the BER curves of $pr = 3, 5, 11$ appear to get shifted down by that gain, compared to no precoder curve.

Figures 4 and 5 show more simulation plots with different code rate. For a rate 2/3 outer code in Figure 4, again a puncturing patterns listed in [13] is used. These plots show similar BER curve patterns as in Figure 3. The BER curves of $\mathrm{SCC_{ch}}$'s with precoders $pr = 3, 5$ start to get flattened as SNR increases, while other precoders $pr = 7, 13, 15$ exhibits quite steep slope for high SNR's. Another interesting point which we can observe is that the cross-over points in BER curves at which precoders $pr = 13, 15, 17$ start to outperform precoders $pr = 3, 5$ are getting lower and lower in BER as a code rate decreases. This means that as more powerful codes are used in outer codes, multiplicities of error events realized in precoders $pr = 3, 5$ become much smaller, due to increased precoder weight gain. Please note that these curve pattern analysis for each precoder is applied for the given channel $H(D)$. With different choice of a channel, one can analyze its Euclidean distance for each precoder as in table III. But, in general, this peculiar BER

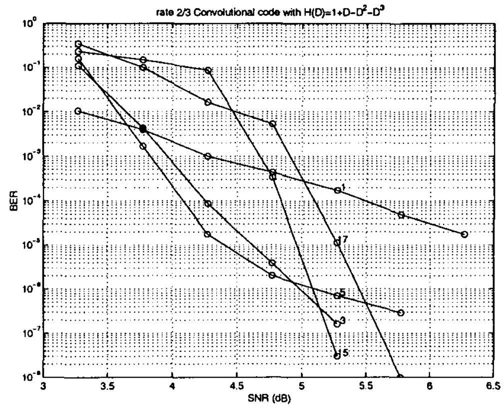


Fig. 6. rate 2/3 convolutional outer code with different precoders

behavior of one group of precoders crossing over the other groups of precoders for high SNR's is observed in other channels as well. Also, one of reasons why BER curves show distinct pattern can be attributed to the nature of iterative decoding algorithm.

We present another simulation plot with a different channel $H(D) = 1 + D - D^2 - D^3$ in Figure 6. Especially for this simulation, 100,000,000 symbols were processed for Monte-Carlo BER count to show very low BER region. Again, similar BER behaviors of different precoders are obtained when compared to previous simulation plots. For example, the BER curve of precoder $pr = 5$ clearly exhibits the precoder weight gain over no precoder case. Based on this plot, we can note that for system applications aiming at BER higher than $10^{-6}$, a simple precoder $pr = 5$ is better suited, while precoder $pr = 15$ is better suited to provide very low error rate performance. The reason why $pr = 15$ has such a steep asymptote can be explained in line with observations made in the previous section. When low weight precoder input words are divided by $Pr(D) = 1 \oplus D \oplus D^3$, many codewords would contain 1110100(11101) where a parenthesized string $(s)$ denotes any positive integer number of repetitions of the string $s$. Codewords containing 1110100(11101) generate very high Euclidean distance for the corresponding error events. In contrast, $Pr(D) = 1 \oplus D^2$ generates codewords which contain 101(01) and this supports the error event $e = +0 + (0+)$ with $d^2(e) = 4$, which is the minimum Euclidean distance for $H(D)$, thus resulting in a lower asymptote slope.

Summarizing a few observations made in this simulation section, we can draw some design considerations:

- The performance of $\mathrm{SCC_{ch}}$'s with no precoder is dominated by $d^o_{\mathrm{free}}$ error events $+$. In other words, the asymptotic slope of no precoder case is determined by $d^2(+) = \|h\|^2$ for $H(D)$
- When a precoder is employed, the asymptotic BER performance is improved by precoder weight gain, which results from transforming low weights outer codewords into ones with high weights by precoders. This precoder weight gain lowers multiplicities of the minimum Euclidean dis-

tance error event. It was shown in simulations that the more powerful outer codes are employed, the greater the precoder weight gain becomes in comparison to no precoder case.

- Based on the BER curve behavior, precoders can be divided into two groups: one group of precoders performs well for low SNR's, while the other group of precoders performs well for high SNR's. For conveniences, we refer to the former and the latter as the small weight precoders and the large weight precoders.[1] Generally, the small weight precoders such as $pr = 3, 5$ exhibits a low slope of the asymptotic curve (even lower than that of no precoder case), while the large weight precoders such as $pr = 15, 17$ shows a steep slope of the asymptote.
- Normally, due to a relatively small Euclidean distance, the BER curves of the small weight precoders are getting flat as SNR increases, and the BER curves of the large weight precoders eventually cross those of the small weight precoders for high SNR's because of their high Euclidean distances.
- The cross-over point in BER at which the large weight precoders start to outperform the small weight precoders becomes lower as more powerful outer codes are used. Therefore, the choice of precoders depends upon the target BER relative to this cross-over point. For example, when one needs to use simple outer decoders for complexity reasons or should employ a high code rate system, the large weight precoders are suitable in these applications, since the BER cross-over point would be normally higher than the target BER.
- It is clear from simulations that primitive polynomials are not necessarily the best choice for precoders.

In conclusion, we can improve the BER performance of $SCC_{ch}$'s by the careful choice of precoders. To do this end, one should first identify Euclidean distances of each precoder for a given $H(D)$ and an interleaver. After that, the target BER is considered to determine the best precoder. In general, when emploied outer codes are powerful such that the cross-over point in BER is much lower than the target BER, precoders with small Euclidean distances could be prefered. In contrast, precoders which yields much higher Euclidean distance are more suitable when trying to provide near error-free performance, because of a steep asymptote.

## IV. Conclusions

We have presented the serially concatenated code system which takes the ISI channel as rate 1 inner code. Through the union bound analysis, we have identified precoder weight gain which explain much smaller multiplicities of error events in precoders compared to no precoder case.

The effect of the choice of different precoders is also analyzed. Through several simulations and Euclidean distance analysis, some important design considerations regarding the choice of precoders are drawn. Based on analysis and guidelines derived in this paper, we can better understand and predict the BER behavior of the serially concatenated code systems and can choose precoders which better serve the system performance requirement.

[1]Note that weight of precoder polynomials does not necessarily determine the BER behavior. Criteria of dividing precoders into two groups is dependent upon $H(D)$.

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